Dynamics of spinor condensates: dipolar interactions and more

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Outline

Dipolar interactions in spinor condensates
Larmor precession and dipolar interactions. Roton instabilities.
Following experiments of D. Stamper-Kurn

Superexchange interaction in double well systems
Towards quantum magnetism of ultracold atoms.
Collaboration with I. Bloch’s group.

Many-body decoherence and Ramsey interferometry
Luttinger liquids and non-equilibrium dynamics.
Collaboration with I. Bloch’s group.
Dipolar interactions in spinor condensates. Introduction
Theory of the Superfluidity of Helium II

L. Landau

From these properties of the energy spectrum the heat capacity of helium II must consist of two parts: the "phonon part," i.e., the normal Debye heat capacity proportional to $T^4$, and the "roton part," depending on the temperature exponentially ($\sim e^{-\Delta/kT}$).
Energy Spectrum of the Excitations in Liquid Helium

R. P. Feynman and Michael Cohen

A wave function previously used to represent an excitation (phonon or roton) in liquid helium, inserted into a variational principle for the energy, gave an energy-momentum curve having the qualitative shape suggested by Landau; but the value computed for the minimum energy $\Delta$ of a roton was $19.1^\circ K$, while thermodynamic data require $\Delta = 9.6^\circ K$. A new wave function is proposed here. The new value computed for $\Delta$ is $11.5^\circ K$. Qualitatively, the wave function suggests that the roton is a kind of quantum-mechanical analog of a microscopic vortex ring, of diameter about equal to the atomic spacing. A forward motion of single atoms through the center of the ring is accompanied by a dipole distribution of returning flow far from the ring.

Fig. 6. The energy spectrum of excitations. Curve $A$ is the spectrum $E_2(k)$ computed from Eq. (61). Curve $B$ is the spectrum $E_1(k)$ computed with the simpler wave function (5). Curve $C$ is the Landau-type spectrum used by de-Klerk et al.\textsuperscript{1} to fit the second sound and specific heat data. Curve $D$ is a Landau-type spectrum with $\rho_0$ taken the same as in $A$, and $\mu$ and $\Delta$ chosen to fit the specific heat data. For small $k$, all curves are asymptotic to the line $E = \hbar c k$. 
Fig. 8. The energy spectrum of the excitations in liquid helium at 1.1°K. The dashed line joining the origin and the first measured point has a slope corresponding to a first sound velocity of 239±5 meters/sec. The maximum occurs at $p/\hbar = 1.11 \pm 0.04$ A$^{-1}$, $E/k = 13.92 \pm 0.10$°K. The region of the minimum is shown in greater detail in Fig. 9.
Possible supersolid phase in $^4$He


Also
G. Chester (1970); A.J. Leggett (1970)

Formation of the supersolid phase due to softening of roton excitations.
Probable observation of a supersolid helium phase

E. Kim & M. H. W. Chan

Resonant period as a function of T
Interlayer coherence in bilayer quantum Hall systems at $\nu = 1$

Hartree-Fock predicts roton softening and transition into a state with both interlayer coherence and stripe order. Transport experiments suggest first order transition into a compressible state.

Eisenstein, Boebinger et al. (1994)

L. Brey and H. Fertig (2000)
Roton spectrum in pancake polar condensates

Santos, Shlyapnikov, Lewenstein (2000)
Fischer (2006)

Origin of roton softening

Repulsion at long distances
Attraction at short distances

Stability of the supersolid phase is a subject of debate
Magnetic dipolar interactions in spinor condensates

\[ U_{\text{contact}}(r) = \frac{4\pi\hbar^2 a}{m} \delta(r) \quad \text{and} \quad V_{dd} = \frac{\mu_0 \mu^2}{4\pi r^3} \left(1 - \cos \theta\right) \]

Comparison of contact and dipolar interactions.
Typical value \( a = 100a_B \)

For \(^{87}\text{Rb} \) \( \mu = \mu_B \) and \( \epsilon = 0.007 \)

For \(^{52}\text{Cr} \) \( \mu = 6\mu_B \) and \( \epsilon = 0.16 \)

Bose condensation of \(^{52}\text{Cr} \).
T. Pfau et al. (2005)

Review:
Menotti et al.,
arXiv 0711.3422
Magnetic dipolar interactions in spinor condensates

Interaction of F=1 atoms

\[ V_S = c_0 + c_2 \vec{f}_1 \cdot \vec{f}_2 \]

\[ c_2 = (4\pi \hbar^2 / M) \times (a_2 - a_0) / 3 \]

Ferromagnetic Interactions for $^{87}$Rb

\[ a_2 - a_0 = -1.07 \ a_B \]


Spin-depenent part of the interaction is small. Dipolar interaction may be important (D. Stamper-Kurn)
Spontaneously modulated textures in spinor condensates

Vengalattore et al. PRL (2008)

Fourier spectrum of the fragmented condensate
This talk: Instabilities of F=1 spinor condensates due to dipolar interactions. New phenomena due to averaging over Larmor precession

Theory: unstable modes in the regime corresponding to Berkeley experiments

Results of Berkeley experiments

Wide range of instabilities tuned by quadratic Zeeman, AC Stark shift, initial spiral spin winding
Instabilities of $F=1$ spinor condensates due to dipolar interactions and roton softening

Earlier theoretical work on dipolar interactions in spinor condensates: Meystre et al. (2002), Ueda et. al. (2006), Lamacraft (2007).

New phenomena: interplay of finite transverse size and dipolar interaction in the presence of fast Larmor precession
Dipolar interactions after averaging over Larmor precession
Energy scales

**Magnetic Field**
- Larmor Precession (100 kHz)
- Quadratic Zeeman (0-20 Hz)

**S-wave Scattering**
- Spin independent ($g_0 n = \text{kHz}$)
- Spin dependent ($g_s n = 10 \text{ Hz}$)

**Dipolar Interaction**
- Anisotropic ($g_d n = 10 \text{ Hz}$)
- Long-ranged

**Reduced Dimensionality**
- Quasi-2D geometry
**Dipolar interactions**

**Static interaction**

\[ \mathbf{F}_b \parallel \mathbf{F}_a \] is preferred. “Head to tail” component dominates.

**Averaging over Larmor precession**

\[ \mathbf{F}_b' \perp \mathbf{F}_a \] is preferred. “Head to tail” component is averaged with the “side by side”
Instabilities: qualitative picture
Stability of systems with static dipolar interactions

Ferromagnetic configuration is robust against small perturbations. Any rotation of the spins conflicts with the “head to tail” arrangement.

Large fluctuation required to reach a lower energy configuration.
Dipolar interaction averaged after precession

“Head to tail” order of the transverse spin components is violated by precession. Only need to check whether spins are parallel.

XY components of the spins can lower the energy using modulation along z.

Z components of the spins can lower the energy using modulation along x.

Strong instabilities of systems with dipolar interactions after averaging over precession.
Instabilities: technical details
From Spinless to Spinor Condensates

\[ \langle \Psi \rangle = \sqrt{n} e^{i\phi} \]

\[ \langle \Psi \rangle = \sqrt{n} [\Phi_x, \Phi_y, \Phi_z] \]

\[ \Psi = \sqrt{n} \begin{bmatrix} ie^{i\eta_t} \cos(\phi + i\chi) \frac{\sin(\rho)}{\sqrt{\cosh(2\chi)}} \\ ie^{i\eta_t} \sin(\phi + i\chi) \frac{\sin(\rho)}{\sqrt{\cosh(2\chi)}} \\ e^{i\eta} \cos(\rho) \end{bmatrix} \]

**Charge mode:**

\( n \) is density and \( \eta \) is the overall phase

**Spin mode:**

\( \phi \) determines spin orientation in the XY plane

\( \chi \) determines longitudinal magnetization (Z-component)
Hamiltonian

\[ \mathcal{H} = \int d^3x \Psi_x^\dagger \left[ -\frac{\nabla^2}{2m} - \mu + \frac{1}{2} \omega_n^2 (\hat{n} \cdot \vec{x})^2 + \frac{1}{2} \mathcal{O}_n \left( \hat{B} \cdot \vec{F} \right) + q \left( \hat{B} \cdot \vec{F} \right)^2 \right] \Psi_x \]

\[ + \int d^3x d^3x' \frac{1}{2} g_{3D}^{\mu\nu}(x - x') \left( \Psi_x^\dagger F_\mu \Psi_{x'} \right) \left( \Psi_{x'}^\dagger F_\nu \Psi_x \right) : \]

Dipolar Interaction

\[ g_{3D}^{ij}(\Delta x) = -g_s \delta^{ij} \delta(\Delta x) + g_d \frac{1}{|\Delta x|^3} [\delta^{ij} - 3 \Delta \hat{x}^i \Delta \hat{x}^j] \]

S-wave Scattering

\[ g_{3D}^{00}(\Delta x) = (g_0 + g_s) \delta(\Delta x) \]

\[ (F^0)_{jk} = \delta_{jk} \]

\[ (F^i)_{jk} = -i \epsilon_{ijk} \]
Precessional and Quasi-2D Averaging

**Rotating Frame**

\[ \Psi \rightarrow R(t, x) \Psi \]

\[ R(t, x) = \exp \left[ i(B_0 t + \vec{k} \cdot \vec{x}) \hat{B} \cdot \vec{F} \right] \]

**Gaussian Profile**

\[ \Psi(x_1 \hat{e}_1 + x_1 \hat{e}_2 + x_n \hat{n}) = \frac{e^{-x_n^2/4d_n^2}}{(2\pi d_n^2)^{1/4}} \Psi(x_1 \hat{e}_1 + x_2 \hat{e}_2) \]

**Quasi-2D Time Averaged Dipolar Interaction**

\[ (\delta^{ij} - 3\Delta \hat{x}^i \Delta \hat{x}^j) \rightarrow \int_{-\infty}^{\infty} \frac{d(\hat{n} \cdot \Delta \vec{x})}{2\sqrt{\pi} d_n} e^{-\Delta x^2/4d_n^2} \int_{-\pi/B_0}^{\pi/B_0} B_0 dt \left[ R(t, x)^T \left( \delta^{ij} - 3\Delta \hat{x}^i \Delta \hat{x}^j \right) R(t, x') \right] \]

\[ \left( \delta^{ii'} - 3\Delta \hat{x}^i \Delta \hat{x}^{i'} \right) \to \int_{-\infty}^{\infty} \frac{d(\hat{n} \cdot \Delta \vec{x})}{2\sqrt{\pi} d_n} e^{-\Delta x^2/4d_n^2} \int_{-\pi/B_0}^{\pi/B_0} B_0 dt \left[ R(t, x)^T \left( \delta^{ii'} - 3\Delta \hat{x}^{i'} \Delta \hat{x}^{i'} \right) R(t, x') \right] \]
Quasi-2D Time Averaged Dipolar Interaction

\[
\bar{h}_{2D}^{ij}(\vec{k}) = -\frac{4\pi}{3} \left[ \bar{h}_{2D}^B(\vec{k}) P_{ij}^B + \bar{h}_{2D}^x(\vec{k}) P_{ij}^\perp + \bar{h}_{2D}^\times(\vec{k}) \hat{B} \cdot \hat{F}_{ij} \right]
\]

\[
P_{ij}^B = \hat{B}^i \hat{B}^j \quad P_{ij}^\perp = \delta^{ij} - \hat{B}^i \hat{B}^j
\]
Collective Modes

Mean Field
\[ \Psi = \Psi_0 + \delta \Psi \]

Collective Fluctuations (Spin, Charge)

Equations of Motion
\[ i \partial_t \begin{bmatrix} \delta \Psi_k \\ \delta \Psi^*_{-k} \end{bmatrix} = \begin{bmatrix} M_k & N_k \\ -N^*_{-k} & -M^*_{-k} \end{bmatrix} \begin{bmatrix} \delta \Psi_k \\ \delta \Psi^*_{-k} \end{bmatrix} \]

\[ \Psi(x, t) \sim \exp(i \omega_k - ikx) \]

Spin Mode
\( \delta f_B \) – longitudinal magnetization
\( \delta \phi \) – transverse orientation

Charge Mode
\( \delta n \) – 2D density
\( \delta \eta \) – global phase
Instabilities of collective modes

$Q$ measures the strength of quadratic Zeeman effect

$$Q = -\frac{q}{2g^\perp n_2 D}$$
Instabilities of collective modes
Berkeley Experiments: checkerboard phase

M. Vengalattore, et. al, arXiv:0712.4182
Dipolar interaction averaged after precession

XY components of the spins can lower the energy using modulation along z.

Z components of the spins can lower the energy using modulation along x.
Instabilities of collective modes
Instabilities of collective modes. Spiral configurations

Spiral wavelength

$$\lambda_\kappa = 50 \mu m$$

Spiral spin winding introduces a separate branch of unstable modes
Superexchange interaction in experiments with double wells

Refs:

Two component Bose mixture in optical lattice


\[ | \uparrow \rangle = | F = 1, m_F = -1 \rangle \]

\[ | \downarrow \rangle = | F = 2, m_F = -2 \rangle \]

Two component Bose Hubbard model

\[
\mathcal{H} = - t \sum_{\langle ij \rangle} b_{i \uparrow}^\dagger b_{j \uparrow} - t \sum_{\langle ij \rangle} b_{i \downarrow}^\dagger b_{j \downarrow} + U_{\uparrow \uparrow} \sum_i n_{i \uparrow} (n_{\uparrow} - 1) \\
+ U_{\downarrow \downarrow} \sum_i n_{i \downarrow} (n_{\downarrow} - 1) + U_{\uparrow \downarrow} \sum_i n_{i \uparrow} n_{\downarrow}
\]
Quantum magnetism of bosons in optical lattices

\[ \mathcal{H} = J_z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + J_\perp \sum_{\langle ij \rangle} \left( \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right) \]

\[ J_z = \frac{t_\uparrow^2 + t_\downarrow^2}{2U_{\uparrow\downarrow}} - \frac{t_\uparrow^2}{U_{\uparrow\uparrow}} - \frac{t_\downarrow^2}{U_{\downarrow\downarrow}} \]

\[ J_\perp = -\frac{t_\uparrow t_\downarrow}{U_{\uparrow\downarrow}} \]

- Ferromagnetic
- Antiferromagnetic

\[ U_{\uparrow\downarrow} >> U_{\uparrow\uparrow}, U_{\downarrow\downarrow} \]

\[ U_{\uparrow\downarrow} << U_{\uparrow\uparrow}, U_{\downarrow\downarrow} \]
Observation of superexchange in a double well potential

Theory: A.M. Rey et al., PRL (2007)

Use magnetic field gradient to prepare a state $|\downarrow\uparrow\rangle$

Observe oscillations between $|\downarrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$ states

Experiment: Trotzky et al., Science (2008)
Preparation and detection of Mott states of atoms in a double well potential
Comparison to the Hubbard model

Experiments: I. Bloch et al.

\[ \hbar \omega_{1,2} = \frac{U}{2} \left( \sqrt{\left(\frac{4J}{U}\right)^2 + 1} \pm 1 \right) \]
Beyond the basic Hubbard model

Basic Hubbard model includes only local interaction

Extended Hubbard model takes into account non-local interaction

\[ \hat{H}^{EHM} = \hat{H}^{HM} - \Delta J \sum_{\sigma \neq \sigma'} (\hat{n}_{\sigma L} + \hat{n}_{\sigma R}) \left( \hat{a}_{\sigma' L}^\dagger \hat{a}_{\sigma' R} + \hat{a}_{\sigma' R}^\dagger \hat{a}_{\sigma' L} \right) + U_{LR} \sum_{\sigma \neq \sigma'} \left( \hat{n}_{\sigma L} \hat{n}_{\sigma' R} + \hat{a}_{\sigma L}^\dagger \hat{a}_{\sigma' R}^\dagger \hat{a}_{\sigma' L} \hat{a}_{\sigma R} \right) + \frac{1}{2} \hat{a}_{\sigma L}^\dagger \hat{a}_{\sigma' L}^\dagger \hat{a}_{\sigma' R} \hat{a}_{\sigma R} + \frac{1}{2} \hat{a}_{\sigma R}^\dagger \hat{a}_{\sigma' R}^\dagger \hat{a}_{\sigma' L} \hat{a}_{\sigma L} \right), \]
Beyond the basic Hubbard model

Frequency (kHz)

Short Lattice Depth (Er)

Spin Imbalance

V=9Er

V=11Er
Observation of superexchange in a double well potential. Reversing the sign of exchange interactions
Many-body decoherence and Ramsey interferometry

Collaboration with A. Widera, S. Trotzky, P. Cheinet, S. Fölling, F. Gerbier, I. Bloch, V. Gritsev, M. Lukin

Ramsey interference

Working with $N$ atoms improves the precision by $\sqrt{N}$. Need spin squeezed states to improve frequency spectroscopy.
Squeezed spin states for spectroscopy

Motivation: improved spectroscopy, e.g. Wineland et. al. PRA 50:67 (1994)

Generation of spin squeezing using interactions. Two component BEC. Single mode approximation

\[ \mathcal{H} = \chi_s \left( S_{\text{tot}}^z \right)^2 \]

Kitagawa, Ueda, PRA 47:5138 (1993)

\[ \mathcal{H} = \int dx \left[ g_c (n_1 + n_2)^2 + g_s (n_1 - n_2)^2 + \frac{|\nabla \Psi_1|^2}{2m} + \frac{|\nabla \Psi_2|^2}{2m} \right] \]

In the single mode approximation we can neglect kinetic energy terms

\[ \mathcal{H}_{\text{SMA}} = \frac{g_s}{V} (N_1 - N_2)^2 \]
Interaction induced collapse of Ramsey fringes

Ramsey fringe visibility

\[ t_{\text{collapse}} \sim \frac{1}{\chi \sqrt{N}} \]
\[ t_{\text{revival}} \sim \frac{1}{\chi} \]

Experiments in 1d tubes: A. Widera, I. Bloch et al.

\[ \chi \sim \frac{\hbar^2}{M V} \]

\( V \) - volume of the system
Spin echo. Time reversal experiments

Single mode approximation

\[ \mathcal{H}_{\text{SMA}} = \frac{g_s}{V} (N_1 - N_2)^2 \]

\[ g_s = \frac{g_{11} - g_{12}}{2} \]

The Hamiltonian can be reversed by changing \( a_{12} \)

\[ a_s \rightarrow -a_s \]

\[ \mathcal{H}_{\text{SMA}} \rightarrow -\mathcal{H}_{\text{SMA}} \]

\[ e^{i \int_{\tau=-T}^{\tau=T} \mathcal{H}_{\text{SMA}}(t)dt} \times e^{i \int_{\tau=0}^{\tau=T} \mathcal{H}_{\text{SMA}}(t)dt} = 1 \]

Predicts perfect spin echo
Spin echo. Time reversal experiments

Expts: A. Widera, I. Bloch et al.

No revival?

Experiments done in array of tubes. Strong fluctuations in 1d systems. Single mode approximation does not apply. Need to analyze the full model.

\[ \mathcal{H} = \int dx \left[ g_c (n_1 + n_2)^2 + g_s (n_1 - n_2)^2 + \frac{|\nabla \Psi_1|^2}{2m} + \frac{|\nabla \Psi_2|^2}{2m} \right] \]
Interaction induced collapse of Ramsey fringes. Multimode analysis

Low energy effective theory: Luttinger liquid approach

Luttinger model

\[ S^+(x, t) \sim e^{i\phi_s(x,t)} \]

\[ [S^z(x), \phi_s(x')] = -i\delta(x - x') \]

\[ \mathcal{H}_s = \int_0^L dx \left[ g_s(S^z_s)^2 + \frac{\rho}{2m}(\nabla \phi_s)^2 \right] \]

Changing the sign of the interaction reverses the interaction part of the Hamiltonian but not the kinetic energy

\[ \mathcal{H}_s = \sum_q \left[ g_s(t)S^z_q S^z_q + \frac{\rho q^2}{m} \phi_{sq} \phi_{sq}^* \right] \]

\[ [S^z_{q'}, \phi_{sq}] = -i\delta_{qq'} \]

Time dependent harmonic oscillators can be analyzed exactly
Time-dependent harmonic oscillator

\[ \mathcal{H} = \frac{p^2}{2m(t)} + k \frac{q^2}{2} \]

Explicit quantum mechanical wavefunction can be found

\[ \psi(p, t) = \frac{\Phi \left( \frac{p}{c(t)} \right)}{\sqrt{c(t)}} e^{i\alpha(t)p^2 + i\gamma(t)} \]

From the solution of classical problem

\[ \ddot{c} + \omega^2(t) c = \frac{\omega_0^2}{c^3} \]

We solve this problem for each momentum component

\[ \mathcal{H}_s = \sum_{q} \left[ g_s(t) S_{q^z}^z S_{q^z}^{z*} + \frac{\rho q^2}{m} \phi_{sq} \phi_{sq}^* \right] \]

See e.g. Lewis, Riesengeld (1969)

Malkin, Man’ko (1970)
Interaction induced collapse of Ramsey fringes in one dimensional systems

Only q=0 mode shows complete spin echo.
Finite q modes continue decay.

The net visibility is a result of competition between q=0 and other modes.

Conceptually similar to experiments with dynamics of split condensates. T. Schumm’s talk.

Fundamental limit on Ramsey interferometry.
Summary

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