Learning about order from noise

Quantum noise studies of ultracold atoms

Eugene Demler
Harvard University

Collaborators:
Ehud Altman, Robert Cherng, Adilet Imambekov,
Vladimir Gritsev, Mikhail Lukin, Anatoli Polkovnikov

Funded by NSF, Harvard-MIT CUA, AFOSR, DARPA, MURI
Outline

Introduction. Historical review

Quantum noise analysis of time-of-flight experiments with atoms in optical lattices: HBT experiments and beyond

Quantum noise in interference experiments with independent condensates

Adiabaticity of creating many-body fermionic states in optical lattices
Quantum noise

Classical measurement:

collapse of the wavefunction into eigenstates of $x$

\[
\langle x \rangle = \int dx \ x \ |\psi(x)|^2
\]

\[
\langle x^2 \rangle = \int dx \ x^2 \ |\psi(x)|^2
\]

\[\ldots\]

\[
\langle x^n \rangle = \int dx \ x^n \ |\psi(x)|^2
\]

\[\ldots\]
Probabilistic nature of quantum mechanics

Bohr-Einstein debate on spooky action at a distance

Einstein-Podolsky-Rosen experiment

\[ | S = 0 \rangle = | \uparrow \rangle_L | \downarrow \rangle_R - | \downarrow \rangle_L | \uparrow \rangle_R \]

Measuring spin of a particle in the left detector instantaneously determines its value in the right detector
Aspect’s experiments: tests of Bell’s inequalities

Correlation function

\[ E(\theta_1, \theta_2) = \frac{\langle (I_1^+ - I_1^-)(I_2^+ - I_2^-) \rangle}{\langle (I_1^+ + I_1^-)(I_2^+ + I_2^-) \rangle} \]

Classical theories with hidden variable require

\[ B = E(\theta_1, \theta_2) - E(\theta_1, \theta'_2) + E(\theta'_1, \theta'_2) - E(\theta'_1, \theta_2) \leq 2 \]

Quantum mechanics predicts B=2.7 for the appropriate choice of \( \theta \)'s and the state

\[ |\psi\rangle = |+\rangle_L |+\rangle_R + |-\rangle_L |-\rangle_R \]

Hanbury-Brown-Twiss experiments

Classical theory of the second order coherence

\[ \langle I(\vec{r}_1) I(\vec{r}_2) \rangle = A + B \cos \left( (\vec{k} - \vec{k}') (\vec{r}_1 - \vec{r}_2) \right) \]


Measurements of the angular diameter of Sirius
Quantum theory of HBT experiments

Glauber, *Quantum Optics and Electronics* (1965)

HBT experiments with matter

For bosons

\[ A = A_1 + A_2 \]

For fermions

\[ A = A_1 - A_2 \]

Experiments with neutrons

Experiments with electrons

Experiments with 4He, 3He
Westbrook et al., Nature (2007)

Experiments with ultracold atoms
Shot noise in electron transport

Proposed by Schottky to measure the electron charge in 1918

Spectral density of the current noise

\[ S_\omega = \int \langle \{ \delta I(t), \delta I(0) \}^+ \rangle e^{i\omega t} dt \]

Related to variance of transmitted charge

\[ S_0 = \frac{2}{\tau} \langle \delta q^2(\tau) \rangle \]

When shot noise dominates over thermal noise

\[ S_0 = 2eI \]

Poisson process of independent transmission of electrons
Shot noise in electron transport

Current noise for tunneling across a Hall bar on the 1/3 plateau of FQE

Etien et al. PRL 79:2526 (1997)
see also Heiblum et al. Nature (1997)
Quantum noise analysis of time-of-flight experiments with atoms in optical lattices: Hanbury-Brown-Twiss experiments and beyond


Experiment: Folling et al., Nature 434:481 (2005);
Spielman et al., PRL 98:80404 (2007);
Atoms in optical lattices

Theory: Jaksch et al. PRL (1998)

Experiment: Kasevich et al., Science (2001);
            Greiner et al., Nature (2001);
            Phillips et al., J. Physics B (2002);
            Esslinger et al., PRL (2004);
            Ketterle et al., PRL (2006)
Bose Hubbard model

\[ \mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1) - \mu \sum_i n_i \]

- \( t \) — tunneling of atoms between neighboring wells
- \( U \) — repulsion of atoms sitting in the same well
Bose Hubbard model

$\mu / U$

$N=3$ Mott

$N=2$ Mott

$N=1$ Mott

Superfluid

$t / U$


$U \ll Nt$

Superfluid phase
Weak interactions

$U \gg Nt$

Mott insulator phase
Strong interactions
Superfluid to insulator transition in an optical lattice

Why study ultracold atoms in optical lattices
Fermionic atoms in optical lattices

\[
\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i
\]

Experiments with fermions in optical lattice, Kohl et al., PRL 2005
Antiferromagnetic and superconducting $T_c$ of the order of 100 K

YBa$_2$Cu$_3$O$_7$

Antiferromagnetism and pairing at sub-micro Kelvin temperatures

Same microscopic model

$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i$$
Positive U Hubbard model


Antiferromagnetic insulator

D-wave superconductor
Atoms in optical lattice

YBa$_2$Cu$_3$O$_7$

Same microscopic model

Quantum simulations of strongly correlated electron systems using ultracold atoms

Detection?
Quantum noise analysis as a probe of many-body states of ultracold atoms
Time of flight experiments

Quantum noise interferometry of atoms in an optical lattice

Second order coherence

\[ G(r_1, r_2) = \langle n(r_1)n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle \]
Second order coherence in the insulating state of bosons. Hanbury-Brown-Twiss experiment

Hanbury-Brown-Twiss stellar interferometer

\[ \langle I(\vec{r}_1) I(\vec{r}_2) \rangle = A + B \cos \left( (\vec{k} - \vec{k}') (\vec{r}_1 - \vec{r}_2) \right) \]
Second order coherence in the insulating state of bosons

Bosons at quasimomentum $\vec{k}$ expand as plane waves with wavevectors $\vec{k}, \vec{k} + \vec{G}_1, \vec{k} + \vec{G}_2$

First order coherence: $\langle \rho(\vec{r}) \rangle$

Oscillations in density disappear after summing over $\vec{k}$

Second order coherence: $\langle \rho(\vec{r}_1) \rho(\vec{r}_2) \rangle$

Correlation function acquires oscillations at reciprocal lattice vectors

$$\langle \rho(\vec{r}_1) \rho(\vec{r}_2) \rangle = A_0 + A_1 \cos \left( \vec{G}_1(\vec{r}_1 - \vec{r}_2) \right) + A_2 \cos \left( \vec{G}_2(\vec{r}_1 - \vec{r}_2) \right) + \ldots$$
Second order coherence in the insulating state of bosons. Hanbury-Brown-Twiss experiment

Second order coherence in the insulating state of fermions.
Hanbury-Brown-Twiss experiment

How to detect antiferromagnetism
Probing spin order in optical lattices

Correlation Function Measurements

\[ G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle_{\text{TOF}} - \langle n(r_1) \rangle_{\text{TOF}} \langle n(r_2) \rangle_{\text{TOF}} \]

\[ \sim \langle n(k_1) n(k_2) \rangle_{\text{LAT}} - \langle n(k_1) \rangle_{\text{LAT}} \langle n(k_2) \rangle_{\text{LAT}} \]

Extra Bragg peaks appear in the second order correlation function in the AF phase
How to detect fermion pairing

Quantum noise analysis of TOF images is more than HBT interference
Second order interference from the BCS superfluid


\[ \Delta n(\mathbf{r}, \mathbf{r}') \equiv n(\mathbf{r}) - n(\mathbf{r}') \]

\[ \Delta n(\mathbf{r}, -\mathbf{r}) \left| \Psi_{BCS} \right> = 0 \]
Momentum correlations in paired fermions

Greiner et al., PRL 94:110401 (2005)
Fermion pairing in an optical lattice

Second Order Interference In the TOF images

\[ G(r_1, r_2) = \langle n(r_1)n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle \]

Normal State

\[ G_N(r_1, r_2) = \delta(r_1 - r_2)\rho(r_1) - \rho^2(r_1) \sum_{G} \delta(r_1 - r_2 - \frac{G\hbar t}{m}) \]

Superfluid State

\[ G_S(r_1, r_2) = G_N(r_1, r_2) + \Psi(r_1) \sum_{G} \delta(r_1 + r_2 + \frac{G\hbar t}{m}) \]

\[ \Psi(r) = |u(Q(r))v(Q(r))|^2 \text{ measures the Cooper pair wavefunction} \]

One can identify unconventional pairing

\[ Q(r) = \frac{mr}{\hbar t} \]
Interference experiments with cold atoms
Interference of independent condensates


Theory: Javanainen, Yoo, PRL 76:161 (1996)
and many more
INTERFERENCE FRINGES PRODUCED BY SUPERPOSITION OF TWO INDEPENDENT MASER LIGHT BEAMS

By G. MAGYAR and Dr. L. MANDEL
Department of Physics, Imperial College of Science and Technology, London

Fig. 2. An example of fringes recorded: (a) photograph; (b) microphotometer tracing
Experiments with 2D Bose gas


Experiments with 1D Bose gas
S. Hofferberth et al. arXiv0710.1575
Interference of two independent condensates

$$\psi(r) = \psi_1(r) + \psi_2(r)$$

$$\rho_{\text{int}}(r) = \psi_1^\dagger(r) \psi_2(r) + \text{c.c.}$$

$$\psi_1(r) = e^{i \phi_1 + i k_1 r} \quad k_1 = \frac{m v_1}{\hbar} = \frac{m r}{\hbar t}$$

$$\psi_2(r) = e^{i \phi_2 + i k_2 r} \quad k_2 = \frac{m v_2}{\hbar} = \frac{m (r + d)}{\hbar t}$$

$$\rho_{\text{int}}(r) = e^{i (k_2 - k_1) r} e^{i (\phi_2 - \phi_1)} + \text{c.c.}$$

$$\rho_{\text{int}}(r) = e^{i \frac{m d r}{\hbar t}} e^{i (\phi_2 - \phi_1)} + \text{c.c.}$$

Clouds 1 and 2 do not have a well defined phase difference. However each individual measurement shows an interference pattern:

$$\langle \rho_{\text{int}}(r) \rangle = 0$$

$$\langle \rho_{\text{int}}(r) \rho_{\text{int}}(r') \rangle = e^{i \frac{m d}{\hbar t} (r - r')} + \text{c.c.}$$
Interference of fluctuating condensates

Polkovnikov, Altman, Demler, PNAS 103:6125(2006)

Amplitude of interference fringes, \( A_{fr} \)

\[
|A_{fr}| e^{i\Delta \phi} = \int_0^L dx \ e^{i(\phi_1(x)-\phi_2(x))}
\]

For independent condensates \( A_{fr} \) is finite but \( \Delta \phi \) is random

\[
\langle |A_{fr}|^2 \rangle = \int_0^L dx_1 \int_0^L dx_2 \ \langle e^{i(\phi_1(x_1)-\phi_2(x_1))} e^{-i(\phi_1(x_2)-\phi_2(x_2))} \rangle
\]

\[
\langle |A_{fr}|^2 \rangle \approx L \int_0^L dx \ \langle e^{i(\phi_1(x)-\phi_1(0))} \rangle \langle e^{-i(\phi_2(x)-\phi_2(0))} \rangle
\]

For identical condensates

\[
\langle |A_{fr}|^2 \rangle = L \int_0^L dx \ (G(x))^2
\]

Instantaneous correlation function

\[
G(x) = \langle e^{i\phi(x)} e^{-\phi(0)} \rangle
\]
Fluctuations in 1d BEC

Thermal fluctuations

Thermally energy of the superflow velocity

\[
\langle e^{i\phi(x_1)} e^{i\phi(x_2)} \rangle \sim e^{-|x_1 - x_2|/\xi_T}
\]

\[
\xi_T = \sqrt{\frac{\hbar^2 m}{T}}
\]

Quantum fluctuations

\[
\langle e^{i\phi(x_1)} e^{i\phi(x_2)} \rangle \sim \left(\frac{\xi_h}{|x_1 - x_2|}\right)^{1/2K}
\]

\[
K = \sqrt{\frac{n}{g m}}
\]
Interference between Luttinger liquids

Luttinger liquid at $T=0$

$$
\langle |A_{fr}|^2 \rangle \sim (\rho \xi_h)^{1/K} (L\rho)^{2-1/K}
$$

$K$ – Luttinger parameter

For non-interacting bosons
$$
K = \infty \quad \text{and} \quad A_{fr} \sim L
$$

For impenetrable bosons
$$
K = 1 \quad \text{and} \quad A_{fr} \sim \sqrt{L}
$$

Finite temperature

Experiments: Hofferberth, Schumm, Schmiedmayer

$$
\langle |A_{fr}|^2 \rangle \sim L \rho^2 \xi_h \left( \frac{\hbar^2}{m \xi_h^2} \frac{1}{T} \right)^{1-1/K}
$$

$n_{1d} = 60 \mu m^{-1}$

$K = 47$

$T_{fit} = 84 \pm 22 \text{ nK}$
Distribution function of fringe amplitudes for interference of fluctuating condensates

\[ A_{fr} \] is a quantum operator. The measured value of \[ |A_{fr}| \] will fluctuate from shot to shot.

\[
\langle |A_{fr}|^{2n} \rangle = \\
\int_0^L dz_1 \ldots dz_n \left| \left< e^{i\phi(z_1)} \ldots e^{i\phi(z_n)} e^{-i\phi(z'_1)} \ldots e^{-i\phi(z'_n)} \right> \right|^2
\]

Higher moments reflect higher order correlation functions.

We need the full distribution function of \[ |A_{fr}| \]

Gritsev, Altman, Demler, Polkovnikov, Nature Physics 2006
Imambekov, Gritsev, Demler, cond-mat/0612011
Distribution function of interference fringe contrast

Experiments: Hofferberth et al., arXiv0710.1575
Theory: Imambekov et al., cond-mat/0612011

Quantum fluctuations dominate:
asymmetric Gumbel distribution
(low temp. T or short length L)

Thermal fluctuations dominate:
broad Poissonian distribution
(high temp. T or long length L)

Intermediate regime:
double peak structure

Comparison of theory and experiments: no free parameters
Higher order correlation functions can be obtained
Interference of two dimensional condensates

              Gati et al., PRL (2006)

\[ \langle |A_{fr}|^2 \rangle = L_x L_y \int_0^{L_x} \int_0^{L_y} d^2 \vec{r} \ (G(\vec{r})^2) \]

\[ G(\vec{r}) = \langle a(\vec{r}) a^\dagger(0) \rangle \]
Interference of two dimensional condensates. Quasi long range order and the KT transition

Above KT transition

\[ G(r) \sim e^{-r/\xi} \]
\[ \langle |A_{fr}|^2 \rangle \sim L_x L_y \]
\[ \log \xi(T) \sim 1/\sqrt{T - T_{KT}} \]

Below KT transition

\[ G(r) \sim \rho \left( \frac{\xi_h}{r} \right)^\alpha \]
\[ \alpha(T) = \frac{mT}{2\pi \rho_s(T) \hbar^2} \]
\[ \langle |A_{fr}|^2 \rangle \sim (L_x L_y)^{2-\alpha} \]
Experiments with 2D Bose gas


Typical interference patterns

low temperature

higher temperature
Experiments with 2D Bose gas

Hadzibabic et al., Nature 441:1118 (2006)

Contrast after integration

integration over x axis
integration over x axis
integration over x axis

Integration distance $D_x$ (pixels)

Contrast

low $T$
middle $T$
high $T$
Experiments with 2D Bose gas

Hadzibabic et al., Nature 441:1118 (2006)

fit by:

\[ C^2 \sim \frac{1}{D_x} \int_0^D [g_1(0,x)]^2 dx \sim \left( \frac{1}{D_x} \right)^{2\alpha} \]

Exponent \(\alpha\)

if \(g_1(r)\) decays exponentially with \(\ell_{\text{coh}} \ll D_x\):

\[ \alpha = 1/2 \]

if \(g_1(r)\) decays algebraically or exponentially with a large \(\ell_{\text{coh}}\):

\[ \alpha < 1/2 \]
Experiments with 2D Bose gas. Proliferation of thermal vortices

Hadzibabic et al., Nature 441:1118 (2006)

The onset of proliferation coincides with $\alpha$ shifting to 0.5!
Adiabaticity of creating many-body fermionic states in optical lattices
Formation of molecules with increasing interaction strength

Strohmaier et al., arXiv:0707.314

\[ \mathcal{H} = -t \sum_{\langle ij \rangle} c_{i\sigma}^\dagger c_{j\sigma} + U(t) \sum_i n_{i\uparrow} n_{i\downarrow} \]

Saturation in the number of molecules created is related to the finite rate of changing interaction strength \( U(t) \)
Formation of molecules with increasing interaction strength

During adiabatic evolution with increasing attractive U, all single atoms should be converted to pairs. Entropy is put into the kinetic energy of bound pairs.

As U is increased, the excess energy of two unpaired atoms should be converted to the kinetic energy of bound pairs.

The kinetic energy of a single molecule is set by \( t_{\text{mol}} = \frac{4t^2}{U} \). When \( U >> t \) many particles will have to be involved in the relaxation process.
Hubbard model with repulsion: dynamics of breaking up pairs

Energy of on-site repulsion

Energy of spin domain walls
Hubbard model with repulsion: dynamics of breaking up pairs

Energy of on-site repulsion $U$

Energy of spin domain wall $N \times 4t^2/U$

Stringent requirements on the rate of change of the interaction strength to maintain adiabaticity at the level crossing
Hubbard model with repulsion: dynamics of breaking up pairs

\[ U \approx (z - 2) \frac{J_{\text{ex}}}{2} N \]

\[ J_{\text{ex}} = \frac{4t^2}{U} \]
Hubbard model with repulsion: dynamics of breaking up pairs

Dynamics of recombination: a moving pair pulls out a spin domain wall

High order perturbation theory

$V = t$
Hubbard model with repulsion: dynamics of breaking up pairs

\[ V = t \]

\[ V_{\text{eff}} \sim V \prod_{n=1}^{N} \frac{V}{(E_n - E_0)} \]

\[ E_n - E_0 \approx (\varepsilon - 2) \frac{J_{\text{ex}}}{2} n = \frac{n}{N} \times U \]

\[ V_{\text{eff}} \sim \frac{t}{N!} \left( \frac{2t}{(\varepsilon - 2)J_{\text{ex}}n} \right)^N \sim t \left( \frac{t}{U} \right)^N \]

\( N \) itself is a function of \( U/t \):

\[ U \approx (\varepsilon - 2) \frac{J_{\text{ex}}}{2} N \]
Hubbard model with repulsion: dynamics of breaking up pairs

Probability of nonadiabatic transition

$$P_{12} \propto e^{-2\pi \omega_{12} \tau_d}$$

$$\omega_{12}$$ – Rabi frequency at crossing point

$$\tau_d$$ – crossing time

Extra geometrical factor to account for different configurations of domain walls

$$V_{\text{eff}} \sim t \left( \frac{t}{U} \right) \frac{U^2}{(z-2)t^2}$$
Formation of molecules with increasing interaction strength

\[ t_{\text{mol}} = \frac{4 t^2}{U} \]

Value of \( U/t \) for which one finds saturation in the production of molecules

\[ V_0/E_R = 10, 7.5, 5.0, 2.5 \]
Summary

Experiments with ultracold atoms provide a new perspective on the physics of strongly correlated many-body systems. Quantum noise is a powerful tool for analyzing many body states of ultracold atoms.

Thanks to: