Part II

New challenges in quantum many-body theory: non-equilibrium coherent dynamics

Non-equilibrium dynamics of many-body systems of ultracold atoms

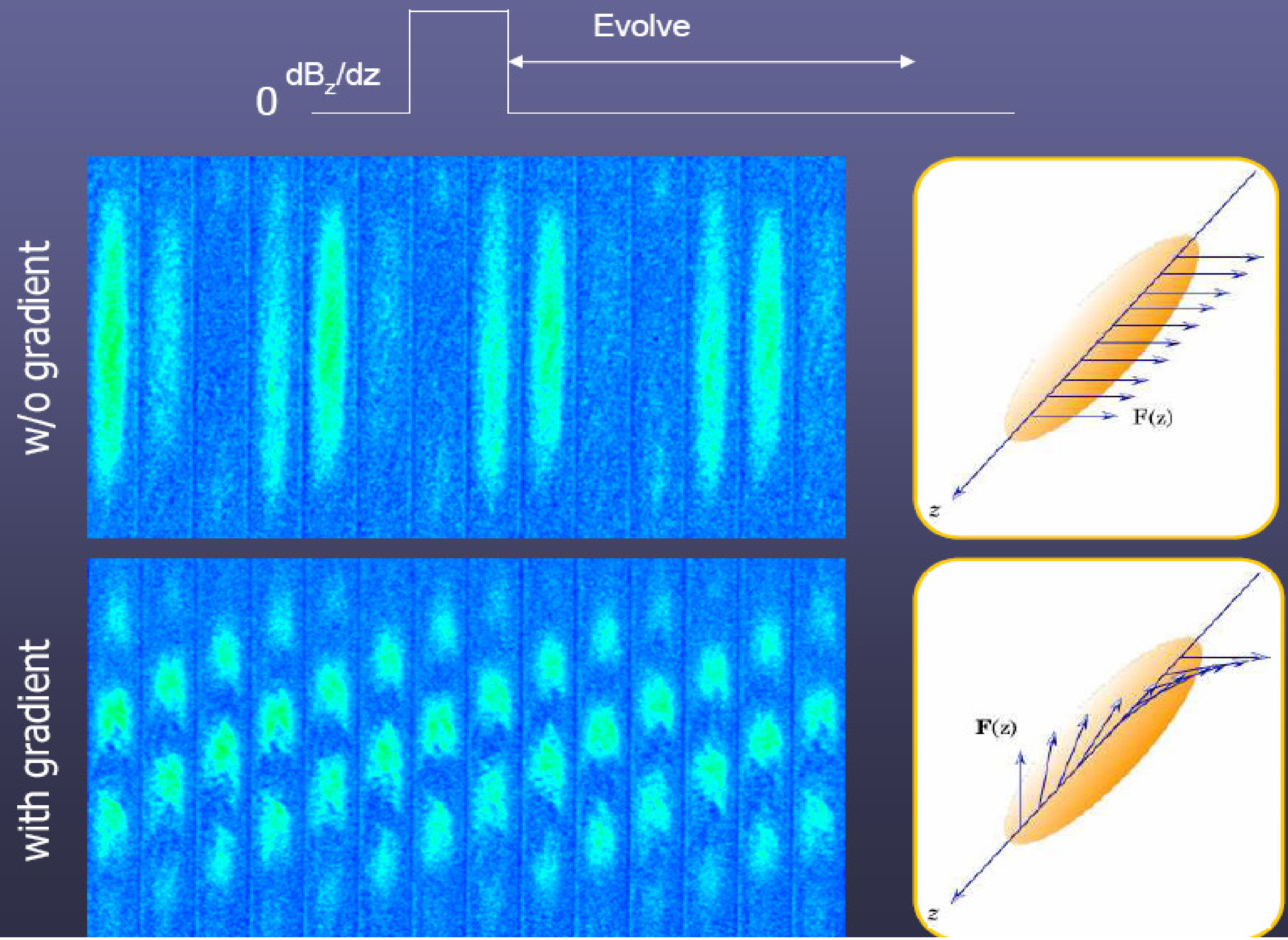
1. Dynamical instability of strongly interacting bosons in optical lattices
2. Adiabaticity of creating many-body fermionic states in optical lattices
3. Dynamical instability of the spiral state of F=1 ferromagnetic condensate
4. Dynamics of coherently split condensates
5. Many-body decoherence and Ramsey interferometry
6. Quantum spin dynamics of cold atoms in an optical lattice

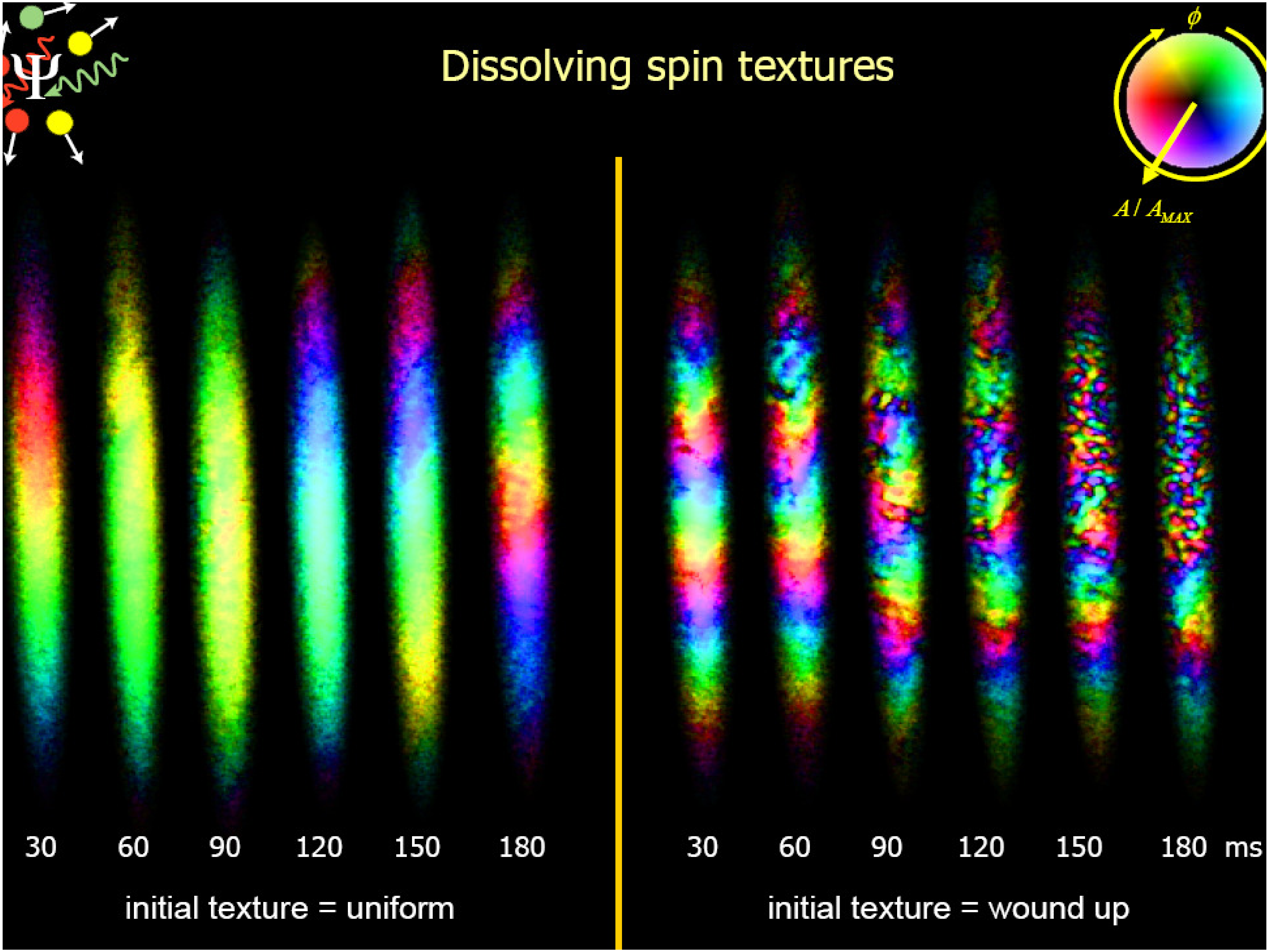
Dynamical Instability of the Spiral State of F=1 Ferromagnetic Condensate

Ref:
R. Cherng et al, arXiv:0710.2499

Ferromagnetic spin textures created by D. Stamper-Kurn et al.

- generate helical spin pattern (uniform spin current) using inhomogeneous fie




Dissolving spin textures
ϕ
A / A_{MAX}
30 60 90 120 150 180
initial texture = uniform
30 60 90 120 150 180 ms
initial texture = wound up

F=1 condensates

Spinor order parameter
Vector representation

$$\vec{\Psi} = \begin{pmatrix} \Psi_x \\ \Psi_y \\ \Psi_z \end{pmatrix}$$

Ferromagnetic State

$S_x = \cos\phi \quad S_y = \sin\phi$

$$\vec{\Psi} = \frac{\Psi_0}{\sqrt{2}} \times \begin{pmatrix} i\ \sin\phi \\ i\ \cos\phi \\ 1 \end{pmatrix}$$

Polar (nematic) state

$$\vec{\Psi} = \Psi_0 \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathcal{H} = \frac{1}{2m}\nabla\Psi_\alpha^\dagger\nabla\Psi_\alpha + qF_z^2 + \frac{g_0}{2}\Psi_\alpha^\dagger\Psi_\beta^\dagger\Psi_\beta\Psi_\alpha + \frac{g_s}{2}\Psi_\alpha^\dagger\Psi_\alpha^\dagger\Psi_\beta\Psi_\beta$$

Ferromagnetic state realized for $g_s > 0$

Spiral Ferromagnetic State of F=1 condensate

Gross-Pitaevski equation

$$i\frac{\partial \Psi_\alpha}{\partial t} = -\frac{\nabla^2}{2m}\Psi_\alpha + g_0 \Psi^\dagger_\beta \Psi_\beta \Psi_\alpha + g_s \Psi^\dagger_\alpha \Psi_\beta \Psi_\beta$$

Mean-field spiral state

The nature of the mean-field state depends on the system preparation.

Sudden twisting

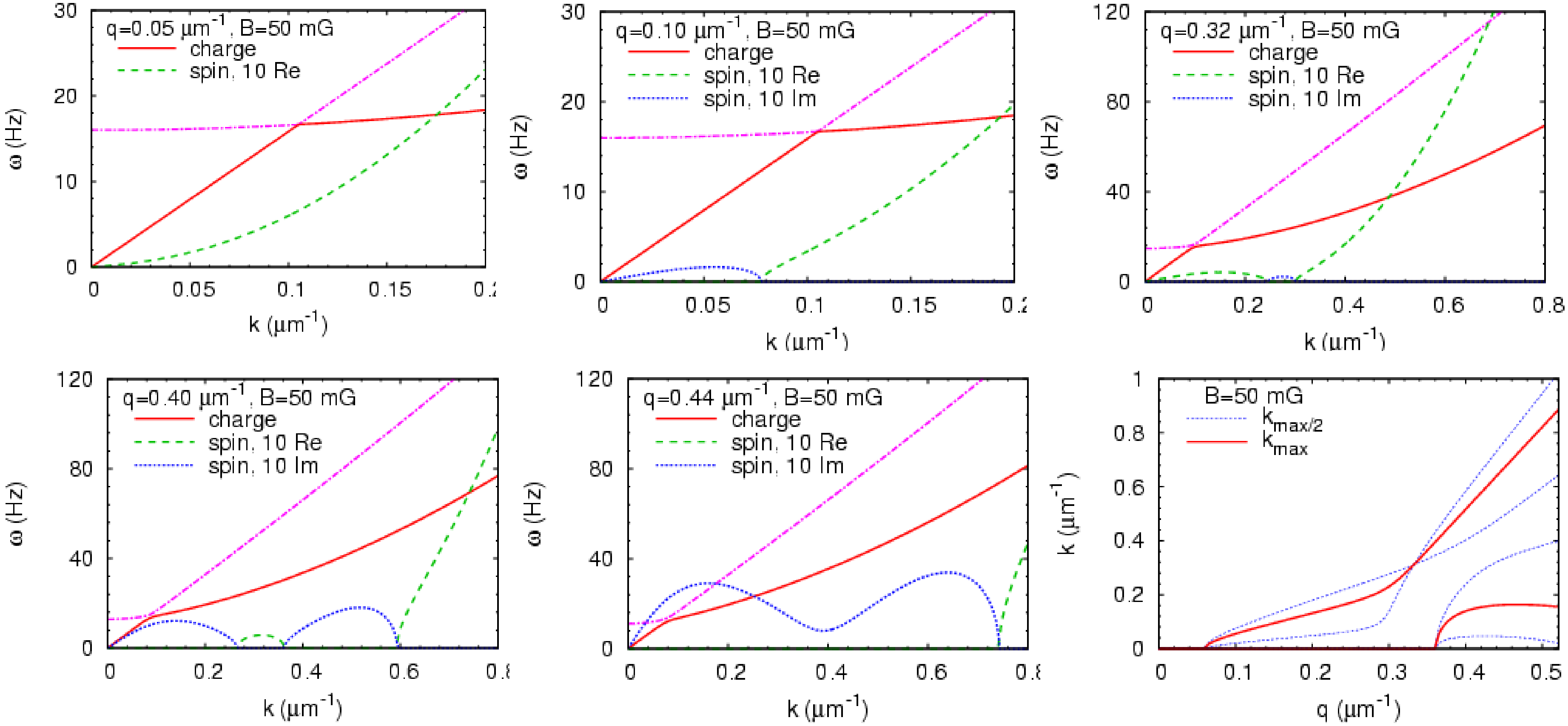
$$\Psi(x,t) = \Psi_0 \begin{pmatrix} i\,\cos qx\,\frac{1}{\sqrt{2}}\,e^{-i\omega_1 t} \\ i\,\sin qx\,\frac{1}{\sqrt{2}}\,e^{-i\omega_1 t} \\ \frac{1}{\sqrt{2}}\,e^{-i\omega_2 t} \end{pmatrix}$$

Adiabatic limit: θ determined from the condition of the stationary state.

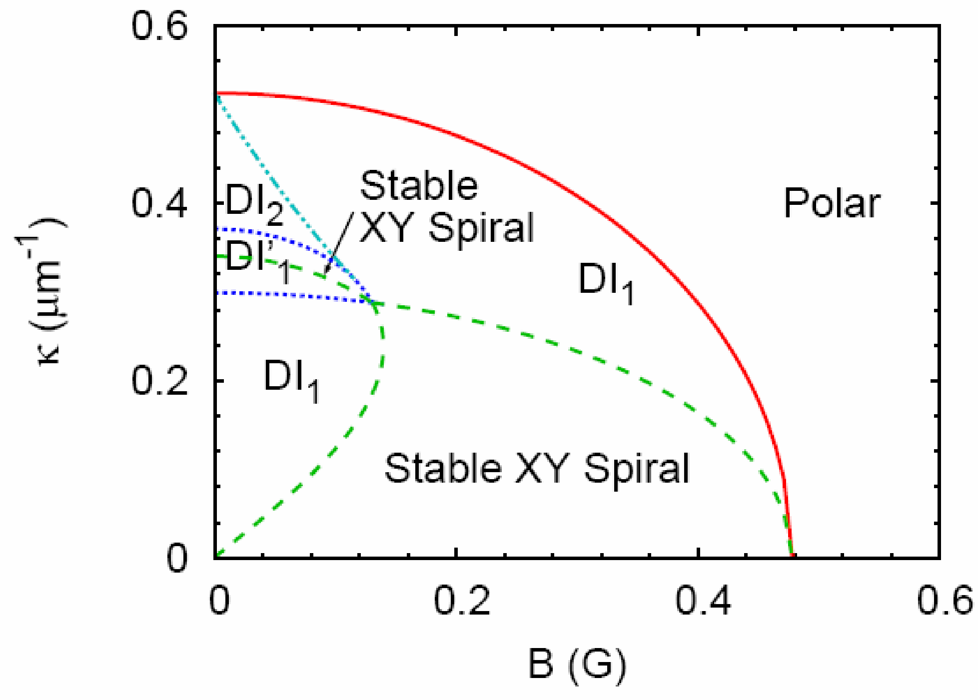
$$\vec{\Psi}(x) = \Psi_0 \begin{pmatrix} i\cos qx\cos\theta \\ i\sin qx\cos\theta \\ \sin\theta \end{pmatrix} e^{-i\mu t}$$

Instabillities can be obtained from the analysis of collective modes

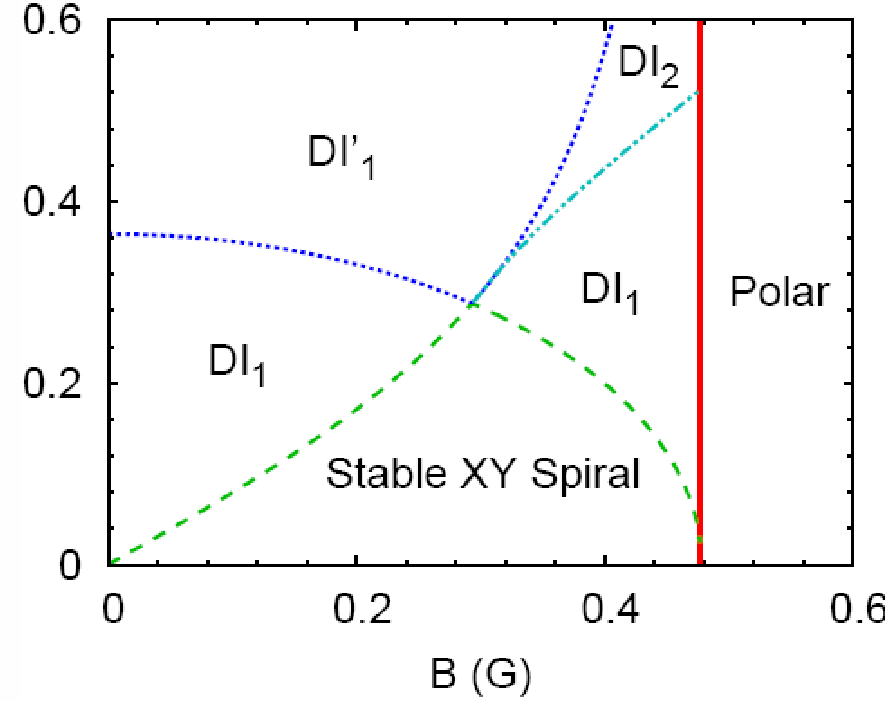
Collective modes



Instabilities of the spiral state

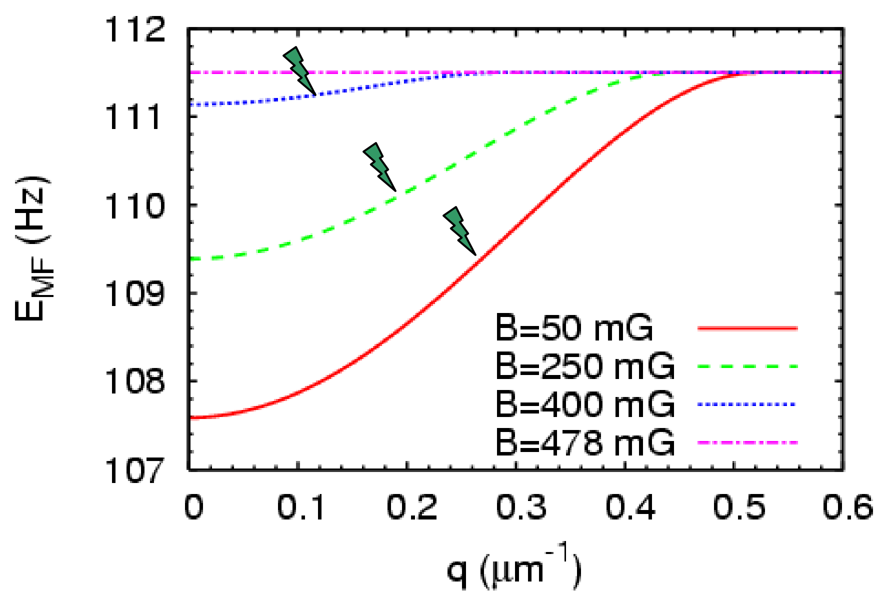


Adiabatic limit



Sudden limit

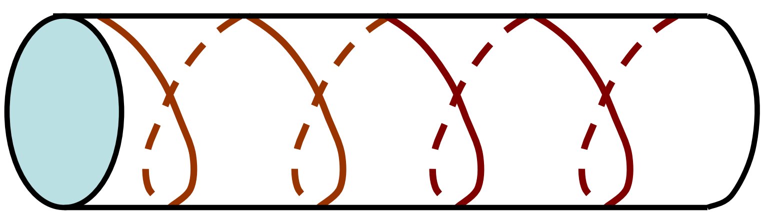
Mean-field energy



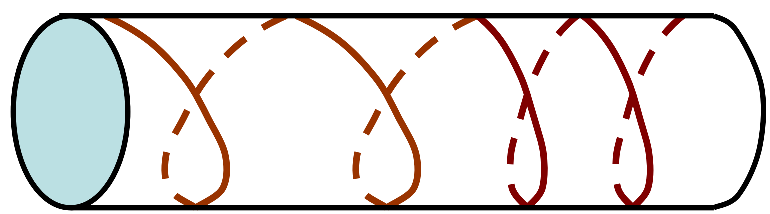
Inflection point suggests instability

Negative value of $\partial^2 E_{\mathrm{MF}}/\partial q^2$ shows that the system can lower its energy by making a non-uniform spiral winding

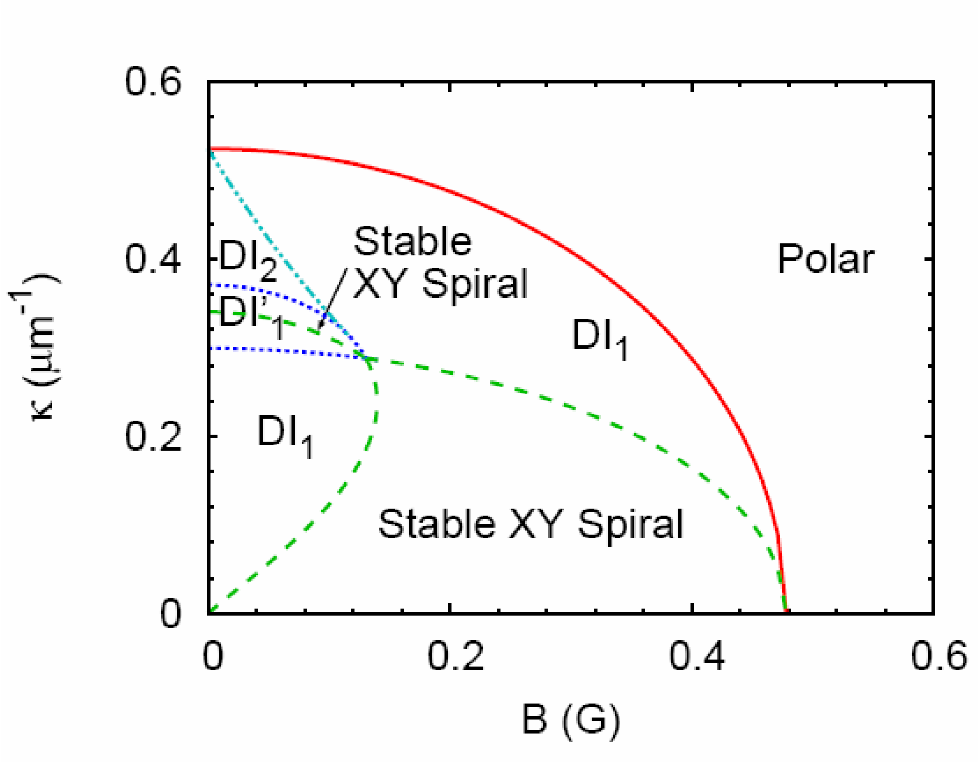
Uniform spiral



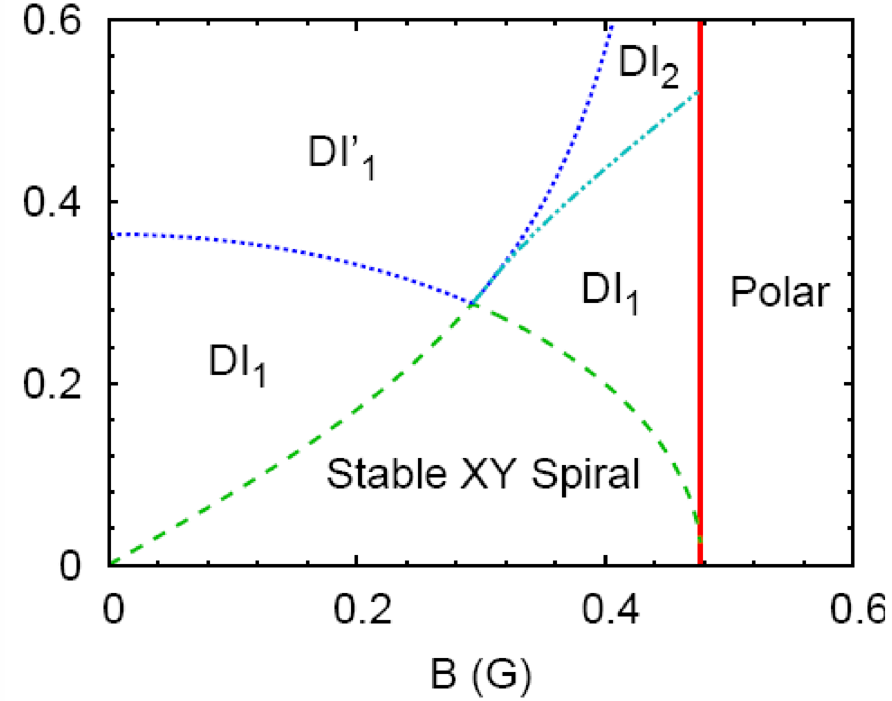
Non-uniform spiral



Instabilities of the spiral state



Adiabatic limit



Sudden limit

Beyond mean-field: thermal and quantum phase slips?

Dynamics of coherently split condensates. Interference experiments

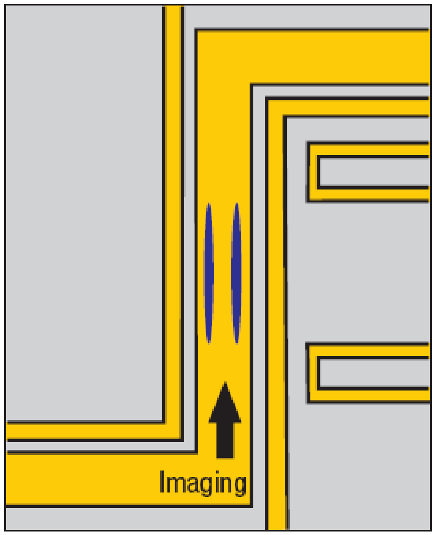
Refs:

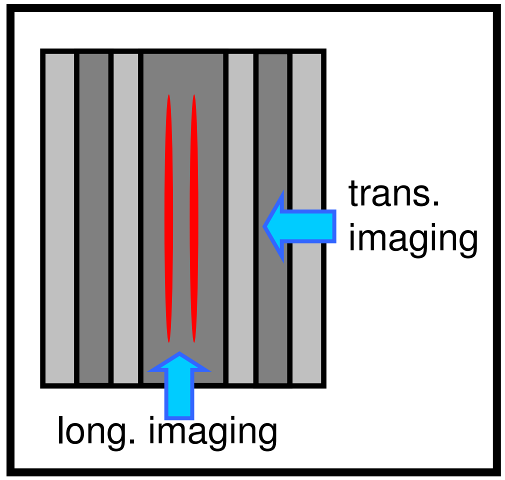
Bistrizer, Altman, PNAS 104:9955 (2007)

Burkov, Lukin, Demler, Phys. Rev. Lett. 98:200404 (2007)

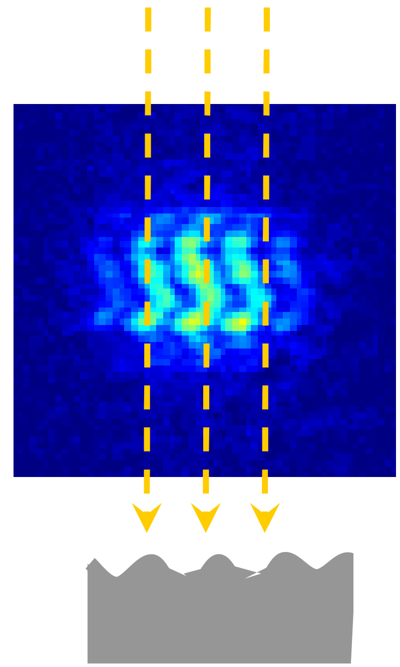
Interference of one dimensional condensates

Experiments: Schmiedmayer et al., Nature Physics (2005,2006)

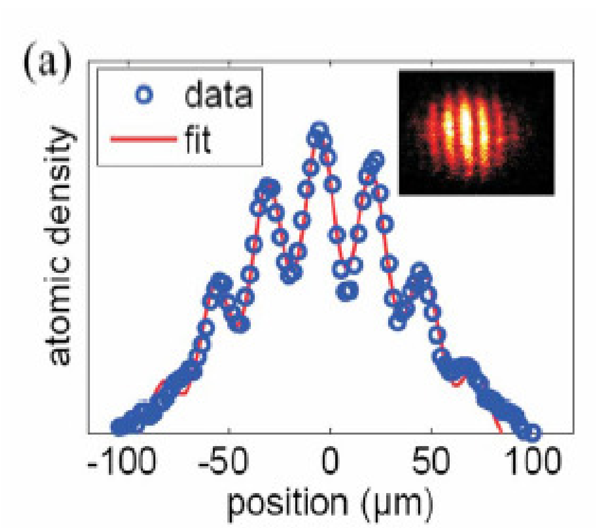




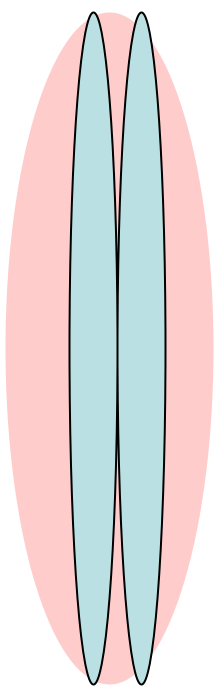
Transverse imaging



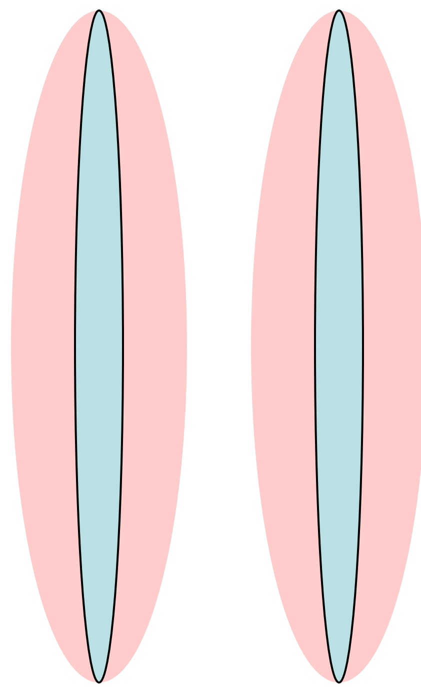
Longitudial imaging



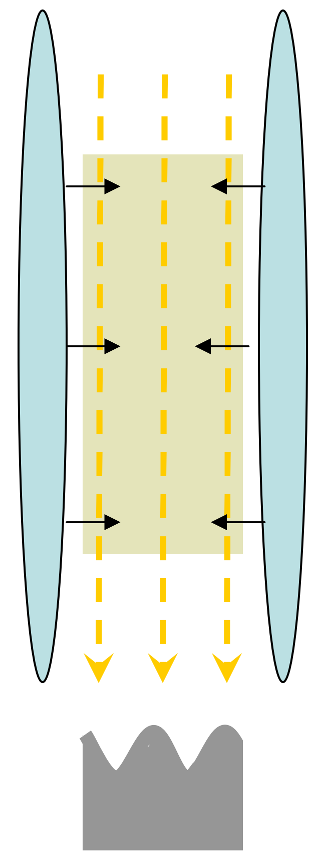
Studying dynamics using interference experiments



Prepare a system by
splitting one condensate

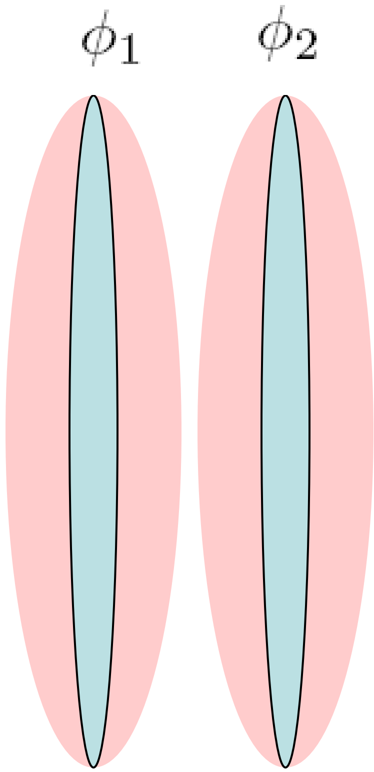


Take to the regime of
zero tunneling



Measure time evolution
of fringe amplitudes

Finite temperature phase dynamics



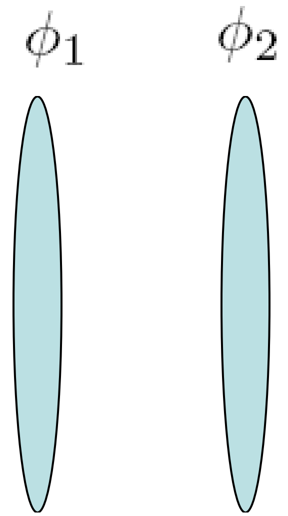
$$\mathcal{H}_0 = \int dx\ \left[g\, n_1^2(x) + \rho\, (\partial_x \phi_1)^2\right] + \int dx\ \left[g\, n_2^2(x) + \rho\, (\partial_x \phi_2)^2\right]$$

Temperature leads to phase fluctuations within individual condensates

Interference experiments measure only the relative phase

$$\phi_{av} = \frac{\phi_1 + \phi_2}{2} \qquad\qquad \phi = \phi_1 - \phi_2$$

Relative phase dynamics



$$\phi = \phi_1 - \phi_2$$

$$\Delta n = (\, n_1 - n_2 \,)/2$$

Conjugate variables

$$\boxed{\mathcal{H} = \int d^d r \, [\, \frac{g}{2} \, (\Delta n)^2 \, + \, \frac{\rho}{2} \, (\nabla \phi)^2 \,]}$$

Hamiltonian can be diagonalized in momentum space

A collection of harmonic oscillators with $\omega_q \; = \; \sqrt{g\rho} \, | \, q \, |$

Initial state $\phi_q = 0$

Need to solve dynamics of harmonic oscillators at finite T

Coherence $\langle \, \Psi(t) \, | \, e^{i\phi} \, | \, \Psi(t) \, \rangle \; = \; e^{-\frac{1}{2} \sum_q \langle \phi_q^2(t) \rangle}$

Relative phase dynamics

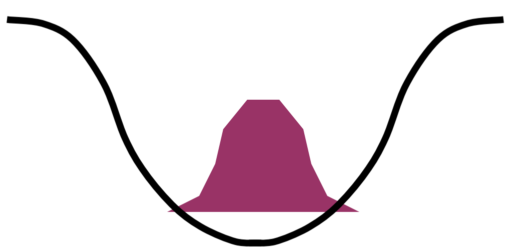
High energy modes, $\hbar\omega_{\rm osc} > k_{\rm B}T$, quantum dynamics

Low energy modes, $\hbar\omega_{\rm osc} < k_{\rm B}T$, classical dynamics

Combining all modes

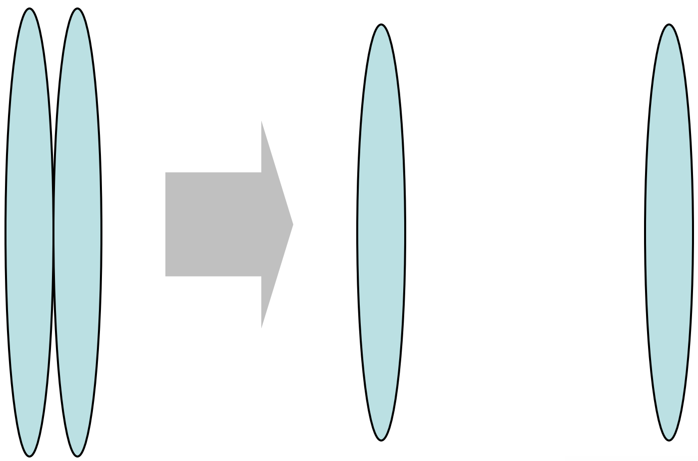
$t < \dfrac{h}{k_{\rm B}T}$ Quantum dynamics

$t > \dfrac{h}{k_{\rm B}T}$ Classical dynamics



For studying dynamics it is important to know the initial width of the phase

Relative phase dynamics



$$\frac{N}{2} \pm \delta N \qquad \frac{N}{2} \mp \delta N$$

Naive estimate

$$\delta N \sim \sqrt{N}$$

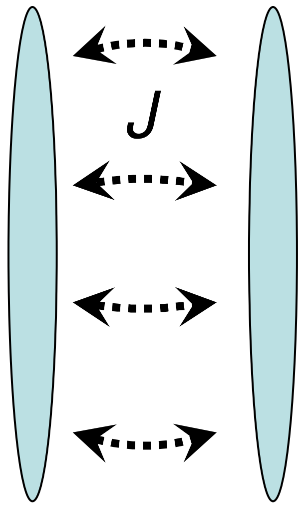
Long Phase Coherence Time and Number Squeezing of two Bose-Einstein Condensates on an Atom Chip

G.-B. Jo, Y. Shin, S. Will, T. A. Pasquini, M. Saba, W. Ketterle, and D. E. Pritchard*
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

M. Vengalattore, M. Prentiss
MIT-Harvard Center for Ultracold Atoms, Jefferson Laboratory, Physics Department, Harvard University, Cambridge, MA 02138, USA

(Dated: August 27, 2006)

Relative phase dynamics



$$\mathcal{H} = \frac{U}{2} (\Delta n)^2 - J \cos\phi$$

$$J(t) = J_0 \, e^{-t/\tau_s}$$

Separating condensates at finite rate

Instantaneous Josephson frequency

$$\omega_J = \sqrt{U \, J}$$

Adiabatic regime $\dot{\omega}_J < \omega_J^2$

Instantaneous separation regime $\dot{\omega}_J > \omega_J^2$

Adiabaticity breaks down when $\omega_J \sim 1/\tau_s$

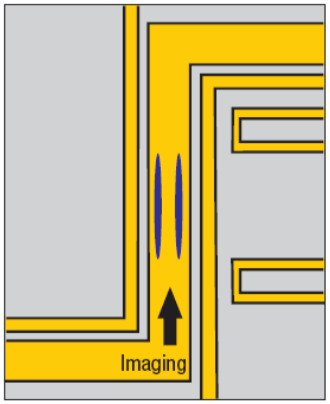
Charge uncertainty at this moment

$$U \, \delta N^2 \sim \omega_J \sim 1/\tau_s$$

Squeezing factor $$\frac{\delta N}{\sqrt{N}} \sim \sqrt{\frac{1}{N \, U \, \tau_s}} \sim \sqrt{\frac{1}{\mu \, \tau_s}}$$

Relative phase dynamics

Bistrizer, Altman, PNAS (2007)
Burkov, Lukin, Demler, PRL (2007)

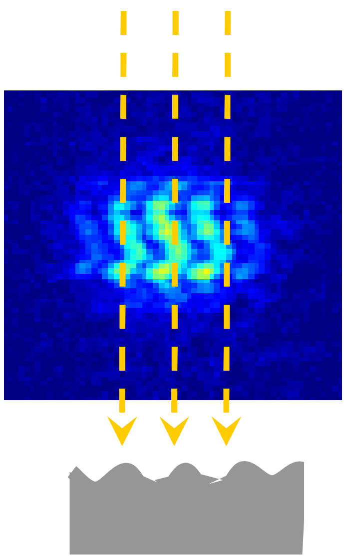


Quantum regime $\frac{h}{\mu} < t < \frac{h}{k_B T}$

1D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} e^{-t/2\pi K\tau_s}$

2D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} \left(\frac{t_0}{t}\right)^{1/16 T_{KT}\tau_s}$

Different from the earlier theoretical work based on a single mode approximation, e.g. Gardiner and Zoller, Leggett



Classical regime $t > \frac{h}{k_B T}$

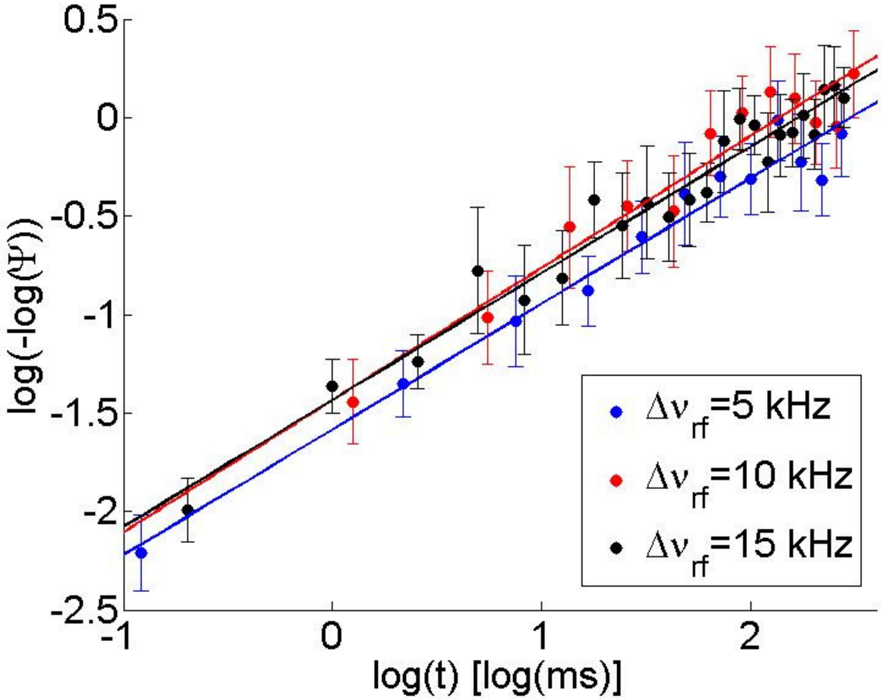
$\boxed{\text{1D systems} \quad \langle e^{i\phi(t)} \rangle \sim e^{-\left(\frac{t}{t_T}\right)^{2/3}}}$

$t_T \sim \frac{\mu K}{T^2}$

2D systems $\langle e^{i\phi(t)} \rangle \sim \left(\frac{t_0}{t}\right)^{\frac{T}{8T_{KT}}}$

1d BEC: Decay of coherence

Experiments: Hofferberth, Schumm, Schmiedmayer, Nature (2007)



$$\Psi(t) \propto \exp[-(t/t_0)^{2/3}]$$

double logarithmic plot of the coherence factor

slopes:

0.64 ± 0.08
0.67 ± 0.1
0.64 ± 0.06

get t_0 from fit with fixed slope 2/3 and calculate T from

$$t_0 = 2.61\pi K/T^2$$

T_5 = 110 ± 21 nK

T_{10} = 130 ± 25 nK

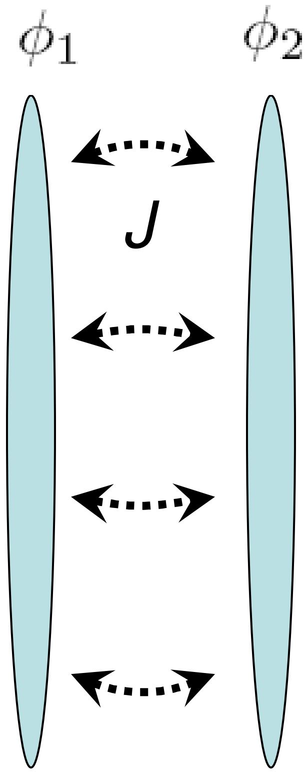
T_{15} = 170 ± 22 nK

Dynamics of partially split condensates. From the Bethe ansatz solution of the quantum Sine-Gordon model to quantum dynamics

Refs:

Gritsev, Demler, Lukin, Polkovnikov, Phys. Rev. Lett. 99:200404 (2007)
Gritsev, Polkovnikov, Demler, Phys. Rev. B 75:174511 (2007)

Coupled 1d systems



$$\mathcal{H}_0 = \int dx\ \left[g\, n_1^2(x) + \rho\,(\partial_x \phi_1)^2\right] + \int dx\ \left[g\, n_2^2(x) + \rho\,(\partial_x \phi_2)^2\right]$$

Interactions lead to phase fluctuations within individual condensates

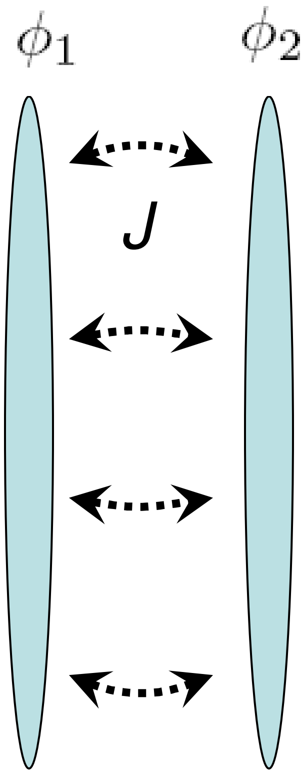
$$\mathcal{H}_{tun} = -J \int dx\ \cos(\phi_1 - \phi_2)$$

Tunneling favors aligning of the two phases

Interference experiments measure only the relative phase

$$\phi_{av} = \frac{\phi_1 + \phi_2}{2} \qquad\qquad \phi = \phi_1 - \phi_2$$

Coupled 1d systems



Conjugate variables

$$\phi = \phi_1 - \phi_2 \qquad \Delta n = (\,n_1 - n_2\,)/2$$

Relative phase

Particle number imbalance

$$[\Delta n(x_1)\,,\phi(x_2)\,] = -i\,\delta(x_1 - x_2)$$

$$\mathcal{H}[\phi] = \int dx\,d\tau \left[\frac{1}{2K}(\Delta n)^2 + \frac{K}{2}\,(\partial_x \phi)^2\right] - J \int dx\,d\tau\,\cos\phi$$

Small K corresponds to strong quantum fluctuations

Quantum Sine-Gordon model

Hamiltonian

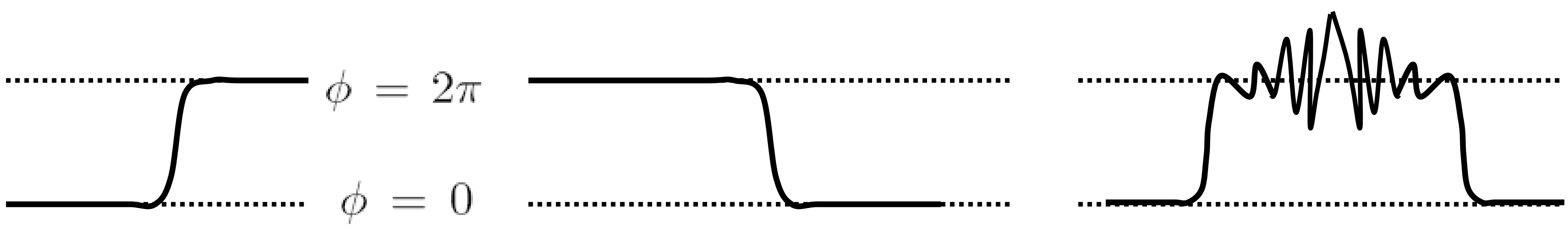
$$\mathcal{H}[\phi] = \int dx\, d\tau \left[\frac{1}{2K} (\Delta n)^2 + \frac{K}{2} (\partial_x \phi)^2 \right] - J \int dx\, d\tau\, \cos\phi$$

Imaginary time action

$$\mathcal{S}[\phi] = \frac{K}{2} \int dx\, d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right] + J \int dx\, d\tau\, \cos\phi$$

Quantum Sine-Gordon model is exactly integrable

Excitations of the quantum Sine-Gordon model



soliton antisoliton many types of breathers

Dynamics of quantum sine-Gordon model

Hamiltonian formalism

$$\mathcal{H}[\phi] = \int dx\, d\tau \left[\frac{1}{2K} (\Delta n)^2 + \frac{K}{2} (\partial_x \phi)^2 \right] - J \int dx\, d\tau\, \cos\phi$$

Initial state $\phi(t=0) = 0$

Quantum action in space-time

$$\mathcal{S}[\phi] = \frac{K}{2} \int dx\, d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right] + J \int dx\, d\tau\, \cos\phi$$

Initial state provides a boundary condition at *t=0*

Solve as a boundary sine-Gordon model

Boundary sine-Gordon model

Exact solution due to Ghoshal and Zamolodchikov (93)
Applications to quantum impurity problem: Fendley, Saleur, Zamolodchikov, Lukyanov,…

$$S = \int_{x\tau} \left[\frac{K}{2} \left(\frac{\partial \phi}{\partial \tau}\right)^2 + \frac{K}{2} \left(\frac{\partial \phi}{\partial x}\right)^2 - m \cos \phi \right] - M \int_\tau cos \frac{\phi(x=0)}{2}$$

Limit $M \to \infty$ enforces boundary condition $\phi\,(\,x = 0\,)\, = \,0$

$\underbrace{\text{Sine-Gordon + boundary condition in space}}_{\text{quantum impurity problem}}$

$\underbrace{\boxed{\text{Boundary Sine-Gordon Model}}}_{\text{space and time enter equivalently}}$

$\underbrace{\text{Sine-Gordon + boundary condition in time}}_{\text{two coupled 1d BEC}}$

Boundary sine-Gordon model

Initial state is a generalized squeezed state

$$|\,\psi(t=0)\,\rangle = e^{\{\sum_\gamma g_\gamma A^\dagger_\gamma(\theta=0)+\sum_{\alpha\beta}\int_\theta K_{\alpha\beta}(\theta)A^\dagger_\alpha(-\theta)A^\dagger_\beta(\theta)\,\}}|\,\mathrm{vac}\,\rangle$$

$A^\dagger_\alpha(\theta)$ creates solitons, breathers with rapidity θ

$A^\dagger_\gamma(\theta=0)$ creates even breathers only

Matrix $K_{\alpha\beta}(\theta)$ and g_γ are known from the exact solution of the boundary sine-Gordon model

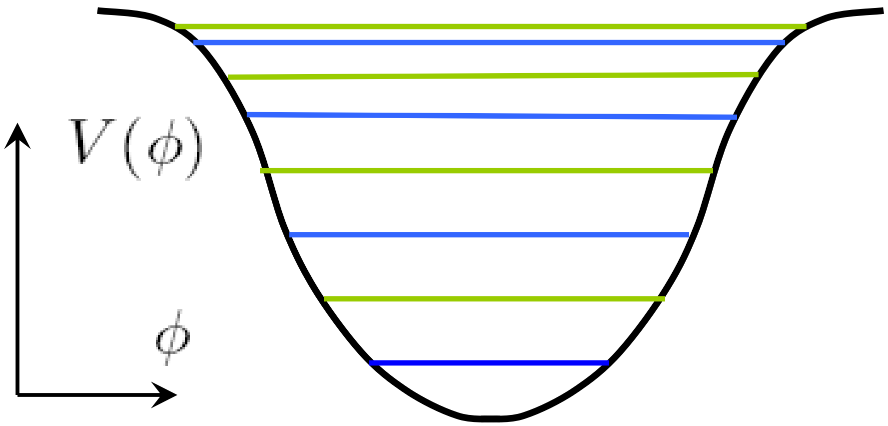
Time evolution $A^\dagger_\alpha(\theta,t) = A^\dagger_\alpha(\theta)\,e^{-iE_\alpha(\theta)t}$

Coherence $\langle\,\psi(t)\,|\,e^{i\phi}\,|\,\psi(t)\,\rangle$

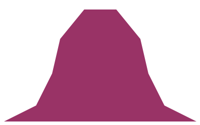
Matrix elements can be computed using form factor approach
Smirnov (1992), Lukyanov (1997)

Quantum Josephson Junction

$$\boxed{\mathcal{H} = \frac{g}{2}\, n^2 \;-\; J\,\cos\phi}$$



Limit of quantum sine-Gordon model when spatial gradients are forbidden

Initial state 

$$|\,\psi(t=0)\,\rangle = \sum_n C_{2n}\,|\,2n\,\rangle$$

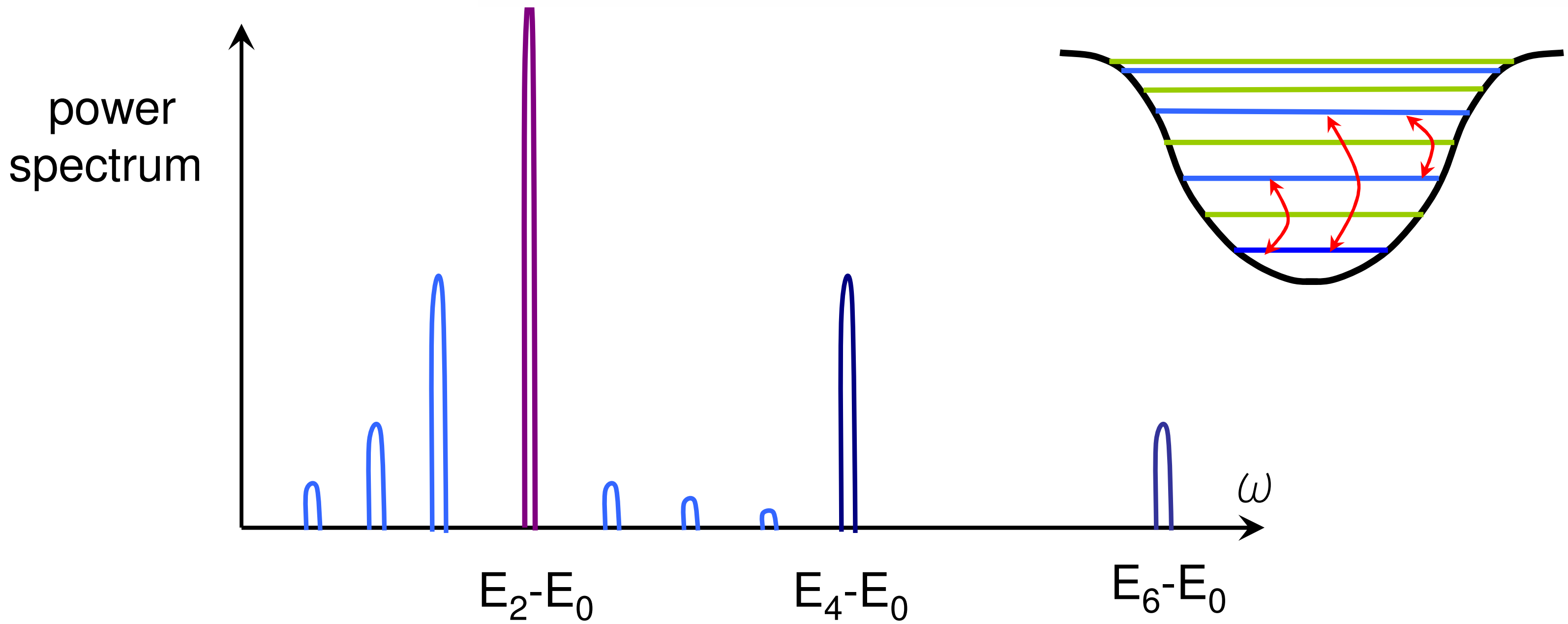
Eigenstates of the quantum Jos. junction Hamiltonian are given by Mathieu's functions

Time evolution $$|\,\psi(t)\,\rangle = \sum_n C_{2n}\, e^{-iE_{2n}t}\,|\,2n\,\rangle$$

Coherence $$\langle\,\psi(t)\,|\,e^{i\phi}\,|\,\psi(t)\,\rangle$$

Dynamics of quantum Josephson Junction

Power spectrum $$P(\omega) = \left| \int_t e^{i\omega t} \langle \psi(t) \,|\, e^{i\phi} \,|\, \psi(t) \rangle \right|^2$$



Main peak $\omega = E_2 - E_0$

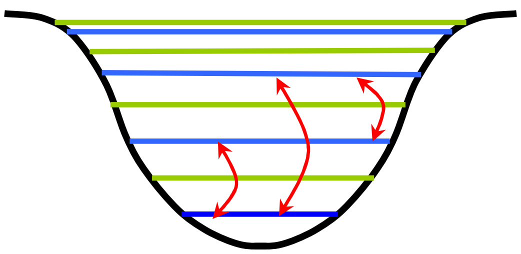
"Higher harmonics" $\omega = E_4 - E_0 \,, E_6 - E_0 \,, \ldots$

Smaller peaks $\omega = E_{2n+2} - E_{2n} \,, E_{2n+4} - E_{2n} \,, \ldots$

Dynamics of quantum sine-Gordon model

$$|\,\psi(t)\,\rangle = e^{\{\sum_\gamma g_\gamma A^\dagger_\gamma(\theta=0)+\sum_{\alpha\beta}\int_\theta K_{\alpha\beta}(\theta)A^\dagger_\alpha(-\theta)A^\dagger_\beta(\theta)\,\}}|\,\mathrm{vac}\,\rangle$$

Coherence $\langle\,\psi(t)\,|\,e^{i\phi}\,|\,\psi(t)\,\rangle$



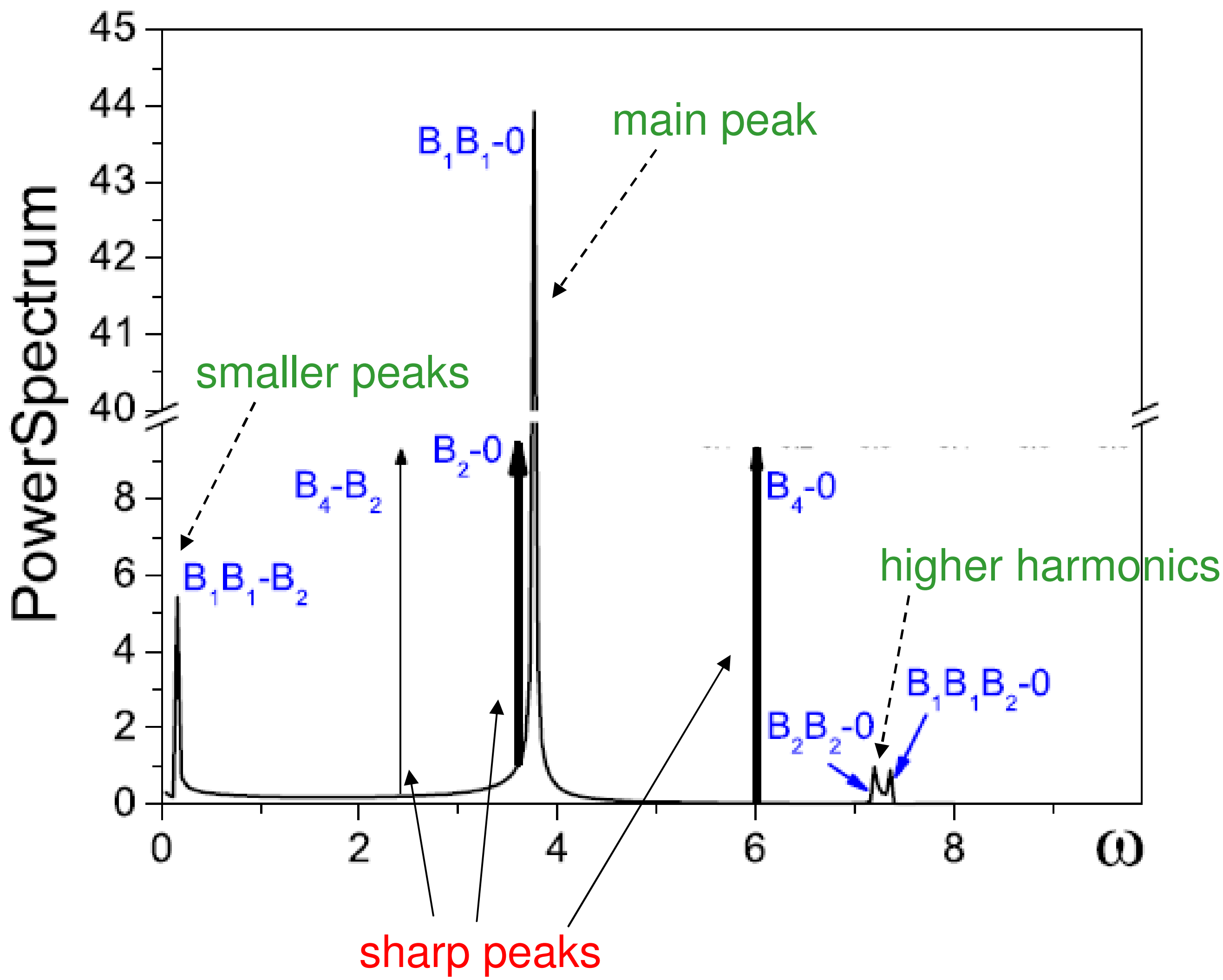
Main peak $\int_\theta \langle\,\mathrm{vac}\,|\,e^{i\phi}\,|\,\mathrm{B}_1(\theta)\,\mathrm{B}_1(-\theta)\,\rangle$

"Higher harmonics" $\int_\theta \langle\,\mathrm{vac}\,|\,e^{i\phi}\,|\,\mathrm{B}_n(\theta)\,\mathrm{B}_n(-\theta)\,\rangle$

Smaller peaks $\int_{\theta\theta'} \langle\,\mathrm{B}_m(\theta')\,\mathrm{B}_m(-\theta')\,|\,e^{i\phi}\,|\,\mathrm{B}_n(\theta)\,\mathrm{B}_n(-\theta)\,\rangle$

Sharp peaks $\langle \mathrm{vac}\,|\,e^{i\phi}\,|\,B_{2n}(\theta=0)\,\rangle$

Dynamics of quantum sine-Gordon model

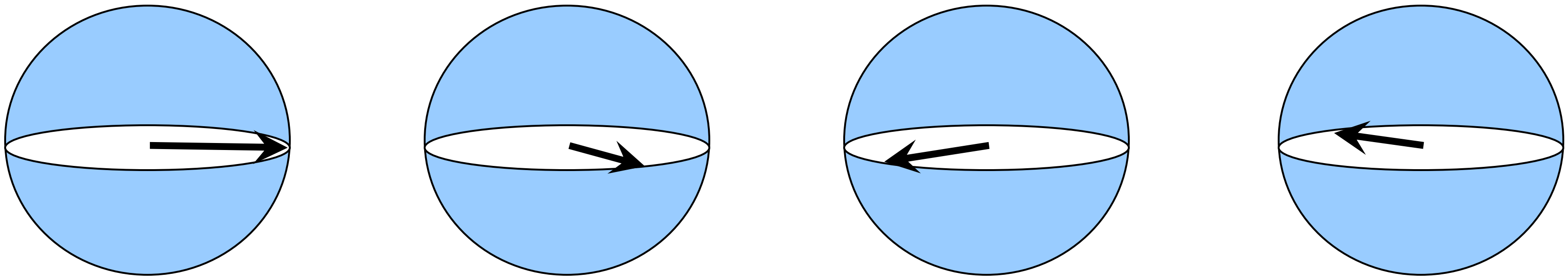


Many-body decoherence and Ramsey interferometry

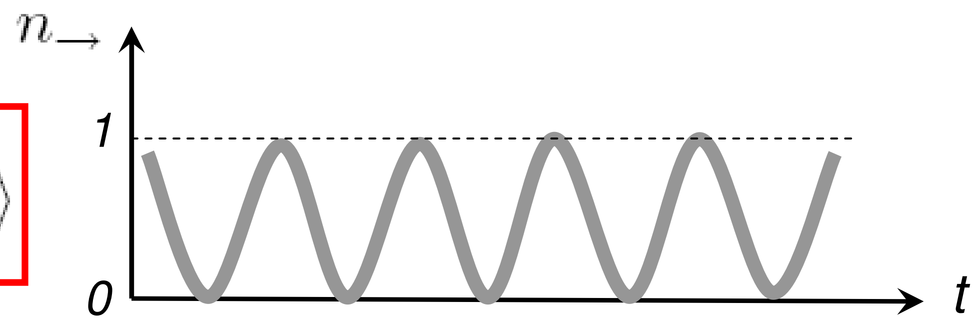
Ref:

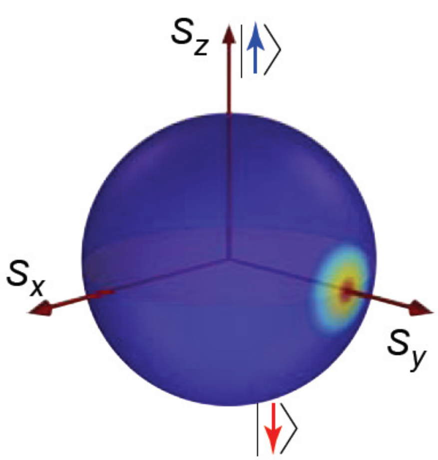
Widera, Trotzky, Cheinet, Fölling, Gerbier, Bloch, Gritsev, Lukin, Demler, arXiv:0709.2094

Ramsey interference



$$|\,\Psi\,\rangle \;=\; e^{-iE_1t}\,|\uparrow\rangle + e^{-iE_2t}\,|\downarrow\rangle$$





Working with *N* atoms improves the precision by $\sqrt{N}$.
Need spin squeezed states to improve frequency spectroscopy

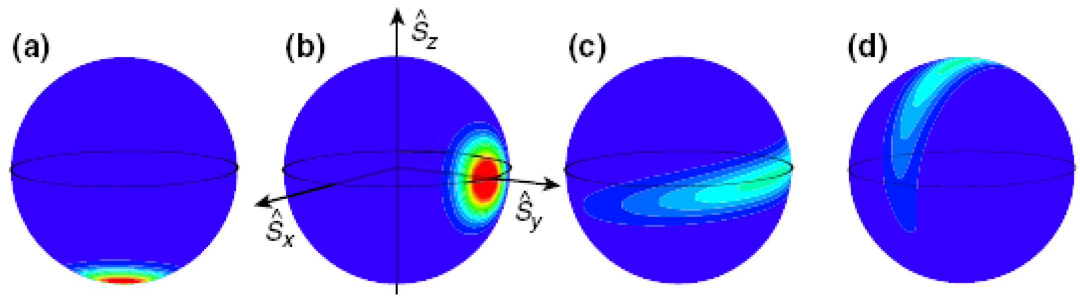
Squeezed spin states for spectroscopy

Motivation: improved spectroscopy, e.g. Wineland et. al. PRA 50:67 (1994)

Generation of spin squeezing using interactions.
Two component BEC. Single mode approximation

$$\mathcal{H} = \chi_s \left(S^z_{\rm tot} \right)^2$$

Kitagawa, Ueda, PRA 47:5138 (1993)



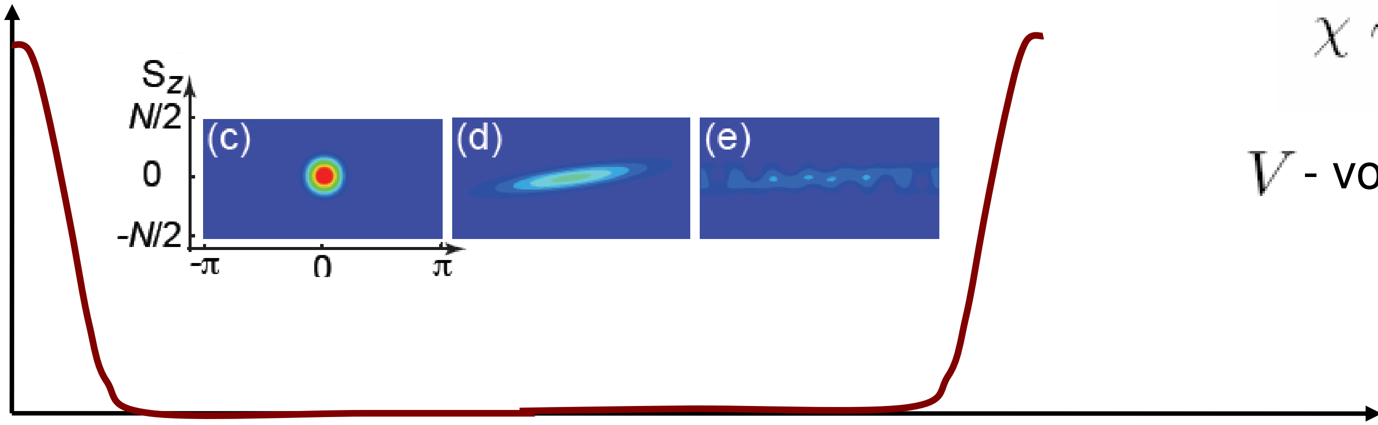
$$\mathcal{H} = \int dx \left[g_c \left(n_1 + n_2 \right)^2 + g_s (n_1 - n_2)^2 + \frac{|\nabla \Psi_1|^2}{2m} + \frac{|\nabla \Psi_2|^2}{2m} \right]$$

In the single mode approximation we can neglect kinetic energy terms

$$\mathcal{H}_{\rm SMA} = \frac{g_s}{V} \left(N_1 - N_2 \right)^2$$

Interaction induced collapse of Ramsey fringes

Ramsey fringe visibility



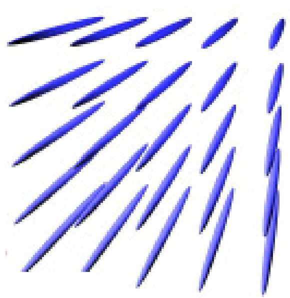
time

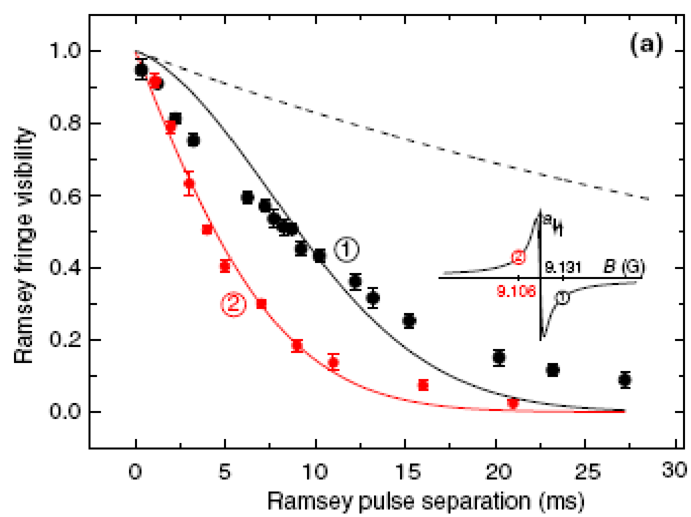
$$\chi \sim \frac{\hbar^2 a_s}{M V}$$

V - volume of the system

$$t_{\text{collapse}} \sim \frac{1}{\chi\sqrt{N}} \qquad t_{\text{revival}} \sim \frac{1}{\chi}$$

Experiments in 1d tubes:
A. Widera, I. Bloch et al.





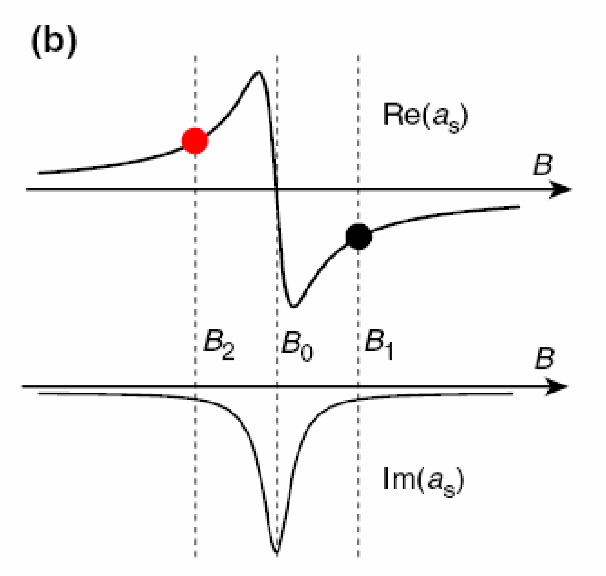
Spin echo. Time reversal experiments

Single mode approximation

$$\mathcal{H}_{\mathrm{SMA}} = \frac{g_s}{V}(N_1 - N_2)^2$$

$$g_s = \frac{g_{11} - g_{12}}{2}$$

The Hamiltonian can be reversed by changing a_{12}



$$a_s \rightarrow -a_s$$

$$\mathcal{H}_{\mathrm{SMA}} \rightarrow -\mathcal{H}_{\mathrm{SMA}}$$

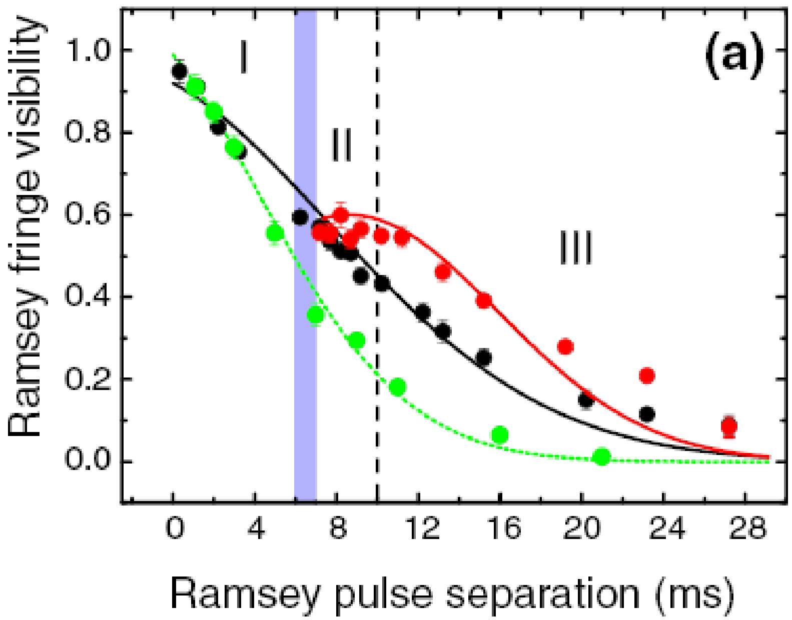
$$e^{i\int_T^{2T}\mathcal{H}_{\mathrm{SMA}}(t)dt} \times e^{i\int_0^T\mathcal{H}_{\mathrm{SMA}}(t)dt} = 1$$

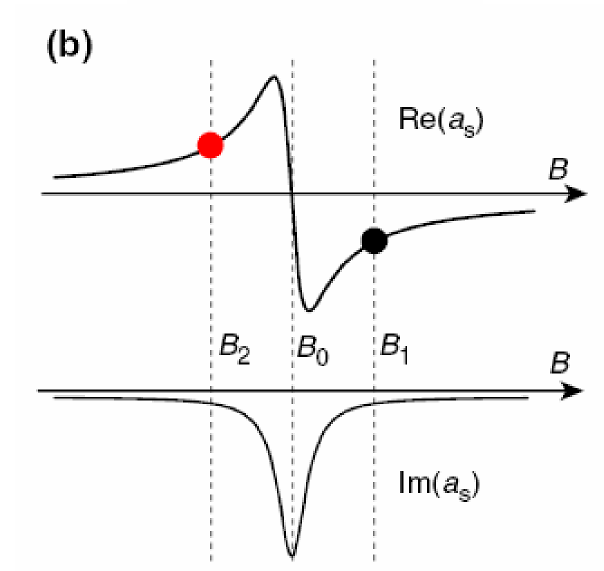
Predicts perfect spin echo

Spin echo. Time reversal experiments

Expts: A. Widera, I. Bloch et al.

$$a_s \longrightarrow -a_s$$





No revival?

Experiments done in array of tubes.
Strong fluctuations in 1d systems.
Single mode approximation does not apply.
Need to analyze the full model

$$\mathcal{H} = \int dx \left[g_c\, (n_1 + n_2)^2 + g_s (n_1 - n_2)^2 + \frac{|\nabla \Psi_1|^2}{2m} + \frac{|\nabla \Psi_2|^2}{2m} \right]$$

Interaction induced collapse of Ramsey fringes. Multimode analysis

Low energy effective theory: Luttinger liquid approach

Luttinger model

$$S^+(x,t) \sim e^{i\phi_s(x,t)} \qquad [S^z(x), \phi_s(x')] = -i\delta(x-x')$$

$$\mathcal{H}_s = \int_0^L dx \left[g_s (S^z)^2 + \frac{\rho}{2m} (\nabla \phi_s)^2 \right]$$

Changing the sign of the interaction reverses the interaction part of the Hamiltonian but not the kinetic energy

$$\mathcal{H}_s = \sum_q \left[g_s(t) S_q^z S_q^{z*} + \frac{\rho\, q^2}{m} \phi_{sq} \phi_{sq}^* \right]$$

$$[S_{q'}^z, \phi_{sq}] = -i\delta_{qq'}$$

Time dependent harmonic oscillators can be analyzed exactly

Time-dependent harmonic oscillator

$$\mathcal{H} = \frac{p^2}{2m(t)} + k\,\frac{q^2}{2}$$

See e.g. Lewis, Riesengeld (1969)
Malkin, Man'ko (1970)

Explicit quantum mechanical wavefunction can be found

$$\psi(p,t) = \frac{\Phi\left(\frac{p}{c(t)}\right)}{\sqrt{c(t)}}\, e^{i\alpha(t)p^2 + i\gamma(t)}$$

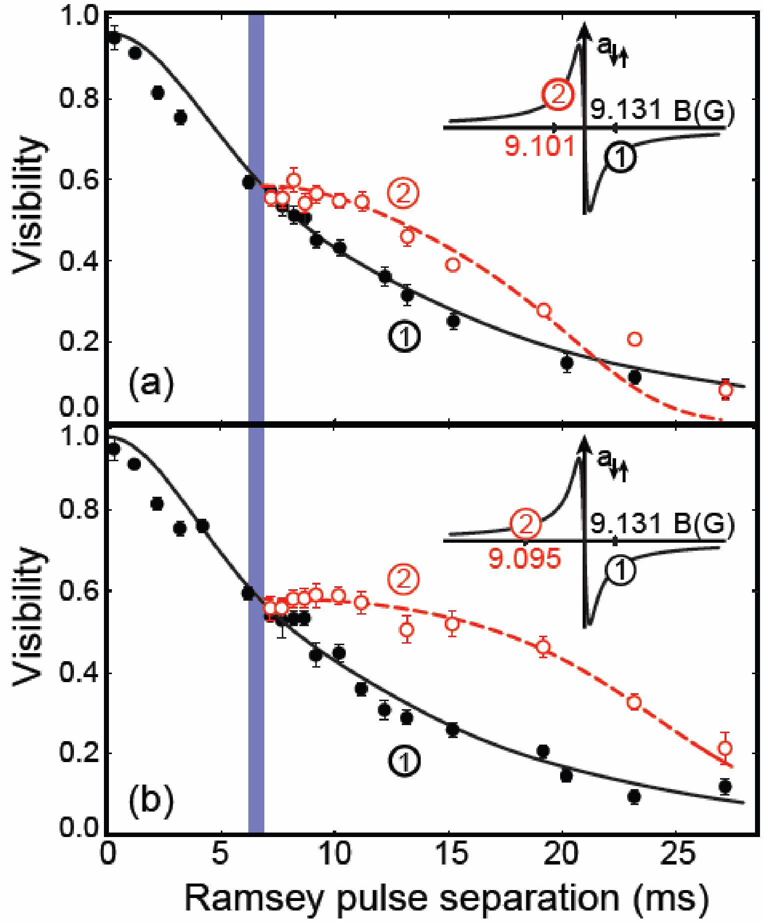
From the solution of classical problem

$$\ddot{c} + \omega^2(t)\, c = \frac{\omega_0^2}{c^3}$$

We solve this problem for each momentum component

$$\mathcal{H}_s = \sum_q \left[g_s(t) S_q^z S_q^{z*} + \frac{\rho\, q^2}{m} \phi_{sq} \phi_{sq}^* \right]$$

Interaction induced collapse of Ramsey fringes in one dimensional systems



Only q=0 mode shows complete spin echo
Finite q modes continue decay

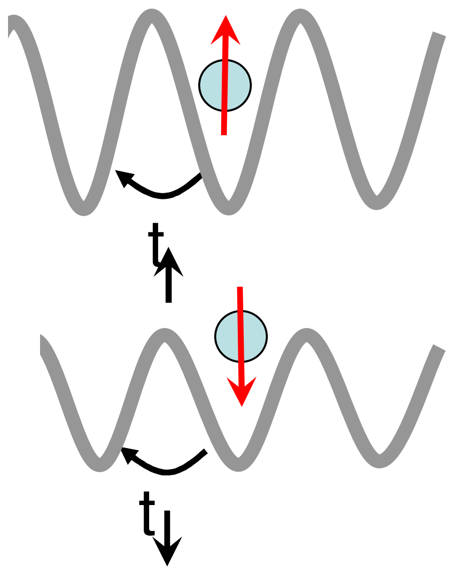
The net visibility is a result of competition between q=0 and other modes

Fundamental limit on Ramsey interferometry

Quantum spin dynamics of cold atoms in an optical lattice

Two component Bose mixture in optical lattice

Example: $^{87}\mathrm{Rb}$. Mandel et al., Nature 425:937 (2003)



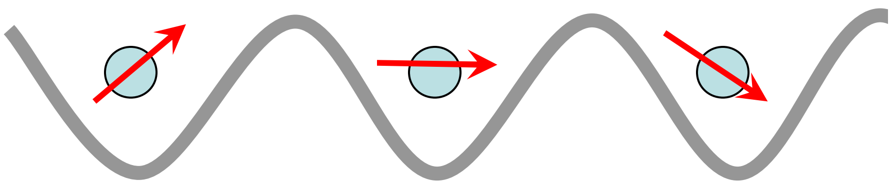
$$|\uparrow\rangle = |\, F=1, m_F=-1\,\rangle$$

$$|\downarrow\rangle = |\, F=2, m_F=-2\,\rangle$$

Two component Bose Hubbard model

$$\begin{aligned}\mathcal{H} = & -\ t_\uparrow \sum_{\langle ij\rangle} b^\dagger_{i\uparrow} b_{j\uparrow} - t_\downarrow \sum_{\langle ij\rangle} b^\dagger_{i\downarrow} b_{j\downarrow} + U_{\uparrow\uparrow} \sum_i n_{i\uparrow}(n_\uparrow - 1) \\ & +\ U_{\downarrow\downarrow} \sum_i n_{i\downarrow}(n_\downarrow - 1) + U_{\uparrow\downarrow} \sum_i n_{i\uparrow} n_\downarrow\end{aligned}$$

Quantum magnetism of bosons in optical lattices



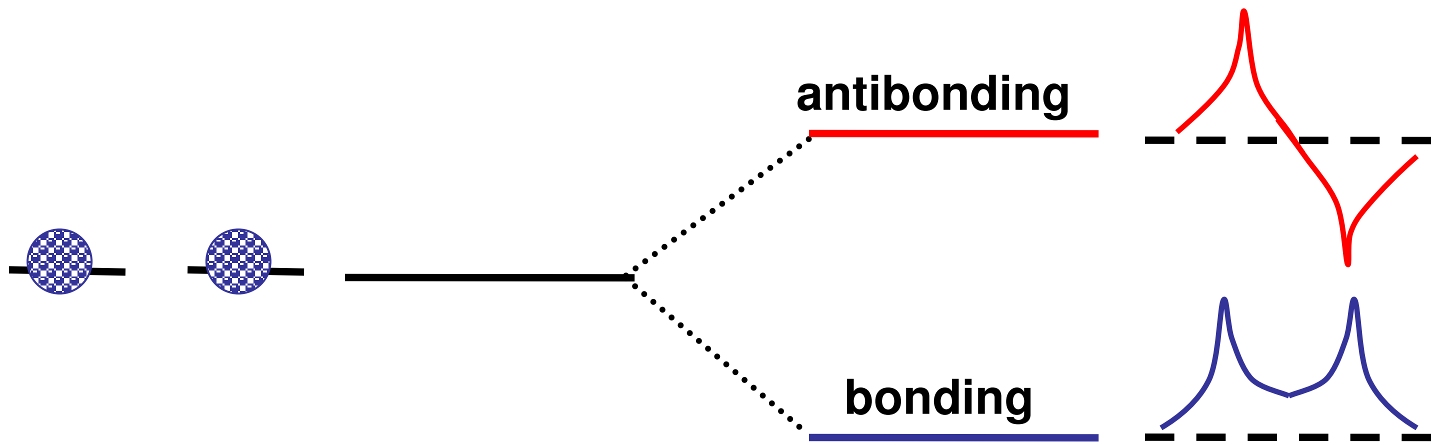
Duan, Demler, Lukin, PRL 91:94514 (2003)

$$\mathcal{H} = J_z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + J_\perp \sum_{\langle ij \rangle} \left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right)$$

$$J_z = \frac{t_\uparrow^2 + t_\downarrow^2}{2U_{\uparrow\downarrow}} - \frac{t_\uparrow^2}{U_{\uparrow\uparrow}} - \frac{t_\downarrow^2}{U_{\downarrow\downarrow}} \qquad J_\perp = -\frac{t_\uparrow t_\downarrow}{U_{\uparrow\downarrow}}$$

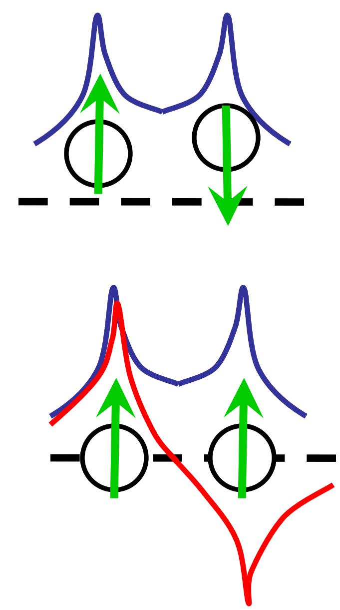
- Ferromagnetic $\quad U_{\uparrow\downarrow} >> U_{\uparrow\uparrow},\ U_{\downarrow\downarrow}$
- Antiferromagnetic $\quad U_{\uparrow\downarrow} << U_{\uparrow\uparrow},\ U_{\downarrow\downarrow}$

Exchange Interactions in Solids



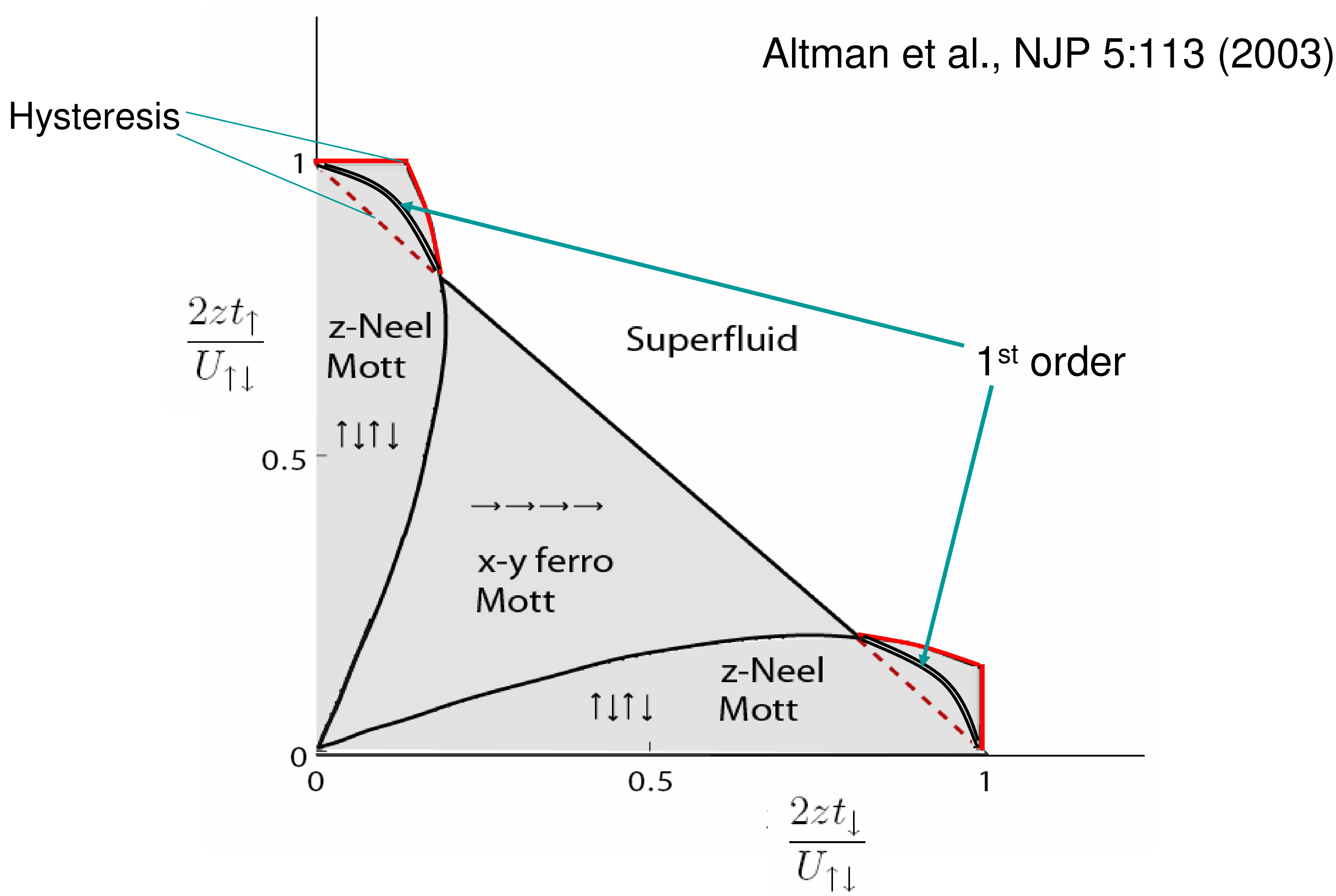
Kinetic energy dominates: antiferromagnetic state

Coulomb energy dominates: ferromagnetic state



Two component Bose mixture in optical lattice.
Mean field theory + Quantum fluctuations

Altman et al., NJP 5:113 (2003)



Superexchange interaction in experiments with double wells

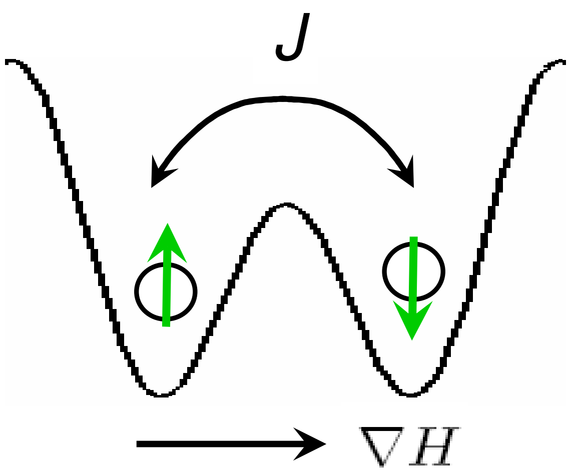
Refs:

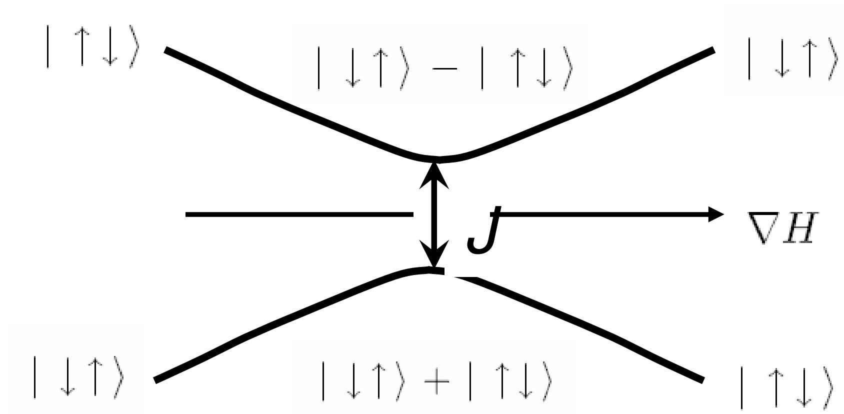
Theory: A.M. Rey et al., arXiv:0704.1413

Experiment: S. Trotzky et al., arXiv:0712.1853

Observation of superexchange in a double well potential

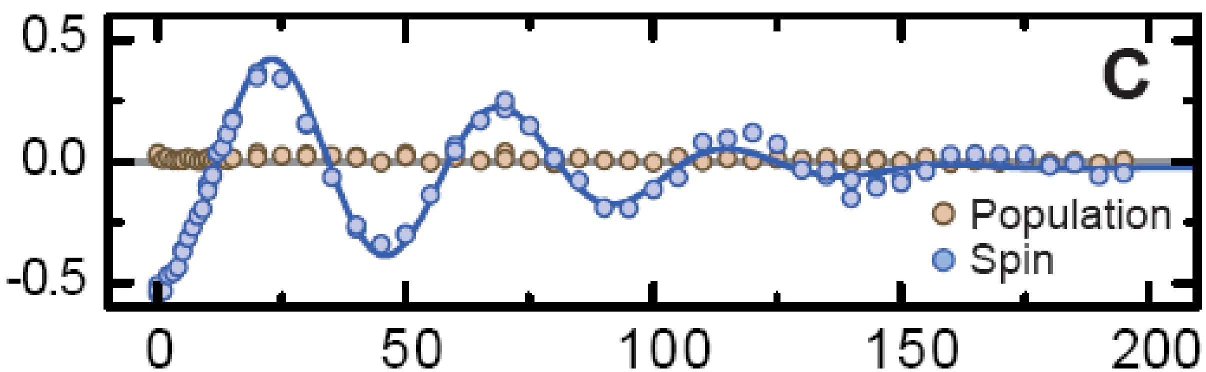
Theory: A.M. Rey et al., arXiv:0704.1413



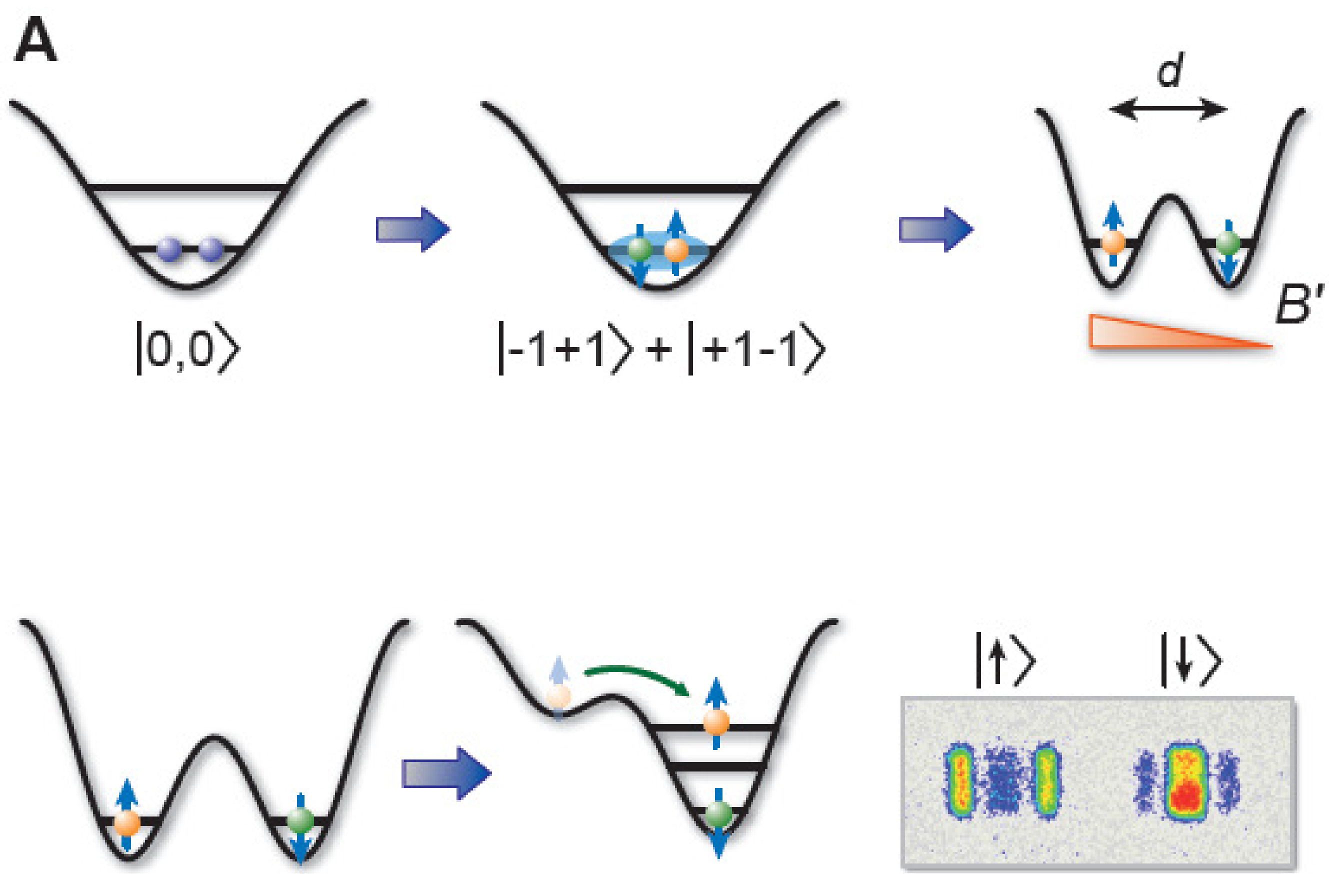


Use magnetic field gradient to prepare a state $|\downarrow\uparrow\rangle$

Observe oscillations between $|\downarrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$ states

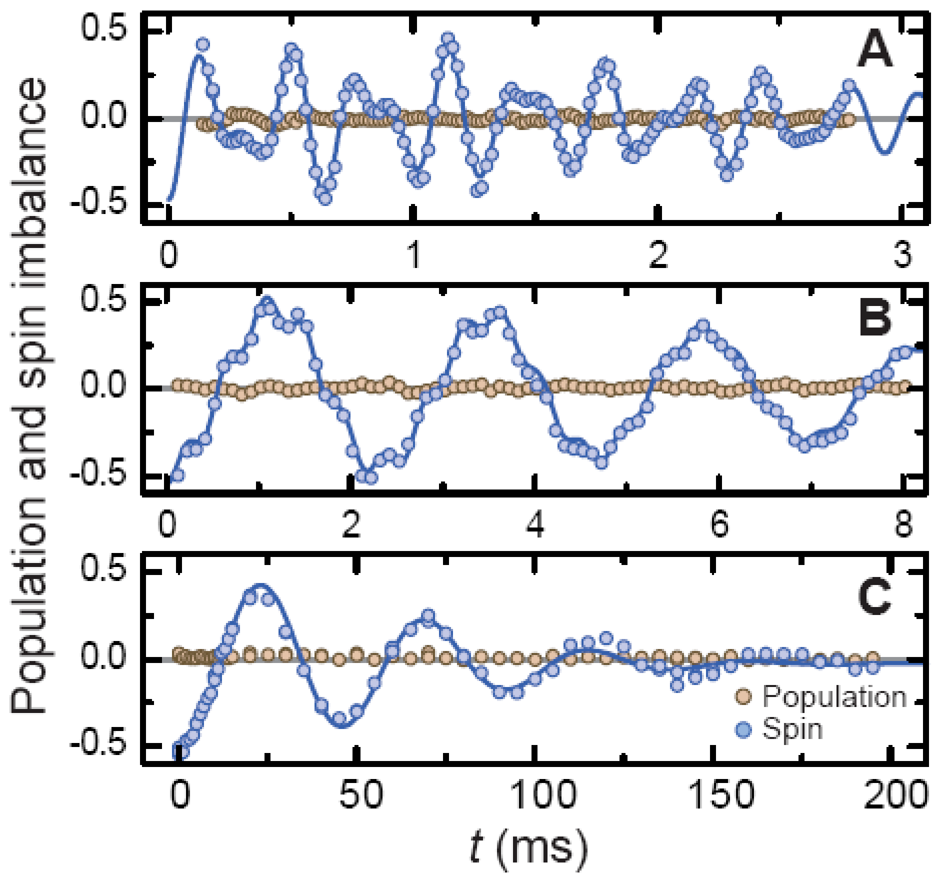


Preparation and detection of Mott states of atoms in a double well potential

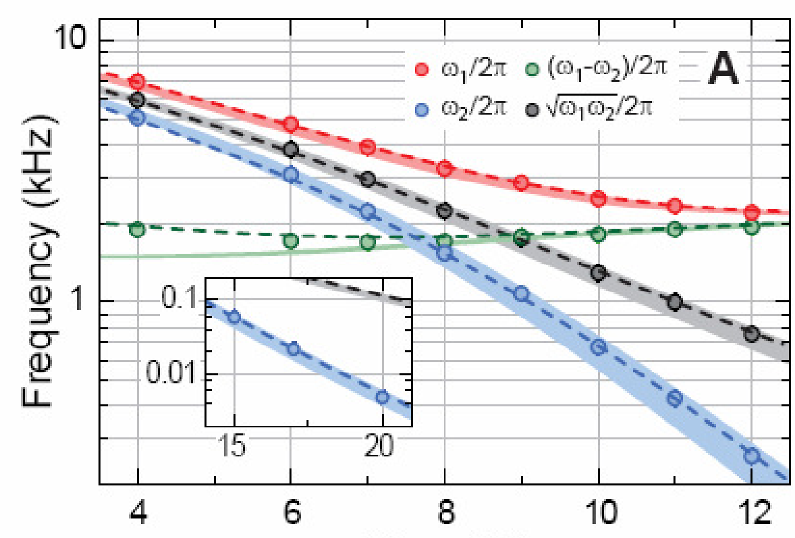


Comparison to the Hubbard model

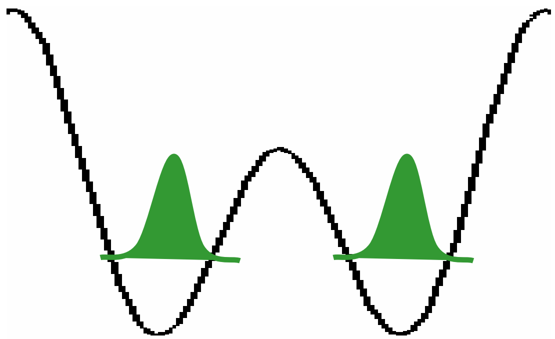
Experiments: I. Bloch et al.



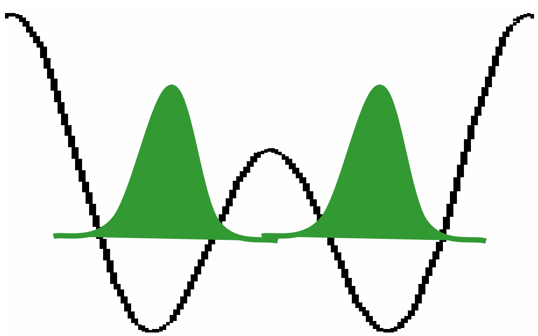
$$\hbar\omega_{1,2} = \frac{U}{2}\left(\sqrt{\left(\frac{4J}{U}\right)^2 + 1} \pm 1\right)$$



Beyond the basic Hubbard model



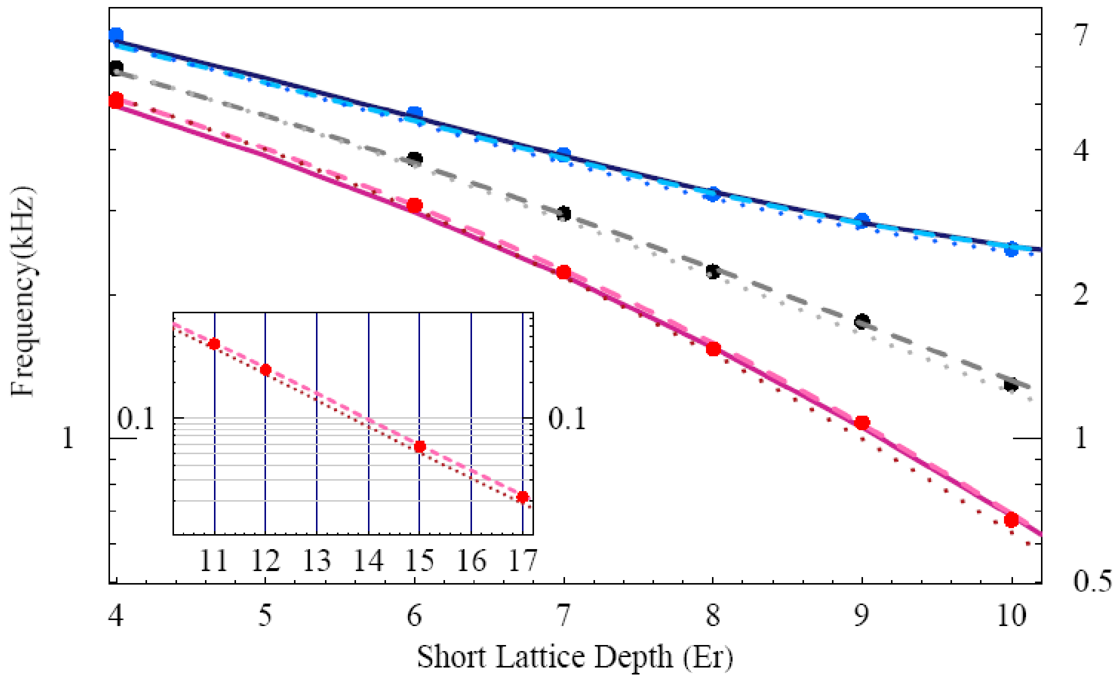
Basic Hubbard model includes only local interaction

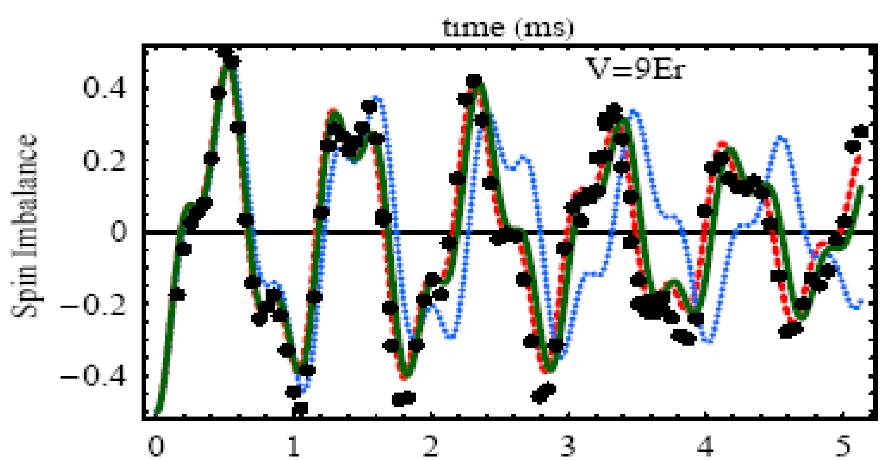


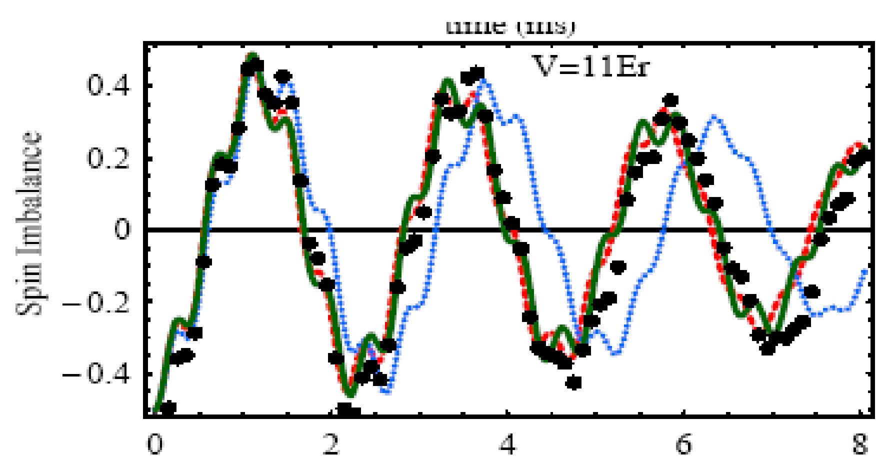
Extended Hubbard model takes into account non-local interaction

$$\begin{aligned}\hat{H}^{\mathrm{EHM}} &= \hat{H}^{\mathrm{HM}} - \Delta J \sum_{\sigma\neq\sigma'} \left(\hat{n}_{\sigma\mathrm{L}} + \hat{n}_{\sigma\mathrm{R}}\right)\left(\hat{a}^{\dagger}_{\sigma'\mathrm{L}}\hat{a}_{\sigma'\mathrm{R}} + \hat{a}^{\dagger}_{\sigma'\mathrm{R}}\hat{a}_{\sigma'\mathrm{L}}\right) \\ &+ U_{\mathrm{LR}} \sum_{\sigma\neq\sigma'} \Big(\hat{n}_{\sigma\mathrm{L}}\hat{n}_{\sigma'\mathrm{R}} + \hat{a}^{\dagger}_{\sigma\mathrm{L}}\hat{a}^{\dagger}_{\sigma'\mathrm{R}}\hat{a}_{\sigma'\mathrm{L}}\hat{a}_{\sigma\mathrm{R}} \\ &\qquad + \frac{1}{2}\hat{a}^{\dagger}_{\sigma\mathrm{L}}\hat{a}^{\dagger}_{\sigma'\mathrm{L}}\hat{a}_{\sigma'\mathrm{R}}\hat{a}_{\sigma\mathrm{R}} + \frac{1}{2}\hat{a}^{\dagger}_{\sigma\mathrm{R}}\hat{a}^{\dagger}_{\sigma'\mathrm{R}}\hat{a}_{\sigma'\mathrm{L}}\hat{a}_{\sigma\mathrm{L}}\Big) ,\end{aligned}$$

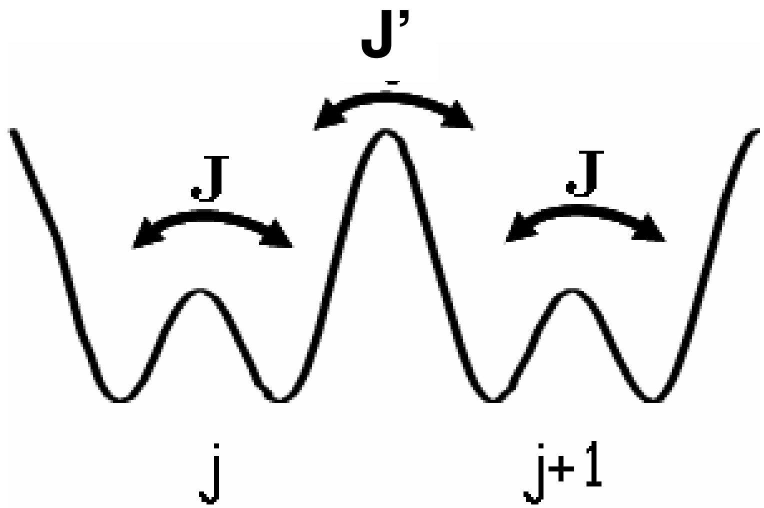
Beyond the basic Hubbard model





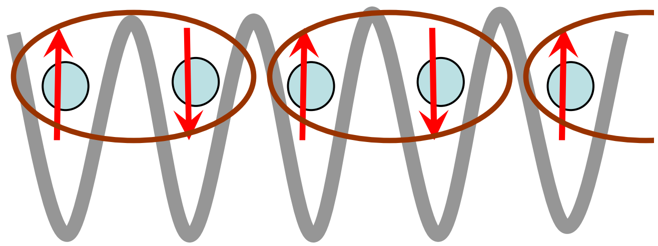


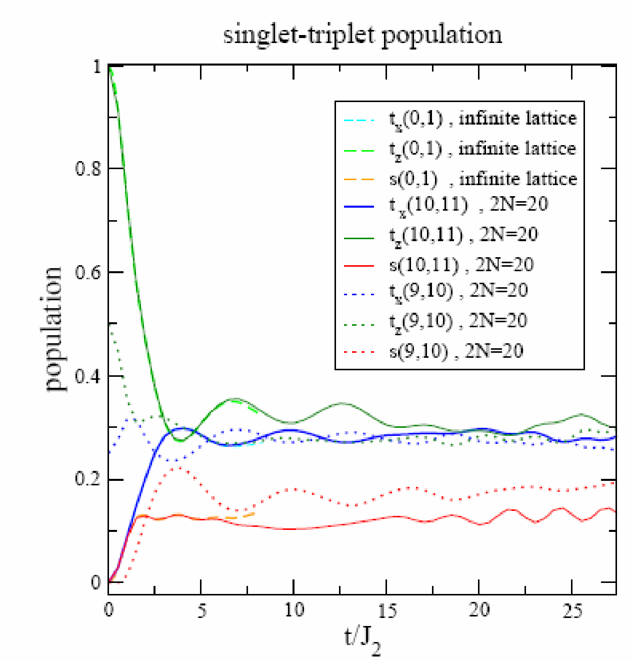
Connecting double wells …

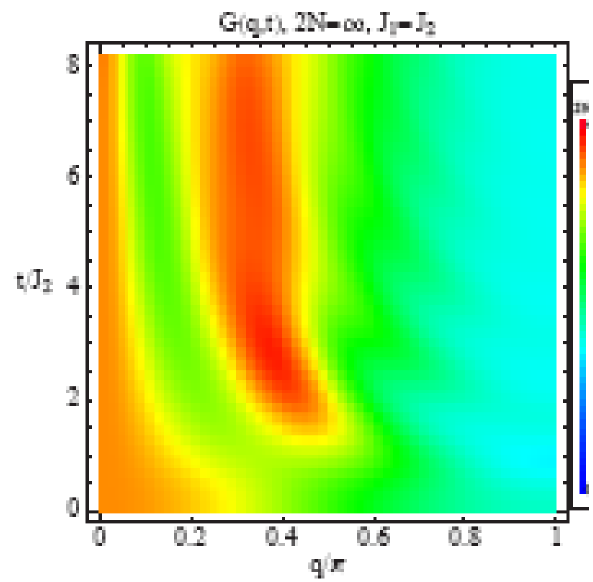


Spin Dynamics of an isotropic 1d Heisenberg model

Initial state: product of triplets







Conclusions

Experiments with ultracold atoms provide a new perspective on the physics of strongly correlated many-body systems. This includes analysis of high order correlation functions, non-equilibrium dynamics, and many more