Dipolar interactions in $F=1$ ferromagnetic spinor condensates. Roton instabilities and possible supersolid phase

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Outline

Introduction

Dipolar interactions averaged over fast Larmor precession

Instabilities: qualitative picture

Instabilities: roton softening and phase diagram

Instabilities of the spiral state
Introduction: Roton excitations and supersolid phase
Theory of the Superfluidity of Helium II

L. Landau

From these properties of the energy spectrum the heat capacity of helium II must consist of two parts: the "phonon part," i.e., the normal Debye heat capacity proportional to $T^4$, and the "roton part," depending on the temperature exponentially ($\sim e^{-\Delta/kT}$).

![Graph showing energy and momentum relationship]

- $E = v I p$
- $v_l = 240 \text{ m/s}$
- $E = \Delta + \frac{(p - p_0)^2}{2\mu}$
- $\Delta/k = 9.6 \text{ °K}$
- $p_0/\hbar = 1.95 \text{ Å}^1$
- $\mu = 0.77 m_{\text{He}}$

MOAMTUM (p/\hbar IN Å⁻¹)
Energy Spectrum of the Excitations in Liquid Helium*

R. P. Feynman and Michael Cohen

A wave function previously used to represent an excitation (phonon or roton) in liquid helium, inserted into a variational principle for the energy, gave an energy-momentum curve having the qualitative shape suggested by Landau; but the value computed for the minimum energy $\Delta$ of a roton was $19.1^\circ$K, while thermodynamic data require $\Delta=9.6^\circ$K. A new wave function is proposed here. The new value computed for $\Delta$ is $11.5^\circ$K. Qualitatively, the wave function suggests that the roton is a kind of quantum-mechanical analog of a microscopic vortex ring, of diameter about equal to the atomic spacing. A forward motion of single atoms through the center of the ring is accompanied by a dipole distribution of returning flow far from the ring.

Fig. 6. The energy spectrum of excitations. Curve $A$ is the spectrum $E_2(k)$ computed from Eq. (61). Curve $B$ is the spectrum $E_1(k)$ computed with the simpler wave function (5). Curve $C$ is the Landau-type spectrum used by de-Klerk et al. to fit the second sound and specific heat data. Curve $D$ is a Landau-type spectrum with $\rho_0$ taken the same as in $A$, and $\mu$ and $\Delta$ chosen to fit the specific heat data. For small $k$, all curves are asymptotic to the line $E=\hbar\omega$.
Fig. 8. The energy spectrum of the excitations in liquid helium at 1.1°K. The dashed line joining the origin and the first measured point has a slope corresponding to a first sound velocity of 239±5 meters/sec. The maximum occurs at $p/\hbar = 1.11\pm0.04$ Å⁻¹, $E/k = 13.92\pm0.10$°K. The region of the minimum is shown in greater detail in Fig. 9.
Possible supersolid phase in $^4$He


Also
G. Chester (1970); A.J. Leggett (1970)

Formation of the supersolid phase due to softening of roton excitations
Probable observation of a supersolid helium phase

E. Kim & M. H. W. Chan

Resonant period as a function of T
Phases of bilayer quantum Hall systems at $\nu=1$

Hartree-Fock predicts roton softening and transition into the QH state with stripe order. Transport experiments suggest first order transition into a compressible state.

Eisenstein, Boebinger et al. (1994)

L. Brey and H. Fertig (2000)
Phases of bilayer quantum Hall systems at ν=1 and roton softening

Roton softening and sharpening observed in Raman experiments. This is in conflict with transport measurements.
Roton spectrum in pancake polar condensates

Santos, Shlyapnikov, Lewenstein (2000)
Fischer (2006)

Origin of roton softening

Repulsion at long distances
Attraction at short distances

Stability of the supersolid phase is a subject of debate
Magnetic dipolar interactions in spinor condensates

Comparison of contact and dipolar interactions.
Typical value $a = 100a_B$

For $^{87}\text{Rb}$ $\mu = 1/2\mu_B$ and $\varepsilon = 0.007$

For $^{52}\text{Cr}$ $\mu = 6\mu_B$ and $\varepsilon = 0.16$

Bose condensation of $^{52}\text{Cr}$.
T. Pfau et al. (2005)

Review:
Menotti et al.,
arXiv 0711.3422
Magnetic dipolar interactions in spinor condensates

Interaction of F=1 atoms

\[ V_S = c_0 + c_2 \mathbf{f}_1 \cdot \mathbf{f}_2 \]

\[ c_2 = (4\pi\hbar^2 / M) \times (a_2 - a_0) / 3 \]

Ferromagnetic Interactions for \(^{87}\text{Rb}\)

\[ a_2 - a_0 = -1.07 \ a_B \]


Spin-dependent part of the interaction is small.
Dipolar interaction may be important (D. Stamper-Kurn)
Spontaneously modulated textures in spinor condensates

Vengalattore et al. PRL (2008)

Fourier spectrum of the fragmented condensate
This talk: Instabilities of F=1 spinor condensates due to dipolar interactions. New phenomena due to averaging over Larmor precession

Theory: unstable modes in the regime corresponding to Berkeley experiments

Results of Berkeley experiments

Wide range of instabilities tuned by quadratic Zeeman, AC Stark shift, initial spiral spin winding
Instabilities of $F=1$ spinor condensates due to dipolar interactions and roton softening

Earlier theoretical work on dipolar interactions in spinor condensates:
Meystre et al. (2002), Ueda et. al. (2006), Lamacraft (2007).

New phenomena: interplay of finite transverse size and dipolar interaction in the presence of fast Larmor precession
Dipolar interactions after averaging over Larmor precession
Energy scales

**Magnetic Field**
- Larmor Precession (10 kHz)
- Quadratic Zeeman (0-20 Hz)

**S-wave Scattering**
- Spin independent \( (g_0 n = \text{kHz}) \)
- Spin dependent \( (g_s n = 10 \text{ Hz}) \)

**Dipolar Interaction**
- Anisotropic \( (g_d n = 10 \text{ Hz}) \)
- Long-ranged

**Reduced Dimensionality**
- Quasi-2D geometry

\[ d < \xi_{\text{spin}} \]
Dipolar interactions

Static interaction

parallel to \( \vec{F}_b \) is preferred

“Head to tail” component dominates

Averaging over Larmor precession

perpendicular to \( \vec{F}_a \) is preferred. “Head to tail” component is averaged with the “side by side”
Instabilities: qualitative picture
Stability of systems with static dipolar interactions

Ferromagnetic configuration is robust against small perturbations. Any rotation of the spins conflicts with the “head to tail” arrangement.

Large fluctuation required to reach a lower energy configuration.
Dipolar interaction averaged after precession

“Head to tail” order of the transverse spin components is violated by precession. Only need to check whether spins are parallel.

XY components of the spins can lower the energy using modulation along z.

Z components of the spins can lower the energy using modulation along x.

Strong instabilities of systems with dipolar interactions after averaging over precession.
Instabilities: technical details
From Spinless to Spinor Condensates

\[ \langle \Psi \rangle = \sqrt{n} e^{i\phi} \]

\[ \langle \Psi \rangle = \sqrt{n} [\Phi_x, \Phi_y, \Phi_z] \]

\[ \Psi = \sqrt{n} \begin{bmatrix}
    i e^{i\eta + i\eta_\perp} \cos(\phi + i\chi) \frac{\sin(\rho)}{\sqrt{\cosh(2\chi)}} \\
    i e^{i\eta + i\eta_\perp} \sin(\phi + i\chi) \frac{\sin(\rho)}{\sqrt{\cosh(2\chi)}} \\
    e^{i\eta} \cos(\rho)
\end{bmatrix} \]

**Charge mode:**
\( n \) is density and \( \eta \) is the overall phase

**Spin mode:**
\( \phi \) determines spin orientation in the XY plane
\( \chi \) determines longitudinal magnetization (Z-component)
Hamiltonian

\[ \mathcal{H} = \int d^3 x \Psi_x^\dagger \left[ -\frac{\nabla^2}{2m} - \mu + \frac{1}{2} \omega_n^2 (\hat{n} \cdot \vec{x})^2 + (-p + B_0) \left( \hat{B} \cdot \vec{F} \right) + q \left( \hat{B} \cdot \vec{F} \right)^2 \right] \Psi_x \]

\[ + \int d^3 x d^3 x' \frac{1}{2} g_{3D}^{\mu\nu}(x - x') : \left( \Psi_x^\dagger F^{\mu} \Psi_x \right) \left( \Psi_{x'}^\dagger F^{\nu} \Psi_{x'} \right) : \]

Dipolar Interaction

\[ g_{3D}^{ij}(\Delta x) = -g_s \delta^{ij} \delta(\Delta x) + g_d \frac{1}{|\Delta x|^3} \left[ \delta^{ij} - 3 \Delta \hat{x}^i \Delta \hat{x}^j \right] \]

S-wave Scattering

\[ g_{3D}^{00}(\Delta x) = (g_0 + g_s) \delta(\Delta x) \]

\[ (F^0)_{jk} = \delta_{jk} \]

\[ (F^i)_{jk} = -i \epsilon_{ijk} \]
Precessional and Quasi-2D Averaging

**Rotating Frame**

\[ \Psi \rightarrow R(t, x)^{\Psi} \]

\[ R(t, x) = \exp \left[ i(B_0 t + \vec{\kappa} \cdot \vec{x}) \hat{B} \cdot \vec{F} \right] \]

**Gaussian Profile**

\[ \Psi(x_1 \hat{e}_1 + x_1 \hat{e}_2 + x_n \hat{n}) = \]

\[ e^{-x_n^2/4d_n^2} \left( \frac{1}{(2\pi d_n^2)^{1/4}} \right) \Psi(x_1 \hat{e}_1 + x_2 \hat{e}_2) \]

**Quasi-2D Time Averaged Dipolar Interaction**

\[ (\delta^{i j} - 3 \Delta \hat{x}^i \Delta \hat{x}^j) \rightarrow \int_{-\infty}^{\infty} \frac{d(\hat{n} \cdot \Delta \vec{x})}{2\sqrt{\pi}d_n} e^{-(\hat{n} \cdot \Delta \vec{x})^2/4d_n^2} \int_{-\pi/B_0}^{\pi/B_0} B_0 dt \left[ R(t, x)^{T}_{ii'} \left( \delta^{i' j'} - 3 \Delta \hat{x}^{i'} \Delta \hat{x}^{j'} \right) R(t, x')_{j' j} \right] \]
Collective Modes

Mean Field

\[ \Psi = \Psi_0 + \delta \Psi \]

Collective Fluctuations (Spin, Charge)

Equations of Motion

\[ i \partial_t \begin{bmatrix} \delta \Psi_k \\ \delta \Psi_{-k} \end{bmatrix} = \begin{bmatrix} M_k & N_k \\ -N_{-k}^* & -M_{-k}^* \end{bmatrix} \begin{bmatrix} \delta \Psi_k \\ \delta \Psi_{-k} \end{bmatrix} \]

\[ \delta \Psi(x, t) \sim \exp(i \omega_k - ikx) \]

Spin Mode

\( \delta f_B \) – longitudinal magnetization
\( \delta \varphi \) – transverse orientation

Charge Mode

\( \delta n \) – 2D density
\( \delta \eta \) – global phase
Instabilities of collective modes

\[ Q = -\frac{q}{2g^\perp n_{2D}} \]

Q measures the strength of quadratic Zeeman effect

\[ \alpha=0.24\pi, \, Q=0.0, \, \theta=0.5\pi \]

\[ \alpha=0.24\pi, \, Q=-0.1, \, \theta=0.5\pi \]

\[ \alpha=0.24\pi, \, Q=-0.2, \, \theta=0.5\pi \]

\[ \alpha=0.24\pi, \, Q=-0.3, \, \theta=0.5\pi \]
Instabilities of collective modes
Berkeley Experiments: checkerboard phase

M. Vengalattore, et. al, arXiv:0712.4182
Dipolar interaction averaged after precession

XY components of the spins can lower the energy using modulation along z.

Z components of the spins can lower the energy using modulation along x.
Instabilities of collective modes
Instabilities of collective modes. Spiral configurations

Spiral wavelength

\[ \lambda_\kappa = 50 \mu m \]

Spiral spin winding introduces a separate branch of unstable modes
Instabilities of the spiral state

Adiabatic limit

Sudden limit
Mean-field energy

Inflection point suggests instability

Negative value of $\partial^2 E_{\text{MF}}/\partial q^2$ shows that the system can lower its energy by making a non-uniform spiral winding.
Conclusions

• Dipolar interactions crucial for spinor condensates…
• But effectively modified by quasi-2D and precession

• Variety of instabilities (ring, stripe, checkerboard)
• But what about the ground state?
Nature of transition and ordered phases