Interference experiments with ultracold atoms

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Outline

Introduction.
Interference of fluctuating low dimensional condensates.
Systems of mixed dimensionality

Interference of fermions: probing paired states
Detection of s-wave pairing
Detection of FFLO
Detection of d-wave pairing

Interference experiments and non-equilibrium dynamics
Decoherence of uniformly split condensates
Ramsey interference of one dimensional systems
Splitting condensates on Y-junctions
Interference of independent condensates


Theory: Javanainen, Yoo, PRL 76:161 (1996)
and many more
Experiments with 2D Bose gas


Experiments with 1D Bose gas

Hofferberth et al. arXiv0710.1575
Amplitude of interference fringes, \( A_{fr} \)

\[
|A_{fr}| e^{i\Delta \phi} = \int_{0}^{L} dx \ a_{1}^{\dagger} (x) \ a_{2} (x)
\]

For independent condensates \( A_{fr} \) is finite but \( \Delta \phi \) is random

\[
\langle |A_{fr}|^2 \rangle = \int_{0}^{L} \int_{0}^{L} dx \ dy \ \langle a_{1}^{\dagger} (x) \ a_{2} (x) \ a_{2}^{\dagger} (y) \ a_{1} (y) \rangle
\]

\[
\simeq L \int_{0}^{L} dx \ \langle a_{1}(x) \ a_{1}^{\dagger}(0) \rangle \ \langle a_{2}(0) a_{2}^{\dagger}(x) \rangle
\]

For identical condensates

\[
\langle |A_{fr}|^2 \rangle = L \int_{0}^{L} dx \ (G(x))^2
\]

Instantaneous correlation function

\[
G(x) = \langle a(x) a^{\dagger}(0) \rangle
\]
Interference between fluctuating condensates

1d: Luttinger liquid, Hofferberth et al., 2007

2d: BKT transition, Hadzibabic et al, 2006
Distribution function of interference fringe contrast

Experiments: Hofferberth et al., arXiv0710.1575
Theory: Imambekov et al., cond-mat/0612011

Quantum fluctuations dominate:
- asymmetric Gumbel distribution
  (low temp. T or short length L)

Thermal fluctuations dominate:
- broad Poissonian distribution
  (high temp. T or long length L)

Intermediate regime:
- double peak structure

Comparison of theory and experiments: no free parameters
Higher order correlation functions can be obtained
Systems of mixed dimensionality
Weakly coupled 2D condensates

Interplay of two dimensional physics of the BKT transition and coupling along the 3\textsuperscript{rd} direction

Experiments: M. Kasevich et al.,

Connection to quasi-2D and 1D condensed matter systems:

quantum magnets, organic superconductors, high Tc cuprates, and many more
Berezinskii – Kosterlitz – Thouless transition

Fisher & Hohenberg, PRB 37, 4936 (1988)
Temperature scales for weakly coupled pancakes

\[ E_k = k_{\perp}^2 \]

\[ E_k = c |k_{\perp}| \]

\[ E_k = 1 - t \cos k_z \]
Interference of a stack of coupled pancakes

Pekker, Gritsev, Demler
Interference experiments with fermions: probing paired states
Interference of fermionic systems

A pair of independent fermionic systems

\[ \rho_{\text{int}}(x, y, z) = A(x, y) e^{iQz} + \text{c.c.} \]

\[ A(x, y) = c_1^\dagger(x, y) c_2(x, y) \]

\[ Q = \frac{md}{\hbar t} \]
Interference as a probe of fermionic pairing

Pairing correlations

\[
A_{\Omega \uparrow} = \int_{\Omega} d^2r \ c_{1\uparrow}^\dagger (r) \ c_{2\uparrow} (r)
\]

\[
A_{\Omega \downarrow} = \int_{\Omega} d^2r \ c_{1\downarrow}^\dagger (r) \ c_{2\downarrow} (r)
\]

\[
\Delta_{1}^\dagger (r - r') = \langle c_{1\uparrow}^\dagger (r) c_{1\downarrow}^\dagger (r') \rangle
\]

\[
\Delta_{2}^\dagger (r - r') = \langle c_{2\uparrow}^\dagger (r) c_{2\downarrow}^\dagger (r') \rangle
\]

\[
A_{\Omega \uparrow} A_{\Omega \downarrow} = \int_{\Omega} \int_{\Omega} \Delta_{1}^\dagger (r - r') \Delta_{2} (r - r')
\]

Expectation value vanishes for independent systems due to random relative phase between $\Delta_{1}$ and $\Delta_{2}$
Interference as a probe of pairing

Polkovnikov, Gritsev, Demler

Experimental procedure
Interfere two independent systems
Measure $\rho_{\uparrow}(r, z)$ and $\rho_{\downarrow}(r, z)$ in the same shot
Extract $A_{\uparrow}(r)$ and $A_{\downarrow}(r)$ from Fourier transforms in the z-direction
Calculate $A_I$ and $A_{II}$
Find $\langle A_I^\dagger A_{II} \rangle$ from averaging over many shots

$$A_I = A_{I\uparrow} A_{I\downarrow} = \int_{\Omega_I} \int_{\Omega_I} \Delta_{I}^{\dagger}(r - r') \Delta_2(r - r')$$

$$A_{II} = A_{II\uparrow} A_{II\downarrow} = \int_{\Omega_{II}} \int_{\Omega_{II}} \Delta_{I}^{\dagger}(r - r') \Delta_2(r - r')$$

$$\langle A_I^\dagger A_{II} \rangle \neq 0$$
FFLO phase

Pairing at finite center of mass momentum

\[ \Delta(r) = \sum_n \Delta_n e^{i\vec{q}_n \cdot \vec{r}} \]

\[ \left| \vec{q}_n \right| = k_{F\uparrow} - k_{F\downarrow} \]

Theory:
Fulde, Ferrell (1964); Larkin, Ovchinnikov (1965); Bowers, Rajagopal (2002); Liu, Wilczek (2003); Sheehy, Radzihovsky (2006); Combescot (2006); Yang, Sachdev (2006); Pieri, Srinati (2006); Parish et al., (2007); and many others

Experiments:
Zwierlein, Ketterle et al., (2006)
Hulet et al., (2006)
Interference as a probe of FFLO phase

1. Integration by a laser beam

2. Manual integration

\[
A_{I\uparrow}(q) = \int_{\Omega} d^2r \ e^{i q \cdot r / 2} c_{1\uparrow}^\dagger(\vec{r}) \ c_{2\uparrow}(\vec{r})
\]

\[
A_{I\downarrow}(q) = \int_{\Omega} d^2r \ e^{i q \cdot r / 2} c_{1\downarrow}^\dagger(\vec{r}) \ c_{2\downarrow}(\vec{r})
\]

\[
A_I(q) = A_{I\uparrow}(q) \ A_{I\downarrow}(q)
\]

\[
\langle A_{I\uparrow}^\dagger(q) \ A_{I\downarrow}(q) \rangle \neq 0 \quad \text{when } q \text{ matches one of the wavevectors of } \Delta(r) \text{ of FFLO phase}
\]
d-wave pairing

Fermionic Hubbard model

\[ \mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i \]

Possible phase diagram of the Hubbard model
Phase sensitive probe of d-wave pairing in high Tc superconductors

Van Harlingen, Leggett et al, PRL 71:2134 (93)

Superconducting quantum interference device (SQUID)
Other signatures of d-wave pairing: dispersion of quasiparticles

Superconducting gap

\[ \Delta_k = \Delta_0 \left( \cos k_x - \cos k_y \right) \]

Normal state dispersion of quasiparticles \( \epsilon_k \)

Quasiparticle energies

\[ E_k = \sqrt{\epsilon_k^2 + \Delta_k^2} \]

Low energy quasiparticles correspond to four Dirac nodes

Observed in:

- Photoemission
- Raman spectroscopy
- T-dependence of thermodynamic and transport properties, \( c_V, \kappa, \lambda_L \)
- STM
- and many other probes
Phase sensitive probe of d-wave pairing in high Tc superconductors

Superconducting quantum interference device (SQUID)

Van Harlingen, Leggett et al, PRL 71:2134 (93)
Interference as a probe of d-wave pairing

System 1 is an s-wave superfluid
System 2 is a d-wave superfluid
Regions II and III differ only by $90^\circ$ rotation

Phase sensitive probe of d-wave pairing

$$\langle A_1^\dagger(q) A_{II}(q) \rangle = - \langle A_1^\dagger(q) A_{III}(q) \rangle$$
Interference experiments and non-equilibrium dynamics
Uniform splitting of the condensates

Prepare a system by splitting one condensate

Take to the regime of zero tunneling

Measure time evolution of fringe amplitudes
Long phase coherence implies squeezing factor of 10.
Squeezing due to finite time of splitting. Leggett, Sols, PRL (1998)
Burkov et al., PRL (2007)

Squeezing factor

$$\frac{\delta N}{\sqrt{N}} \sim \frac{1}{\sqrt{\mu T_s}}$$
1d BEC: Decay of coherence


\[ \Psi(t) \propto \exp[-(t/t_0)^{2/3}] \]

double logarithmic plot of the coherence factor

slopes: \[0.64 \pm 0.08\]
\[0.67 \pm 0.1\]
\[0.64 \pm 0.06\]

get \(t_0\) from fit with fixed slope 2/3 and calculate \(T\) from

\[ t_0 = 2.61 \pi K / T^2 \]

\[ T_5 = 110 \pm 21 \text{ nK} \]
\[ T_{10} = 130 \pm 25 \text{ nK} \]
\[ T_{15} = 170 \pm 22 \text{ nK} \]
Relative phase dynamics beyond single mode approximation

\[ \phi = \phi_1 - \phi_2 \]
\[ \Delta n = (n_1 - n_2)/2 \]

Conjugate variables

Hamiltonian can be diagonalized in momentum space

\[ \mathcal{H} = \int d^d r \left[ \frac{g}{2} (\Delta n)^2 + \frac{\rho}{2} (\nabla \phi)^2 \right] \]

A collection of harmonic oscillators with \( \omega_q = \sqrt{g\rho} \mid q \mid \)

Initial state \( \phi_q = 0 \)

Need to solve dynamics of harmonic oscillators at finite \( T \)

Coherence

\[ \langle \Psi(t) \mid e^{i\phi} \mid \Psi(t) \rangle = e^{-\frac{1}{2} \sum_q \langle \phi_q^2(t) \rangle} \]
Relative phase dynamics beyond single mode approximation

Quantum regime

\[ \frac{\hbar}{\mu} < t < \frac{\hbar}{k_B T} \]

1D systems

\[ \langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} e^{-t/2\pi K \tau_s} \]

2D systems

\[ \langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} \left( \frac{t_0}{t} \right)^{1/16T_KT \tau_s} \]

Classical regime

\[ t > \frac{\hbar}{k_B T} \]

1D systems

\[ \langle e^{i\phi(t)} \rangle \sim e^{-\left(\frac{t}{t_T}\right)^2/3} \quad t_T \sim \frac{\mu K}{T^2} \]

2D systems

\[ \langle e^{i\phi(t)} \rangle \sim \left( \frac{t_0}{t} \right)^{\frac{T}{8T_KT}} \]
Dynamics of condensate splitting and Ramsey interference
Ramsey interference

\[ |\Psi\rangle = e^{-iE_1 t} |\uparrow\rangle + e^{-iE_2 t} |\downarrow\rangle \]

Working with \( N \) atoms improves the precision of frequency spectroscopy by \( \sqrt{N} \).
Interaction induced collapse of Ramsey fringes

Spin echo. Time reversal experiments

Single mode approximation predicts full revival

Experiments in 1d tubes:

Need to analyze multi-mode model in 1d

Only q=0 mode shows complete spin echo
Finite q modes continue decay

The net visibility is a result of competition between q=0 and other modes
Splitting condensates on Y-junctions: quantum zipper problem
Splitting condensates on Y-junctions

Partial splitting stage: new physics

Full splitting stage: same as before

Earlier work:
Non-interacting atoms: Scully and Dowling, PRA (1993)
Tonks-Girardeau regime: Girardeau et al. PRA (2002)
Splitting condensates on Y-junctions: beyond mean-field

Wave equation in both arms of the interferometer. $c$ is the speed of sound

$$\left( \frac{1}{c} \frac{\partial^2}{\partial t^2} - c \frac{\partial^2}{\partial x^2} \right) \phi_1 = 0$$

$$\left( \frac{1}{c} \frac{\partial^2}{\partial t^2} - c \frac{\partial^2}{\partial x^2} \right) \phi_2 = 0$$

Relative phase $\phi = \phi_1 - \phi_2$

$$\left( \frac{1}{c} \frac{\partial^2}{\partial t^2} - c \frac{\partial^2}{\partial x^2} \right) \phi = 0$$

Time dependent boundary conditions in the frame of the condensate. $v$ is the condensate velocity

$$\frac{\partial \phi}{\partial x} (x = 0) = 0$$

$$\phi(x_B(t) = vt) = 0$$
Moving mirror problem in optics


Exciting photons in a cavity with a moving mirror

\[
\begin{align*}
\left( \frac{1}{c} \frac{\partial^2}{\partial t^2} - c \frac{\partial^2}{\partial x^2} \right) \phi &= 0 \\
\phi(x_B(t)) &= 0
\end{align*}
\]

\(C\) is the speed of light

Experimentally always in the adiabatic regime

\[
\begin{align*}
\omega_n &= \frac{2 \pi n c}{L} \\
\dot{\omega}_n &= \omega_n \frac{\dot{L}}{L} \\
\frac{\dot{\omega}_n}{\omega_n^2} &\sim \frac{\dot{L}}{c} \ll 1
\end{align*}
\]
Splitting condensate on Y-junction: beyond mean-field

\[ \langle e^{i\phi(d)} \rangle \sim \left( 1 - \frac{v^2}{c^2} \frac{\xi_h}{d} \right)^{\frac{1}{2K}} \]

\( K \) – Luttinger parameter
\( v \) is the condensate velocity

This is similar to the usual 1d condensates
\[ \langle e^{i\phi(d)} e^{-i\phi(0)} \rangle \sim \left( \frac{\xi_h}{d} \right)^{\frac{1}{2K}} \]

Extra suppression due to finite velocity of splitting
Splitting with acceleration: Unruh like effect

Fagnocchi, Altman, Demler

\[ x_B(t) = ct - A e^{-\kappa t} \]

Splitting condensates with relativistic acceleration gives rise to thermal correlations. This is analogous to the Unruh effect in field theory and quantum gravity Unruh (1974), Fulling and Davies (1976)

\[ \langle e^{i\phi(\Delta x, \Delta t)} e^{-i\phi(0)} \rangle \sim \left( \frac{\xi_h}{\Delta x_+} \right)^\frac{1}{2\kappa} e^{-\frac{\kappa}{2\kappa} \Delta x_-} \]

\[ \Delta x_\pm = \Delta x \pm c \Delta t \]
Interference experiments with ultracold atoms provide a powerful tool for analyzing equilibrium properties and dynamics of many-body systems. Analysis beyond mean-field and single mode approximation is needed.