Quantum Simulation MURI Review

Theoretical work by groups lead by

Luming Duan (Michigan)
Mikhail Lukin (Harvard)
Subir Sachdev (Harvard)
Peter Zoller (Innsbruck)
Hans-Peter Buchler (Stuttgart)
Eugene Demler (Harvard)
MURI quantum simulation – map of achievements

**d-wave superfluidity**
- Extended Hubbard long range int. (Molecules: Doyle, cavity: Kasevich)
- d-wave in plaquettes, noise corr. Detection (Rey, Demler, Lukin)
- Fermionic superfluid in opt. lattice (Ketterle, Bloch, Demler, Lukin, Duan)
- Fermi Hubbard model, Mott (Bloch, Ketterle, Demler, Lukin, Duan)
- Polaron physics (Zwierlein)
- QS of BCS-BEC crossover, imb. Spin mix. (Ketterle, Zwierlein)

**Quantum magnetism**
- Itinerant ferromagnetism (Ketterle, Demler)
- Super-exchange interaction (Bloch, Demler, Lukin, Duan)
- Spinor gases in optical lattices (Ketterle, Demler)
- Quantum gas microscope (Greiner, Thywissen)
- Single atom single site detection (Greiner)
- new Hamiltonian manipulation cooling

**Fermionic superfluidity**
- Bose-Hubbard QS validation (Bloch)
- Bose-Hubbard precision QS (Ketterle)

**Bose-Hubbard model**
- Itinerant ferromagnetism (Ketterle, Demler)
- Super-exchange interaction (Bloch, Demler, Lukin, Duan)
- Spinor gases in optical lattices (Ketterle, Demler)
- Quantum gas microscope (Greiner, Thywissen)
- Single atom single site detection (Greiner)
- new Hamiltonian manipulation cooling

- d-wave superfluidity
- Quantum magnetism

- Fermionic superfluidity
- Bose-Hubbard model
Quantum magnetism
with ultracold atoms
Dynamics of magnetic domain formation near Stoner transition

Experiments:
G. B. Jo et al.,
Science 325:1521 (2009)

Theory:
David Pekker, Rajdeep Sensarma,
Mehrtash Babadi, Eugene Demler,
arXiv:0909.3483
Stoner model of ferromagnetism

Mean-field criterion

\[ U N(0) = 1 \]

\( U \) – interaction strength
\( N(0) \) – density of states at Fermi level

Observation of Stoner transition by G.B. Jo et al., Science (2009)

Signatures of ferromagnetic correlations in particles losses, molecule formation, cloud radius

Magnetic domains could not be resolved. Why?
Stoner Instability

New feature of cold atoms systems: non-adiabatic crossing of $U_c$

Two timescales in the system: screening and magnetic domain formation

Screening of $U$ (Kanamori) occurs on times $1/E_F$

$$U_{\text{eff}} = \frac{U}{1 + U \chi_0} \sim \frac{U}{1 + \frac{U}{E_F}}$$

Magnetic domain formation takes place on much longer time scales: critical slowing down
Quench dynamics across Stoner instability

\[ \chi_{\text{RPA}}(q, \omega) = \frac{\chi_0(q, \omega)}{1 - U \chi_0(q, \omega)} \]

\[ \chi_0(q, \omega) = \int d^3k \frac{n_{k+q} - n_k}{\omega - (\epsilon_{k+q} - \epsilon_k)} \]

Find collective modes

For \( U < U_c \) damped collective modes \( \omega_q = \omega' - i \omega'' \)

For \( U > U_c \) unstable collective modes \( \omega_q = + i \omega'' \)

Unstable modes determine characteristic lengthscale of magnetic domains
Dynamics of magnetic domain formation near Stoner transition

Quench dynamics in D=3

Moving across transition at a finite rate

\[ \frac{du}{dt} = \beta \]

slow domains
growth freeze
coarsen

Domains freeze when

\[ \Gamma(u(t)) t \sim 1 \]

Domain size at “freezing” point

\[ \frac{\xi}{\lambda_F} \sim \left( \frac{1}{\beta} \right)^{\frac{\nu}{\gamma+1}} \]

For MIT experiments domain sizes of the order of a few \( \lambda_F \)

Growth rate of magnetic domains

\[ u = \frac{U}{U_c} - 1 \]

Domains freeze when

\[ \Gamma(u(t)) t \sim 1 \]

Domain size at “freezing” point

\[ \frac{\xi}{\lambda_F} \sim \left( \frac{1}{\beta} \right)^{\frac{\nu}{\gamma+1}} \]

For MIT experiments domain sizes of the order of a few \( \lambda_F \)
Superexchange interaction in experiments with double wells

Theory: A.M. Rey et al., PRL 2008
Experiments: S. Trotzky et al., Science 2008
Quantum magnetism of bosons in optical lattices

\[ \mathcal{H} = J_z \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + J_\perp \sum_{\langle i,j \rangle} \left( \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right) \]

\[ J_z = \frac{t_\uparrow^2 + t_\downarrow^2}{2U_{\uparrow\downarrow}} - \frac{t_\uparrow^2}{U_{\uparrow\uparrow}} - \frac{t_\downarrow^2}{U_{\downarrow\downarrow}} \]

\[ J_\perp = -\frac{t_\uparrow t_\downarrow}{U_{\uparrow\downarrow}} \]

- Ferromagnetic
- Antiferromagnetic

For spin independent lattice and interactions we find ferromagnetic exchange interaction. Ferromagnetic ordering is favored by the boson enhancement factor.
Observation of superexchange in a double well potential

Theory: A.M. Rey et al., PRL 2008

Experiments: S. Trotzky et al. Science 2008

Use magnetic field gradient to prepare a state $| \downarrow \uparrow \rangle$

Observe oscillations between $| \downarrow \uparrow \rangle$ and $| \uparrow \downarrow \rangle$ states
Comparison to the Hubbard model

\[ \hbar \omega_{1,2} = \frac{U}{2} \left( \sqrt{\left(\frac{4J}{U}\right)^2 + 1} \pm 1 \right) \]
Two-Orbital SU(N) Magnetism with Ultracold Alkaline-Earth Atoms

A. Gorshkov, et al., arXiv:0905.2963 (poster)

Ex: $^{87}$Sr (I = 9/2)

$|e\rangle = \begin{pmatrix} 3 \end{pmatrix}_0$

$|g\rangle = \begin{pmatrix} 1 \end{pmatrix}_0$

Nuclear spin decoupled from electrons  \( SU(N=2I+1) \) symmetry
\( \rightarrow \) SU(N) spin models \( \bigoplus \) valence-bond-solid & spin-liquid phases

- orbital degree of freedom \( \bigoplus \) spin-orbital physics
  \( \rightarrow \) Kugel-Khomskii model [transition metal oxides with perovskite structure]
  \( \rightarrow \) SU(N) Kondo lattice model [for N=2, colossal magnetoresistance in manganese oxides and heavy fermion materials]
Mott state of the fermionic Hubbard model
Signatures of incompressible Mott state of fermions in optical lattice

Suppression of double occupancies  Compressibility measurements  
Fermions in optical lattice. Next challenge: antiferromagnetic state

\[ T_N \sim U \]

Mott

\[ T_N \sim \frac{t^2}{U} \]

current experiments
Lattice modulation experiments with fermions in optical lattice

Probing the Mott state of fermions

Pekker, Sensarma, Lukin, Demler (2009)
Pekker, Pollet, unpublished

Measure number of doubly occupied sites

Modulate lattice potential $V_0$

Doublon/hole production rate determined by the spectral functions of excitations

Experiments by ETH Zurich group and others
“High” Temperature $T_N << T << U$

Experiment:

All spin configurations are equally likely. Can neglect spin dynamics

Spectral Function for doublons/holes

Rate of doublon production

Temperature dependence comes from probability of finding nearest neighbors

Double occupancy

Rate of doublon production

Spectral Function $A^h(\omega)$

$V_o = 10 E_R$

$U/(6J) = 13.6$
“Low” Temperature \( T \ll T_N \)

Spins are antiferromagnetically ordered

Rate of doublon production

- Sharp absorption edge due to coherent quasiparticles
- Spin wave shake-off peaks
Doublon decay in a compressible state

How to get rid of the excess energy $U$?

Doublon can decay into a pair of quasiparticles with many particle-hole pairs

\[
P \sim \left( \frac{t}{U} \right)^{\text{const} \cdot \frac{U}{6t}} \sim e^{-\text{const} \cdot \frac{U}{6t} \cdot \log\left(\frac{U}{t}\right)}
\]

Consider processes which maximize the number of particle-hole excitations

Perturbation theory to order $n=U/6t$

d-wave pairing in the fermionic Hubbard model
MURI quantum simulation – map of achievements

**d-wave superfluidity**

- Extended Hubbard long range int. 
  (Molecules: Doyle, cavity: Kasevich)

- **Fermionic superfluid in opt. lattice**
  (Ketterle, Bloch, Demler, Lukin, Duan)

- **Fermi Hubbard model, Mott**
  (Bloch, Ketterle, Demler, Lukin, Duan)

- **d-wave in plaquettes, noise corr. Detection**
  (Rey, Demler, Lukin)

- Polaron physics 
  (Zwierlein)

- **QS of BCS-BEC crossover, imb. Spin mix.**
  (Ketterle, Zwierlein)

**Quantum magnetism**

- **Itinerant ferromagnetism**
  (Ketterle, Demler)

- **Super-exchange interaction**
  (Bloch, Demler, Lukin, Duan)

- **Spinor gases in optical lattices**
  (Ketterle, Demler)

- **Quantum gas microscope**
  (Greiner, Thywissen)

- **New Hamiltonian manipulation cooling**

- **Single atom single site detection**
  (Greiner)

- **Bose-Hubbard model precision QS**
  (Ketterle)

- **Bose-Hubbard model validation**
  (Bloch)

- **Fermi Hubbard model, Mott**
  (Bloch, Demler, Lukin, Duan)

**Fermionic superfluidity**
Using plaquettes to reach d-wave pairing

A.M. Rey et al., EPL 87, 60001 (2009)

Superlattice was a useful tool for observing magnetic superexchange.

Can we use it to create and observe d-wave pairing?

Minimal system exhibiting d-wave pairing is a 4-site plaquette
Ground State properties of a plaquette

- Unique singlet ($S=0$)
- $d$-wave symmetry

$$\langle 4 | \hat{\Delta} | 2 \rangle \neq 0$$

- Unique singlet ($S=0$)
- $s$-wave symmetry

$$\hat{\Delta} = \left( \hat{S}_{12} + \hat{S}_{34} - \hat{S}_{14} - \hat{S}_{23} \right)/2,$$

$$\hat{S}_{rr'} = \left( \hat{c}_{r \uparrow}^\dagger \hat{c}_{r' \downarrow}^\dagger - \hat{c}_{r \downarrow}^\dagger \hat{c}_{r' \uparrow}^\dagger \right)/\sqrt{2}$$

d-wave pair creation operator

Singlet creation operator

Scalapino, Trugman (1996)
Altman, Auerbach (2002)
Vojta, Sachdev (2004)
Kivelson et al., (2007)
A super-plaquette

For weak coupling there are two possible competing configurations:

![Configuration 1](image1)

vs

![Configuration 2](image2)

Which one is lower in energy is determined by the binding energy

$$\Delta_b = 2E_g(N = 3) - E_g(N = 4) - E_g(N = 2)$$

---

A.F. Tsai et al., PRB 73, 214510 (2006);
S. Trebst et al., PRL 96, 250402 (2006);
E. Altman et al., PRL 65, 104508 (2002)
Using plaquettes to reach d-wave pairing

1. Prepare 4 atoms in a single plaquette: A plaquette is the minimum system that exhibits d-wave symmetry.

2. Connect two plaquettes into a superplaquette and study the dynamics of a d-wave pair. Use it to measure the pairing gap.

3. Weakly connect the 2D array to realize and study a superfluid exhibiting long range d-wave correlations.

4. Melt the plaquettes into a 2D lattice, experimentally explore the regime unaccessible to theory.
Phase sensitive probe of d-wave pairing

From noise correlations to phase sensitive measurements in systems of ultra-cold atoms

T. Kitagawa, A. Aspect, M. Greiner, E. Demler
Second order interference from paired states


\[ \Delta n(r, r') \equiv n(r) - n(r') \]

\[ \Delta n(r, -r) \left| \Psi_{BCS} \right> = 0 \]
Momentum correlations in paired fermions

Experiments: Greiner et al., PRL 94:110401 (2005)
How to measure the molecular wavefunction?

\[ |\Psi\rangle = \sum_p \psi(p) c_{p\uparrow}^\dagger c_{-p\downarrow}^\dagger \]

How to measure the non-trivial symmetry of \( \psi(p) \)?

\[ |\Psi\rangle = \sum_p \psi(p) c_{p\uparrow}^\dagger c_{-p\downarrow}^\dagger + \sum_k \psi(k) c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger + \cdots \]

We want to measure the relative phase between components of the molecule at different wavevectors.
Two particle interference

Coincidence count on detectors measures two particle interference

\[ \langle n_1 n_3 \rangle_c = |\Psi|^2 \sin^2(2\beta) \cos^2 \left( \frac{\Phi_I}{2} \right) \]

\[ \langle n_1 n_4 \rangle_c = |\Psi|^2 \left[ 1 - \sin^2(2\beta) \cos^2 \left( \frac{\Phi_I}{2} \right) \right] \]

\[ \Phi_I = \tilde{\phi}_k - \tilde{\phi}_p + \chi_\uparrow - \chi_\downarrow \]

\[ \chi_\uparrow - \chi_\downarrow \]

phase controlled by beam splitters and mirrors
Two particle interference

Implementation for atoms: Bragg pulse before expansion

Bragg pulse mixes states $k$ and $-p = k-G$
-k and $p = -k+G$

\[
\hat{c}_{k\uparrow}^\dagger = \cos \beta c_{k\uparrow}^\dagger - i \sin \beta e^{i\chi} c_{k-G\uparrow}^\dagger \\
\hat{c}_{-k\downarrow}^\dagger = \cos \beta c_{-k\downarrow}^\dagger - i \sin \beta e^{-i\chi} c_{-k+G\downarrow}^\dagger
\]

Coincidence count for states $k\uparrow$ and $p\downarrow$ depends on two particle interference and measures phase of the molecule wavefunction

\[
\langle n_{k\uparrow} n_{p\downarrow} \rangle = |\Psi|^2 \sin^2 (2\beta) \cos^2 \left( \frac{\Phi}{2} \right) \\
\Phi = \phi_k - \phi_p + \chi\uparrow - \chi\downarrow
\]
Research by Luming Duan’s group: poster

- Description of s-wave Feshbach resonance in an optical lattice

\[
H = \sum_i \left[ (\Delta/2) n_i (n_i - 1) - \mu n_i \right] + \sum_{\langle i, j \rangle, \sigma} \left[ t + \delta g (n_i \sigma + n_j \overline{\sigma}) + \delta t n_i \overline{\sigma} n_j \sigma \right] a_{i\sigma}^\dagger a_{j\sigma} + H.c.
\]

- \( p \)-wave Feshbach resonance in an optical lattice and its phase diagram

- Low-dimensional effective Hamiltonian and 2D BEC-BCS crossover

- Anharmonicity induced resonance in a lattice
Other theory projects by the Harvard group

One dimensional systems: nonequilibrium dynamics and noise. T. Kitagawa (poster)

Fermionic Mott states with spin imbalance. B. Wunsch (poster)


Fermions in spin dependent optical lattice. N. Zinner, B. Wunsch (poster).
Bosons in optical lattice with disorder. D. Pekker (poster).
Dynamical preparation of AF state using bosons with ferromagnetic interactions. M. Gullans, M. Rudner
Summary

Quantum magnetism with ultracold atoms
- Dynamics of magnetic domain formation near Stoner transition
- Superexchange interaction in experiments with double wells
- Two-Orbital SU(N) Magnetism with Ultracold Alkaline-Earth Atoms

Mott state of the fermionic Hubbard model
- Signatures of incompressible Mott state of fermions in optical lattice
- Lattice modulation experiments with fermions in optical lattice
- Doublon decay in a compressible state

Making and probing d-wave pairing in the fermionic Hubbard model
- Using plaquettes to reach d-wave pairing
- Phase sensitive probe of d-wave pairing