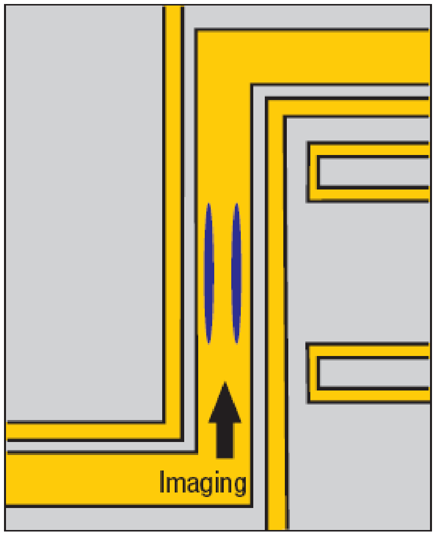
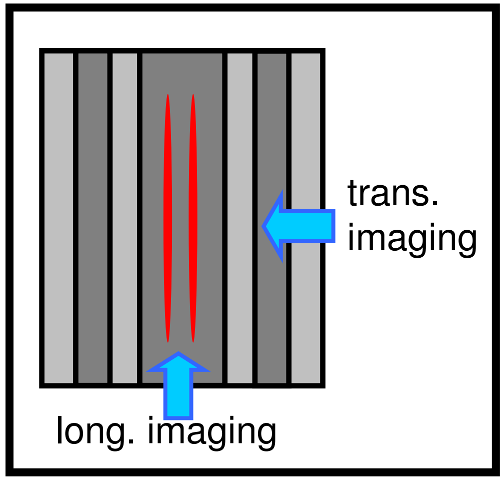
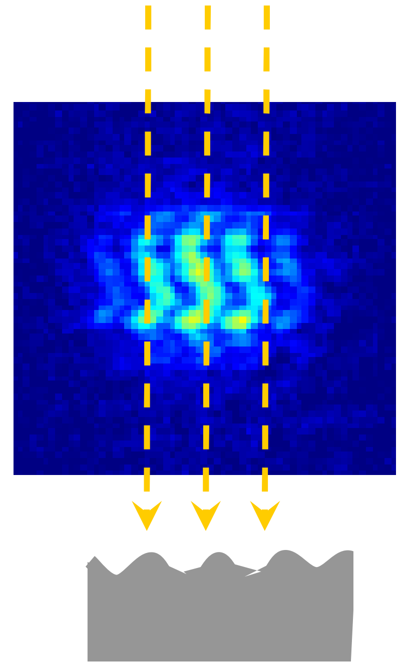
Interference of one dimensional condensates

Experiments: Schmiedmayer et al., Nature Physics (2005,2006)

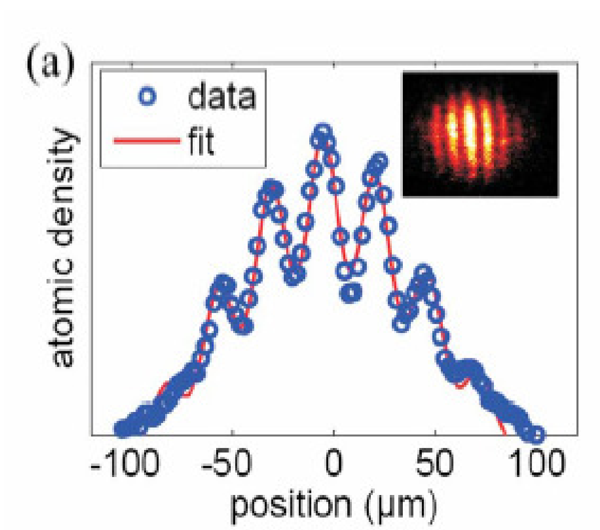




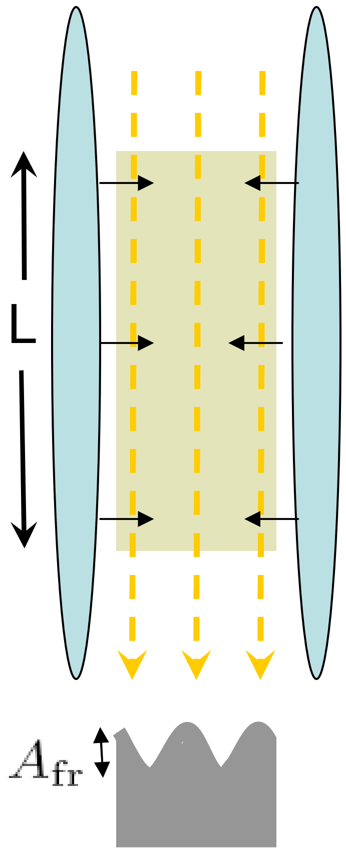
Transverse imaging

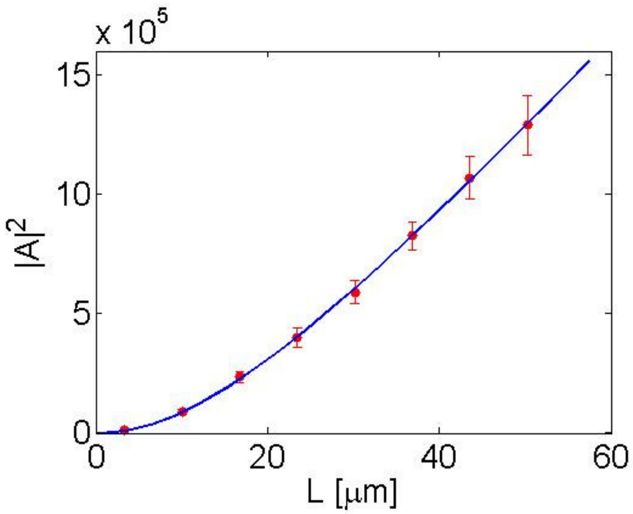


Longitudial
imaging



Interference between Luttinger liquids





Experiments: Hofferberth, Schumm, Schmiedmayer

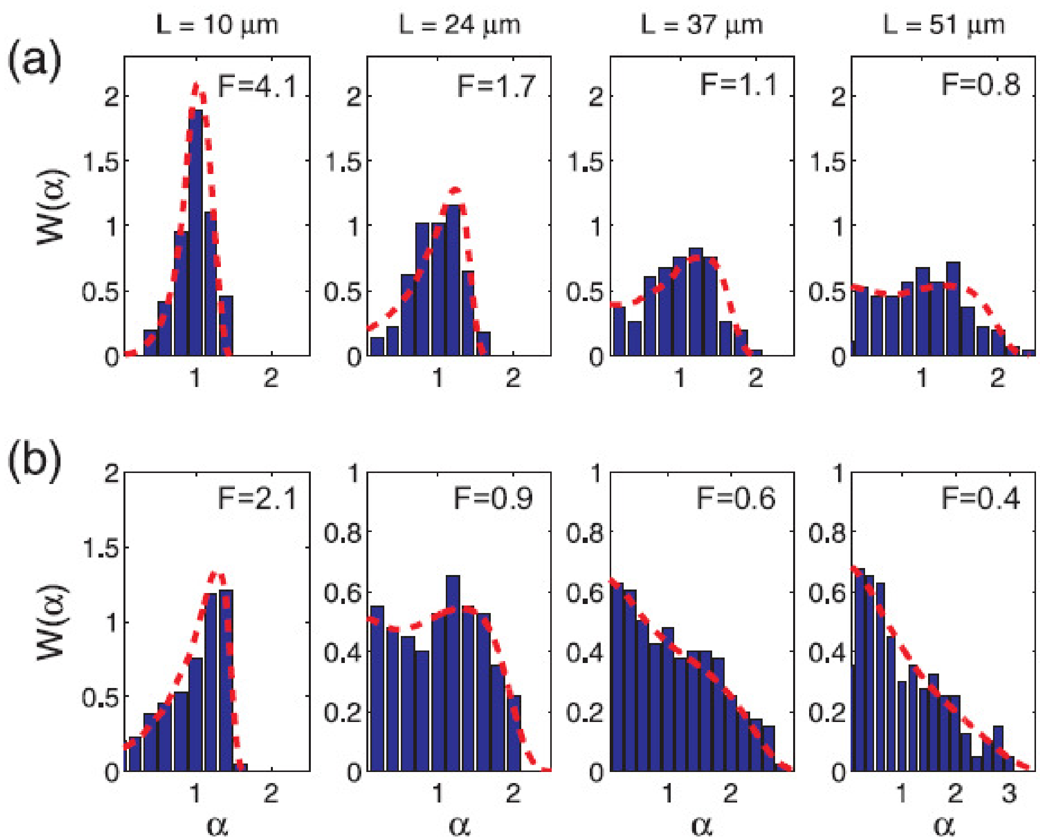
$$n_{1d} = 60\mu m^{-1}$$

$$K = 47$$

$$T_{fit} = 84 \pm 22 \text{ nK}$$

Distribution function of interference fringe contrast

Experiments: Hofferberth et al., arXiv0710.1575
Theory: Imambekov et al. , cond-mat/0612011

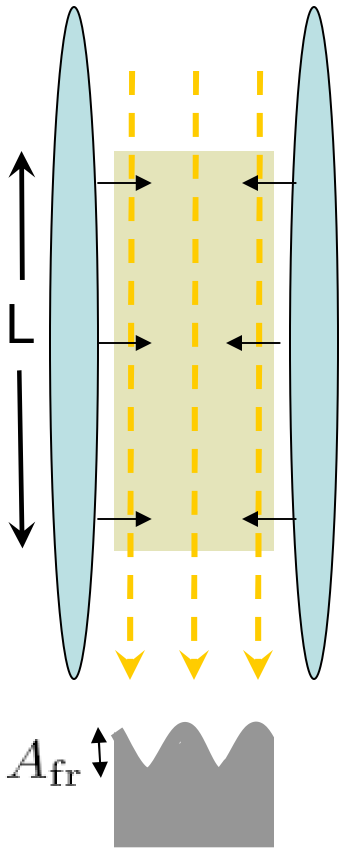


Quantum fluctuations dominate:
asymetric Gumbel distribution
(low temp. T or short length L)

Thermal fluctuations dominate:
broad Poissonian distribution
(high temp. T or long length L)

Intermediate regime:
double peak structure

Comparison of theory and experiments: no free parameters
Higher order correlation functions can be obtained



Calculating distribution function of interference fringe amplitudes

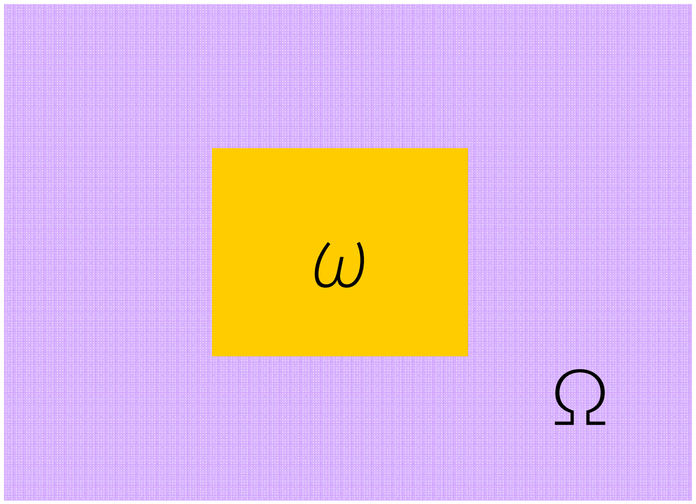
Method II: mapping to inhomogeneous sine-Gordon model

Imambekov, Gritsev, Demler, cond-mat/0612011

Can be used for 1d systems with arbitrary boundary conditions and at finite temperature

Can be used to study interference of 2d condensates

Inhomogeneous Sine-Gordon models



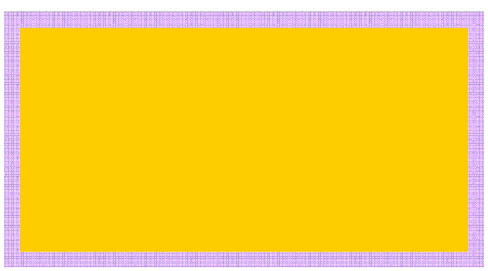
$$S = \frac{K}{2}\int_\Omega d^2x\,(\nabla\phi)^2 \; + \; g\int_\omega d^2x\cos\phi$$

$$Z(g) = \frac{\int \mathcal{D}\phi e^{-S(g)}}{\int \mathcal{D}\phi e^{-S(0)}}$$

Limiting cases

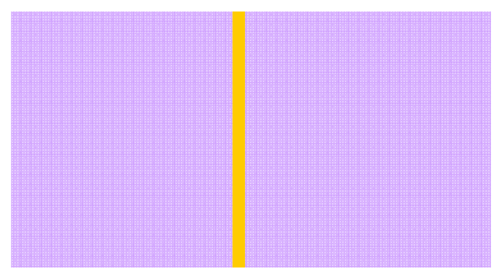
Bulk Sine-Gordon model

$\omega = \Omega$



Boundary Sine-Gordon model

$\omega = \delta\,(\mathbf{x} - \mathbf{x}_0)$



Inhomogeneous Sine-Gordon models

$$S = \frac{K}{2}\int_\Omega d^2x\,(\nabla\phi)^2 \,+\, g\int_\omega d^2x\cos\phi \qquad\qquad Z(g) \,=\, \frac{\int \mathcal{D}\phi e^{-S(g)}}{\int \mathcal{D}\phi e^{-S(0)}}$$

Expand in powers of g

$$Z(g) \,=\, \sum_{n=0}^{n=\infty} \frac{g^{2n}}{(n!)^2}\, Z_{2n}$$

$$Z_{2n} \,=\, \prod_{i=1}^{n}\int_\omega du_i \,\prod_{j=1}^{n}\int_\omega dv_j\, \frac{\prod_{ij}\langle e^{i\phi(u_i)}e^{-i\phi(v_j)}\rangle}{\prod_{i<i'}\langle e^{i\phi(u_i)}e^{-i\phi(u_{i'})}\rangle\,\prod_{j<j'}\langle e^{i\phi(v_j)}e^{-i\phi(v_{j'})}\rangle}$$

Coulomb gas representation

$$Z_{2n} \,=\, \prod_{i=1}^{n}\int_\omega du_i \,\prod_{j=1}^{n}\int_\omega dv_j\, e^{\frac{1}{K}(\sum_{i<i'} f(u_i,u_{i'})+\sum_{j<j'} f(v_i,v_{j'})-\sum_{ij} f(u_i,v_j))}$$

Diagonalize Coulomb gas interaction

$$\int_{\omega} f(x,y)\Psi_m(x)dx = f(m)\Psi_m(y)$$

$$f(x,y) = \sum_{m=1}^{\infty} f(m)\Psi_m(x)\Psi_m(y)$$

Introduce probability distribution function

$$Z_{2n} = \int W(\alpha)\,\alpha^n\,d\alpha$$

$$Z(g) = \sum_{n=0}^{n=\infty} \frac{g^{2n}}{(n!)^2} Z_{2n} = \int \left(\sum_{n=0}^{n=\infty} \frac{g^{2n}}{(n!)^2} \alpha^n \right) W(\alpha)d\alpha = \int I_0\left(2g\sqrt{\alpha}\right) W(\alpha)d\alpha$$

This is the same probability distribution function that we need for describing interference experiments

From SG models to fluctuating surfaces

$$W(\alpha) = \prod_m \frac{\int_{-\infty}^{\infty} dt_m\, e^{-t_m^2/2}}{\sqrt{2\pi}}\, \delta\left(\alpha - \int_\omega dx\, e^{h(x,t_m)+h_0(x)} \int_\omega dx\, e^{h(x,-t_m)+h_0(x)}\right)$$

$$h(x,t_m) = \sum_m t_m \sqrt{\frac{f(m)}{K}}\,\Psi_m(x) \qquad h_0(x) = -\sum_m \frac{f(m)}{2K}\Psi_m(x)^2$$

Simulate by Monte-Carlo!

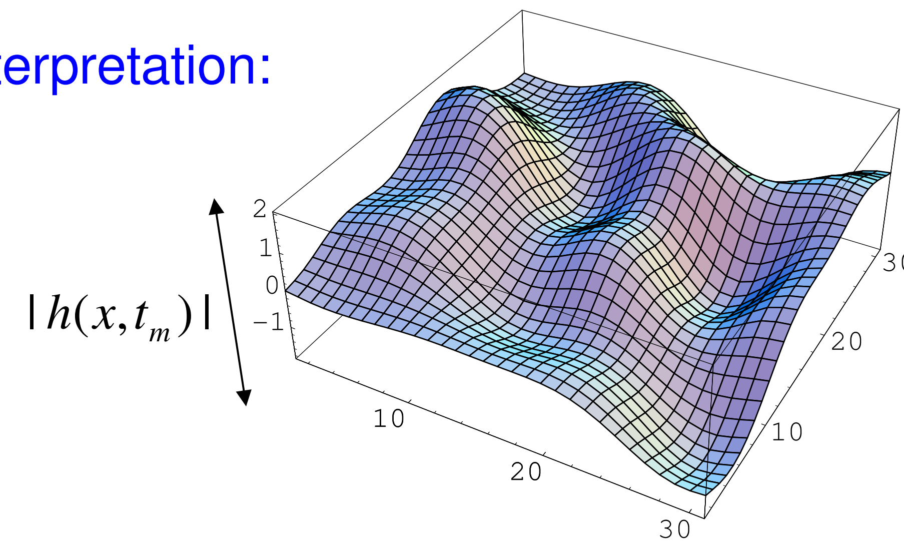
Random surfaces interpretation:

$h(x,t_m)$ fluctuating surface

t_m "noise" variables

$\Psi_m(x)$ eigenmodes } determined by $f(x,y)$

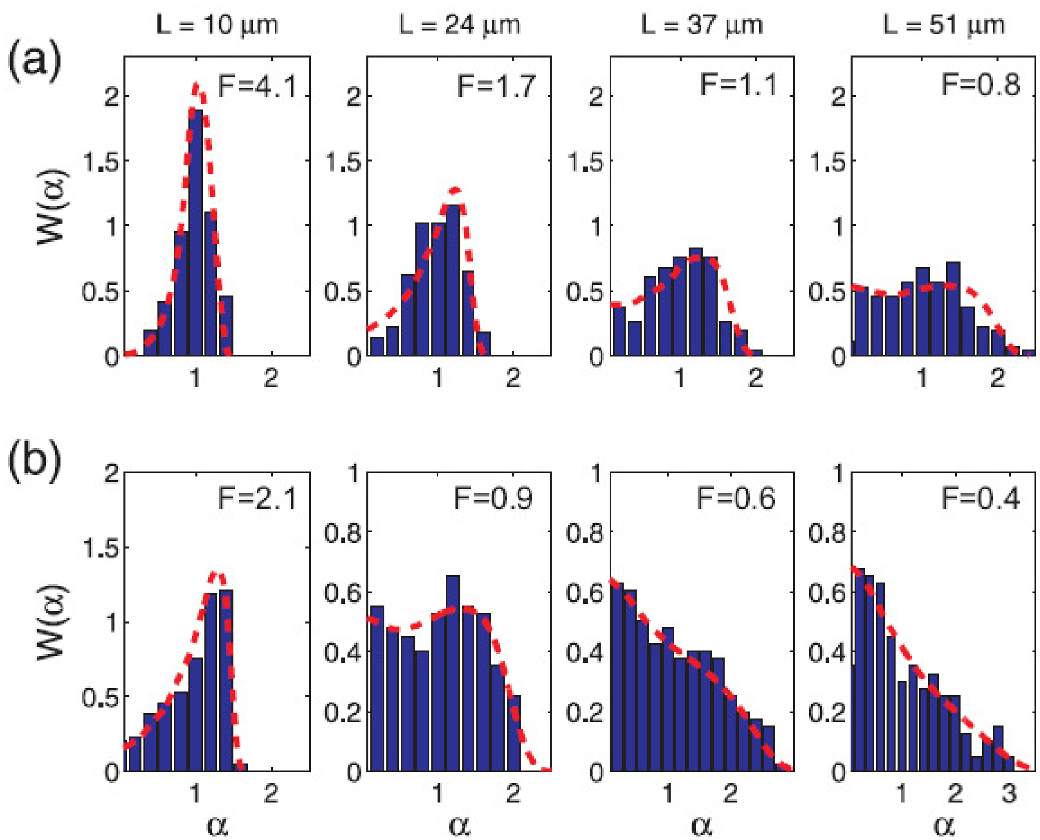
|f(m)| "noise" power }



This method does not rely on the existence of the exact solution

Distribution function of interference fringe contrast

Experiments: Hofferberth et al., arXiv0710.1575
Theory: Imambekov et al. , cond-mat/0612011



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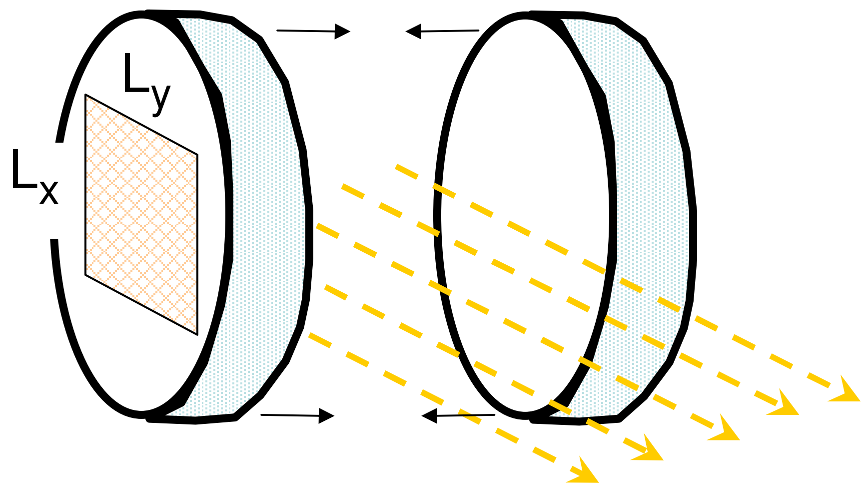
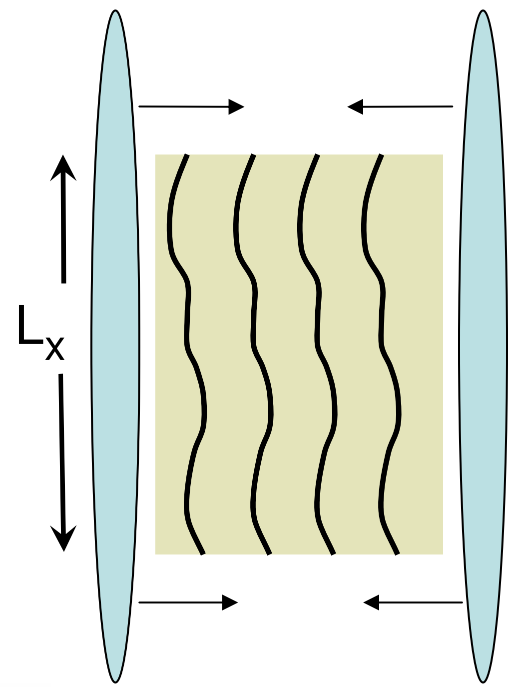
Higher order correlation functions can be obtained

Interference of two dimensional condensates

Experiments: Hadzibabic et al. Nature (2006)
Gati et al., PRL (2006)



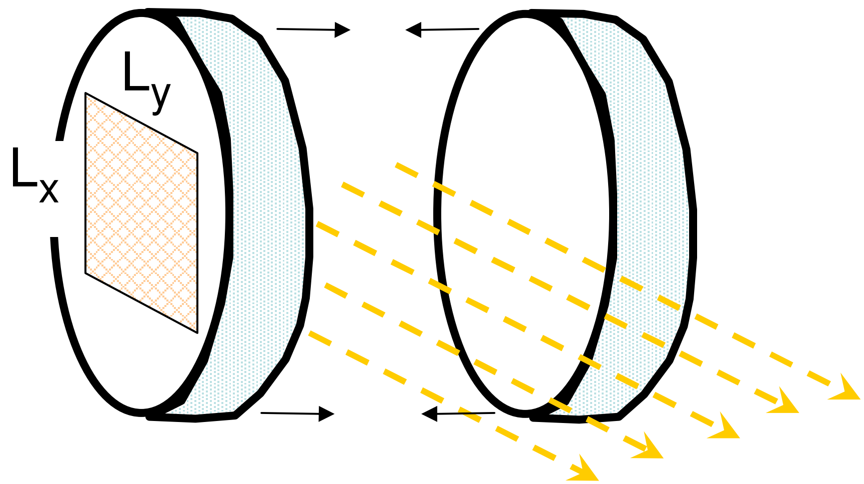
Probe beam parallel to the plane of the condensates



$$\langle |A_{\text{fr}}|^2 \rangle = L_x L_y \int_0^{L_x} \int_0^{L_y} d^2\vec{r}\, (G(\vec{r}))^2$$

$$G(\vec{r}) = \langle a(\vec{r})\, a^\dagger(0) \rangle$$

Interference of two dimensional condensates. Quasi long range order and the BKT transition



Above BKT transition

$$G(r) \sim e^{-r/\xi}$$

$$\langle |A_{\rm fr}|^2 \rangle \sim L_x L_y$$

$$\log \xi(T) \sim 1/\sqrt{T - T_{\rm KT}}$$

Below BKT transition

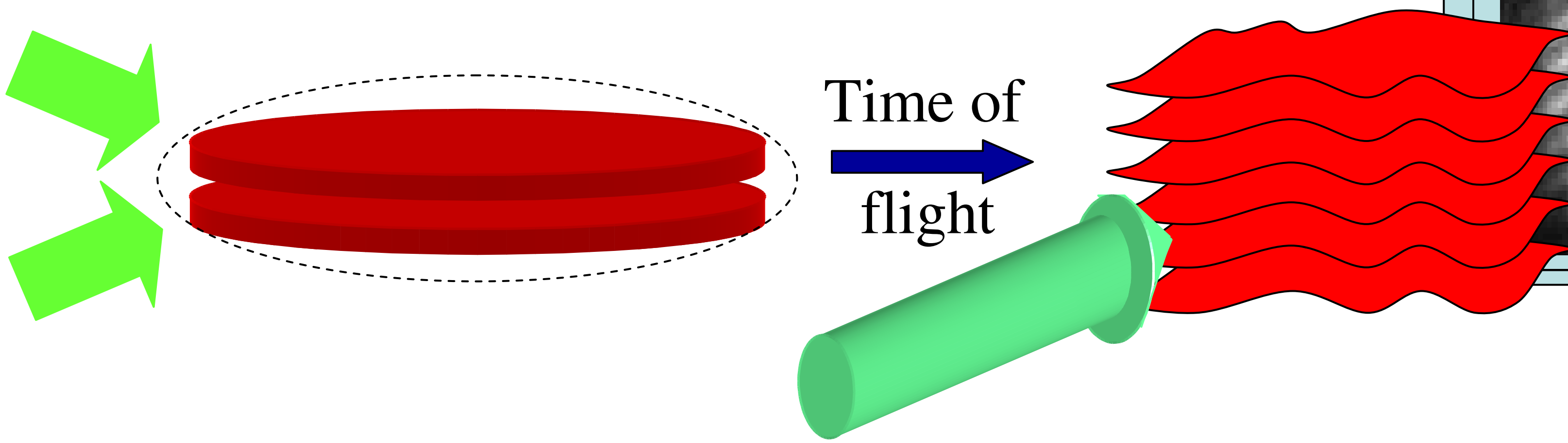
$$G(r) \sim \rho \left(\frac{\xi_h}{r} \right)^{\alpha}$$

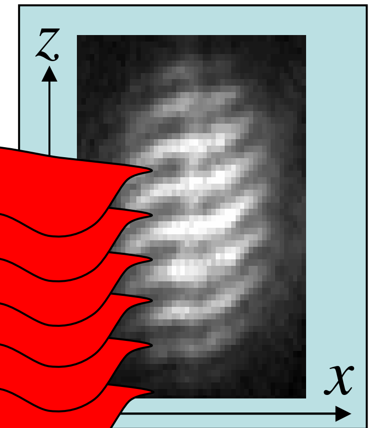
$$\alpha(T) = \frac{m T}{2 \pi \rho_s(T) \hbar^2}$$

$$\langle |A_{\rm fr}|^2 \rangle \sim (L_x L_y)^{2-\alpha}$$

Experiments with 2D Bose gas

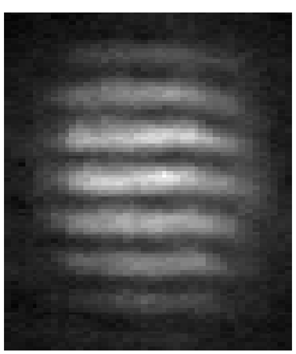
Hadzibabic, Dalibard et al., Nature 441:1118 (2006)



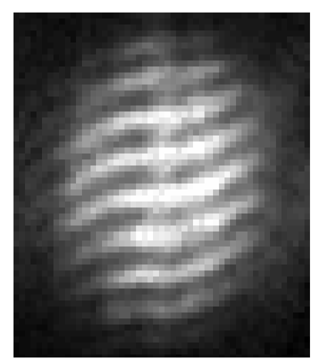


Typical interference patterns

low temperature

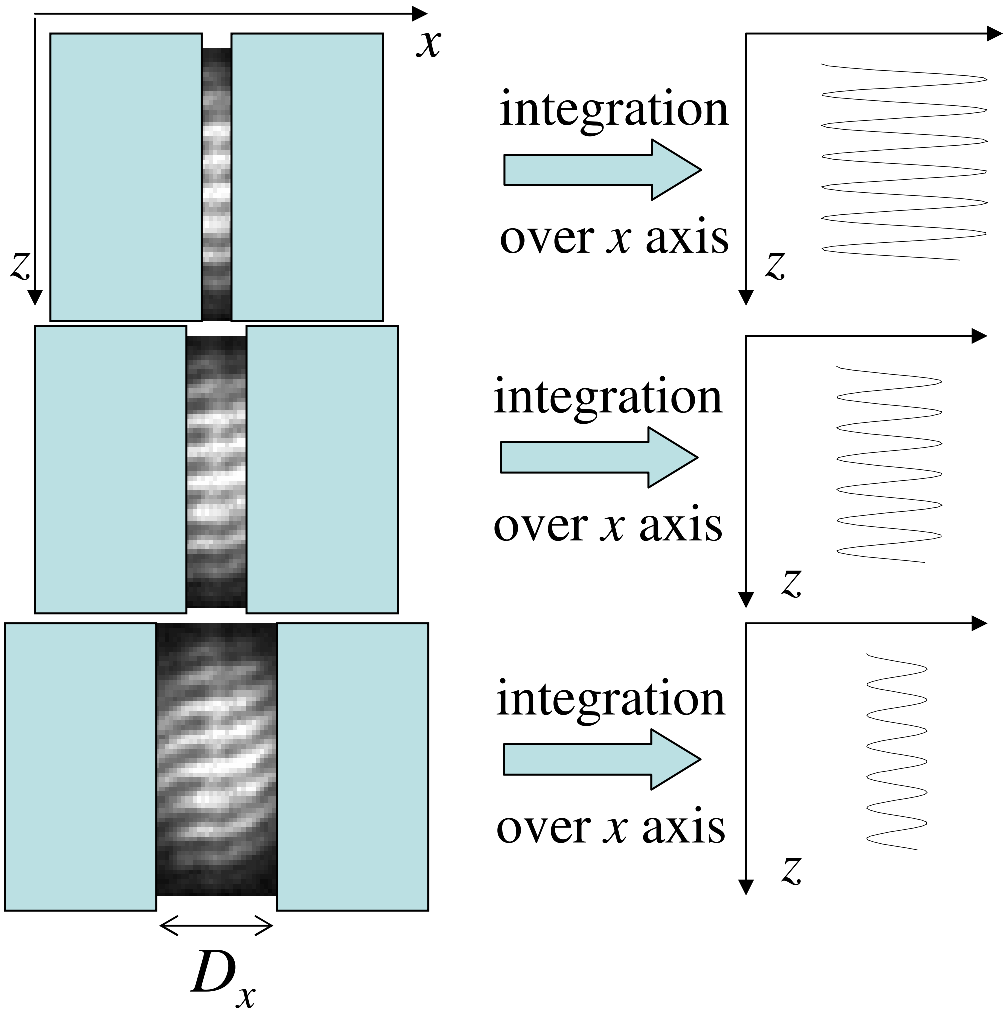


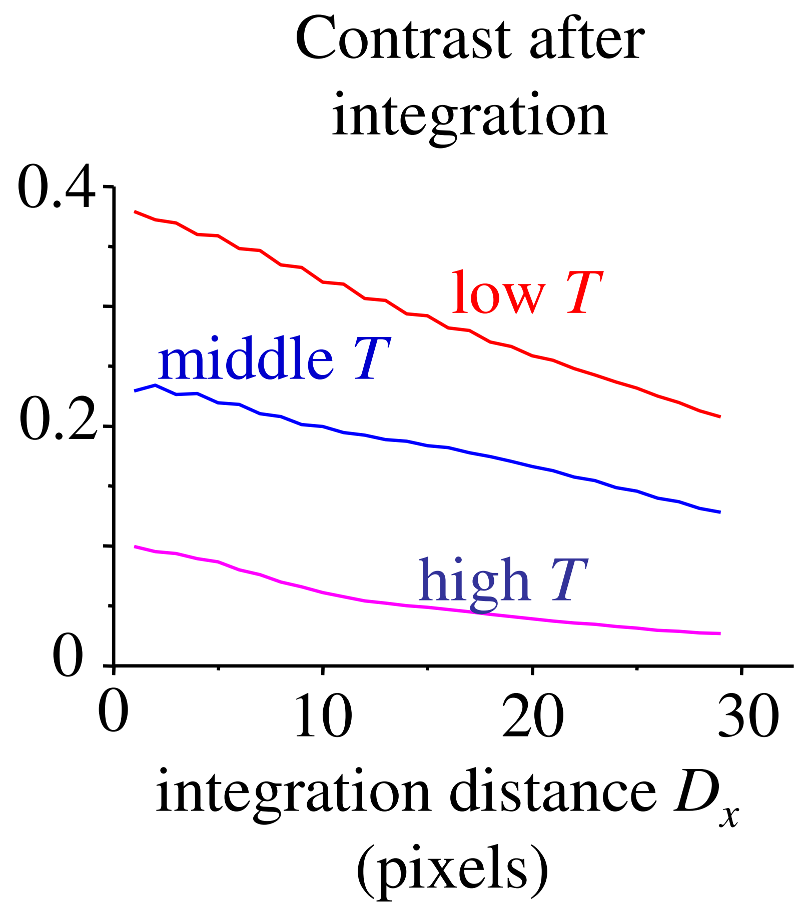
higher temperature



Experiments with 2D Bose gas

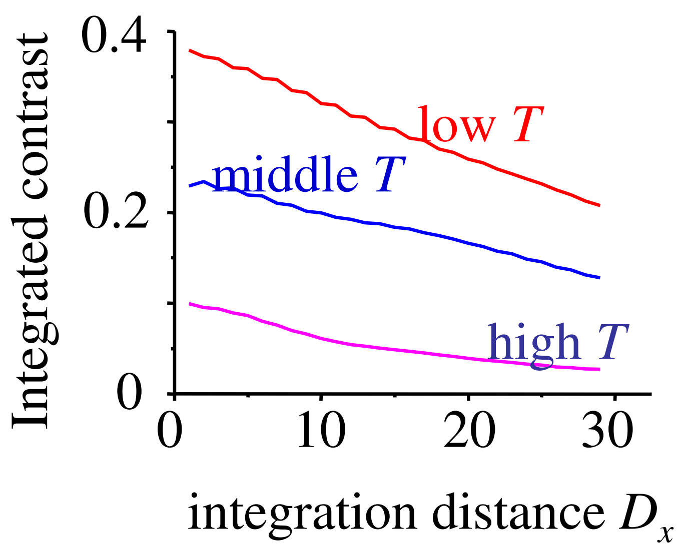
Hadzibabic et al., Nature 441:1118 (2006)



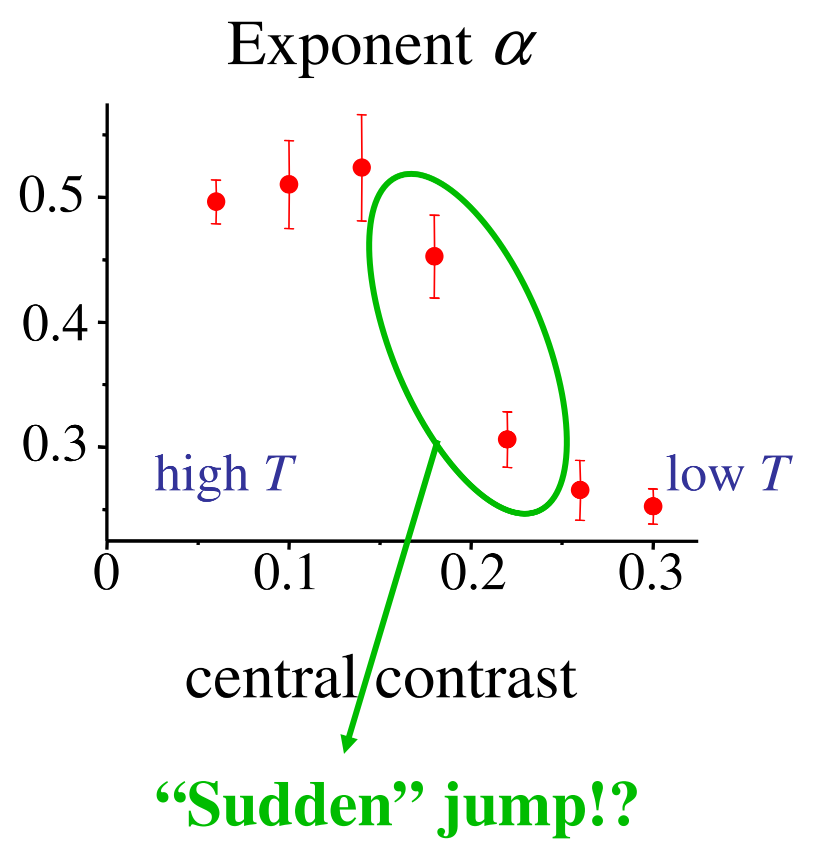


Experiments with 2D Bose gas

Hadzibabic et al., Nature 441:1118 (2006)



fit by: $C^2 \sim \frac{1}{D_x}\int^{D_x}[g_1(0,x)]^2 dx \sim \left(\frac{1}{D_x}\right)^{2\alpha}$



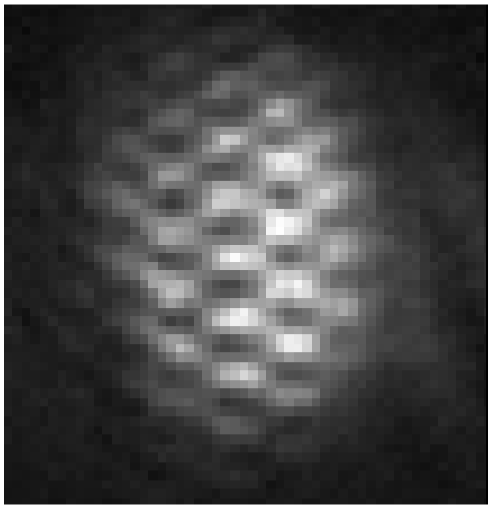
⇒ if $g_1(r)$ decays exponentially with $\ell_{\text{coh}} \ll D_x$: $\alpha = 1/2$

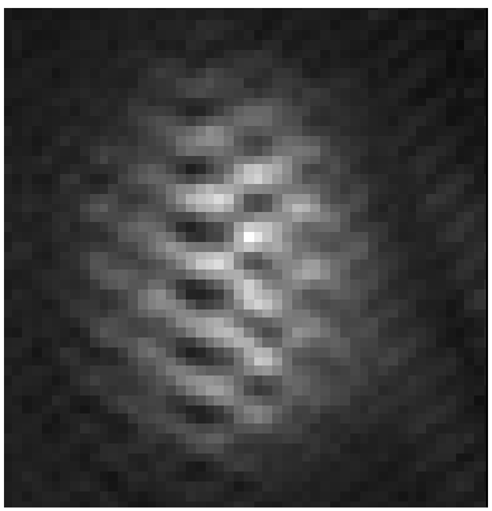
⇒ if $g_1(r)$ decays algebraically or exponentially with a large ℓ_{coh}:

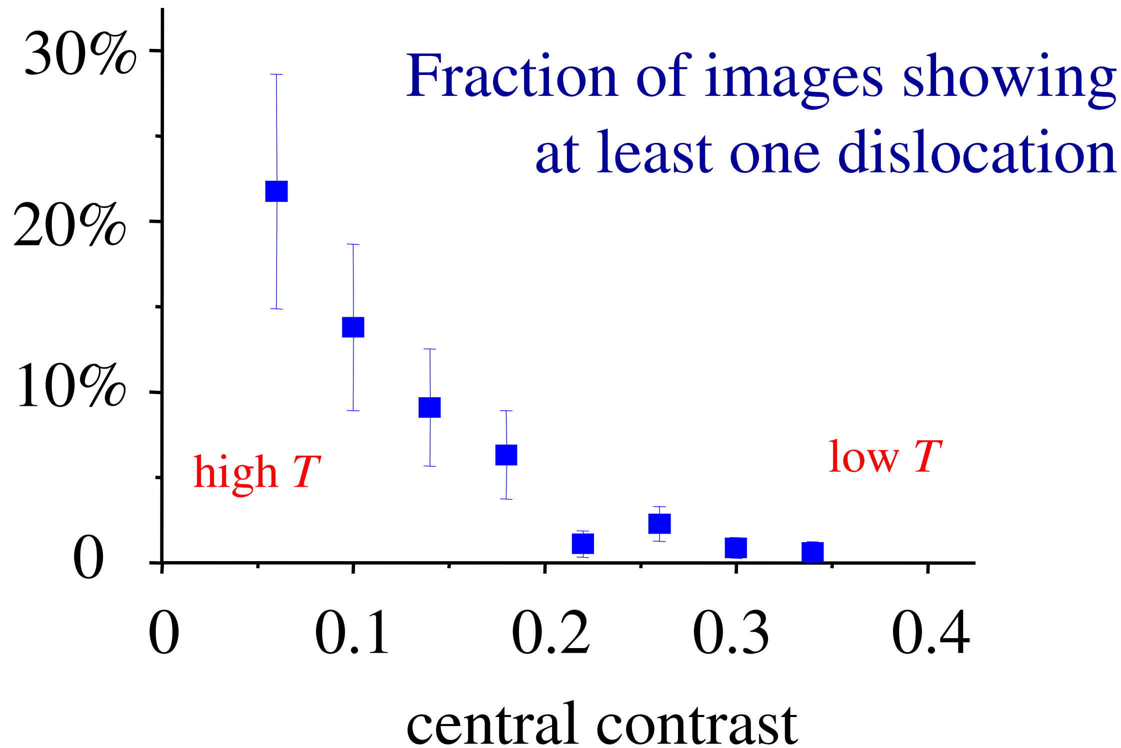
$$\alpha < 1/2$$

Experiments with 2D Bose gas. Proliferation of thermal vortices

Hadzibabic et al., Nature 441:1118 (2006)





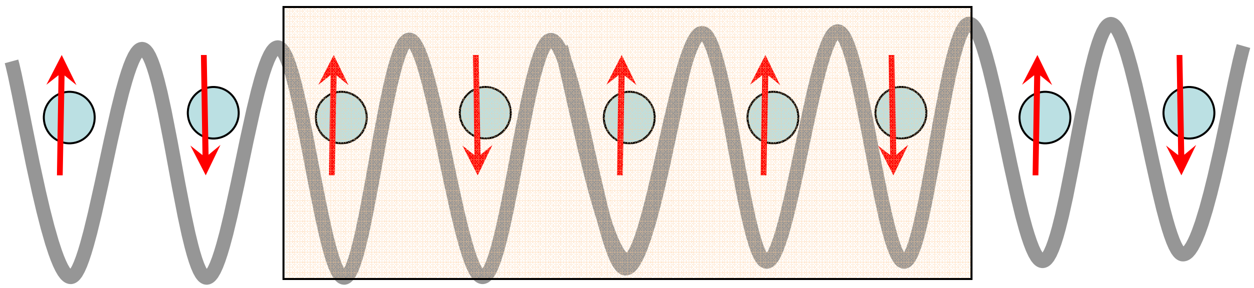


The onset of proliferation coincides with α shifting to 0.5!

Probing spin systems using distribution function of magnetization

Probing spin systems using distribution function of magnetization

Cherng, Demler, New J. Phys. 9:7 (2007)



Magnetization in a finite system $$M^z_{\rm tot} = \sum_{i=1}^{L} M^z(i)$$

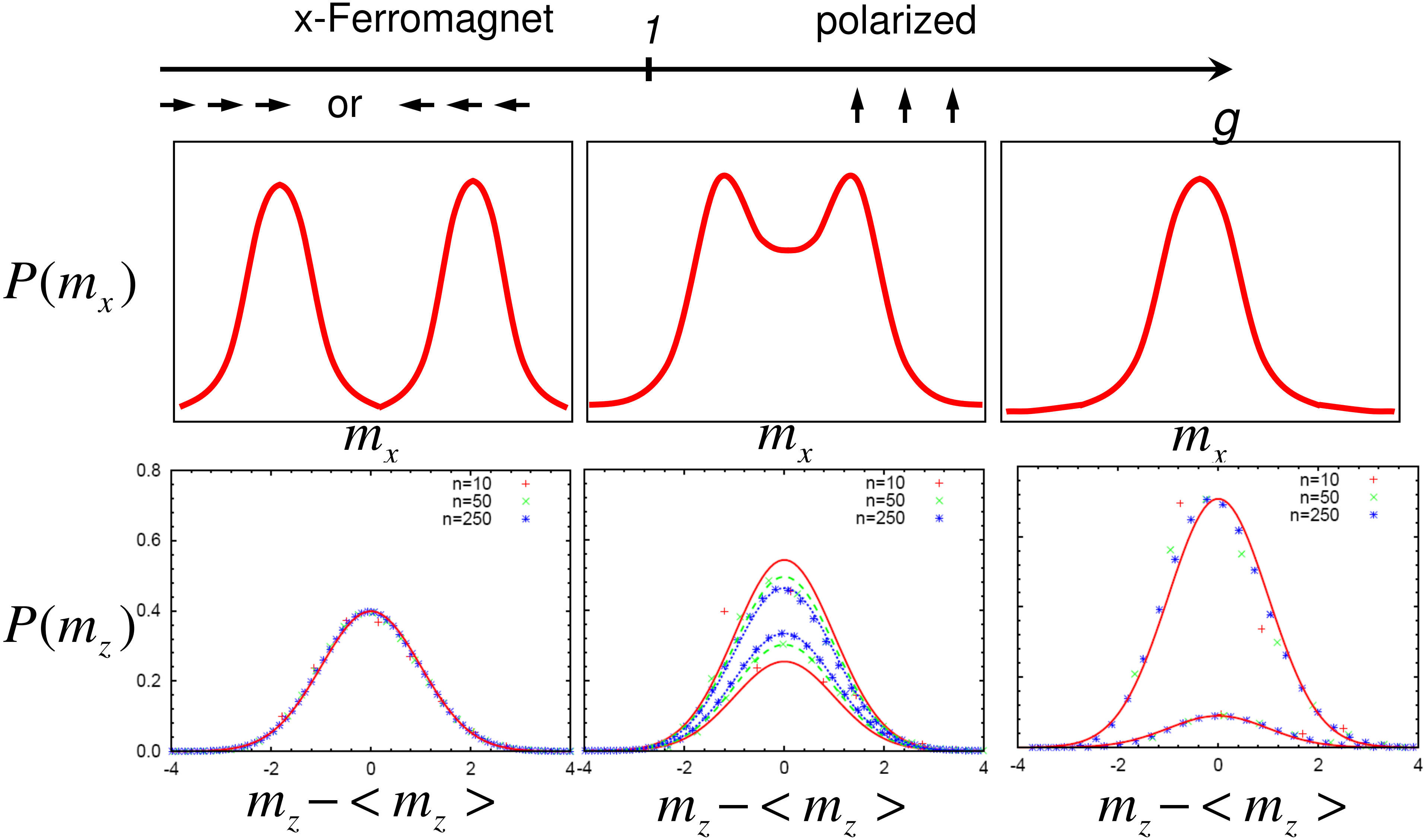
Average magnetization $$\langle M^z_{\rm tot} \rangle = L \langle M^z \rangle$$

Higher moments of $M^z_{\rm tot}$ contain information about higher order correlation functions

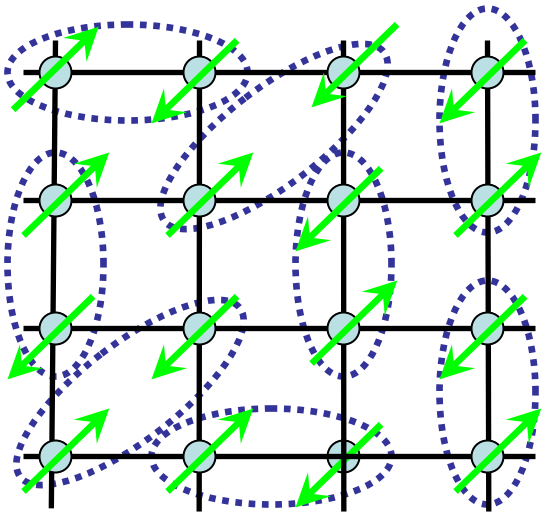
$$\langle (M^z_{\rm tot} - \langle M^z_{\rm tot} \rangle)^2 \rangle = L \sum_{ij} \langle (M_z(i) - \langle M^z \rangle) \, (M^z(j) - \langle M^z \rangle) \rangle$$

Distribution Functions

$$\mathcal{H} = -J\sum_i \left[2S^x(i)S^x(i+1) + gS^z(i)\right]$$



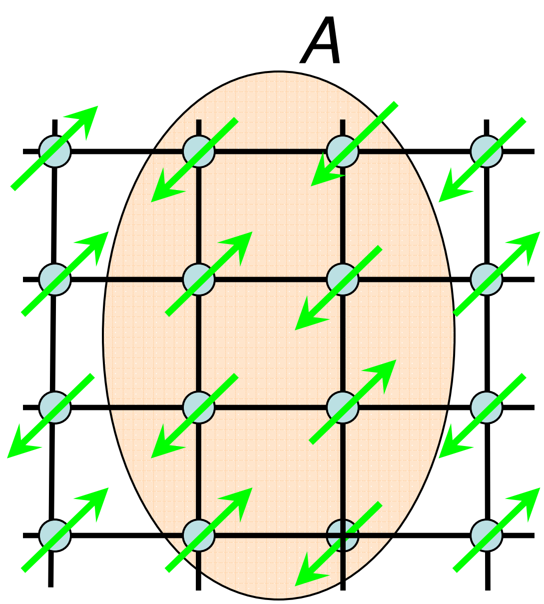
Using noise to detect spin liquids



Spin liquids have no broken symmetries
No sharp Bragg peaks

Algebraic spin liquids have long range
spin correlations

$$\langle S_i \, S_j \rangle \; = \; \frac{e^{i\,Q\,r_{ij}}}{|\, r_i \; - \; r_j \,|^{1+\eta}}$$



No static magnetization $\langle S_{\rm A} \rangle \; = \; 0$

Noise in magnetization exceeds shot noise

$$\langle S_{\rm A}^2 \rangle \; = \sum_{ij} \langle S_i \, S_j \rangle \; \sim \; A \int_{\rm A} \frac{r\, dr}{|\, r \,|^{1+\eta}} \sim A^{1+\frac{1-\eta}{2}}$$

Summary of part I

Experiments with ultracold atoms provide a new perspective on the physics of strongly correlated many-body systems. Quantum noise is a powerful tool for analyzing many body states of ultracold atoms

Outline

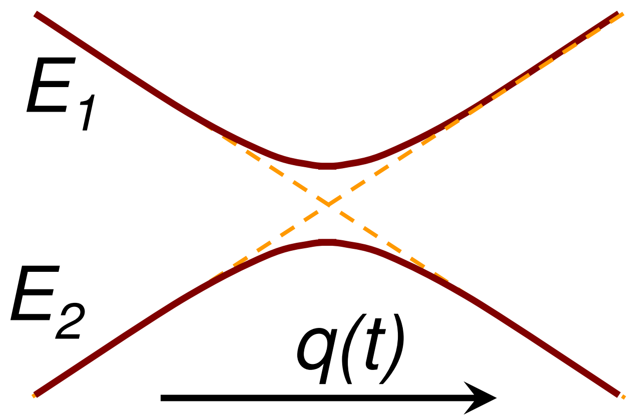
Part I

Detection and characterization of many body states

Part II

New challenges in quantum many-body theory: non-equilibrium coherent dynamics

Landau-Zener tunneling



Landau, Physics of the Soviet Union 3:46 (1932)
Zener, Poc. Royal Soc. A 137:692 (1932)

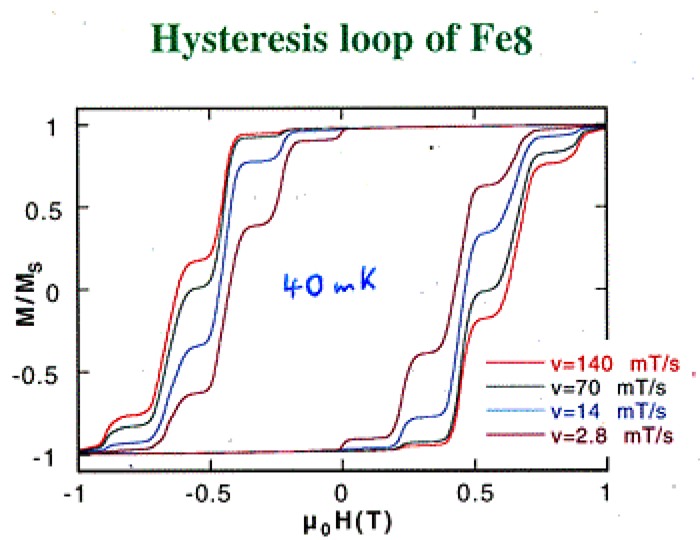
Probability of nonadiabatic transition

$$P_{12} \propto e^{-2\pi\omega_{12}\tau_d}$$

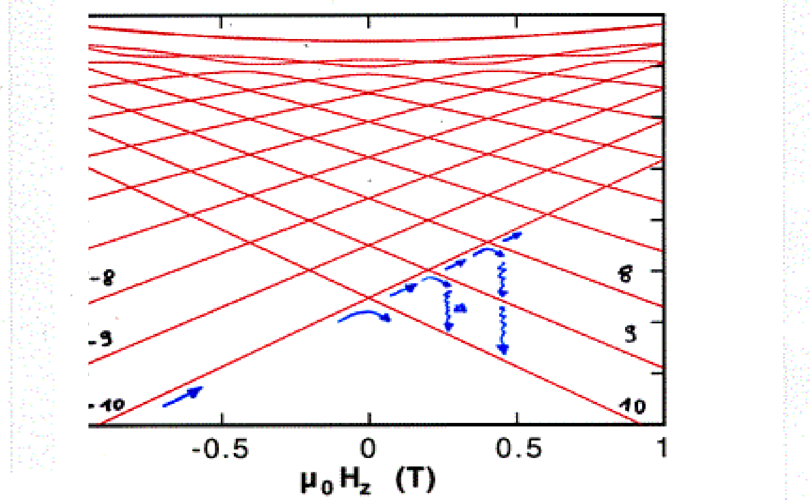
ω_{12} – Rabi frequency at crossing point

τ_d – crossing time

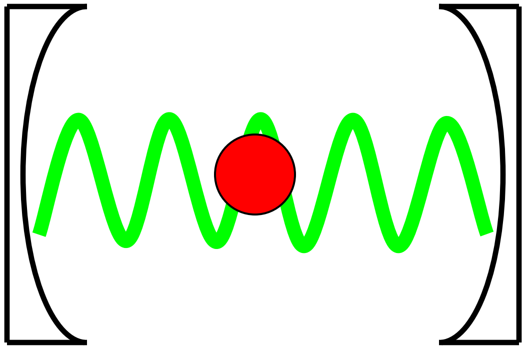
Hysteresis loops of Fe8 molecular clusters



Wernsdorfer et al., cond-mat/9912123



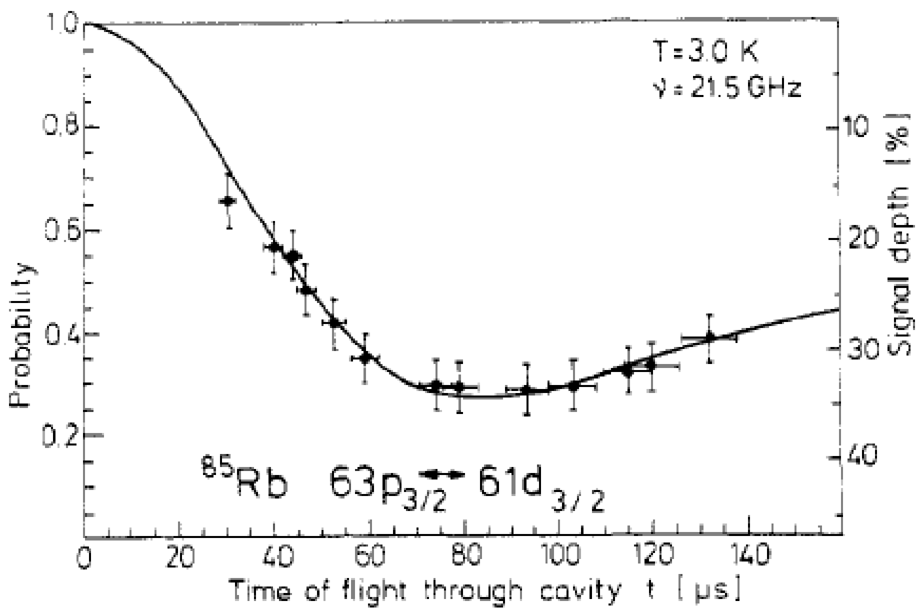
Single two-level atom and a single mode field



Jaynes and Cummings,
Proc. IEEE 51:89 (1963)

$$\mathcal{H} = \hbar\omega_1 a^\dagger a + \hbar\omega_2 \sigma_z + g(a\sigma_+ + a^\dagger\sigma_-)$$

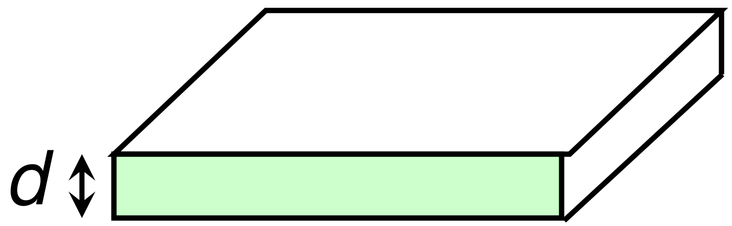
Observation of collapse and revival in a one atom maser



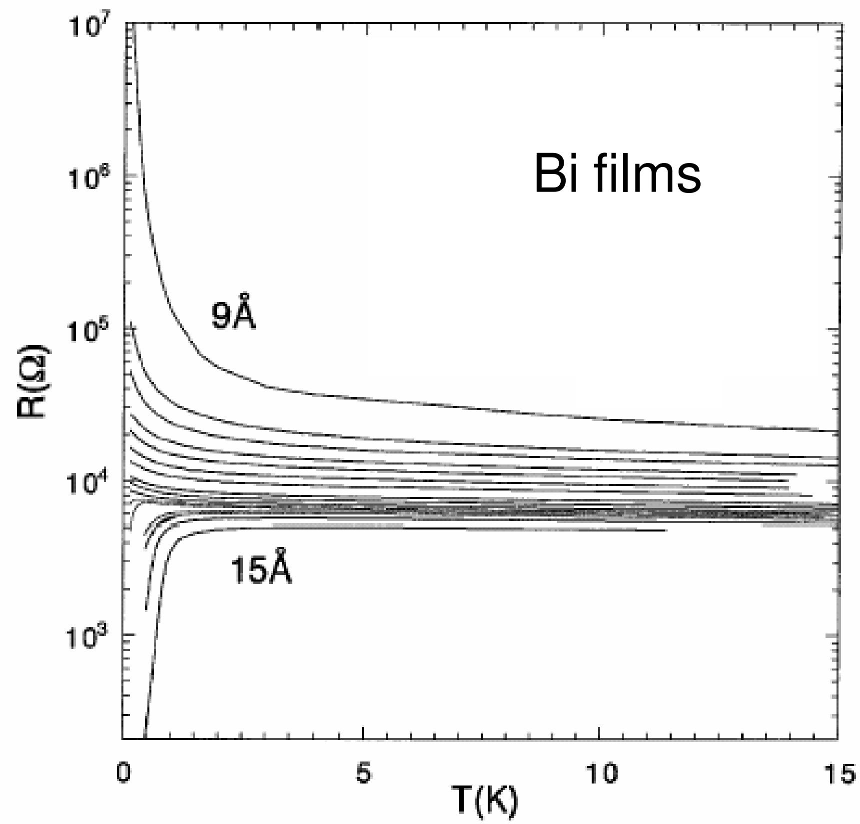
Rempe, Walther, Klein,
PRL 58:353 (87)

See also solid state realizations
by R. Shoelkopf, S. Girvin

Superconductor to Insulator transition in thin films



Superconducting films of different thickness. Transition can also be tuned with a magnetic field

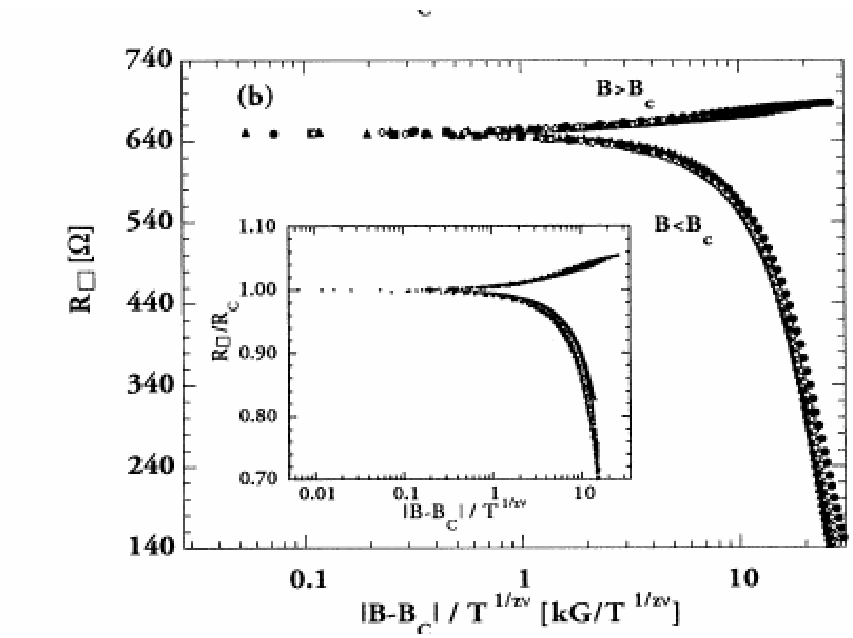


Marcovic et al., PRL 81:5217 (1998)

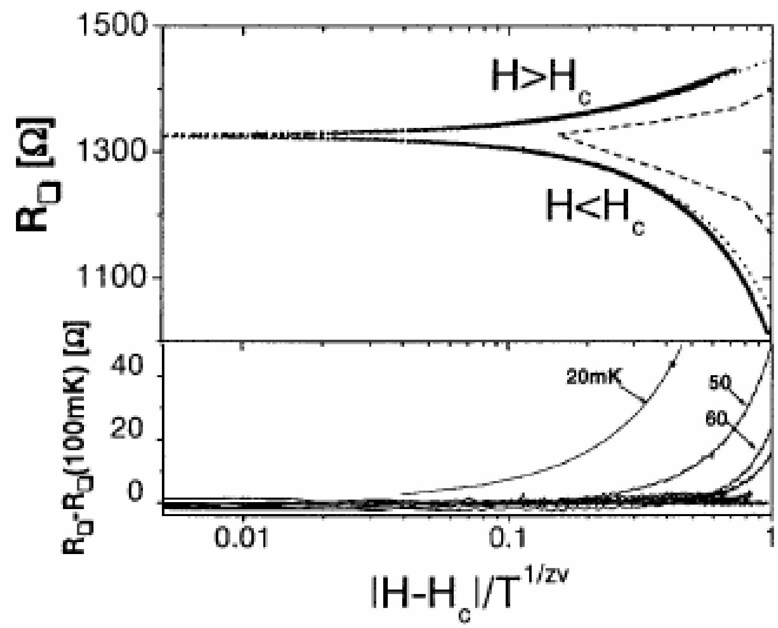
Scaling near the superconductor to insulator transition

Yes at "higher" temperatures

No at lower" temperatures



Yazdani and Kapitulnik
Phys.Rev.Lett. 74:3037 (1995)

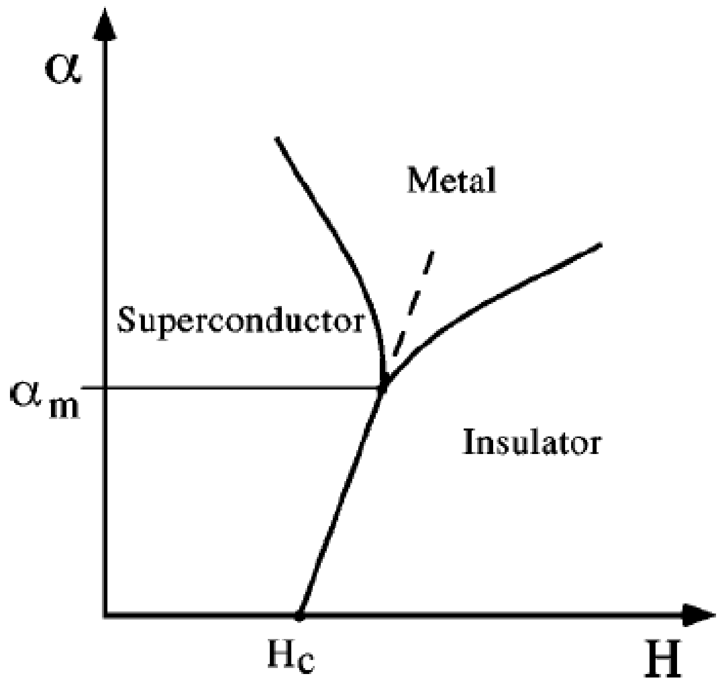


Mason and Kapitulnik
Phys. Rev. Lett. 82:5341 (1999)

Mechanism of scaling breakdown

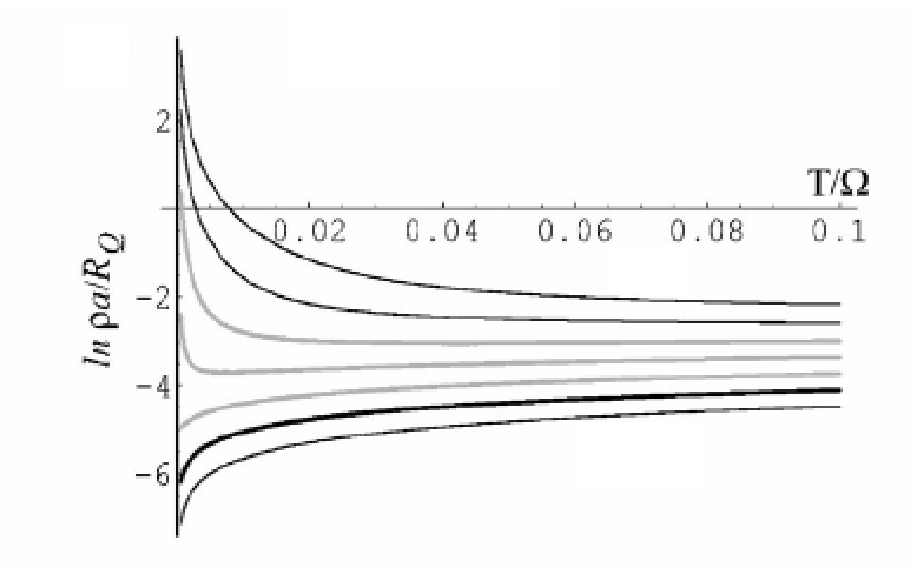
New many-body state

Kapitulnik, Mason, Kivelson, Chakravarty,
PRB 63:125322 (2001)



Extended crossover

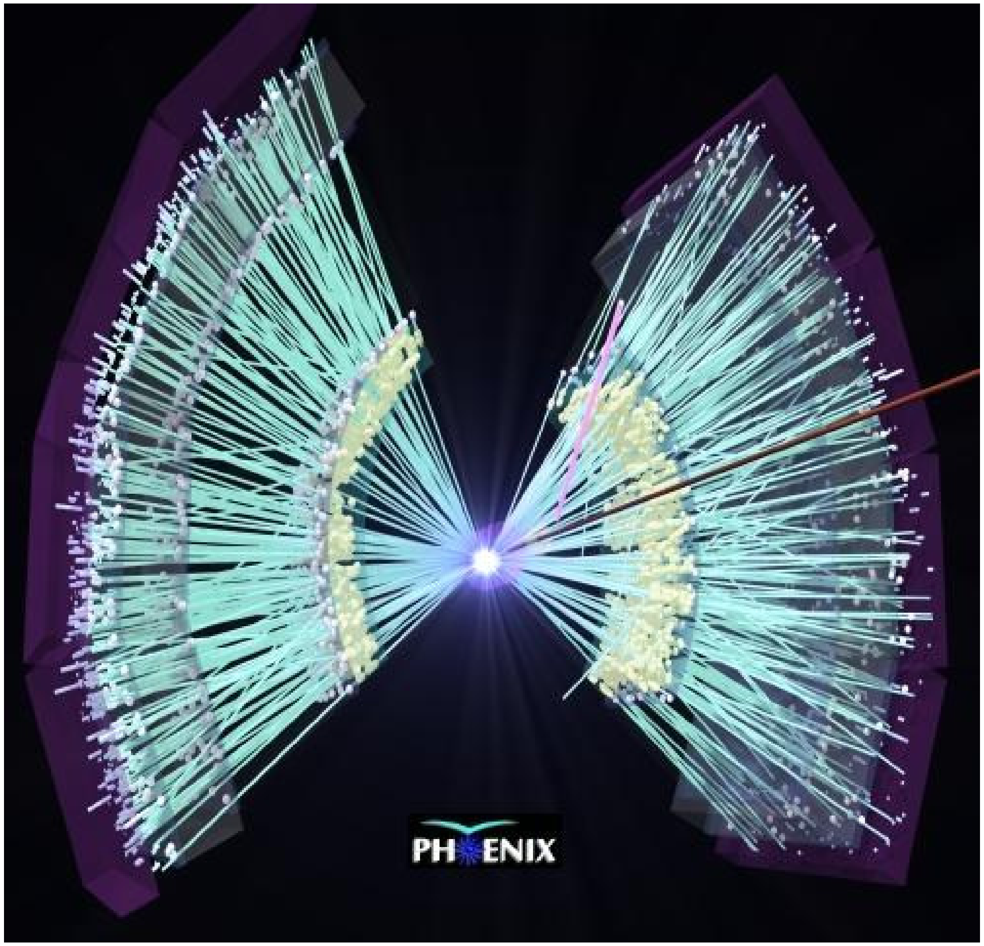
Refael, Demler, Oreg, Fisher
PRB 75:14522 (2007)



Dynamics of many-body quantum systems

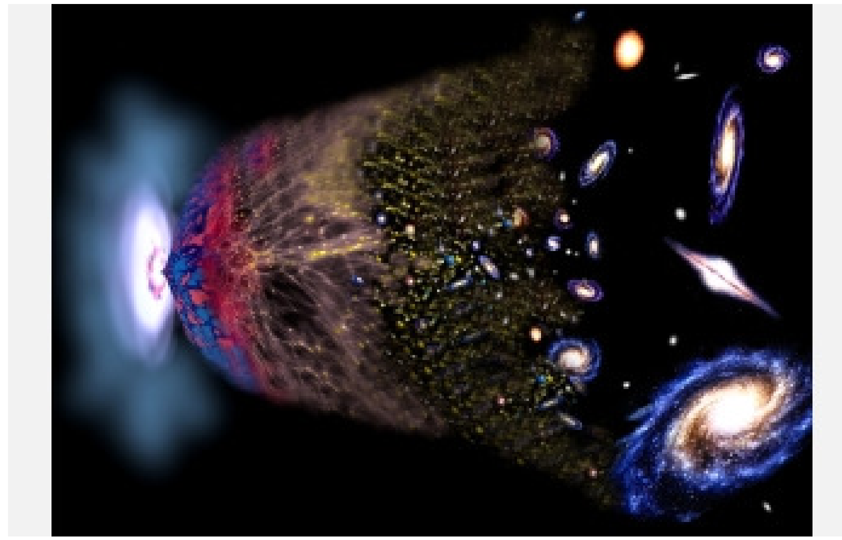
Heavy Ion collisions at RHIC

Signatures of quark-gluon plasma?

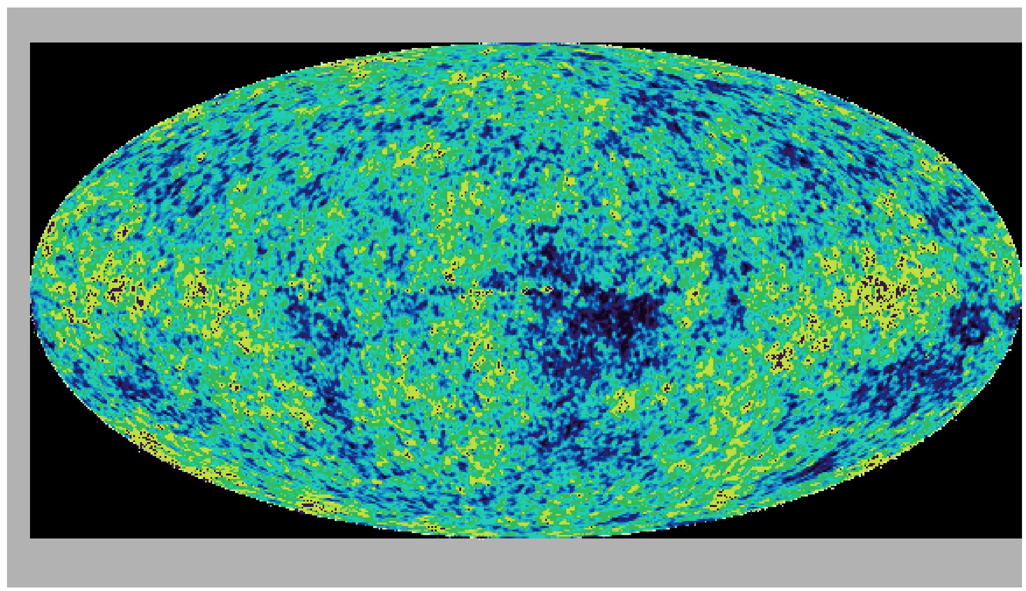


Dynamics of many-body quantum systems

Big Bang and Inflation



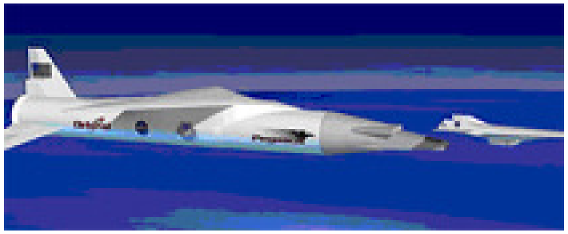
Fluctuations of the cosmic microwave background radiation. Manifestation of quantum fluctuations during inflation

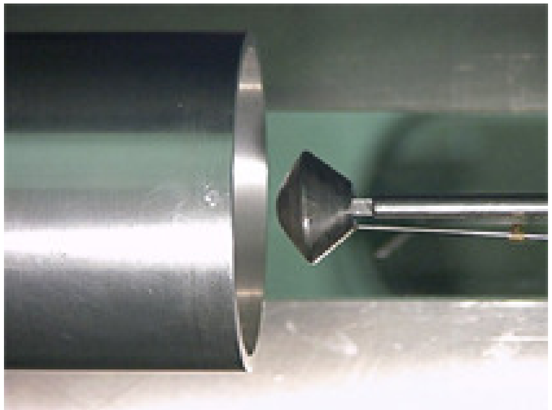


Goal:

Use ultracold atoms to create many-body systems with interesting collective properties

Keep them simple enough to be able to control and understand them





Non-equilibrium dynamics of many-body systems of ultracold atoms

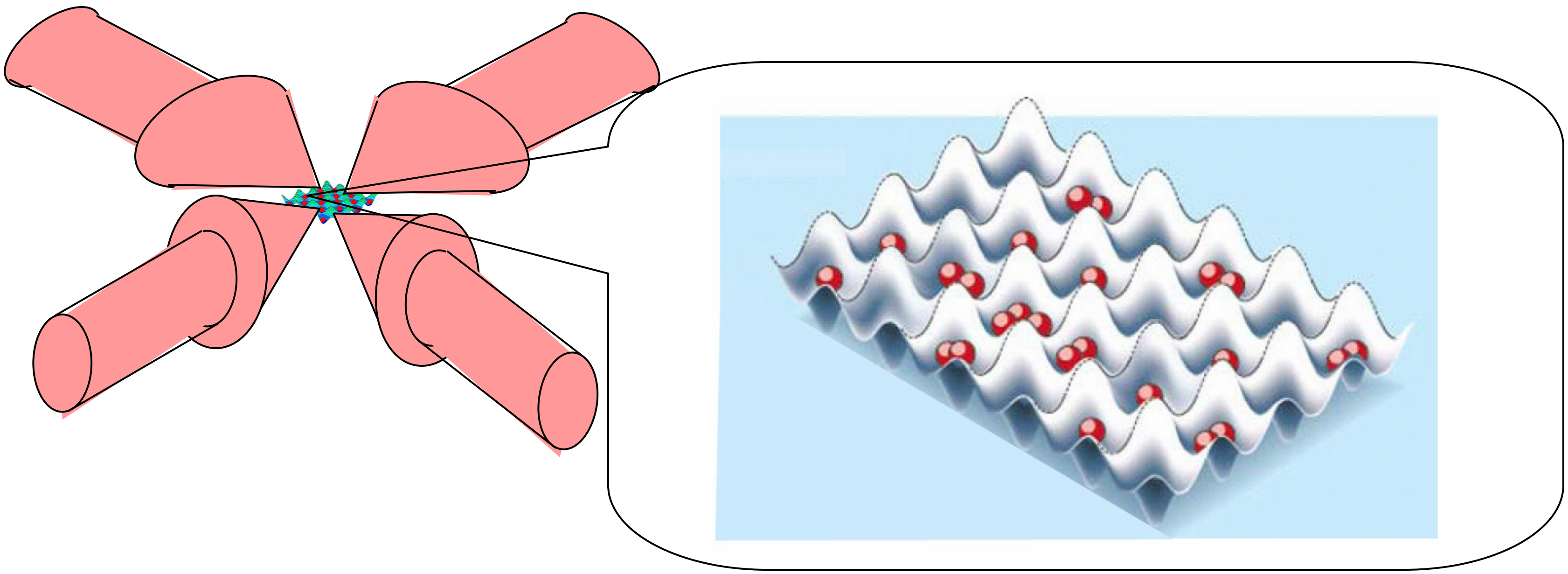
1. Dynamical instability of strongly interacting bosons in optical lattices
2. Adiabaticity of creating many-body fermionic states in optical lattices
3. Dynamical instability of the spiral state of F=1 ferromagnetic condensate
4. Dynamics of coherently split condensates
5. Many-body decoherence and Ramsey interferometry
6. Quantum spin dynamics of cold atoms in an optical lattice

Dynamical Instability of strongly interacting bosons in optical lattices

References:

Altman, Polkovnikov, Demler, Halperin, Lukin,
J. Superconductivity 17:577 (2004)
Phys. Rev. Lett. 95:20402 (2005)
Phys. Rev. A 71:63613 (2005)

Atoms in optical lattices

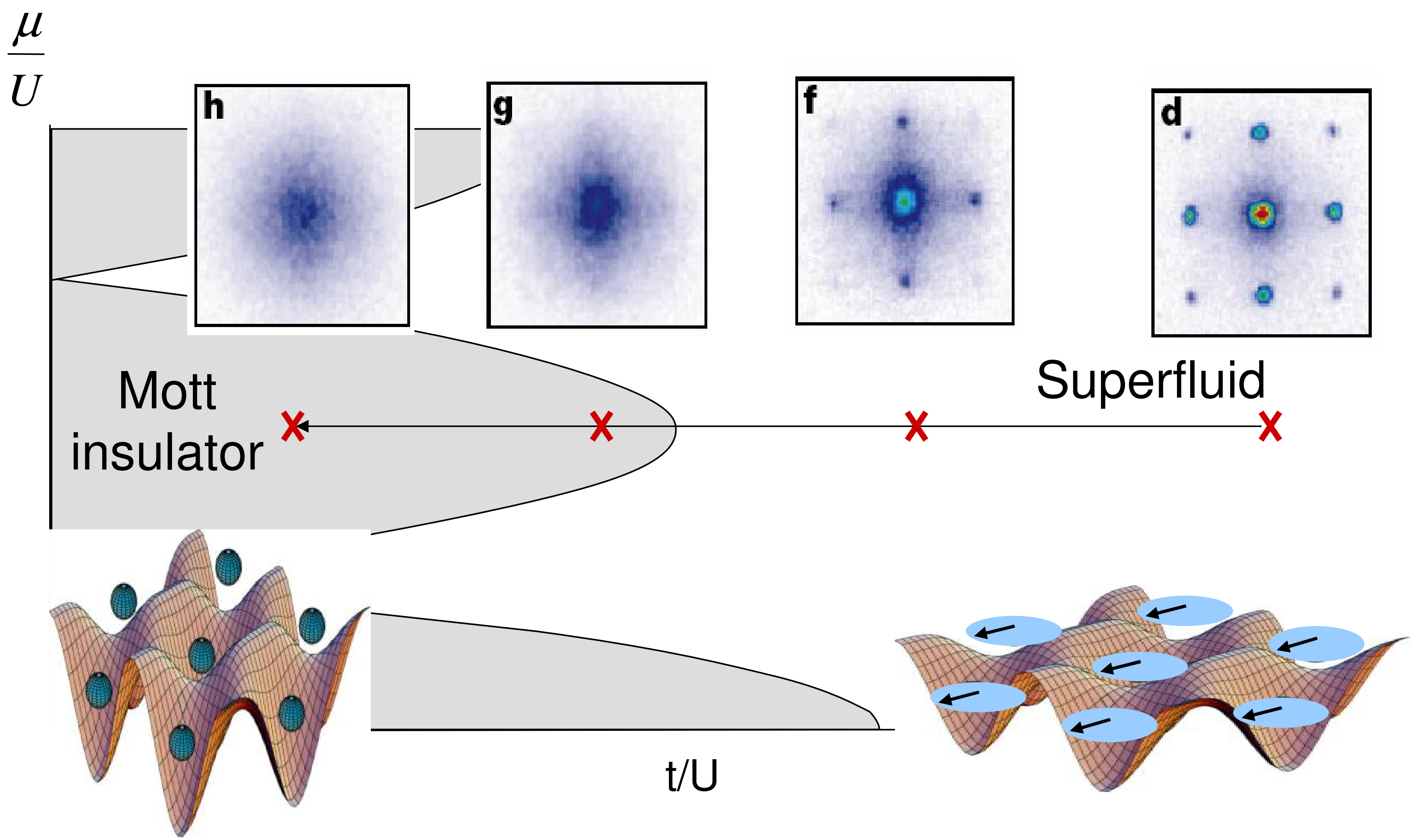


Theory: Zoller et al. PRL (1998)

Experiment: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Phillips et al., J. Physics B (2002)
Esslinger et al., PRL (2004);
Ketterle et al., PRL (2006)

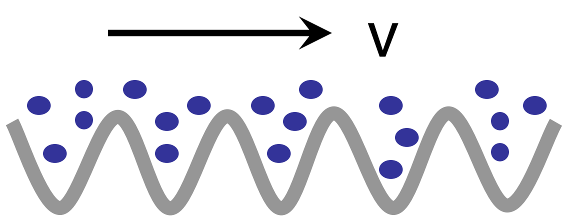
Equilibrium superfluid to insulator transition

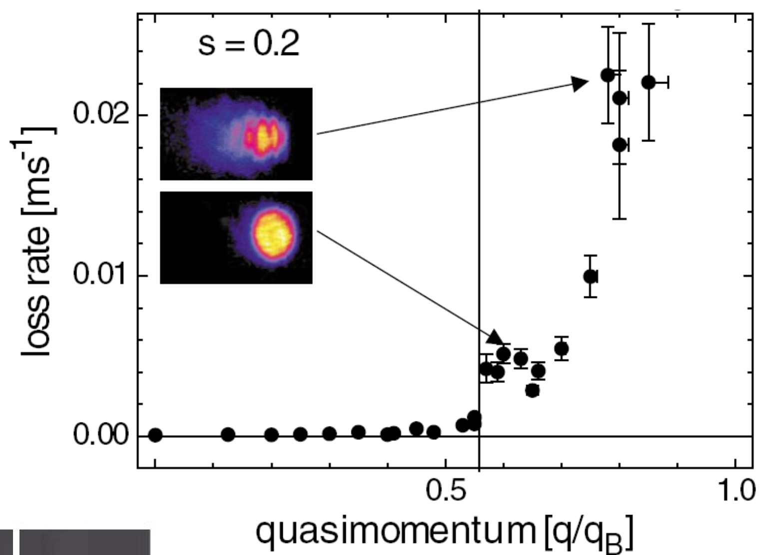
Theory: Fisher et al. PRB (89), Jaksch et al. PRL (98)
Experiment: Greiner et al. Nature (01)

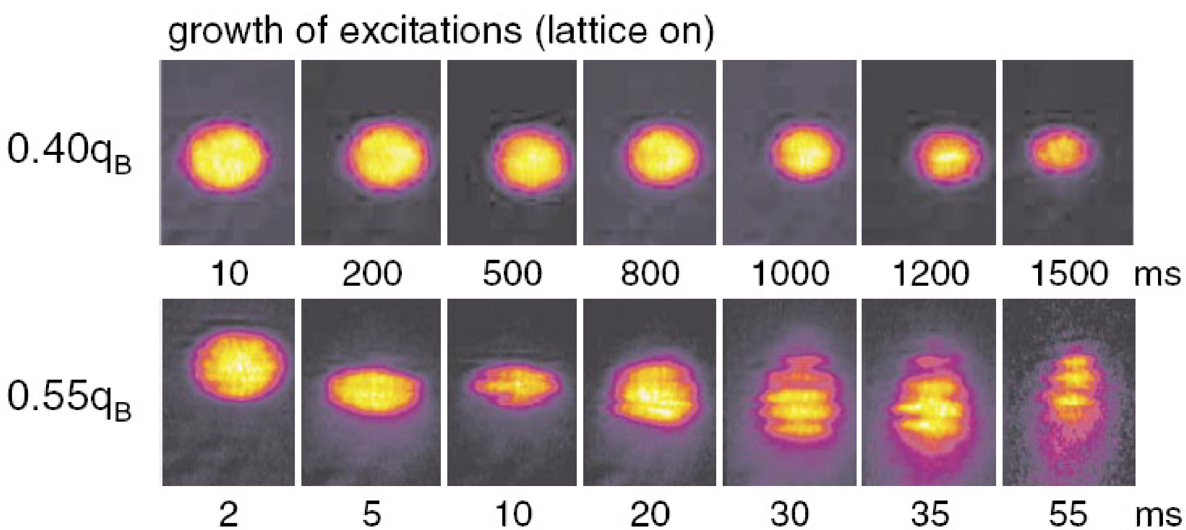


Moving condensate in an optical lattice. Dynamical instability

Theory: Niu et al. PRA (01), Smerzi et al. PRL (02)
Experiment: Fallani et al. PRL (04)

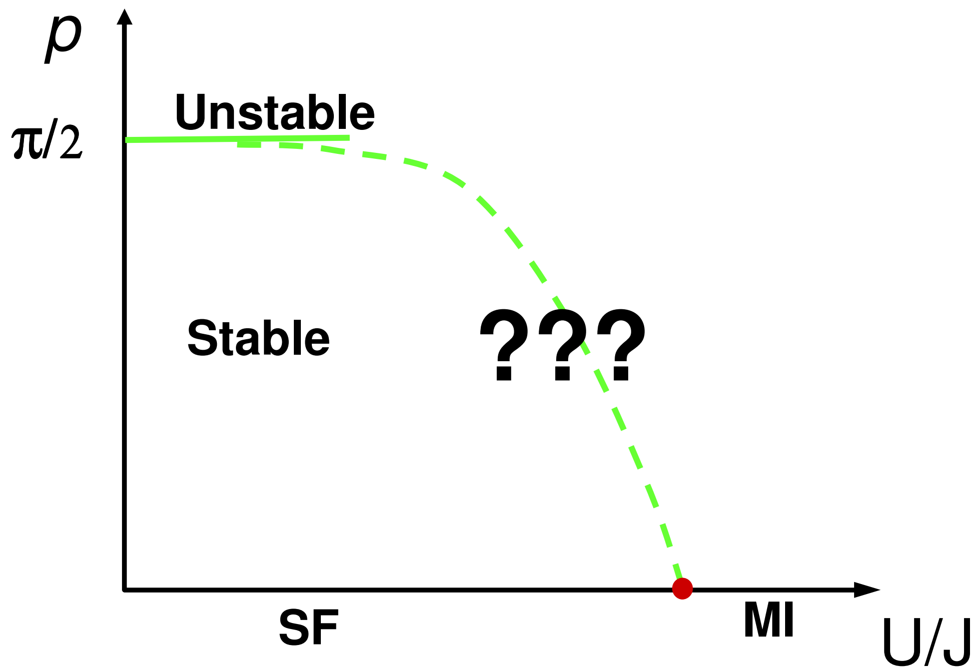




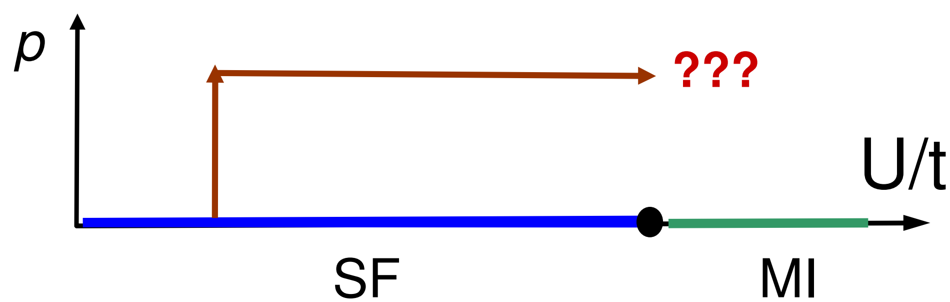


Related experiments by
Eiermann et al, PRL (03)

Question: How to connect
the dynamical instability (irreversible, classical)
to the superfluid to Mott transition (equilibrium, quantum)



Possible experimental sequence:



Dynamical instability

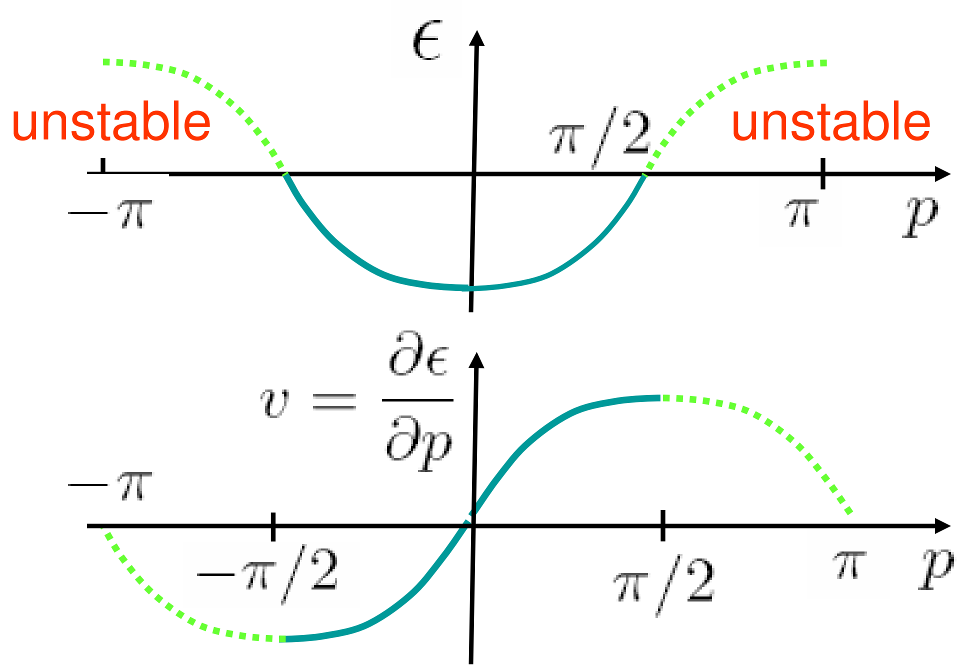
Wu, Niu, New J. Phys. 5:104 (2003)

Classical limit of the Hubbard model. $N\,t >> U$ Discreet GP equation

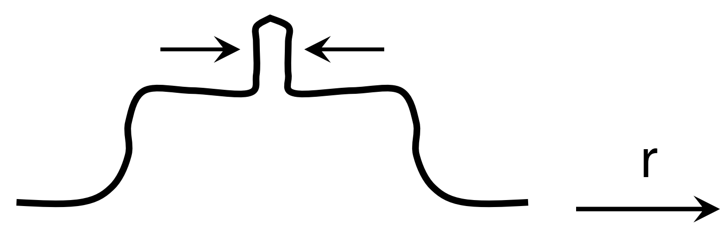
$$i\,\frac{d\,\Psi_j}{d\,t} = -t \sum_{\langle k \rangle} \Psi_k + U\,|\Psi_j|^2\,\Psi_j$$

Current carrying states $\Psi_j \sim e^{ipx_j}$

Linear stability analysis: States with p>$\pi/2$ are unstable



Amplification of density fluctuations



Dynamical instability for integer filling

Order parameter for a current carrying state $\Psi_j(p) = A(p)\ e^{ipx_j}$

Current $J(p) = |A(p)|^2 \sin(p)$

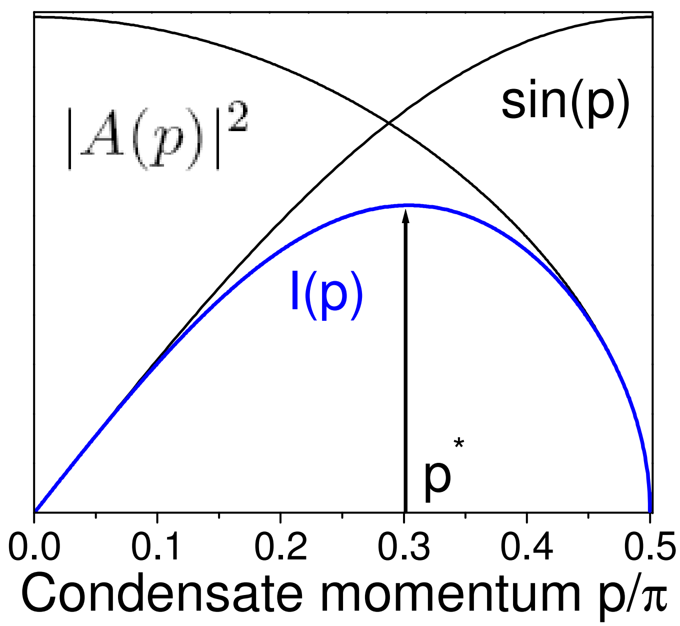
GP regime $A(p) = \sqrt{N}$. Maximum of the current for $p = \pi/2$

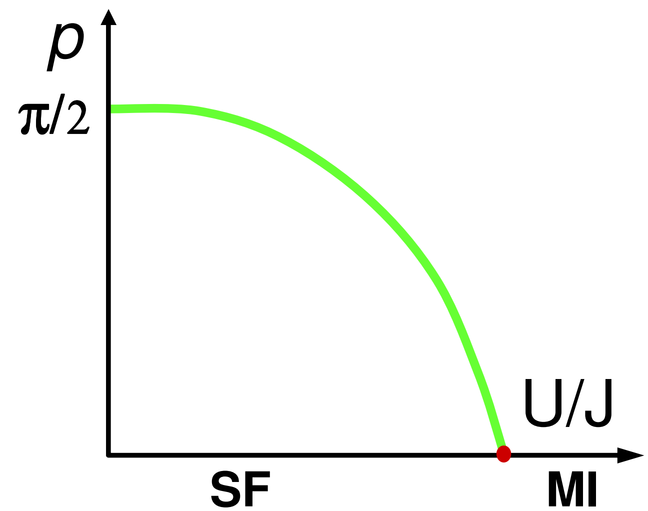
When we include quantum fluctuations, the amplitude of the order parameter is suppressed

$$\frac{A(p=0)}{\sqrt{N}} \approx 1 - \left(\frac{U}{Nt}\right)^{1/2}$$

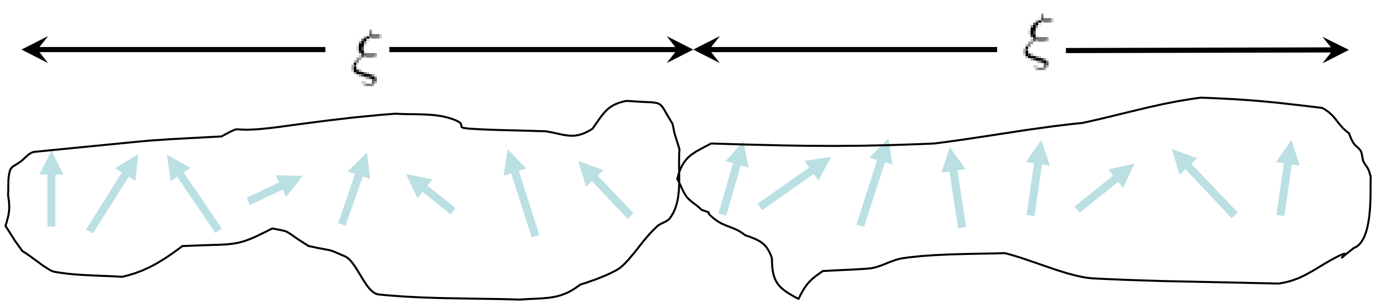
$A(p)$ decreases with increasing phase gradient p

Dynamical instability for integer filling





Vicinity of the SF-I quantum phase transition.
Classical description applies for $L > \xi \sim (U_c - U)^{-1/2}$



Dynamical instability occurs for $p\,\xi \approx \pi/2$

Dynamical instability. Gutzwiller approximation

Wavefunction

$$|\Psi(t)\rangle = \prod_j \left[\sum_{n=0}^{\infty} f_{jn}(t)\,|\,n\,\rangle_j \right]$$

Time evolution

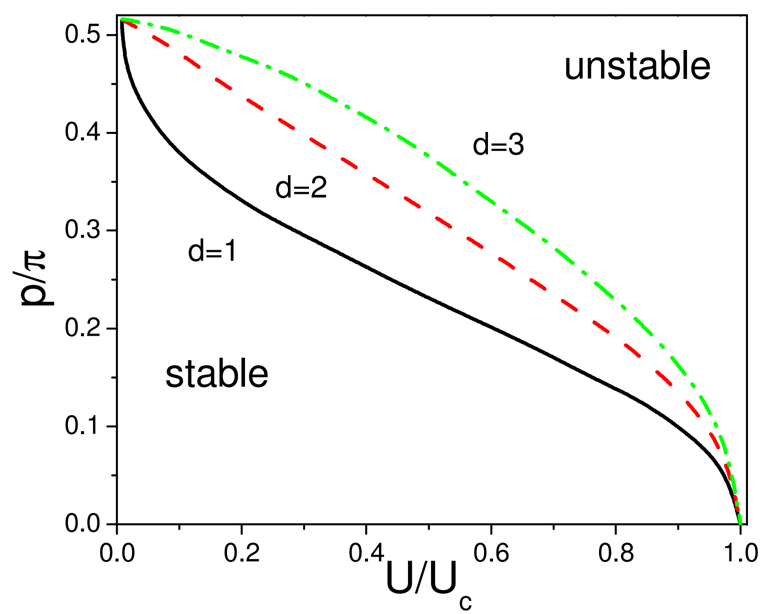
$$-i\frac{df_{jn}}{dt} = -t\,(\,f_{jn-1}\,\phi_j \,+\, f_{jn+1}\,\phi_j^*\,) \,+\, \frac{U}{2}\,n\,(n-1)\,f_{jn}$$

$$\phi_j(t) = \sum_{\langle i \rangle} \langle\,\Psi(t)\,|\,a_i\,|\,\Psi(t)\,\rangle$$

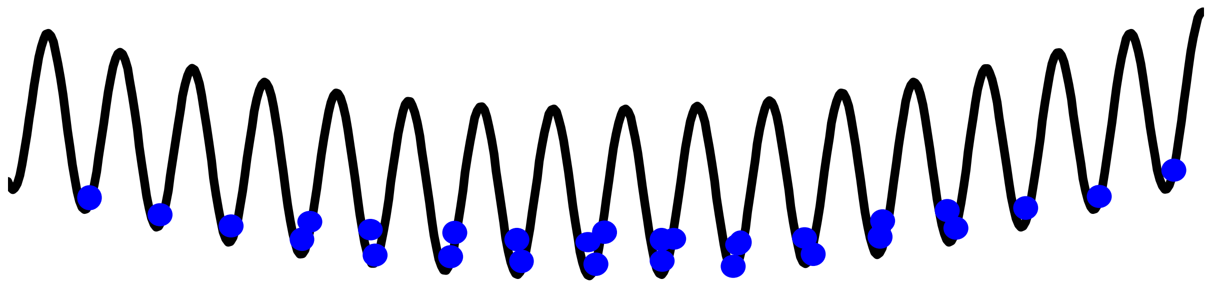
We look for stability against small fluctuations

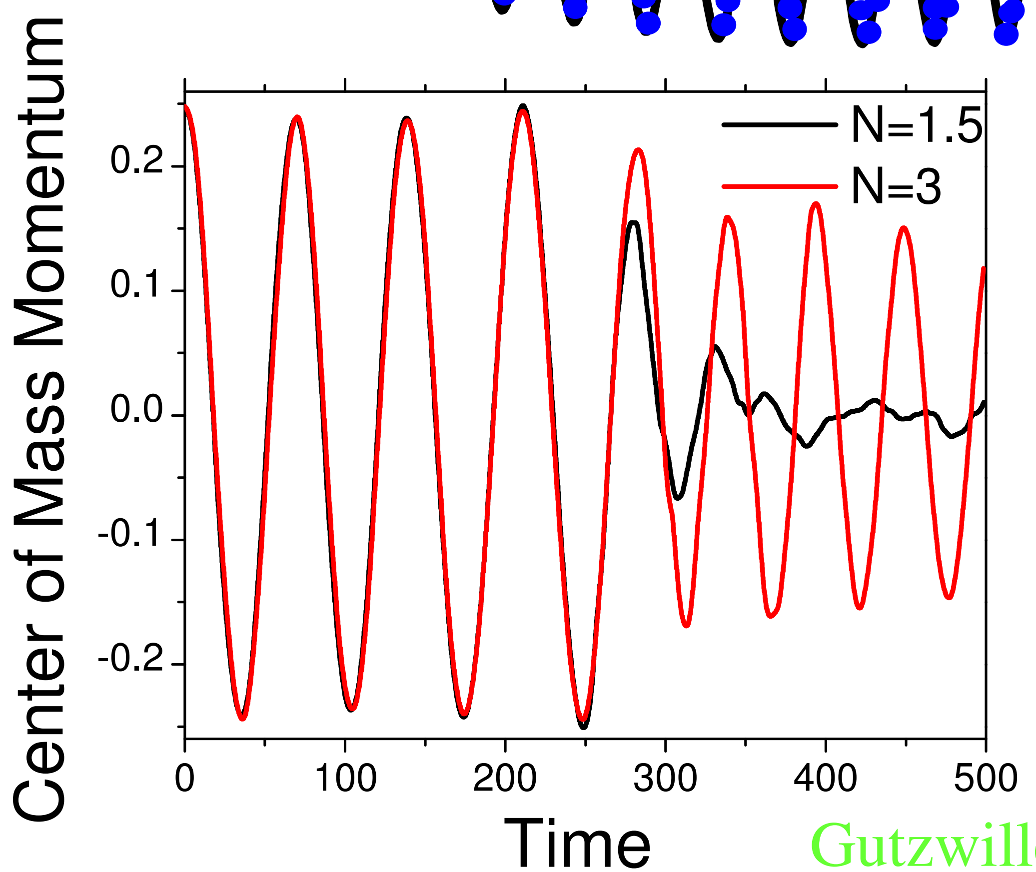
Phase diagram. Integer filling

Altman et al., PRL 95:20402 (2005)



Optical lattice and parabolic trap.
Gutzwiller approximation





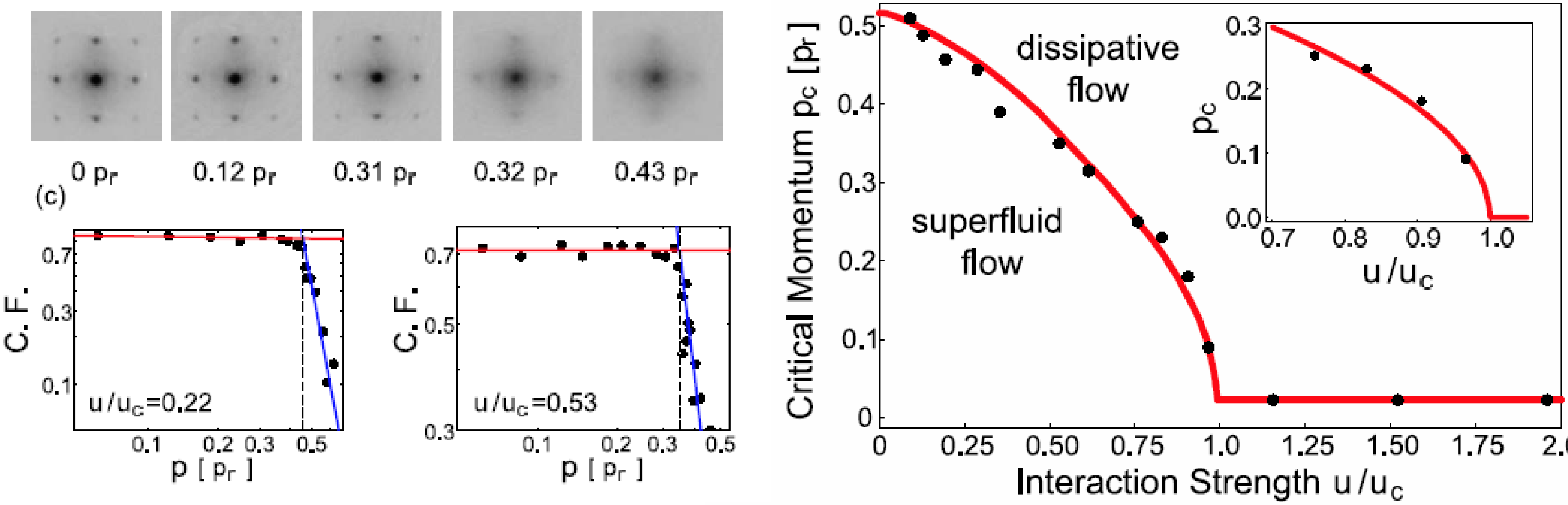
The first instability develops near the edges, where N=1

U=0.01 t
J=1/4

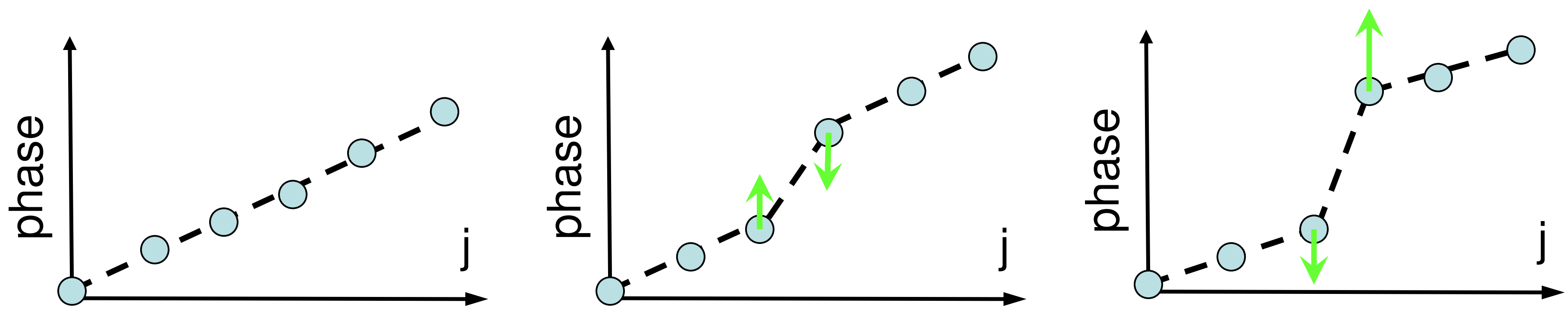
Gutzwiller ansatz simulations (2D)

Phase diagram for a Bose-Einstein condensate moving in an optical lattice

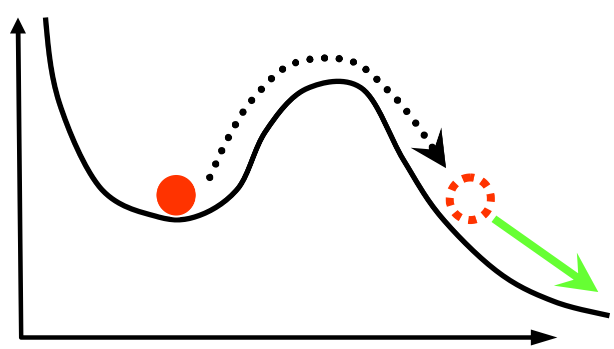
Jongchul Mun, Patrick Medley, Gretchen K. Campbell,* Luis G. Marcassa,† David E. Pritchard, and Wolfgang Ketterle
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics,
and Department of Physics, MIT, Cambridge, Massachusetts 02139, USA.



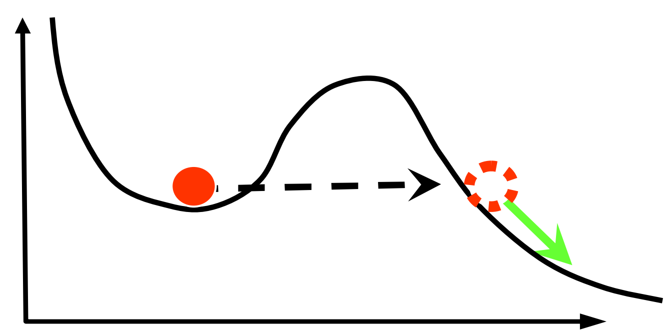
Beyond semiclassical equations. Current decay by tunneling



Current carrying states are metastable.
They can decay by thermal or quantum tunneling

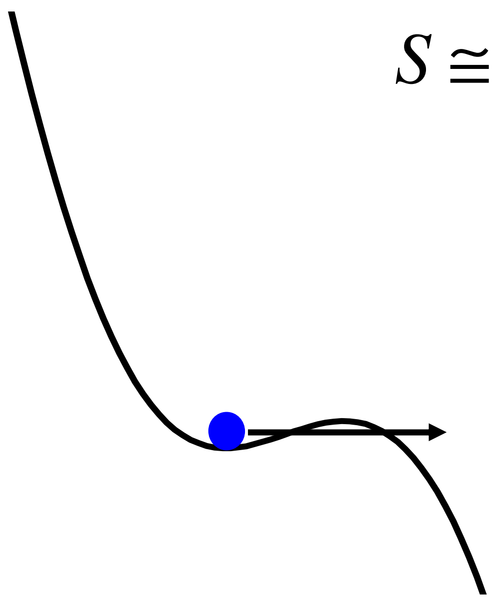


Thermal activation



Quantum tunneling

Decay rate from a metastable state. Example



$$S \cong \int_0^{\tau_0} d\tau \left(\frac{1}{2m} \left(\frac{dx}{d\tau} \right)^2 + \varepsilon x^2 - b x^3 \right)$$

Expansion in small ϵ

$$\Gamma \sim e^{-S}$$

$$S \sim \epsilon^{5/2}$$

Our small parameter of expansion:
proximity to the classical dynamical instability

$$\varepsilon \propto (p_c - p) \to 0$$

Weakly interacting systems. Quantum rotor model. Decay of current by quantum tunneling

$$S = \sum_j \int d\tau \quad \frac{1}{2U}\left(\frac{d\varphi_j}{d\tau}\right)^2 - 2JN\cos\left(\varphi_{j+1} - \varphi_j\right)$$

$$\varphi_j = pj + \bar{\varphi}_j$$

At $p \rightarrow \pi/2$ we get

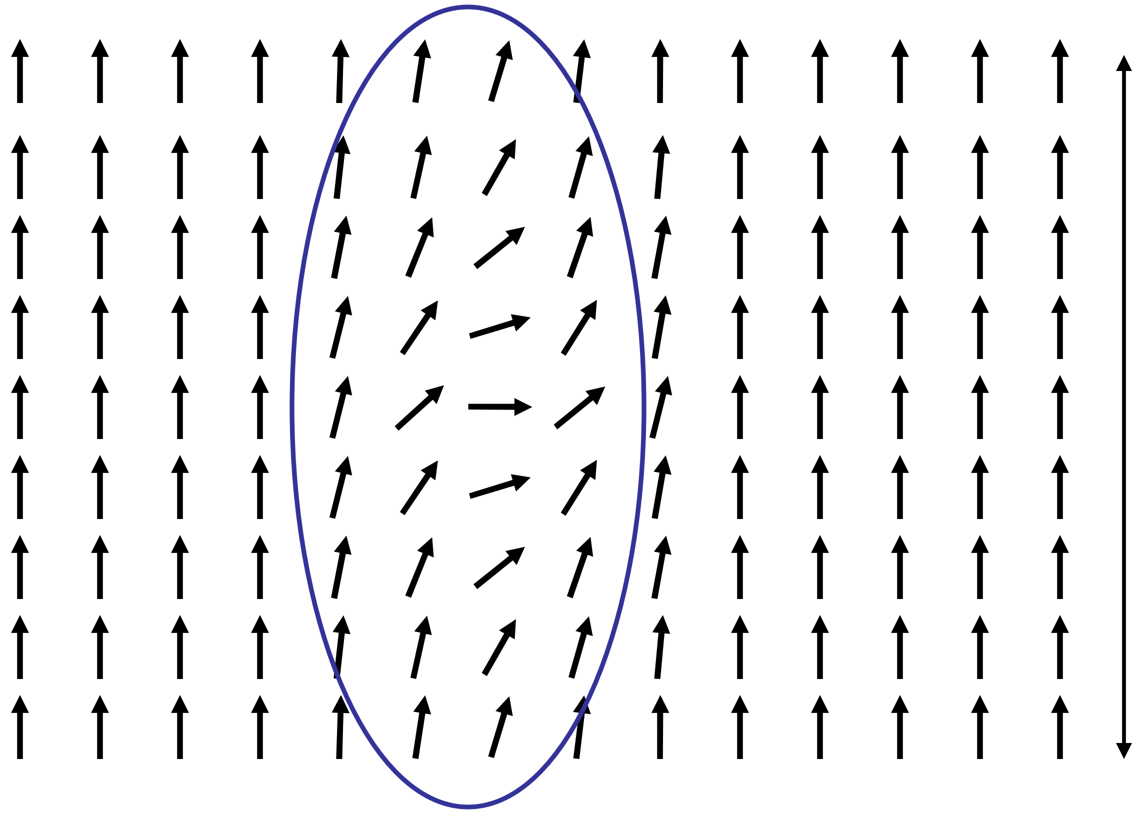
For the link on which the QPS takes place

$$S \cong \sum_j \int d\tau \quad \frac{1}{2U}\left(\frac{d\bar{\varphi}_j}{d\tau}\right)^2 + JN\cos p\left(\bar{\varphi}_{j+1} - \bar{\varphi}_j\right)^2 - \frac{JN}{3}\left(\bar{\varphi}_{j+1} - \bar{\varphi}_j\right)^3$$

d=1. Phase slip on one link + response of the chain.
Phases on other links can be treated in a harmonic approximation

For d>1 we have to include transverse directions.

Need to excite many chains to create a phase slip



$$J_{\parallel} \to J \cos p,$$

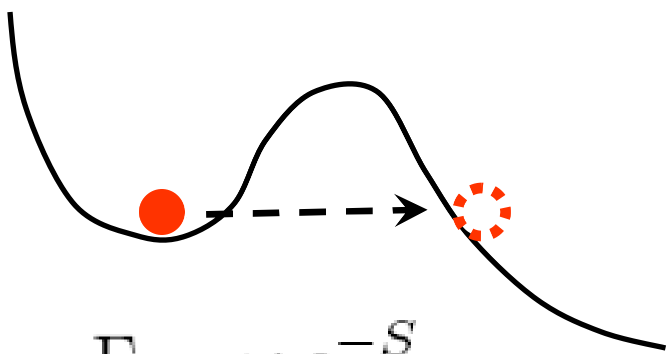
$$J_{\perp} = J$$

Longitudinal stiffness is much smaller than the transverse.

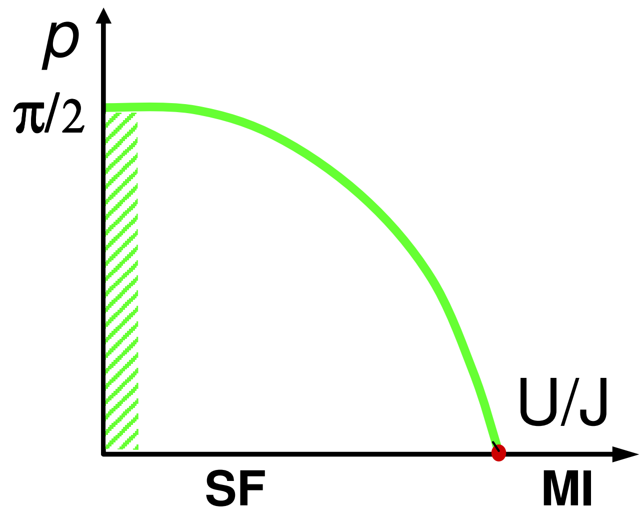
$$L_T \sim (p - p_c)^{-1/2}$$

The transverse size of the phase slip diverges near a phase slip. We can use continuum approximation to treat transverse directions

Weakly interacting systems. Gross-Pitaevskii regime. Decay of current by quantum tunneling

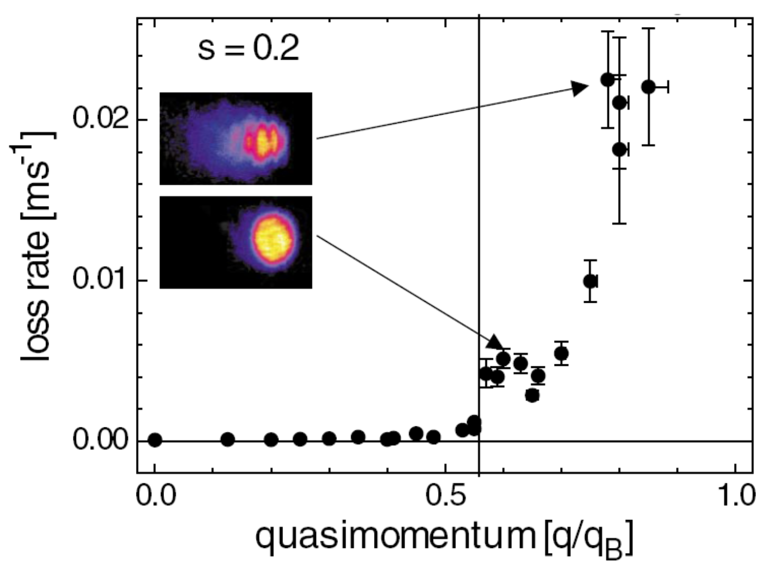


$$\Gamma = \omega\, e^{-S}$$



$$S_d \sim \sqrt{\frac{J\,N}{U}}\,(p_c - p)^{\frac{6-d}{2}}$$

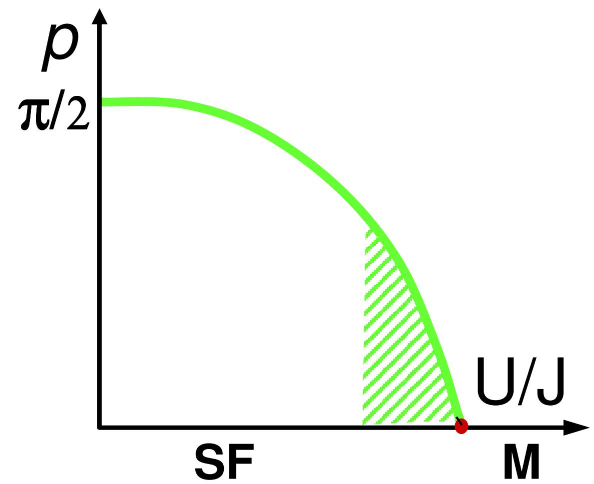
Fallani et al., PRL (04)



Quantum phase slips are strongly suppressed in the GP regime

Strongly interacting regime. Vicinity of the SF-Mott transition

Close to a SF-Mott transition we can use an effective relativistivc GL theory (Altman, Auerbach, 2004)

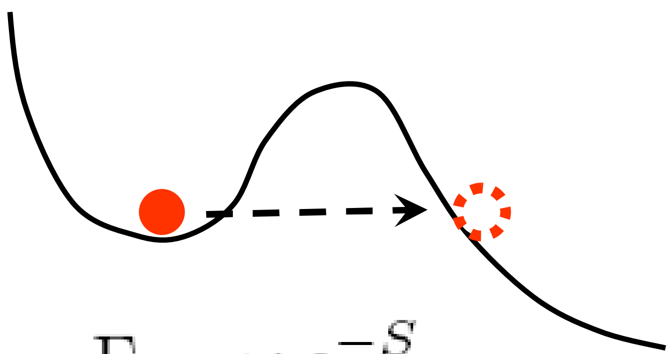


$$S = \int d\tau\, d^dx \left\{ \frac{v^2}{\xi^2} \left| \frac{d\psi}{d\tau} \right|^2 + \frac{1}{\xi^2} |\vec{\nabla}\psi|^2 - |\psi|^2 + |\psi|^4 \right\}$$

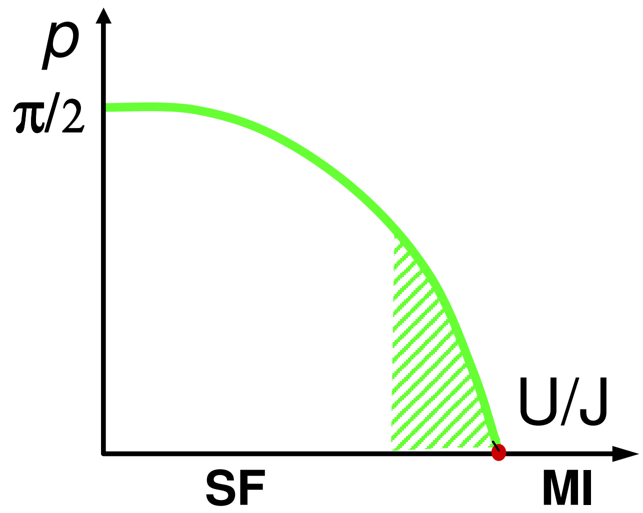
Metastable current carrying state: $\psi = \sqrt{1 - p^2\xi^2}\, e^{ip\xi x}$

This state becomes unstable at $p_c = 1/\xi\sqrt{3}$ corresponding to the maximum of the current: $I \propto p|\psi|^2 = p\left(1 - p^2\xi^2\right)$.

Strongly interacting regime. Vicinity of the SF-Mott transition
Decay of current by quantum tunneling



$$\Gamma = \omega\, e^{-S}$$



Action of a quantum phase slip in d=1,2,3

$$S_d \sim \frac{1}{\xi^2}\,(\,1 \;-\; \sqrt{3}\,p\,\xi\,)^{5/2-d}$$

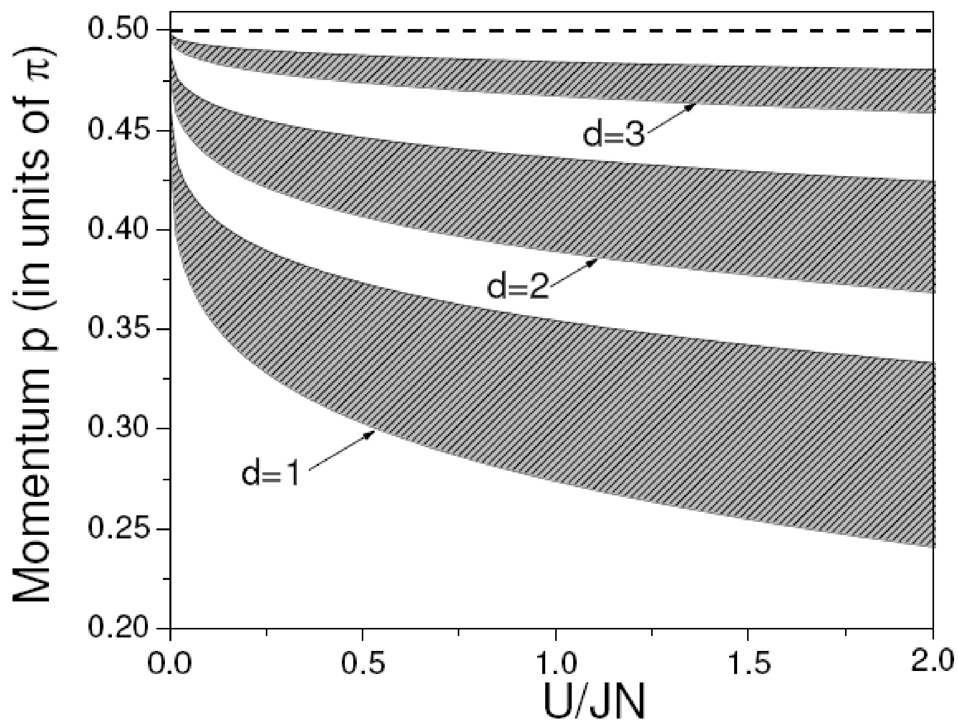
ξ - correlation length

$$\xi \sim (\,U_c - U\,)^{-1/2}$$

Strong broadening of the phase transition in d=1 and d=2

S_{3d} is discontinuous at the transition. Phase slips are not important.
Sharp phase transition

Decay of current by quantum tunneling



Phase diagram for a Bose-Einstein condensate moving in an optical lattice

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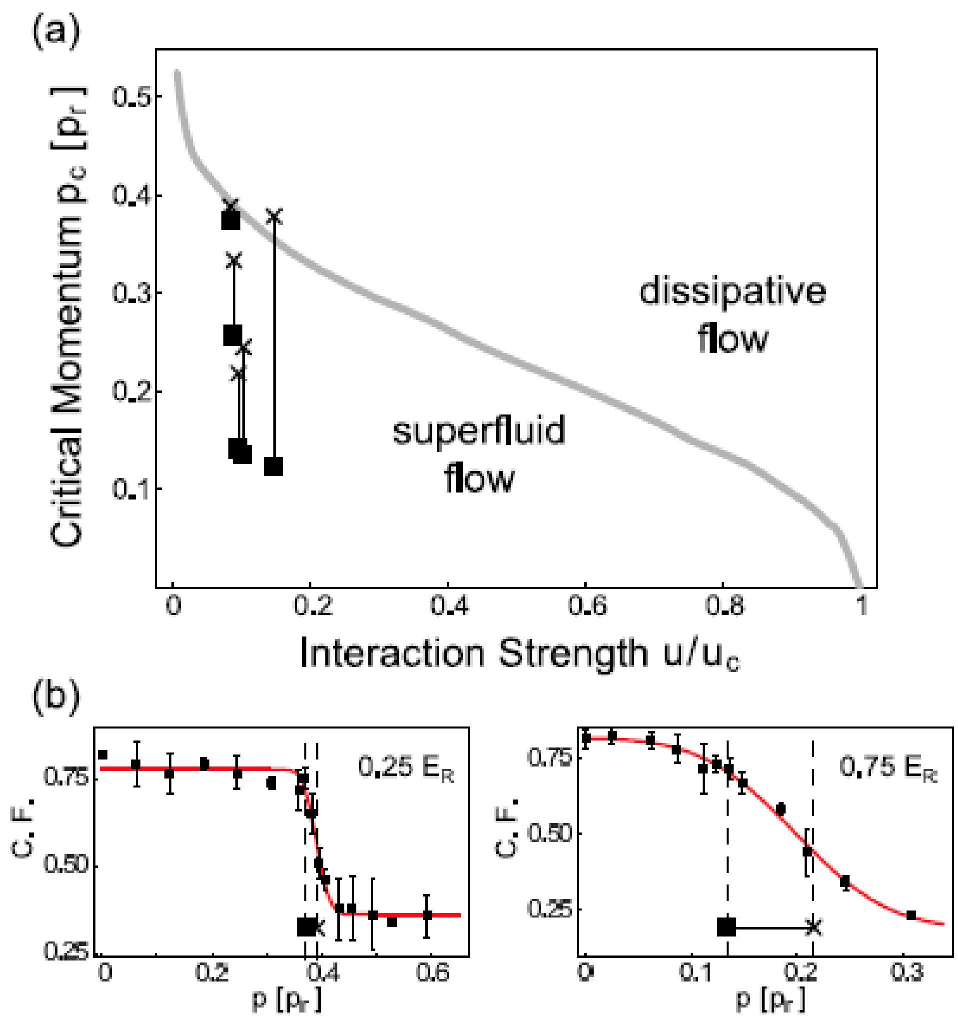
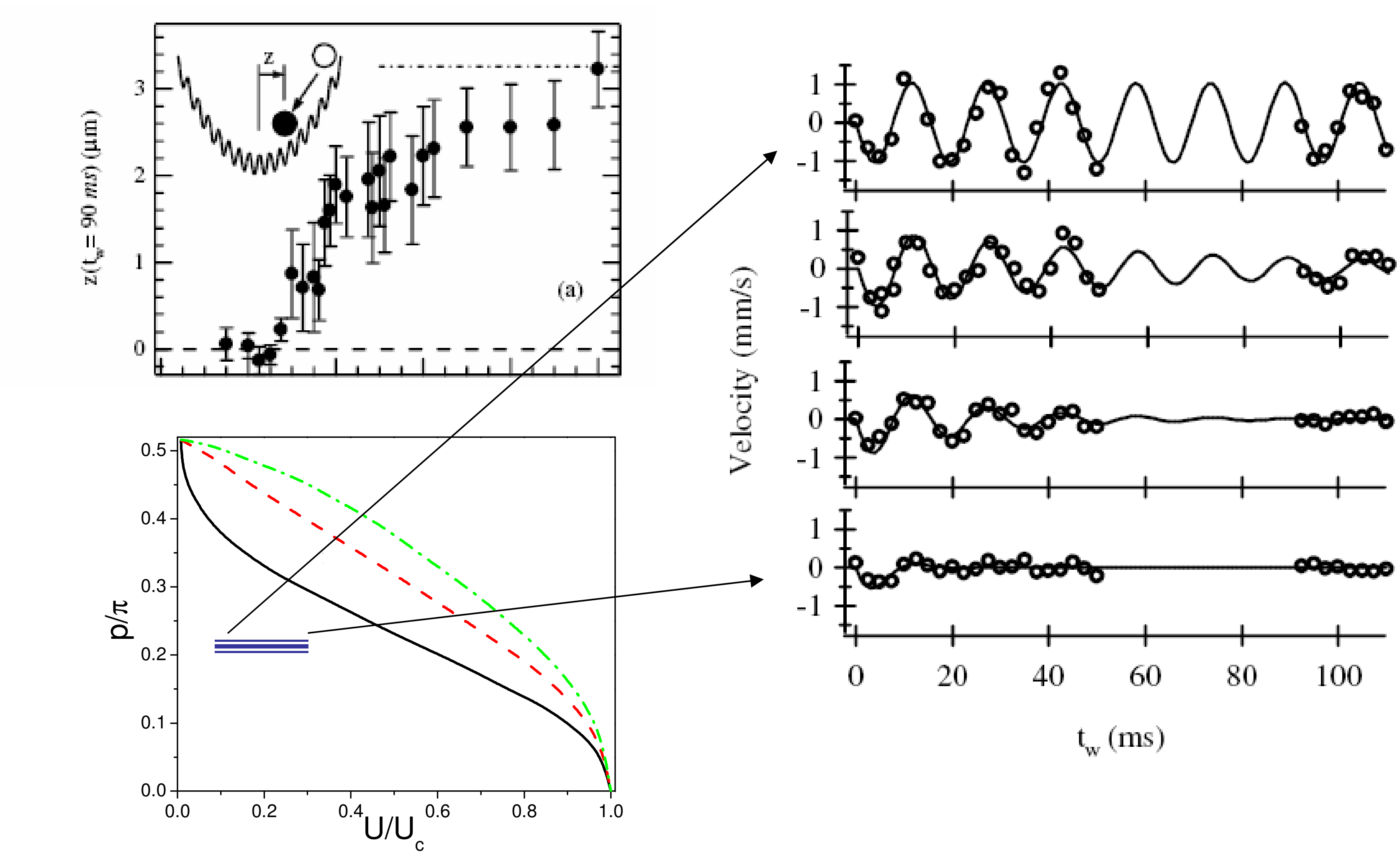
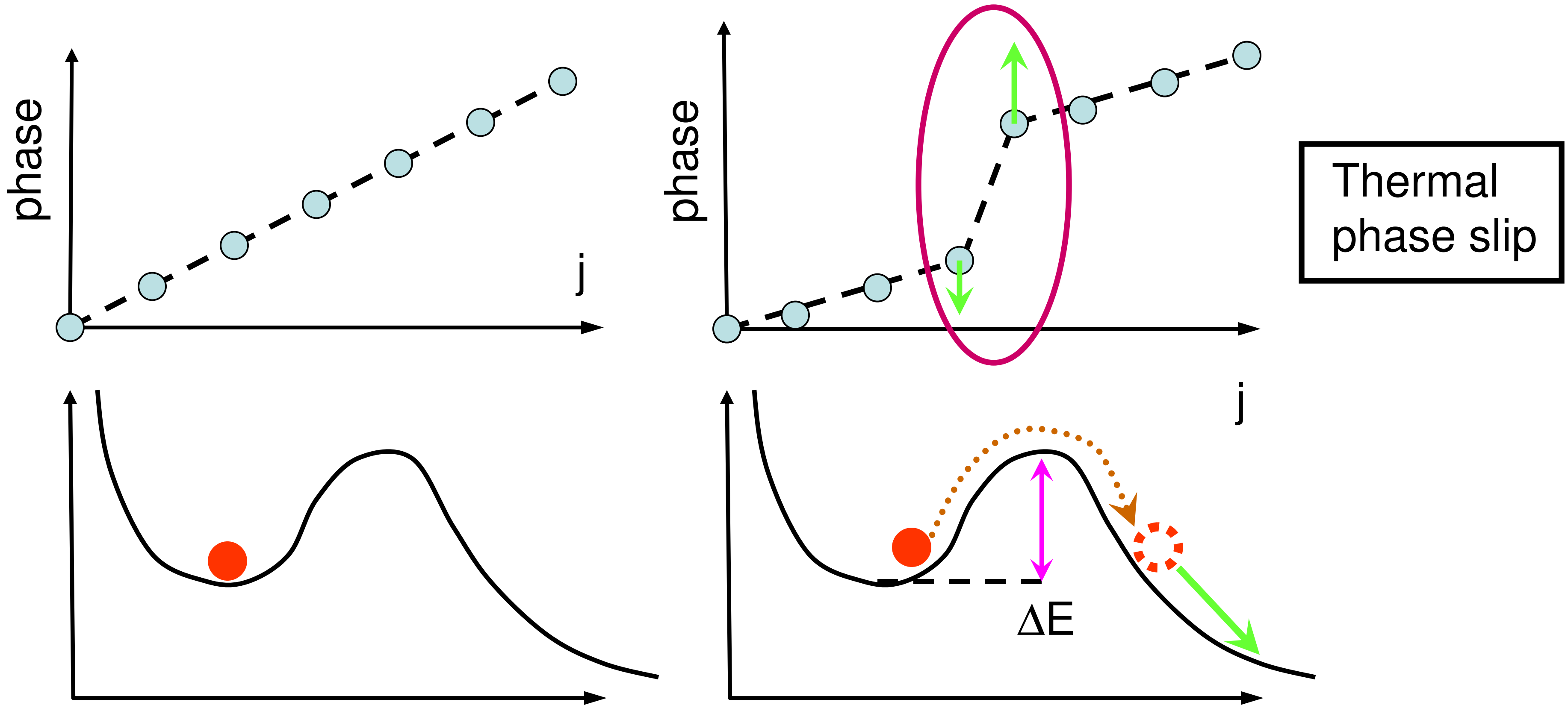


Strongly Inhibited Transport of a Degenerate 1D Bose Gas in a Lattice

C. D. Fertig,[1,2] K. M. O'Hara,[1,*] J. H. Huckans,[1,2] S. L. Rolston,[1,2] W. D. Phillips,[1,2] and J. V. Porto[1]



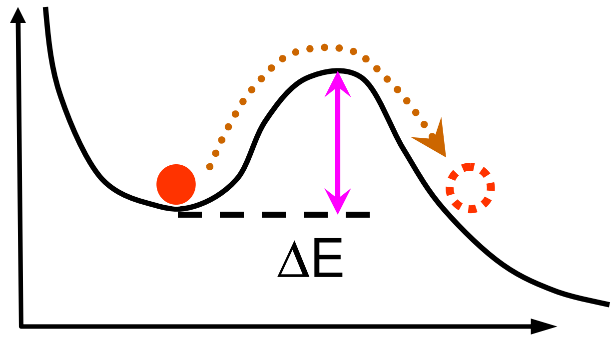
Decay of current by thermal activation



Escape from metastable state by thermal activation

$$\Gamma \sim e^{-\Delta E/T}$$

Thermally activated current decay. Weakly interacting regime



Thermal phase slip

$$\Gamma \sim e^{-\Delta E/T}$$

Activation energy in d=1,2,3

$$\Delta E_1 = 1.3\, J\, N\, (\frac{\pi}{2} - p)^3$$

$$\Delta E_2 = 10\, J\, N\, (\frac{\pi}{2} - p)^{5/2}$$

$$\Delta E_3 = 35\, J\, N\, (\frac{\pi}{2} - p)^2$$

Thermal fluctuations lead to rapid decay of currents

Crossover from thermal to quantum tunneling

$$T_Q \sim \sqrt{N\, J\, U \times (\frac{\pi}{2} - p)}$$

Decay of current by thermal fluctuations

Unstable regimes for a Bose-Einstein condensate in an optical lattice

L. De Sarlo*, L. Fallani, J. E. Lye, M. Modugno[1], R. Saers†, C. Fort and M. Inguscio

Phys. Rev. Lett. (2004)

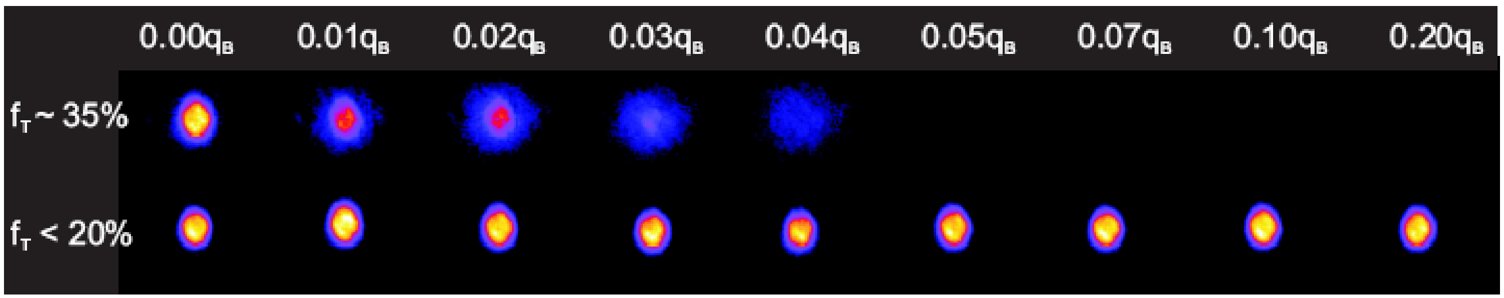
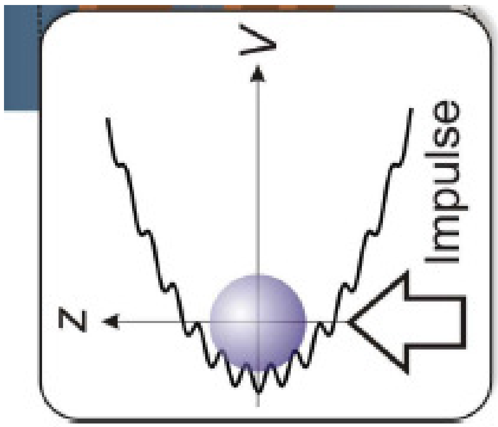


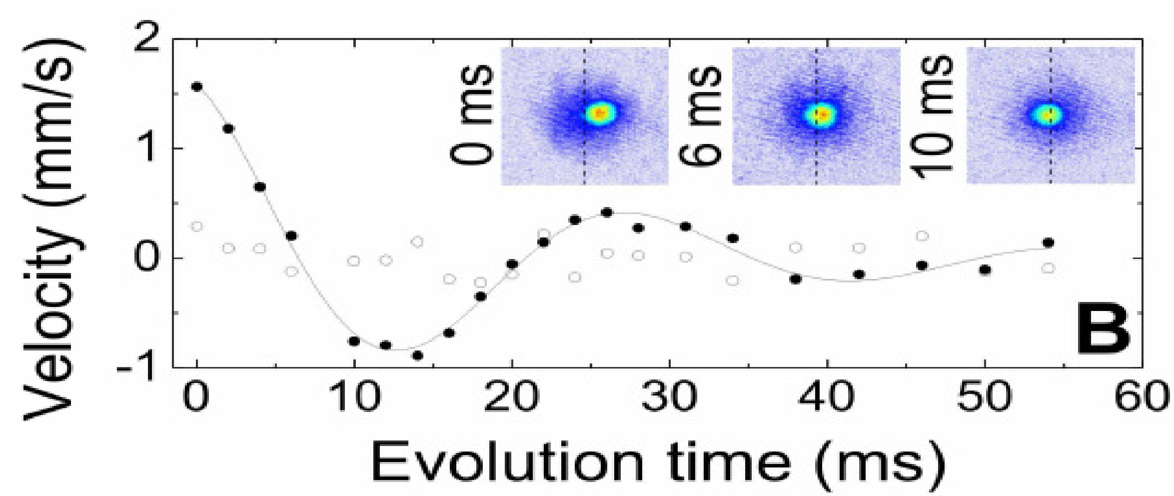
FIG. 2: Absorption images of the condensate interacting for $t = 15$s with a lattice with $s = 0.2$ for different values of quasimomentum ranging from 0 to $0.20\, q_B$ and for respectively a condensed fraction of about 65% (*top*) and no detectable thermal component (*bottom*).

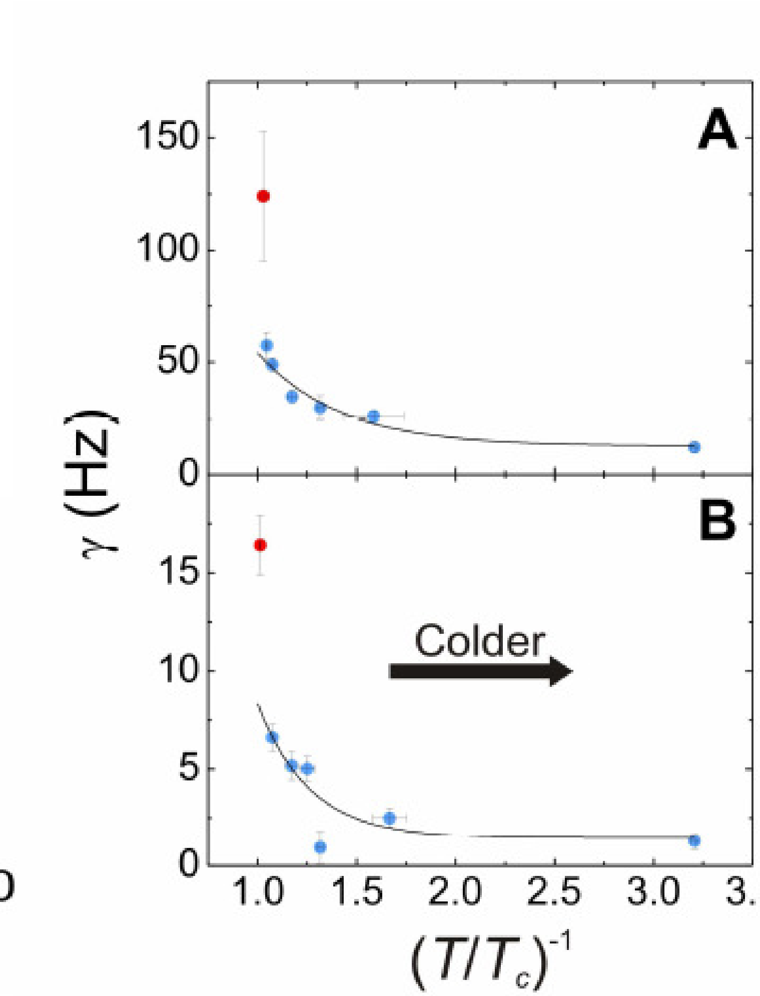
Also experiments by Brian DeMarco et al., arXiv 0708:3074

Decay of current by thermal fluctuations

Experiments: Brian DeMarco et al., arXiv 0708:3074





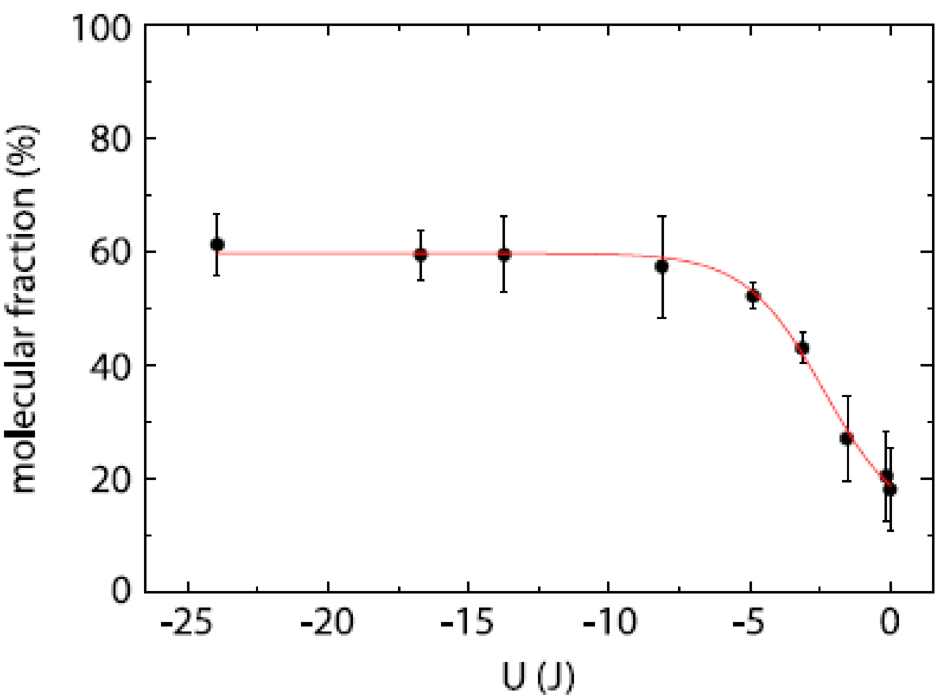


Adiabaticity of creating many-body fermionic states in optical lattices

Formation of molecules with increasing interaction strength

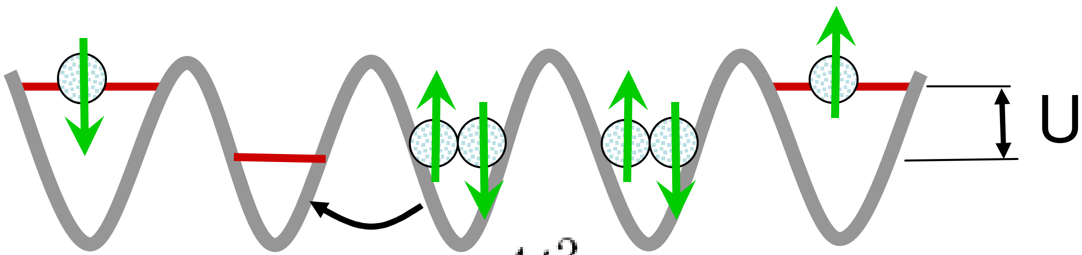
Strohmaier et al., arXiv:0707.314

$$\mathcal{H} = -t \sum_{\langle ij \rangle} c^{\dagger}_{i\sigma} c_{j\sigma} + U(t) \sum_{i} n_{i\uparrow} n_{i\downarrow}$$



Saturation in the number of molecules created is related to the finite rate of changing interaction strength U(t)

Formation of molecules with increasing interaction strength



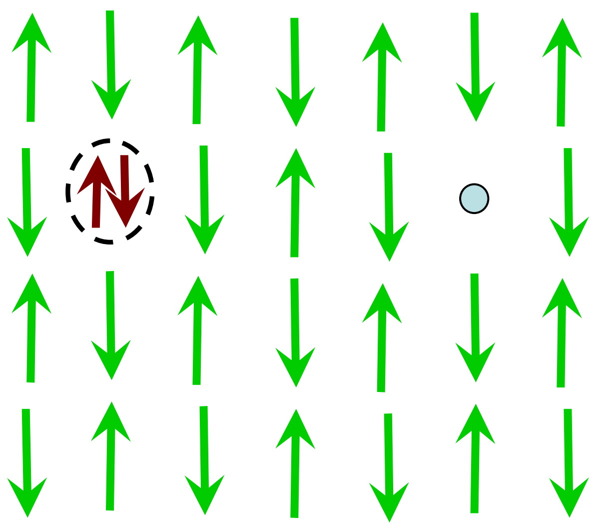
$$t_{\rm mol} = \frac{4t^2}{U}$$

During adiabatic evolution with increasing attractive U, all single atoms should be converted to pairs. Entropy is put into the kinetic energy of bound pairs.

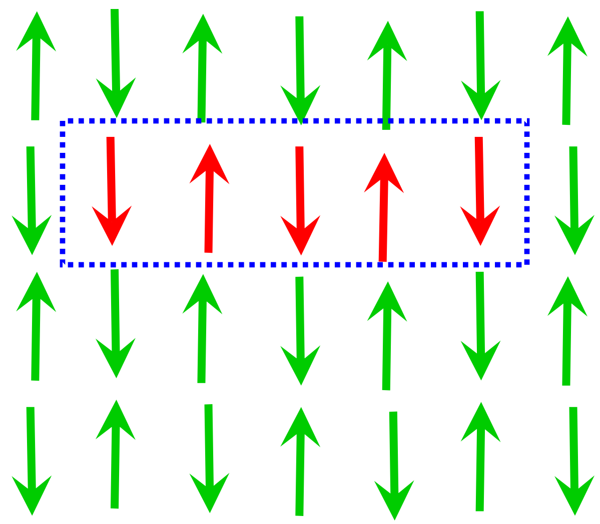
As U is increased, the excess energy of two unpaired atoms should be converted to the kinetic energy of bound pairs.

The kinetic energy of a single molecule is set by $t_{\rm mol} = 4t^2/U$.
When *U>>t* many particles will have to be involved in the relaxation process.

Hubbard model with repulsion: dynamics of breaking up pairs

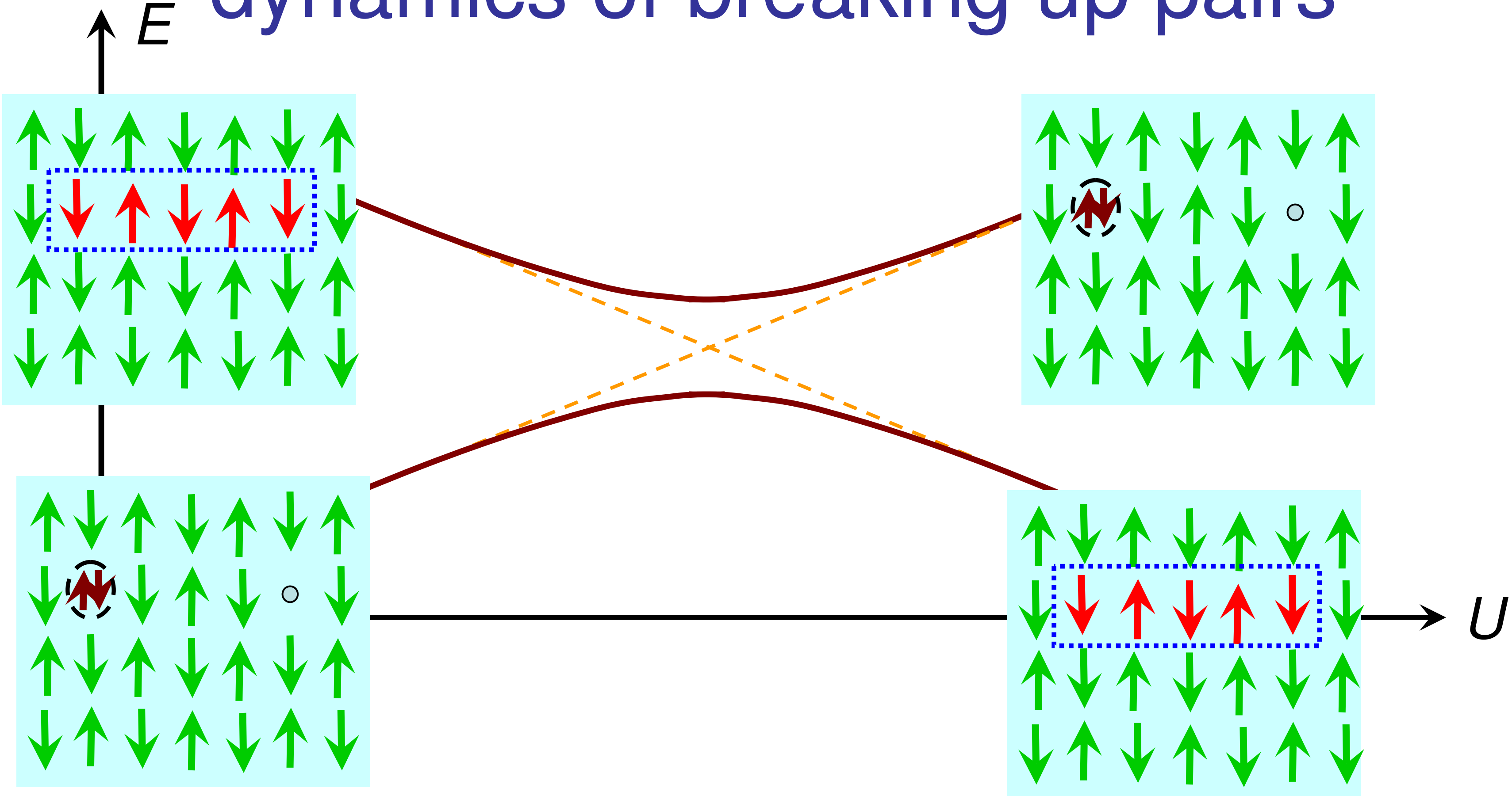


Energy of on-site repulsion



Energy of spin domain walls

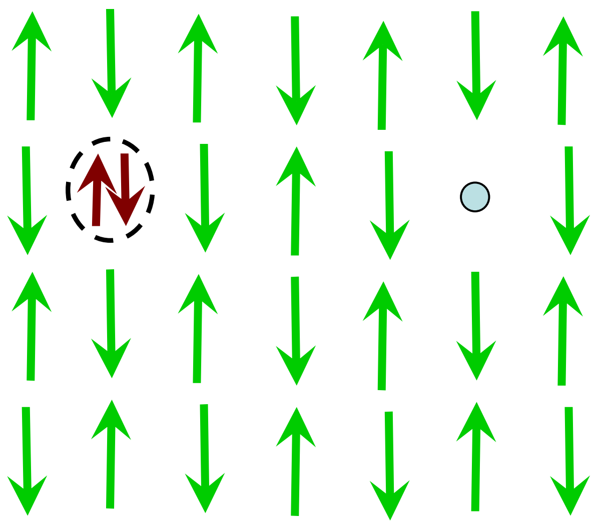
Hubbard model with repulsion: dynamics of breaking up pairs

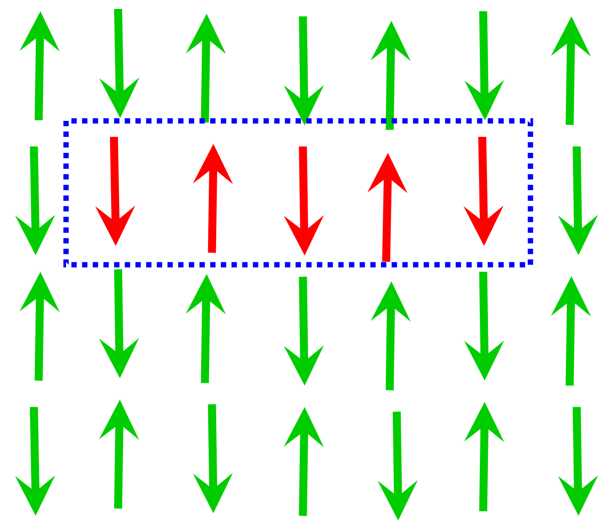


Energy of on-site repulsion U
Energy of spin domain wall $N \times 4t^2/U$

Stringent requirements on the rate of change of the interaction strength to maintain adiabaticity at the level crossing

Hubbard model with repulsion: dynamics of breaking up pairs



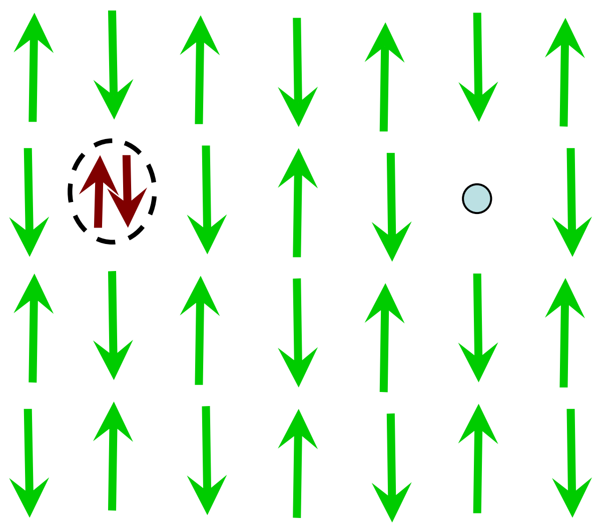


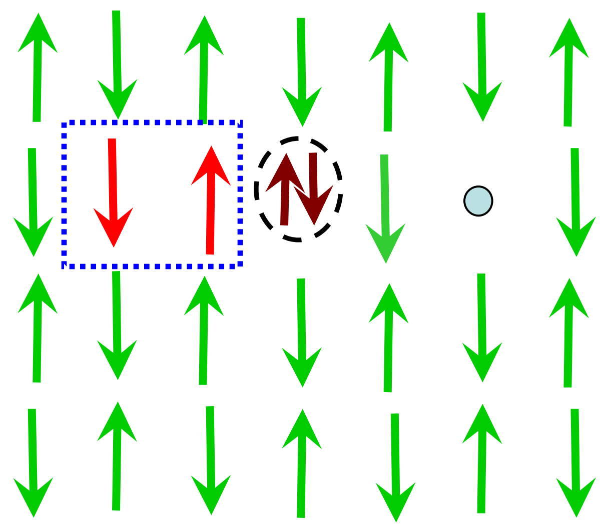
$$U \approx (z-2)\,\frac{J_{\rm ex}}{2}\,N$$

$$J_{\rm ex} = 4t^2/U$$

Hubbard model with repulsion: dynamics of breaking up pairs

Dynamics of recombination: a moving pair pulls out a spin domain wall

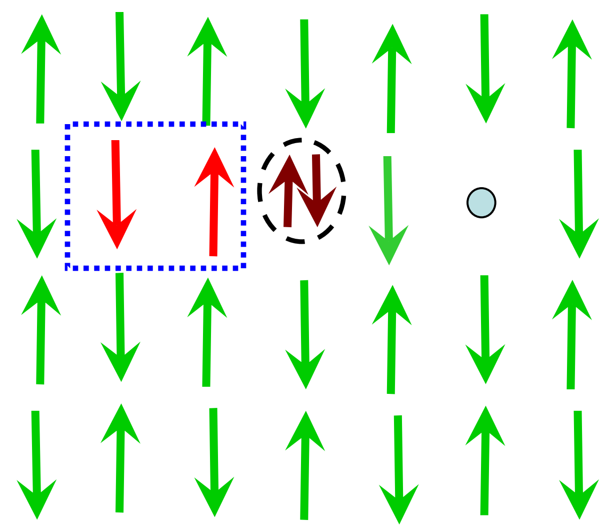




High order perturbation theory $V = t$

Hubbard model with repulsion: dynamics of breaking up pairs

$$V = t$$



$$V_{\text{eff}} \sim V \prod_{n=1}^{N} \frac{V}{(E_n - E_0)}$$

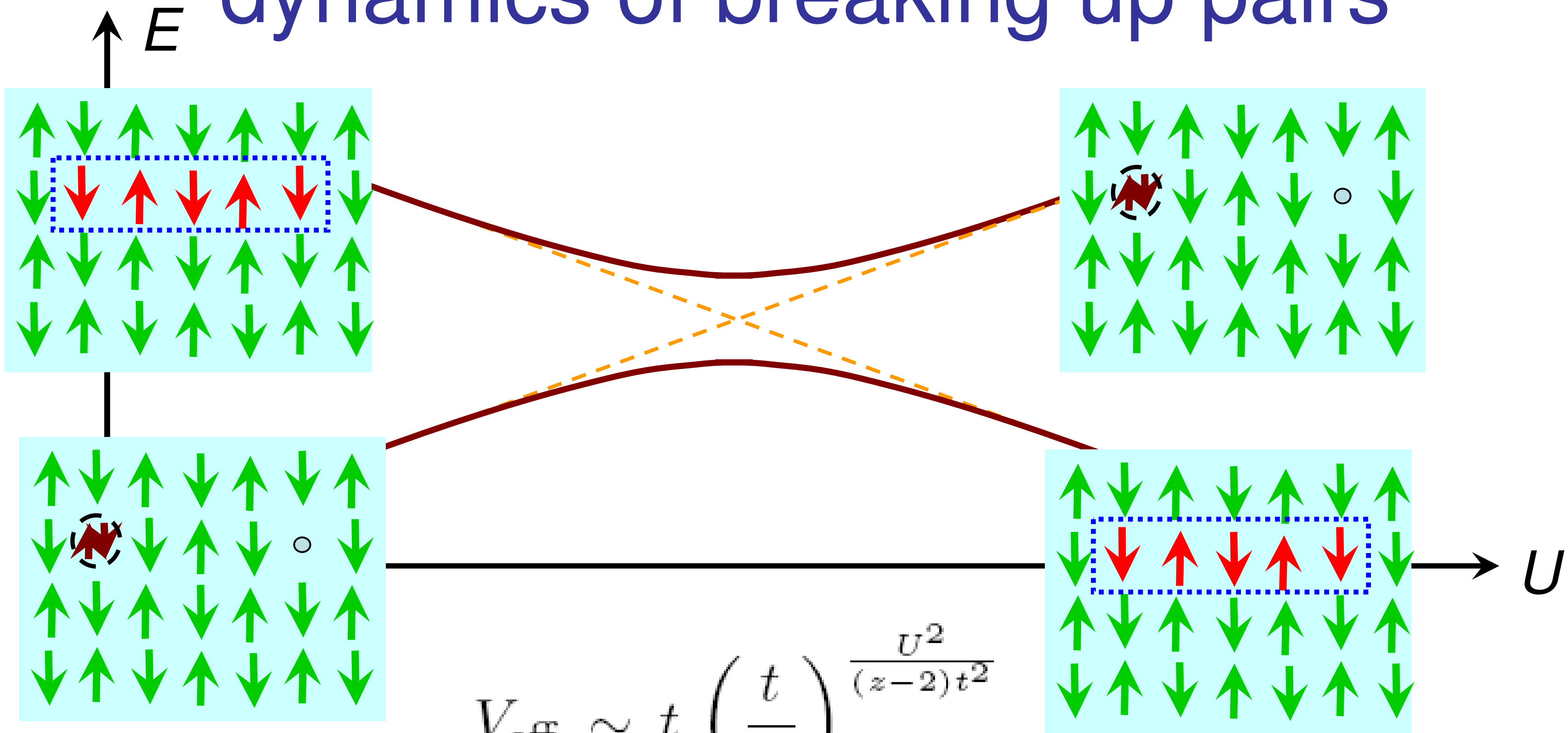
$$E_n - E_0 \approx (z-2)\frac{J_{\text{ex}}}{2} n = \frac{n}{N} \times U$$

$$V_{\text{eff}} \sim \frac{t}{N!}\left(\frac{2t}{(z-2)J_{\text{ex}} n}\right)^N \sim t\left(\frac{t}{U}\right)^N$$

N itself is a function of *U/t* :

$$U \approx (z-2)\frac{J_{\text{ex}}}{2} N$$

Hubbard model with repulsion: dynamics of breaking up pairs



$$V_{\text{eff}} \sim t \left(\frac{t}{U}\right)^{\frac{U^2}{(z-2)t^2}}$$

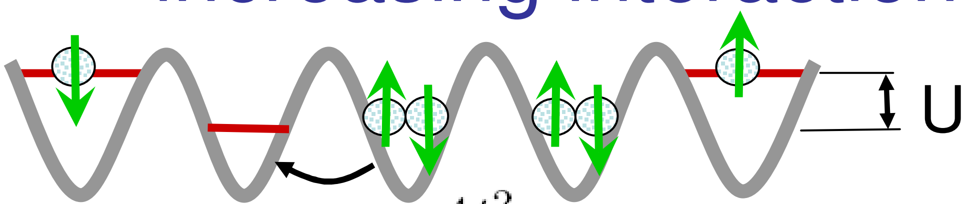
Extra geometrical factor to account for different configurations of domain walls

Probability of nonadiabatic transition $P_{12} \propto e^{-2\pi\omega_{12}\tau_d}$

ω_{12} – Rabi frequency at crossing point

τ_d – crossing time

Formation of molecules with increasing interaction strength



$$t_{\mathrm{mol}} = \frac{4t^2}{U}$$

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Value of U/t for which one finds saturation in the production of molecules