Lattice modulation experiments with fermions in optical lattices and more

Nonequilibrium dynamics of Hubbard model

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Fermionic Hubbard model

From high temperature superconductors to ultracold atoms

\[
\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i
\]

\(YBa_2Cu_3O_7\)

Antiferromagnetic and superconducting \(T_c\) of the order of 100 K

Antiferromagnetism and pairing at sub-micro Kelvin temperatures

Atoms in optical lattice
Fermions in optical lattice

Hubbard model plus parabolic potential

\[ \mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c_i^{\dagger} c_j \sigma + U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_i V(r_i) \]

\[ V(r_i) = \frac{1}{2} m_0 \omega_0^2 r_i^2 \]

Probing many-body states

Electrons in solids

- Thermodynamic probes i.e. specific heat
- X-Ray and neutron scattering
- ARPES
- Optical conductivity
- STM

Fermions in optical lattice

- System size, number of doublons as a function of entropy, $U/t$, $\omega_0$
- Bragg spectroscopy, TOF noise correlations
- ARPES
- Optical conductivity
- STM
Outline

• Introduction. Recent experiments with fermions in optical lattice. Signatures of Mott state
• Lattice modulation experiments in the Mott state. Linear response theory
• Comparison to experiments
• Lifetime of repulsively bound pairs
• Lattice modulation experiments with d-wave superfluids
Mott state of fermions in optical lattice
Signatures of incompressible Mott state

Suppression in the number of double occupancies

Esslinger et al. arXiv:0804.4009

![Diagram showing energy vs position with double occupancies and atom number graphs.](image)
Signatures of incompressible Mott state

Response to external potential

Radius of the cloud as a function of the confining potential

Comparison with DMFT+LDA models suggests that temperature is above the Neel transition

Next step: observation of antiferromagnetic order

However superexchange interactions have already been observed
Radius of the cloud: high temperature expansion

Starting point: zero tunneling.
Expand in $t/T$.
Interaction can be arbitrary

Minimal cloud size for attractive interactions

- Observed experimentally by the Mainz group
- Competition of interaction energy and entropy
- Theory: first two terms in $t/T$ expansion
Lattice modulation experiments with fermions in optical lattice. Mott state

Related theory work: Kollath et al., PRA 74:416049R) (2006)
Huber, Ruegg, arXiv:0808:2350
Lattice modulation experiments
Probing dynamics of the Hubbard model

Modulate lattice potential $V_0$

Measure number of doubly occupied sites

$t \sim \exp\left(-\sqrt{V_0/E_R}\right)$

$U \sim \left(\frac{V_0}{E_R}\right)^{3/4}$

Main effect of shaking: modulation of tunneling

$\mathcal{H}_{\text{pert}}(\tau) = \lambda t \cos \omega \tau \sum_{\langle ij \rangle, \sigma} c_{i \sigma}^{\dagger} c_{j \sigma}$

Doubly occupied sites created when frequency $\omega$ matches Hubbard $U$
Lattice modulation experiments
Probing dynamics of the Hubbard model

R. Joerdens et al., arXiv:0804.4009
Mott state

Regime of strong interactions $U \gg t.$

Mott gap for the charge forms at $T \sim U$

Antiferromagnetic ordering at $T_N \sim J = \frac{4t^2}{U}$

“High” temperature regime $T_N \ll T \ll U$

All spin configurations are equally likely. Can neglect spin dynamics.

“Low” temperature regime $T \leq T_N$

Spins are antiferromagnetically ordered or have strong correlations.
Schwinger bosons and Slave Fermions

\[
\begin{align*}
\uparrow & \quad \downarrow & \quad \uparrow \downarrow & \quad \bullet \\
\quad a_{\uparrow}^\dagger & \quad a_{\downarrow}^\dagger & \quad d_{\uparrow}^\dagger & \quad h_{\uparrow}^\dagger
\end{align*}
\]

**Bosons**

**Fermions**

\[
c_{i\sigma}^\dagger = a_{i\sigma}^\dagger h_i + \sigma a_{i-\sigma} d_i^\dagger
\]

**Constraint:**

\[
a_{i\sigma} a_{i\sigma} + d_i^\dagger d_i + h_i^\dagger h_i = 1
\]

**Singlet Creation**

\[
A_{ij}^\dagger = a_{i\uparrow}^\dagger a_{j\downarrow}^\dagger - a_{i\downarrow}^\dagger a_{j\uparrow}^\dagger
\]

**Boson Hopping**

\[
F_{ij}^\dagger = a_{i\uparrow}^\dagger a_{j\uparrow} + a_{i\downarrow}^\dagger a_{j\downarrow}
\]
Schwinger bosons and slave fermions

Fermion hopping

\[ c_{i\uparrow}^\dagger c_{j\uparrow} + c_{i\downarrow}^\dagger c_{j\downarrow} + \text{h.c.} = (d_{i\uparrow}^\dagger d_{j\uparrow} - h_{i\uparrow}^\dagger h_{j\uparrow}) F_{ij} + d_{i\downarrow}^\dagger h_{j\downarrow}^\dagger A_{ij} + \text{h.c.} \]

Propagation of holes and doublons is coupled to spin excitations. Neglect spontaneous doublon production and relaxation.

Doublon production due to lattice modulation perturbation

\[ \mathcal{H}(\tau) = \lambda t \sin \omega \tau \sum_{\langle ij \rangle} \left( d_{i\uparrow}^\dagger h_{j\downarrow}^\dagger A_{ij} + \text{h.c.} \right) \]

Second order perturbation theory. Number of doublons

\[ N_d(\tau) = t^2 \lambda^2 \int_0^\tau dt' \int_0^\tau dt'' \sin[\omega t'] \sin[\omega t''] \sum_{\langle ij \rangle \langle lm \rangle} \langle A_{ij}^\dagger(t') d_i(t') h_j(t') h_{m}^\dagger(t'') d_{l}^\dagger(t'') A_{lm}(t'') \rangle \]
“Low” Temperature $T \ll T_N$

Propagation of holes and doublons strongly affected by interaction with spin waves

Assume independent propagation of hole and doublon (neglect vertex corrections)

Self-consistent Born approximation

Spectral function for hole or doublon

Sharp coherent part:
dispersion set by $J$, weight by $J/t$

Incoherent part:
dispersion $4t \times \text{dimension}$
Propogation of doublons and holes

Spectral function:
Oscillations reflect shake-off processes of spin waves

Comparison of Born approximation and exact diagonalization: Dagotto et al.

Hopping creates string of altered spins: bound states
“Low” Temperature $T << T_N$

Rate of doublon production

- Low energy peak due to sharp quasiparticles
- Broad continuum due to incoherent part
"High" Temperature

Atomic limit. Neglect spin dynamics. All spin configurations are equally likely.

$A_{ij}(t')$ replaced by probability of having a singlet

$$N_d(\tau) = \frac{1}{4} t^2 \lambda^2 \int_0^\tau dt' \int_0^\tau dt'' \sin[\omega t'] \sin[\omega t'']$$

$$\sum_{\langle ij \rangle \langle lm \rangle} \langle d_i(t') h_j(t') h_m^\dagger(t'') d_l^\dagger(t'') \rangle$$

Assume independent propagation of doublons and holes. Rate of doublon production

$$P_d(\omega) = \frac{\pi^3}{2} t^2 \lambda^2 \sum_{r \delta \delta'} \int d\omega' A^d(r + \delta, \omega') A^h(r + \delta', \omega - U - \omega')$$

$A^{d(h)}$ is the spectral function of a single doublon (holon)
Propogation of doublons and holes

Hopping creates string of altered spins

Retraceable Path Approximation *Brinkmann & Rice, 1970*

Consider the paths with no closed loops

Spectral Fn. of single hole  Doublon Production Rate  Experiments
Lattice modulation experiments. Sum rule

\[ P_d(\omega) = \frac{\pi^3}{2} t^2 \lambda^2 \sum_{\delta} \int d\omega' A^d(r + \delta, \omega') A^h(r + \delta', \omega - U - \omega') \]

\(A^{(d,h)}\) is the spectral function of a single doublon (holon)

Sum Rule:

\[ \int d\omega P_d(\omega) = \frac{\pi^3}{2} t^2 \lambda^2 z \]

Experiments:

The total weight does not scale quadratically with \(t\)

Possible origin of sum rule violation:

- Nonlinearity
- Doublon decay
Lattice modulation experiments
Probing dynamics of the Hubbard model

R. Joerdens et al., arXiv:0804.4009
Doublon decay rate
inspired by experiments in ETH
Relaxation of doublon hole pairs in the Mott state

- Energy released $\sim U$

- Energy carried by spin excitations
  $\sim J = 4t^2/U$

- Relaxation requires creation of $\sim U^2/t^2$ spin excitations

Relaxation rate

$$W \sim t(t/U)^{U^2/t^2}$$

Large $U/t$: Very slow Relaxation
Alternative mechanism of relaxation

- Thermal escape to edges
- Relaxation in compressible edges

Thermal escape time

\[ \Gamma_{esc} \sim e^{-\frac{\Delta V}{k_B T}} \]

Relaxation in compressible edges

\[ \Gamma_{comp} \sim e^{-\text{const.} \frac{U}{t}} \]
Doublon decay in a compressible state

How to get rid of the excess energy $U$?

Compressible state: Fermi liquid description

Doublon can decay into a pair of quasiparticles with many particle-hole pairs

$U$

$p-h$

$p-h$

$p-h$

$p-p$
Doublon decay in a compressible state

Decay amplitude
Doublon decay in a compressible state

Fermi liquid description

Single particle states

\[ \mathcal{H}_t = -t \sum_{\langle ij \rangle \sigma} P c_{i\sigma}^\dagger c_{j\sigma} P \]

Doublons

\[ \mathcal{H}_d = U \sum_i d_i^\dagger d_i \]

Interaction

\[ \mathcal{H}_{\text{int1}} = -t \sum_{\langle ij \rangle \sigma} d_i c_{i\sigma}^\dagger c_{j\sigma}^\dagger \]

Decay

\[ \mathcal{H}_{\text{int2}} = -t \sum_{\langle ij \rangle \sigma} d_i^\dagger c_{j\sigma}^\dagger d_j c_{i\sigma} \]

Scattering
Doublon decay in a compressible state

Decay rate contained in self-energy

Self-consistent equations for doublon

\[
G^{-1}(\omega) = G_0^{-1}(\omega) - \Sigma(\omega)
\]

\[
\Sigma(\omega) = t^2 \gamma_{pp}^2 \chi_{pp}(\omega) + t^2 \gamma_{ph}^2 \int d\omega' G(\omega') \chi_{ph}(\omega - \omega')
\]
Doublon decay in a compressible state
Lattice modulation experiments with fermions in optical lattice. Detecting d-wave superfluid state
Setting: BCS superfluid

- consider a mean-field description of the superfluid

\[ H_0 = \sum_{k,\sigma} (\epsilon_k - \mu) c_{k\sigma}^\dagger c_{k\sigma} + \Delta_k c_{k\uparrow}^\dagger c_{-k\downarrow} + \Delta_k^* c_{-k\downarrow} c_{k\uparrow} \]

- s-wave: \( \Delta_k = \Delta_0 \)

- d-wave: \( \Delta_k = \Delta_0 (\cos k_x - \cos k_y) \)

- anisotropic s-wave: \( \Delta_k = \Delta_0 |\cos k_x - \cos k_y| \)

Can we learn about paired states from lattice modulation experiments? Can we distinguish pairing symmetries?
Lattice modulation experiments

Modulating hopping via modulation of the optical lattice intensity

\[ H_1 = t_1 \sin(\omega t) \sum_{\langle i,j \rangle, \sigma} c_{i\sigma}^\dagger c_{j\sigma} \]

\[ = t_1 \sin(\omega t) \sum_k f_k \left[ (u_k^2 - v_k^2)(\gamma_k^{\uparrow} \gamma_k^{\uparrow} + \gamma_k^{\downarrow} \gamma_k^{\downarrow}) + 2u_kv_k(\gamma_k^{\uparrow} \gamma_{-k}^{\downarrow} + \gamma_{-k}^{\downarrow} \gamma_k^{\uparrow}) \right] \]

where \[ f_k = 2(\cos(k_x) + \cos(k_y)) \]

- Equal energy contours

Resonantly exciting quasiparticles with

\[ 2E_k = \omega \]

Enhancement close to the banana tips due to coherence factors
Lattice modulation as a probe of d-wave superfluids

Distribution of quasi-particles after lattice modulation experiments (1/4 of zone)

Momentum distribution of fermions after lattice modulation (1/4 of zone)

Can be observed in TOF experiments
Lattice modulation as a probe of d-wave superfluids

- Peaks at wave-vectors connecting tips of bananas
- Similar to point contact spectroscopy
- Sign of peak and order-parameter (red=up, blue=down)

\[
\langle \gamma_{q_{\uparrow}} \gamma_{q_{\downarrow}} \rangle
\]

\[
\langle \rho_{q_{\uparrow}} \rho_{-q_{\downarrow}} \rangle - \langle \rho_{0,q_{\uparrow}} \rho_{0,-q_{\downarrow}} \rangle
\]
Scanning tunneling spectroscopy of high Tc cuprates
Conclusions

Experiments with fermions in optical lattice open many interesting questions about dynamics of the Hubbard model

Thanks to: