# Learning about order from noise

## Quantum noise studies of ultracold atoms

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Experiments:
Bloch et al., Dalibard et al., Greiner et al., Schmiedmayer et al.

# Quantum noise

Classical measurement:

collapse of the wavefunction into eigenstates of x

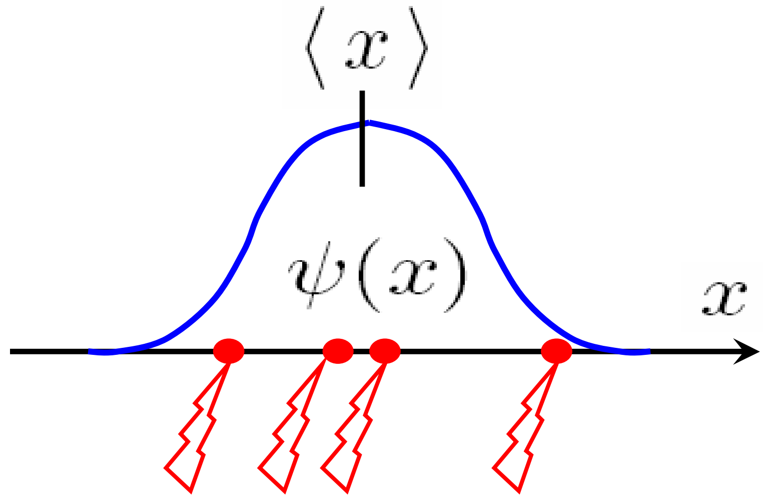
$$\langle x \rangle = \int dx\, x\, |\psi(x)|^2$$

$$\langle x^2 \rangle = \int dx\, x^2\, |\psi(x)|^2$$

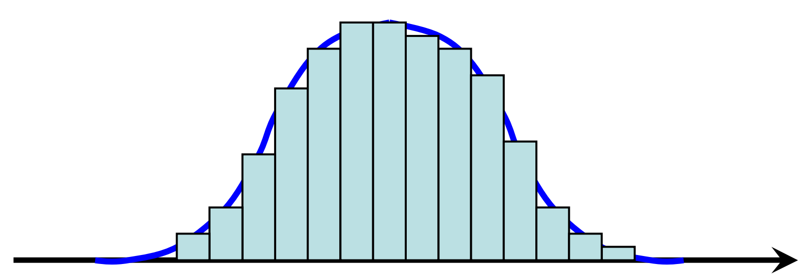
$$\dots$$

$$\langle x^n \rangle = \int dx\, x^n\, |\psi(x)|^2$$

$$\dots$$




Histogram of measurements of x

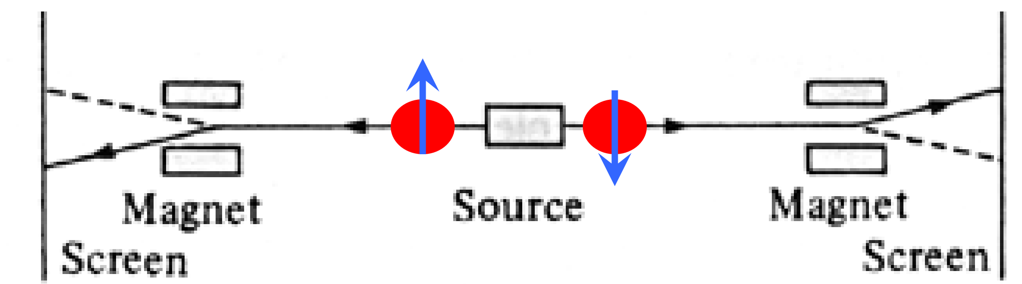


# Probabilistic nature of quantum mechanics

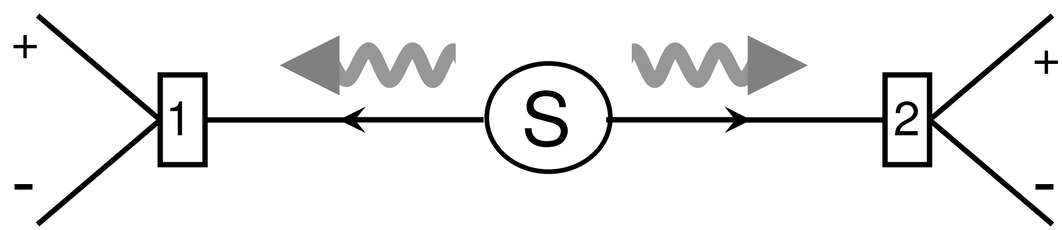
Bohr-Einstein debate:
EPR thought experiment
(1935)

"Spooky action at a distance"

$$|S=0\rangle = |\uparrow\rangle_L |\downarrow\rangle_R - |\downarrow\rangle_L |\uparrow\rangle_R$$




Aspect's experiments with correlated photon pairs:
tests of Bell's inequalities (1982)

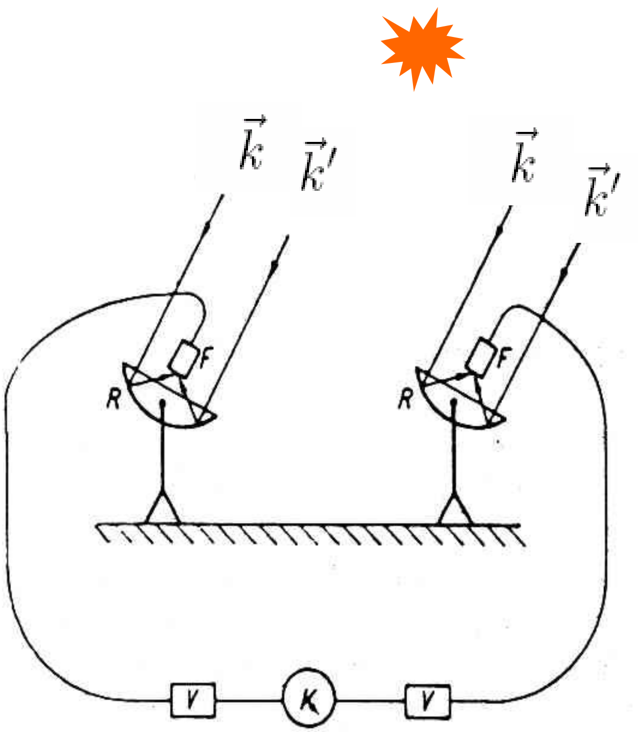



Analysis of correlation functions can be used to rule out hidden variables theories

# Second order coherence: HBT experiments

## Classical theory

Hanburry Brown and Twiss (1954)

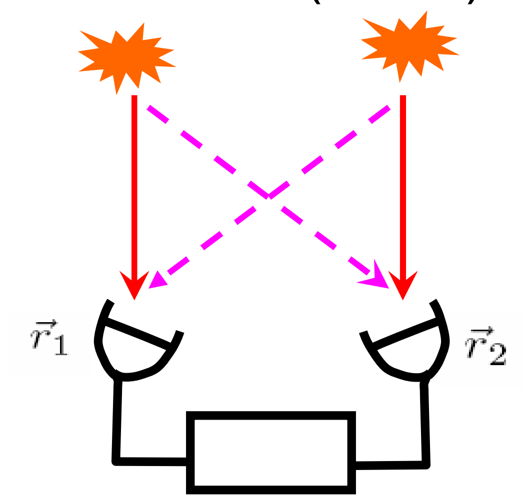



$$\langle I(\vec{r}_1)\ I(\vec{r}_2)\rangle = A + B\ \cos\Big((\vec{k}-\vec{k}')\ (\vec{r}_1-\vec{r}_2)\Big)$$

Used to measure the angular diameter of Sirius

## Quantum theory

Glauber (1963)




For bosons

$$A = A_1 + A_2$$

For fermions

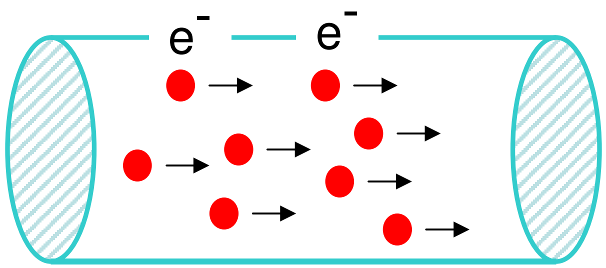
$$A = A_1 - A_2$$

HBT experiments with matter

# Shot noise in electron transport

Variance of transmitted charge

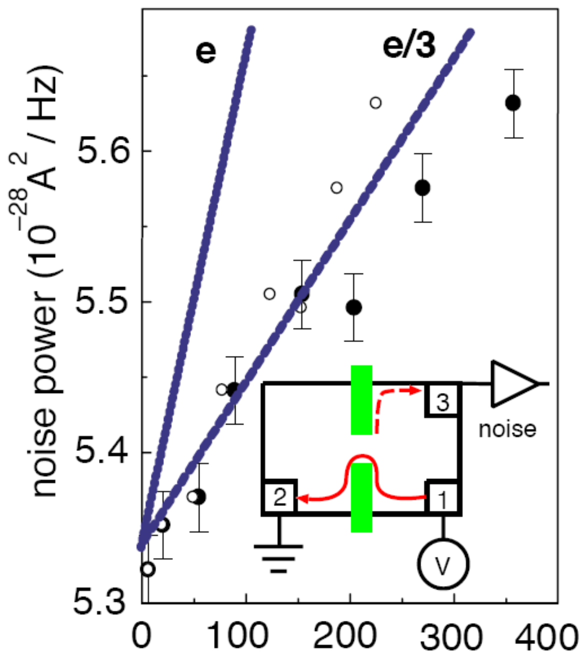
$$S_0 = \frac{2}{\tau} \langle \delta q^2(\tau) \rangle$$




Shot noise
Schottky (1918)

$$\boxed{S_0 = 2\, e\, I}$$

## Measurements of fractional charge




Current noise for tunneling across a Hall bar on the 1/3 plateau of FQE

Etien et al. PRL 79:2526 (1997)
see also Heiblum et al. Nature (1997)

Analysis of quantum noise:
powerful experimental tool

Can we use it for cold atoms?

# Outline

Quantum noise in interference experiments with independent condensates

Quantum noise analysis of time-of-flight experiments with atoms in optical lattices: HBT experiments and beyond

Goal: new methods of detection of quantum many-body phases of ultracold atoms

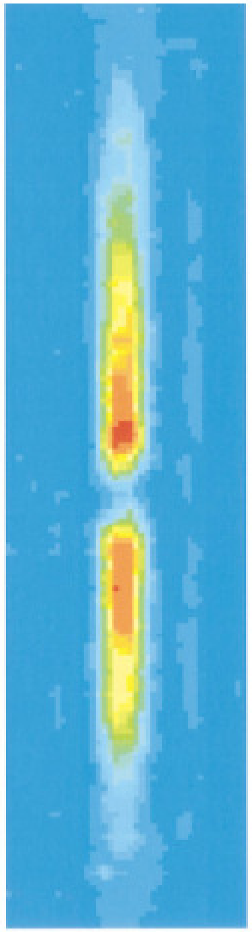
# Interference experiments with cold atoms

## Analysis of thermal and quantum noise in low dimensional systems

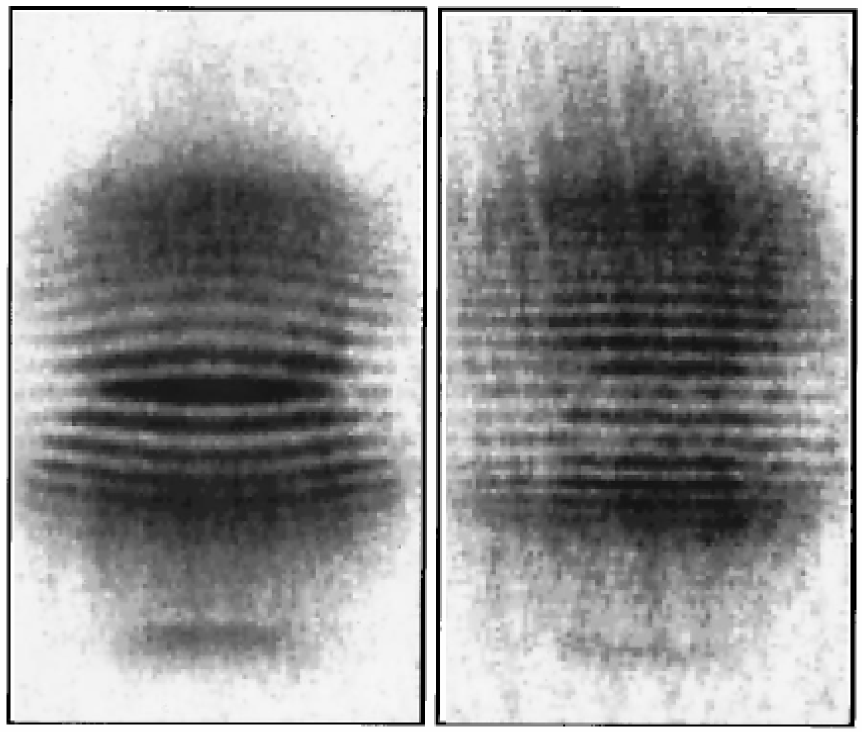
Theory: For review see
Imambekov et al., Varenna lecture notes, c-m/0612011

Experiment
2D: Hadzibabic, Kruger, Dalibard, Nature 441:1118 (2006)
1D: Hofferberth et al., Nature Physics 4:489 (2008)

# Interference of independent condensates



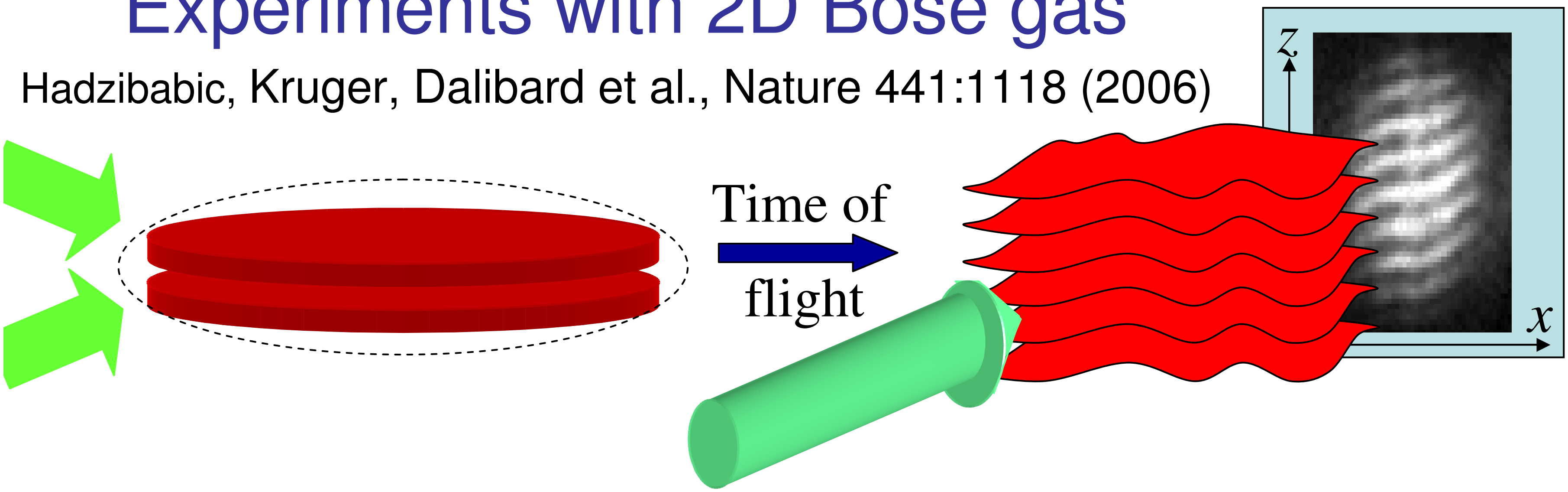
Experiments: Andrews et al., Science 275:637 (1997)



Theory: Javanainen, Yoo, PRL 76:161 (1996)
Cirac, Zoller, et al. PRA 54:R3714 (1996)
Castin, Dalibard, PRA 55:4330 (1997)
and many more

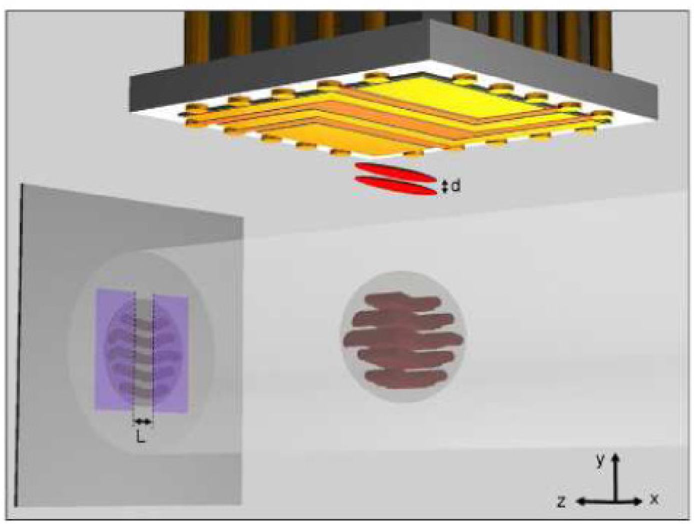
# Experiments with 2D Bose gas

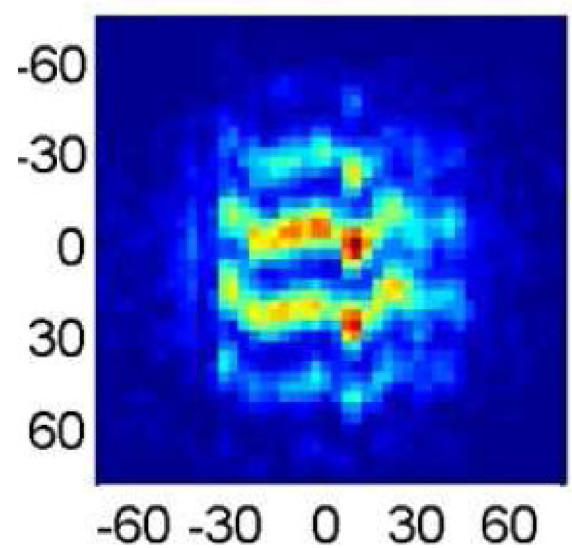
Hadzibabic, Kruger, Dalibard et al., Nature 441:1118 (2006)



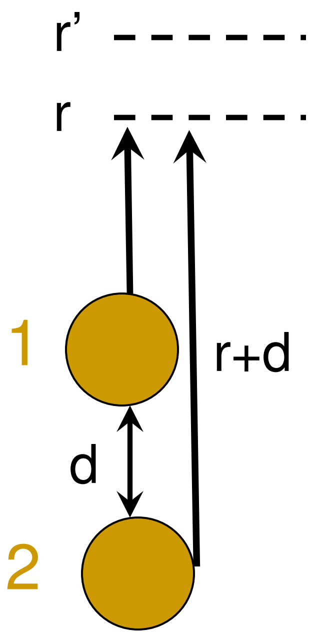

# Experiments with 1D Bose gas

Hofferberth et al., Nature Physics 4:489 (2008)



# Interference of two independent condensates




$$\psi(r) = \psi_1(r) + \psi_2(r)$$

$$\rho_{\rm int}(r) = \psi_1^\dagger(r)\,\psi_2(r) + {\rm c.c.}$$

Assuming ballistic expansion

$$\boxed{\rho_{\rm int}(r) = e^{i\,\frac{m\,d\,r}{\hbar\,t}}\, e^{i\,(\phi_2-\phi_1)} + {\rm c.c.}}$$

Phase difference between clouds 1 and 2
is not well defined

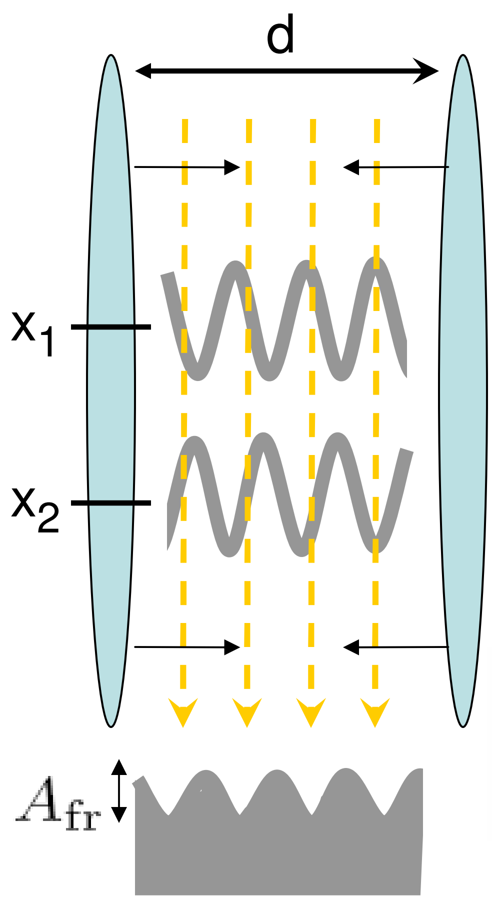
Individual measurements show interference patterns
They disappear after averaging over many shots

$$\langle\, \rho_{\rm int}(r)\,\rangle = 0$$

$$\langle\, \rho_{\rm int}(r)\,\rho_{\rm int}(r')\,\rangle = e^{i\,\frac{m\,d}{\hbar\,t}\,(r-r')} + {\rm c.c.}$$

# Interference of fluctuating condensates

Polkovnikov, Altman, Demler, PNAS 103:6125(2006)




Amplitude of interference fringes, $A_{\rm fr}$

$$|A_{\rm fr}|\, e^{i\Delta\phi} = \int_0^L dx\, e^{i(\phi_1(x)-\phi_2(x))}$$

For independent condensates $A_{fr}$ is finite but $\Delta\phi$ is random

$$\langle\, |A_{\rm fr}|^2 \,\rangle = \int_0^L dx_1 \int_0^L dx_2 \,\langle\, e^{i(\phi_1(x_1)-\phi_2(x_1))}\, e^{-i(\phi_1(x_2)-\phi_2(x_2))} \rangle$$

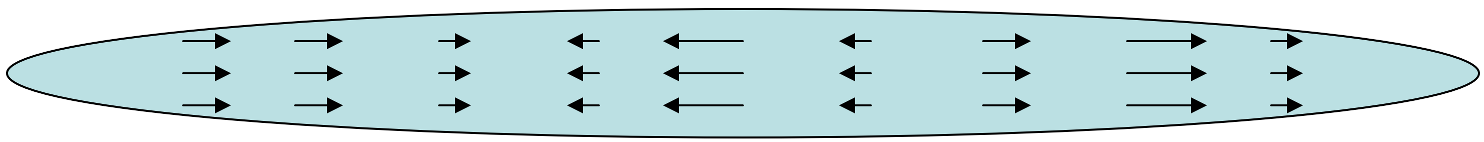
$$\langle\, |A_{\rm fr}|^2 \,\rangle \approx L \int_0^L dx \,\langle\, e^{i(\phi_1(x)-\phi_1(0))} \rangle \,\langle\, e^{-i(\phi_2(x)-\phi_2(0))} \,\rangle$$

For identical condensates $\langle |A_{\rm fr}|^2 \rangle = L \int_0^L dx \,(\, G(x)\,)^2$

Instantaneous correlation function $G(x) = \langle\, e^{i\phi(x)}\, e^{-\phi(0)} \,\rangle$

# Fluctuations in 1d BEC

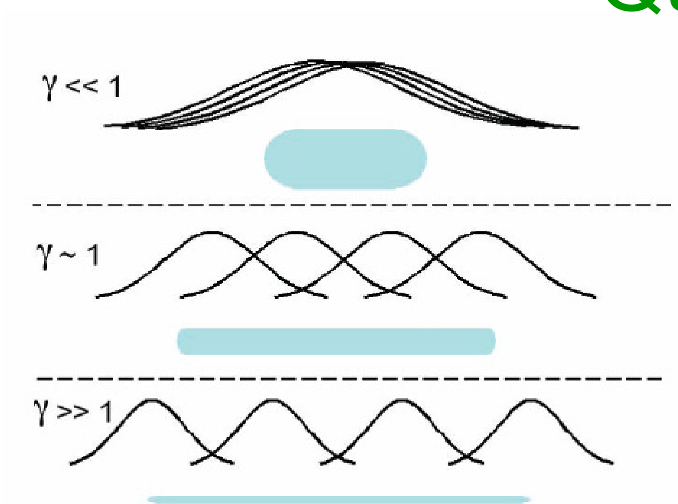
## Thermal fluctuations



Thermally energy of the superflow velocity $\quad \mathrm{v}_s = \nabla\phi(x)$

$$\boxed{\langle\, e^{i\phi(x_1)}\, e^{i\phi(x_2)}\,\rangle \sim e^{-|x_1-x_2|/\xi_T}} \qquad \xi_T = \sqrt{\frac{\hbar^2\, m}{T}}$$

## Quantum fluctuations




$$\boxed{\langle\, e^{i\phi(x_1)}\, e^{i\phi(x_2)}\,\rangle \sim \left(\frac{\xi_h}{|x_1-x_2|}\right)^{1/2K}}$$

Weakly interacting atoms $\qquad K = \sqrt{\dfrac{n}{g\, m}}$

# Interference between Luttinger liquids

Luttinger liquid at T=0 $\quad G(x) \sim \rho \left( \frac{\xi_h}{x} \right)^{1/2K}$

$$\langle |A_{\rm fr}|^2 \rangle \sim (\rho \xi_h)^{1/K} (L\rho)^{2-1/K}$$

*K* – Luttinger parameter

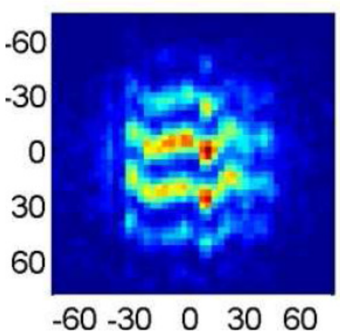
For non-interacting bosons $K = \infty$ and $A_{\rm fr} \sim L$

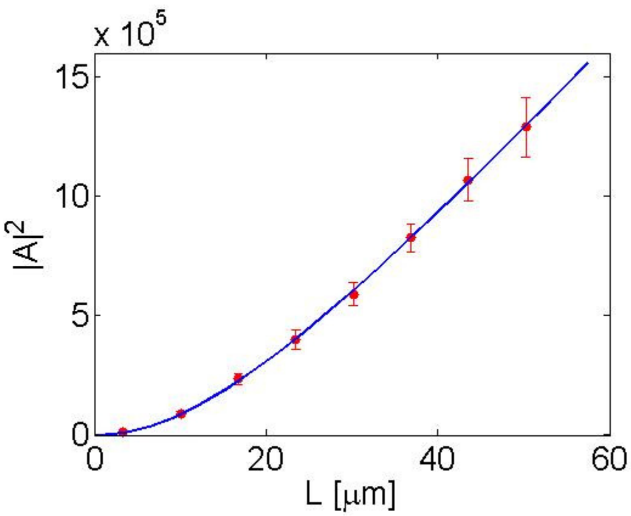
For impenetrable bosons $K = 1$ and $A_{\rm fr} \sim \sqrt{L}$

Finite temperature

$$\langle |A_{\rm fr}|^2 \rangle \sim L \rho^2 \xi_h \left( \frac{\hbar^2}{m \xi_h^2} \frac{1}{T} \right)^{1-1/K}$$

Experiments: Hofferberth, Schumm, Schmiedmayer



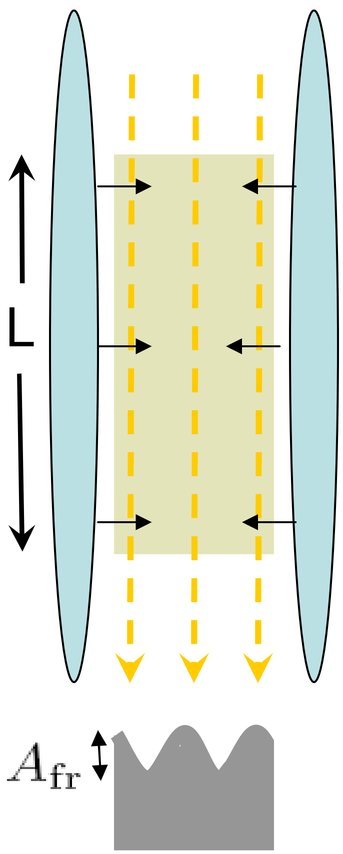

$n_{1d} = 60 \mu\mathrm{m}^{-1}$

$K = 47$

$\boxed{T_{fit} = 84 \pm 22 \text{ nK}}$

# Distribution function of fringe amplitudes for interference of fluctuating condensates

Gritsev, Altman, Demler, Polkovnikov, Nature Physics 2006
Imambekov, Gritsev, Demler, Varenna lecture notes, c-m/0703766




$A_{\mathrm{fr}}$ is a quantum operator. The measured value of $|A_{\mathrm{fr}}|$ will fluctuate from shot to shot.

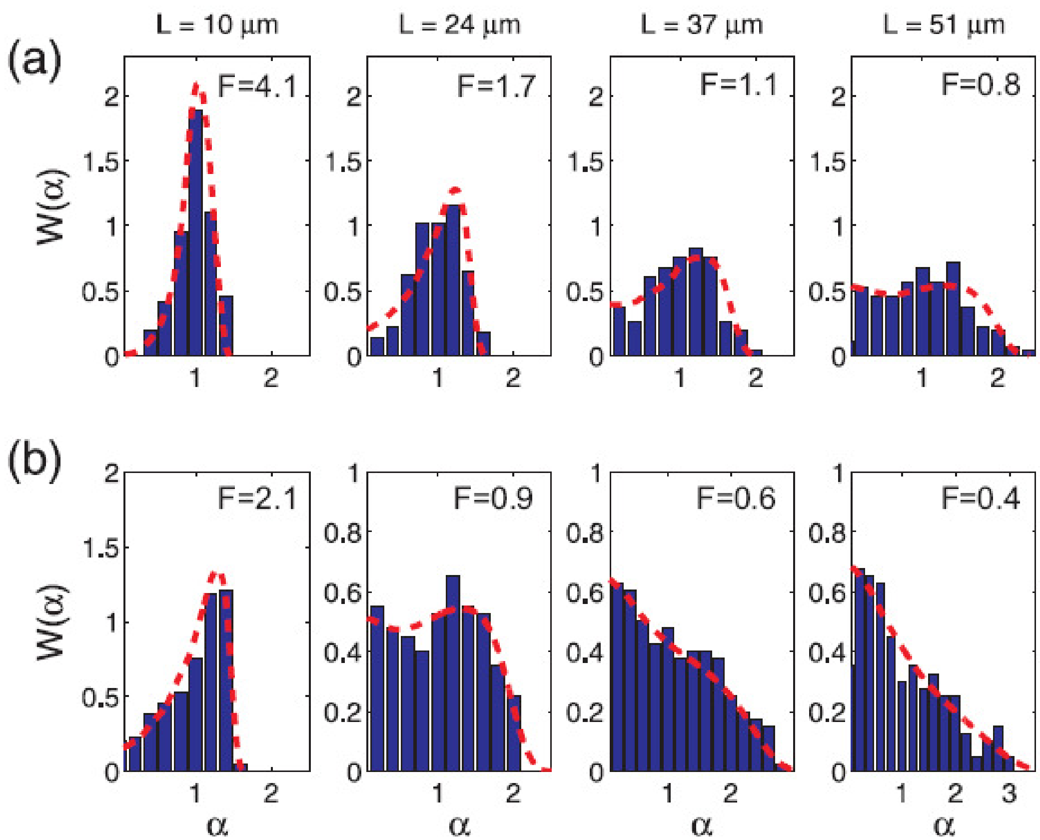
$$\langle\, |A_{\mathrm{fr}}|^{2n} \,\rangle = \int_0^L dz_1 \ldots dz_n' \, |\, \langle\, e^{i\phi(z_1)} \ldots e^{i\phi(z_n)}\, e^{-i\phi(z_1')} \ldots e^{-i\phi(z_n')} \,\rangle|^2$$

Higher moments reflect higher order correlation functions

We need the full distribution function of $|A_{\mathrm{fr}}|$

# Distribution function of interference fringe contrast

Hofferberth et al., Nature Physics 4:489 (2008)




Quantum fluctuations dominate:
asymetric Gumbel distribution
(low temp. T or short length L)

Thermal fluctuations dominate:
broad Poissonian distribution
(high temp. T or long length L)

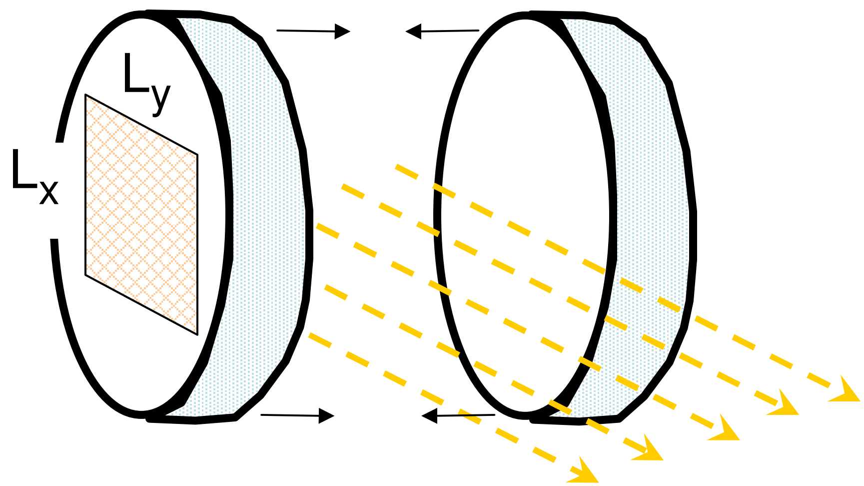
Intermediate regime:
double peak structure

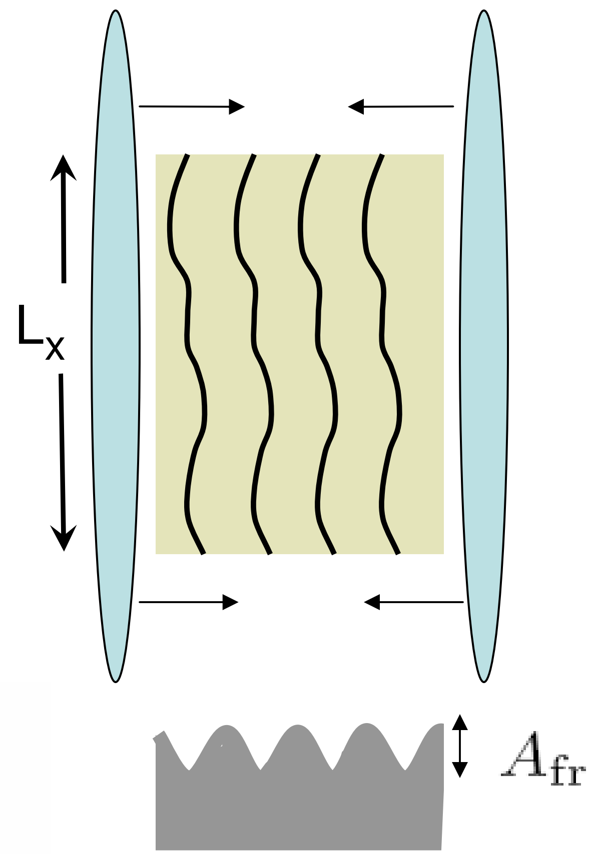
Comparison of theory and experiments: no free parameters
Higher order correlation functions can be obtained

# Interference of two dimensional condensates

Experiments: Hadzibabic et al. Nature (2006)

Gati et al., PRL (2006)




Probe beam parallel to the plane of the condensates

$$\langle\, |A_{\rm fr}|^2 \rangle \,=\, L_x\, L_y \int_0^{L_x} \int_0^{L_y} d^2\vec{r}\, (\, G(\vec{r}\,)\,)^2$$

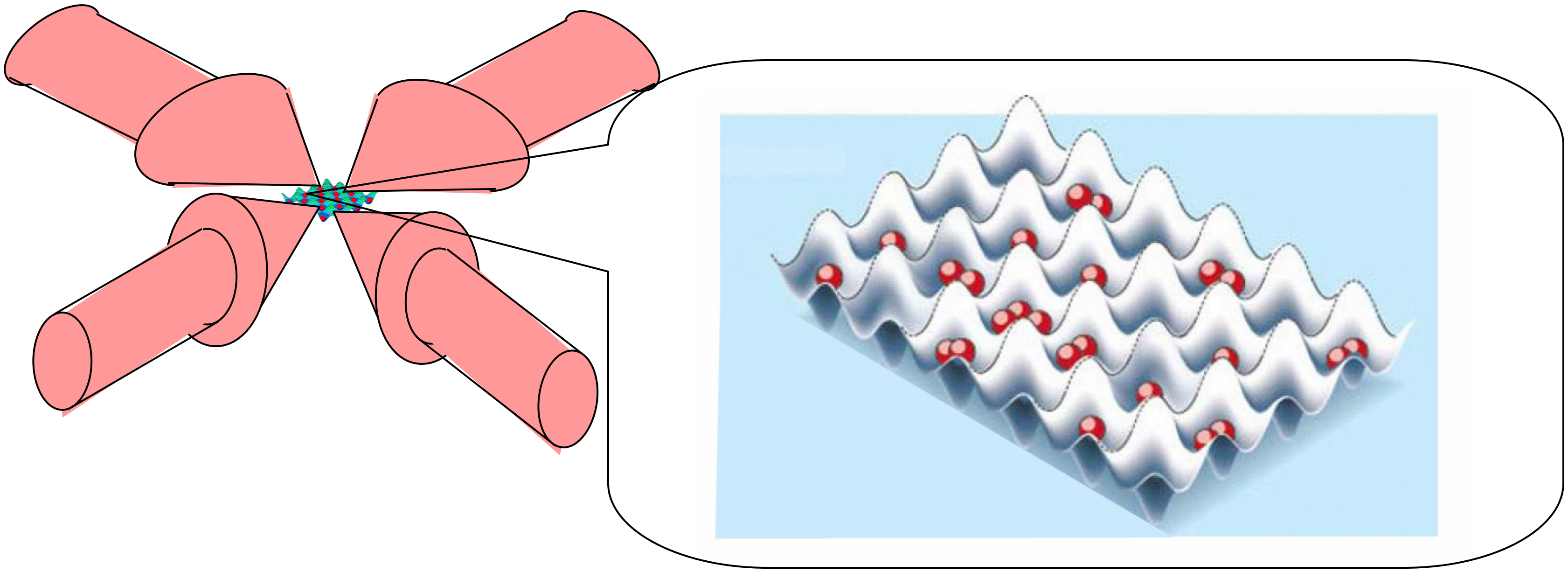
Observation of BTK transition: see talk by Peter Kruger

# Time-of-flight experiments with atoms in optical lattices

Theory: Altman, Demler, Lukin, PRA 70:13603 (2004)

Experiment: Folling et al., Nature 434:481 (2005);
Spielman et al., PRL 98:80404 (2007);
Tom et al. Nature 444:733 (2006);
Guarrera et al., PRL 100:250403 (2008)

# Atoms in optical lattices



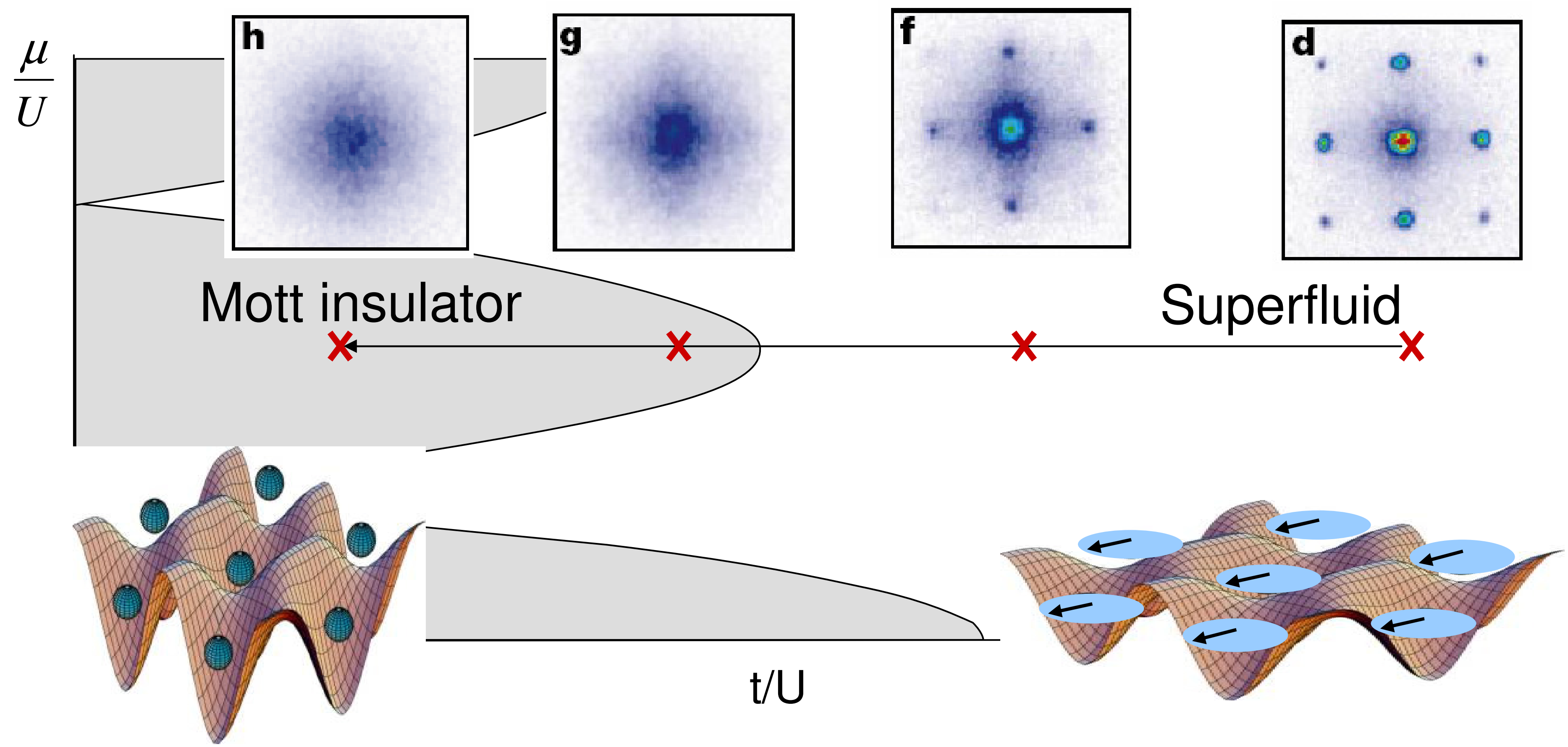
Theory: Jaksch et al. PRL (1998)

Experiment: Greiner et al., Nature (2001) and many more

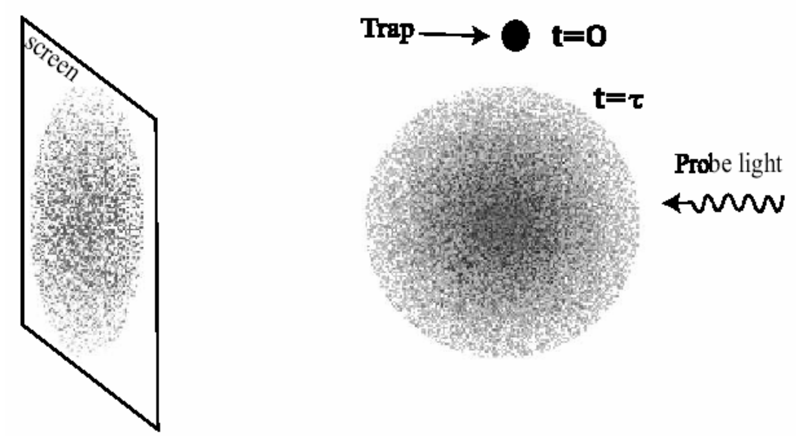
Motivation: quantum simulations of strongly correlated electron systems including quantum magnets and unconventional superconductors. Hofstetter et al. PRL (2002)

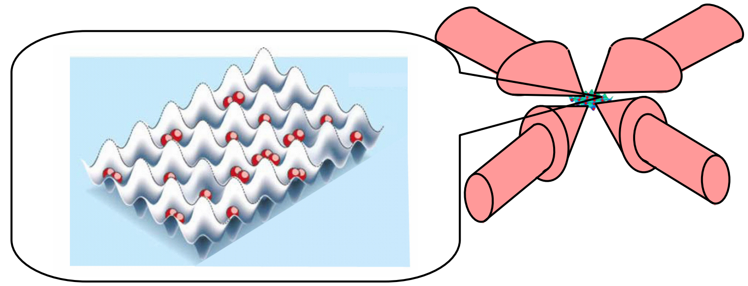
# Superfluid to insulator transition in an optical lattice

M. Greiner et al., Nature 415 (2002)

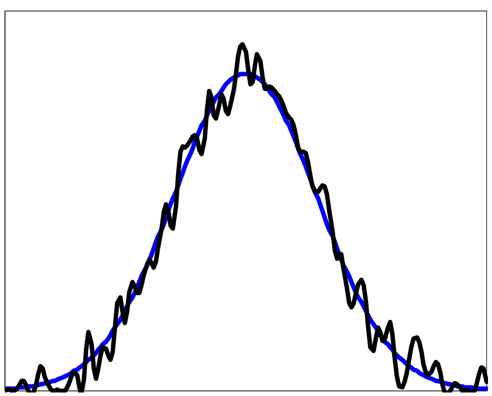


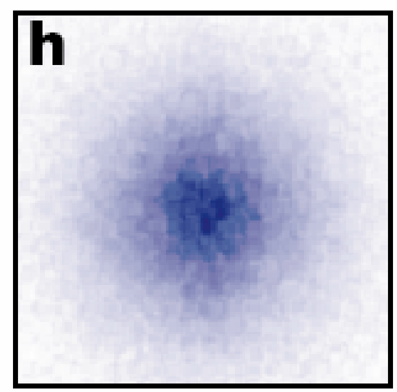
# Time of flight experiments



# Quantum noise interferometry of atoms in an optical lattice

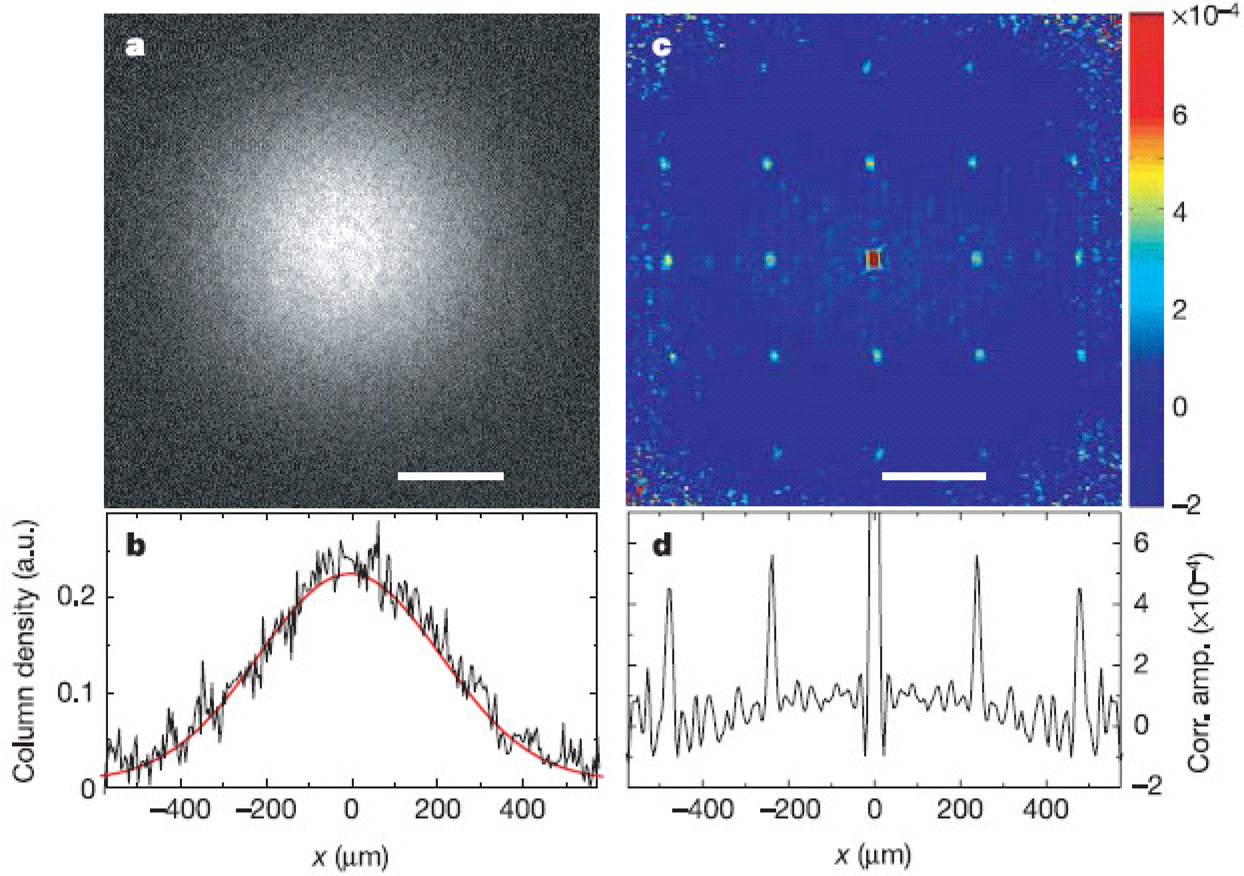




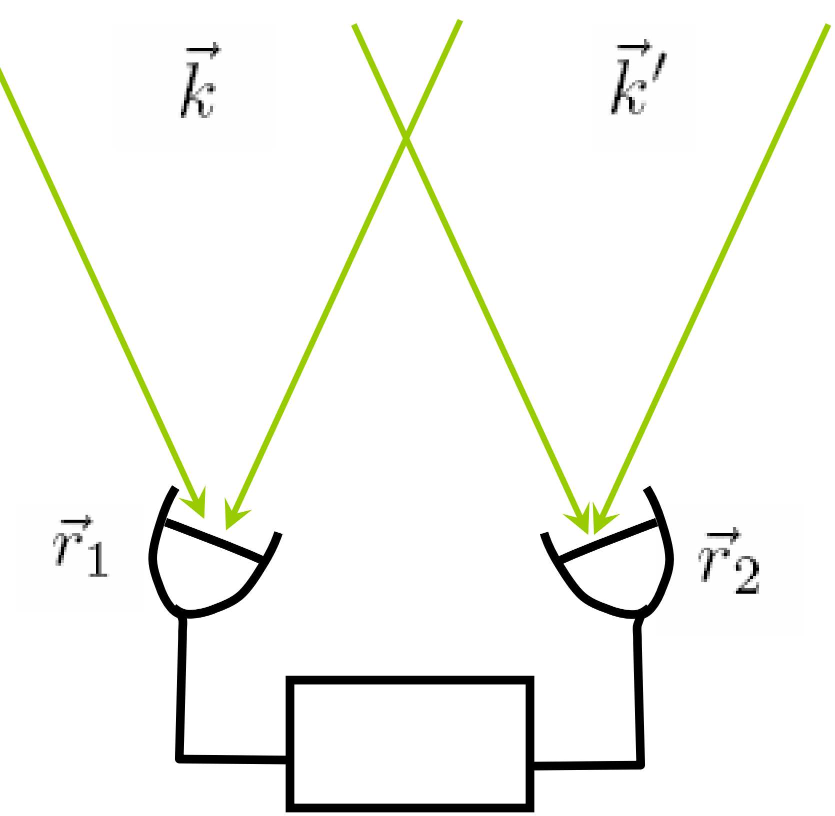

Second order coherence $G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$

# Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

Experiment: Folling et al., Nature 434:481 (2005)

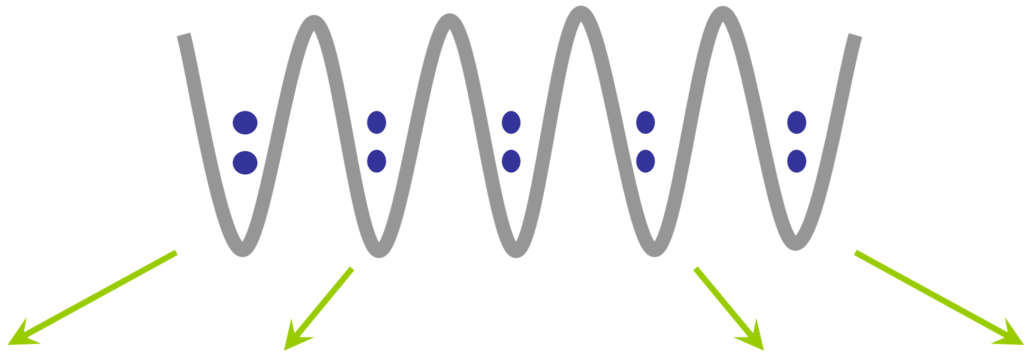


# Hanburry-Brown-Twiss stellar interferometer




$$\langle I(\vec{r}_1)\ I(\vec{r}_2)\rangle = A + B\ \cos\Big((\vec{k}-\vec{k}')\ (\vec{r}_1-\vec{r}_2)\Big)$$

# Second order coherence in the insulating state of bosons



Bosons at quasimomentum $\vec{k}$ expand as plane waves

with wavevectors $\vec{k}, \ \vec{k} + \vec{G}_1, \ \vec{k} + \vec{G}_2$

First order coherence: $\langle \ \rho(\vec{r}) \ \rangle$

Oscillations in density disappear after summing over $\vec{k}$

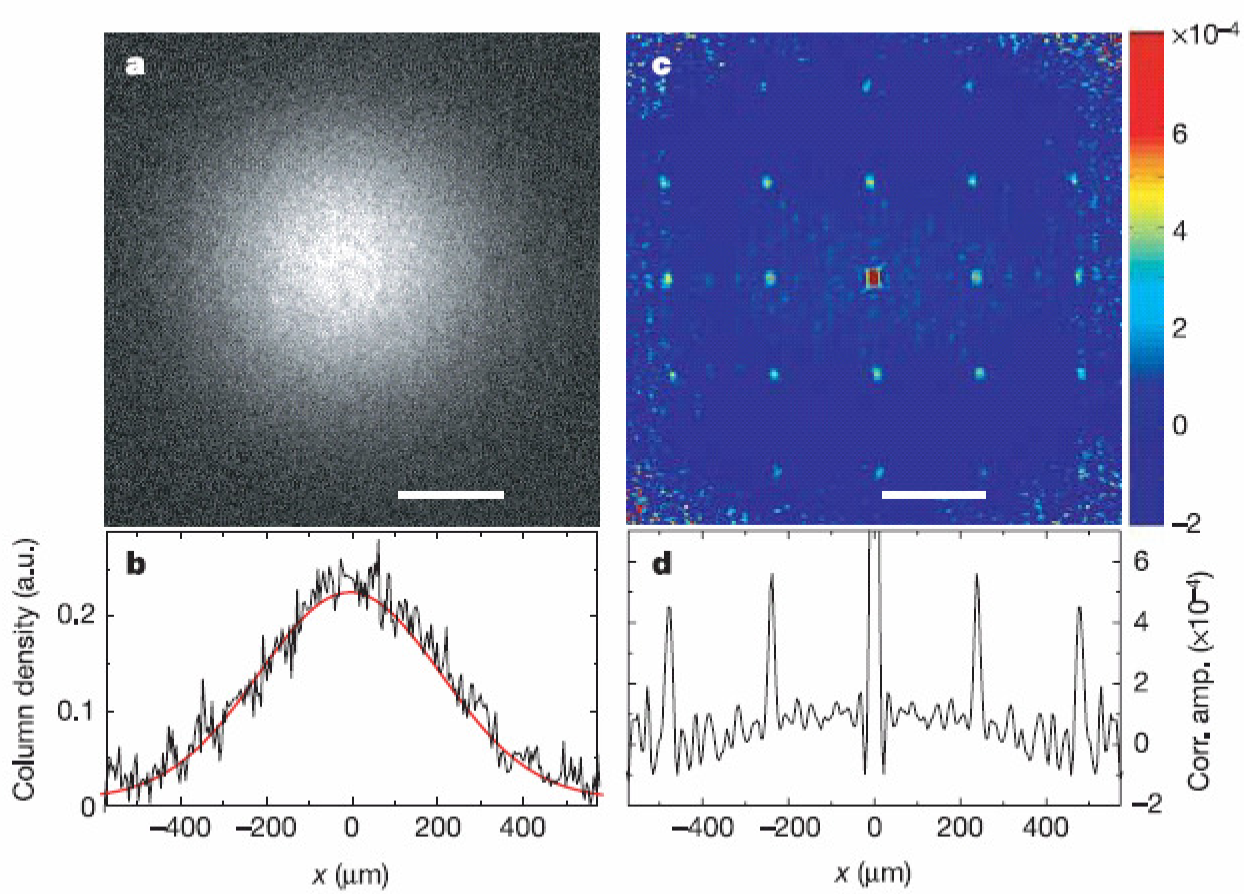
Second order coherence: $\langle \ \rho(\vec{r}_1) \ \rho(\vec{r}_2) \ \rangle$

Correlation function acquires oscillations at reciprocal lattice vectors

$$\langle \ \rho(\vec{r}_1) \ \rho(\vec{r}_2) \ \rangle = A_0 + A_1 \ \cos\Big( \ \vec{G}_1(\vec{r}_1 - \vec{r}_2) \ \Big) + A_2 \ \cos\Big( \ \vec{G}_2(\vec{r}_1 - \vec{r}_2) \ \Big) + \ldots$$

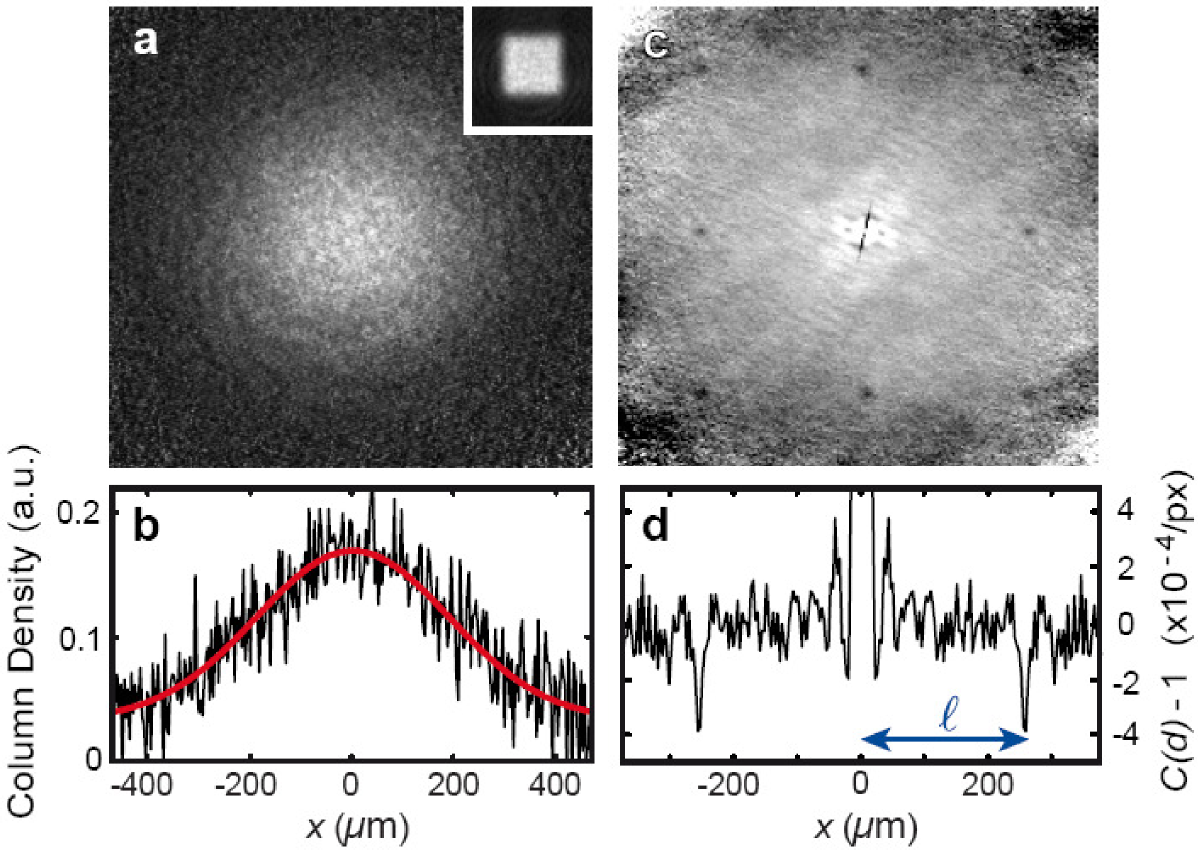
# Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

Experiment: Folling et al., Nature 434:481 (2005)

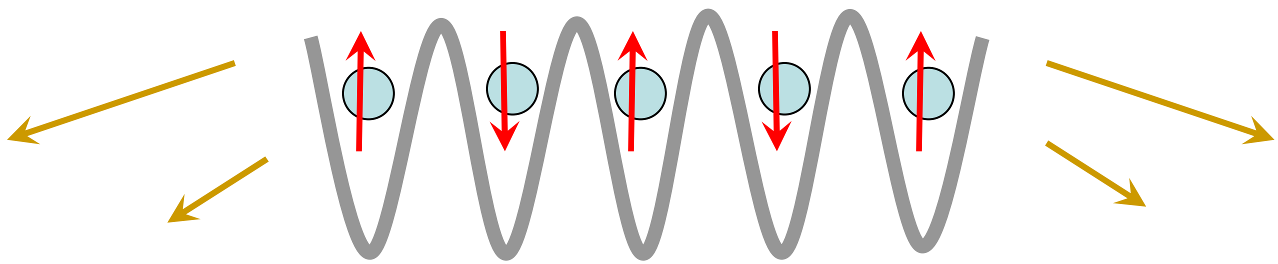


## Second order coherence in the insulating state of fermions. Hanburry-Brown-Twiss experiment

Experiment: Tom et al. Nature 444:733 (2006)



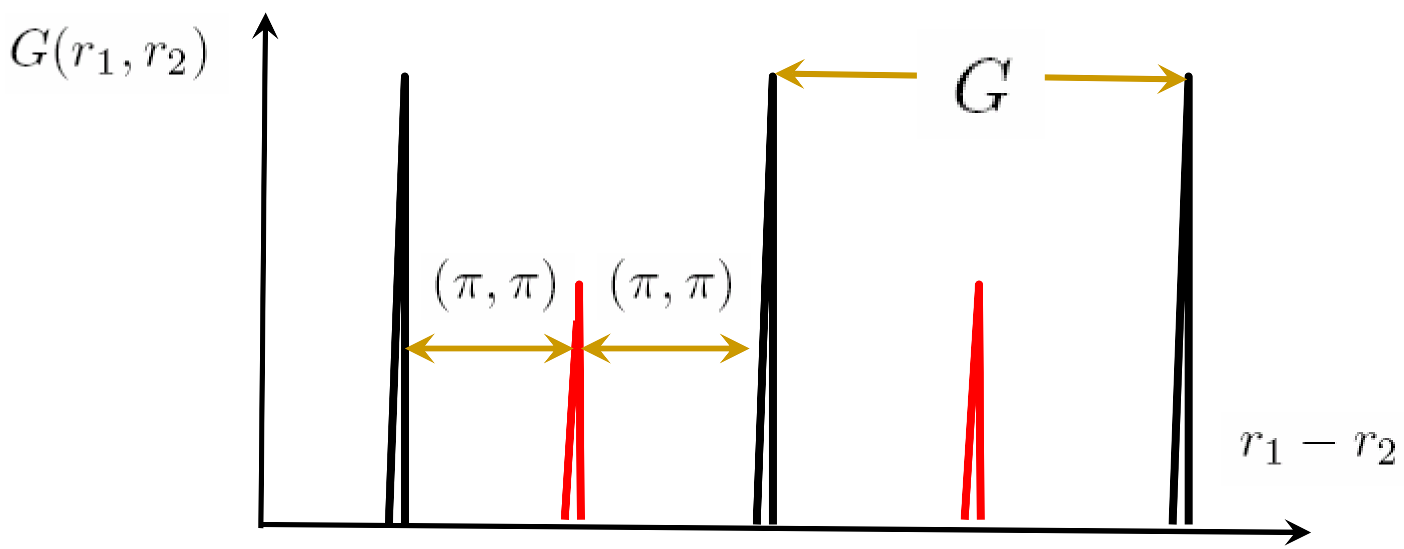
# Probing spin order in optical lattices



$$r = \frac{\hbar\ k\ t}{m}$$

Correlation Function Measurements

$$\begin{aligned} G(r_1, r_2) &= \langle\ n(r_1)\ n(r_2)\ \rangle_{\mathrm{TOF}} - \langle\ n(r_1)\ \rangle_{\mathrm{TOF}}\ \langle\ n(r_2)\ \rangle_{\mathrm{TOF}} \\ &\sim \langle\ n(k_1)\ n(k_2)\ \rangle_{\mathrm{LAT}} - \langle\ n(k_1)\ \rangle_{\mathrm{LAT}}\ \langle\ n(k_2)\ \rangle_{\mathrm{LAT}} \end{aligned}$$



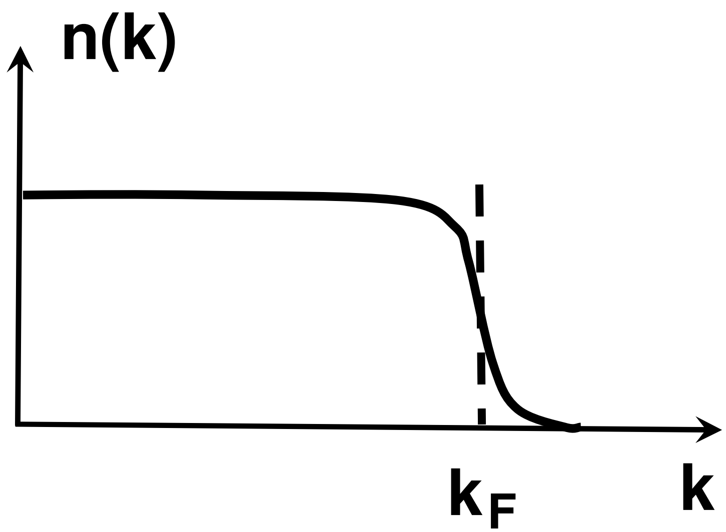

Extra Bragg peaks appear in the second order correlation function in the AF phase

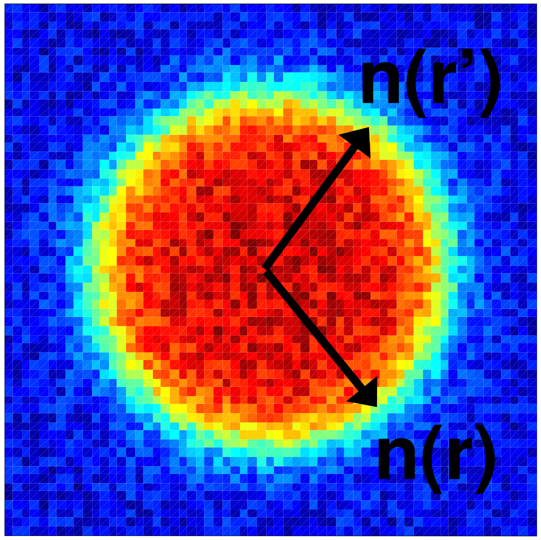
Quantum noise analysis of TOF images is more than HBT interference

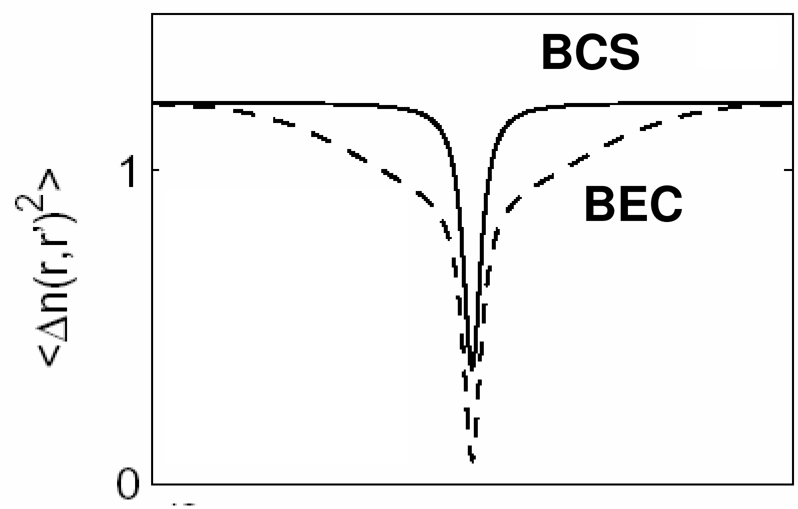
Detection of fermion pairing

# Second order interference from the BCS superfluid

Theory: Altman et al., PRA 70:13603 (2004)



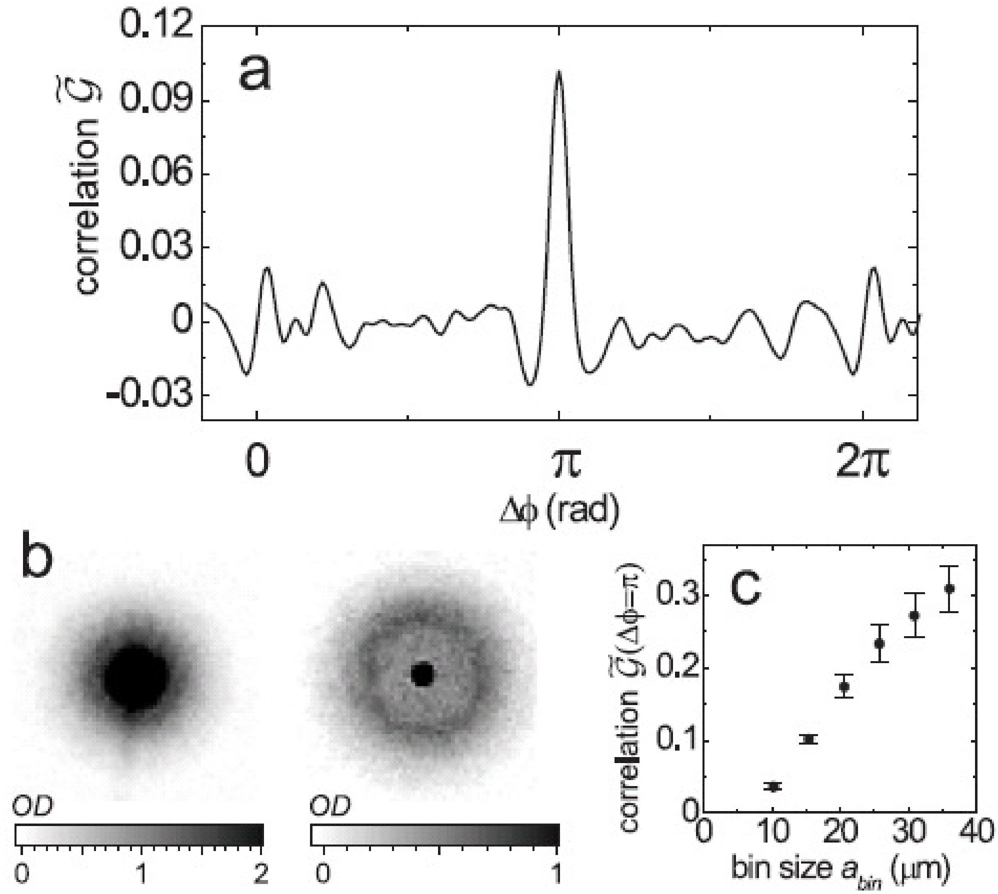




$$\Delta n(\mathbf{r},\mathbf{r}') \equiv n(\mathbf{r}) - n(\mathbf{r}')$$

$$\Delta n(\mathbf{r},-\mathbf{r}) \left| \Psi_{BCS} \right\rangle = 0$$

# Momentum correlations in paired fermions

**Experiments: Greiner et al., PRL 94:110401 (2005)**



# Summary

Experiments with ultracold atoms provide a new perspective on the physics of strongly correlated many-body systems. Quantum noise is a powerful tool for analyzing many body states of ultracold atoms

Thanks to:

NSF

Harvard-MIT CUA

MURI Program in Optical Lattices

DARPA DSO DEFENSE SCIENCES OFFICE

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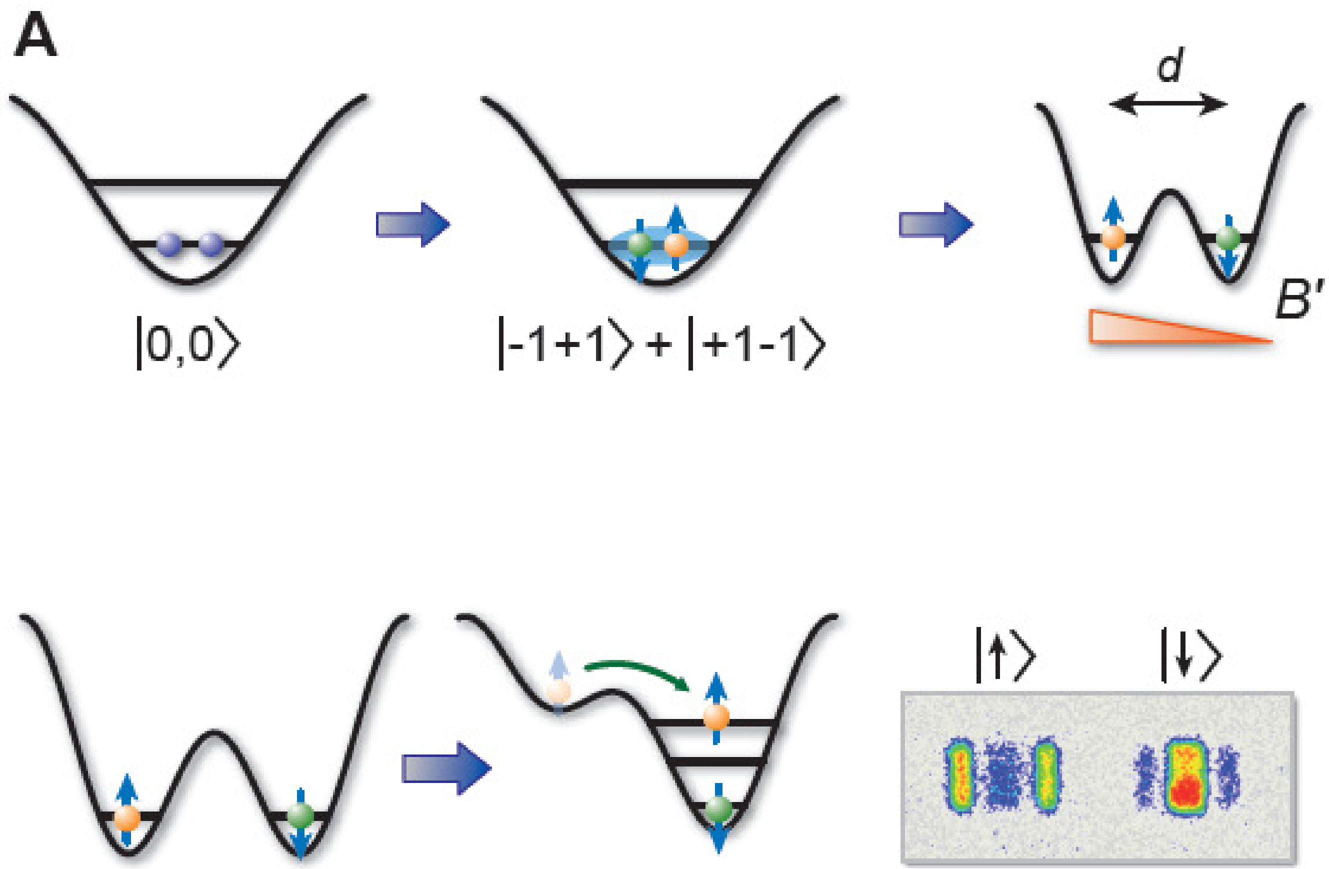
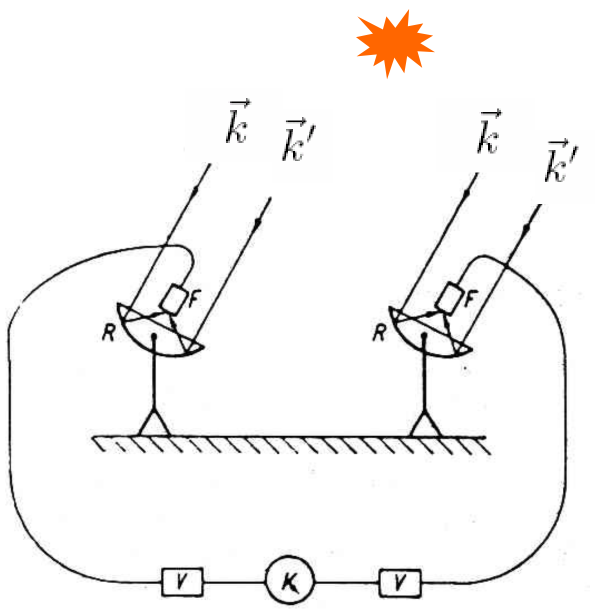
# Preparation and detection of Mott states of atoms in a double well potential



# Second order coherence

## Classical theory

Hanburry-Brown-Twiss

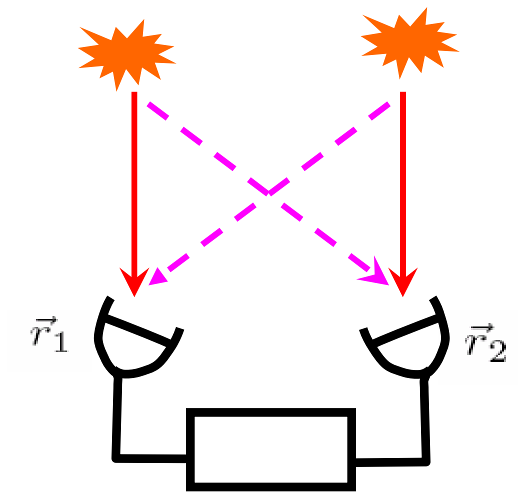



$$\langle I(\vec{r}_1)\ I(\vec{r}_2)\rangle = A + B\ \cos\Big((\vec{k}-\vec{k}')\ (\vec{r}_1-\vec{r}_2)\Big)$$

Measurements of the angular diameter of Sirius
*Proc. Roy. Soc.* (*19XX*)

## Quantum theory

Glauber



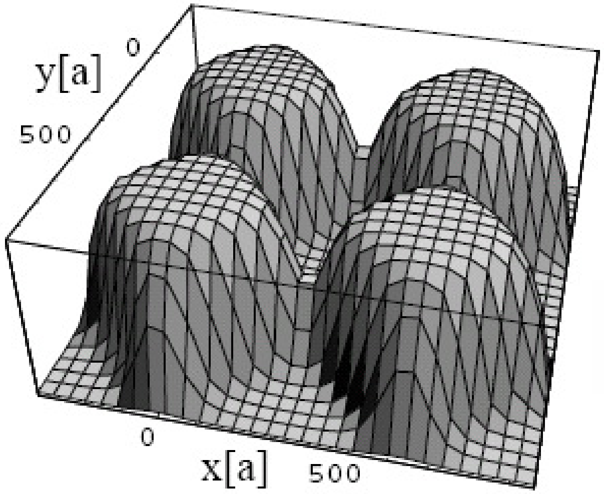

For bosons

$$A = A_1 + A_2$$

For fermions

$$A = A_1 - A_2$$

# Fermion pairing in an optical lattice




**Second Order Interference**
**In the TOF images**

$$G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$$

**Normal State**

$$G_{\rm N}(r_1, r_2) = \delta(r_1 - r_2)\rho(r_1) - \rho^2(r_1) \sum_G \delta(r_1 - r_2 - \frac{G\hbar t}{m})$$

**Superfluid State**

$$G_{\rm S}(r_1, r_2) = G_{\rm N}(r_1, r_2) + \Psi(r_1) \sum_G \delta(r_1 + r_2 + \frac{G\hbar t}{m})$$

$\Psi(r) = |u(Q(r))v(Q(r))|^2$ **measures the Cooper pair wavefunction**

$$Q(r) = \frac{mr}{\hbar t}$$

**One can identify unconventional pairing**