Emergent phenomena in nonequilibrium dynamics of ultracold atoms

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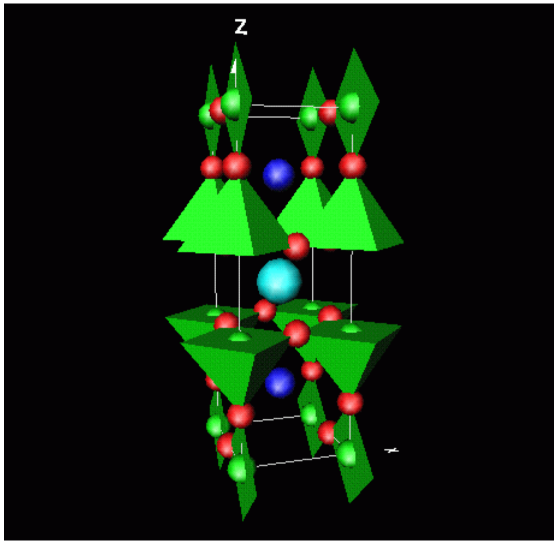
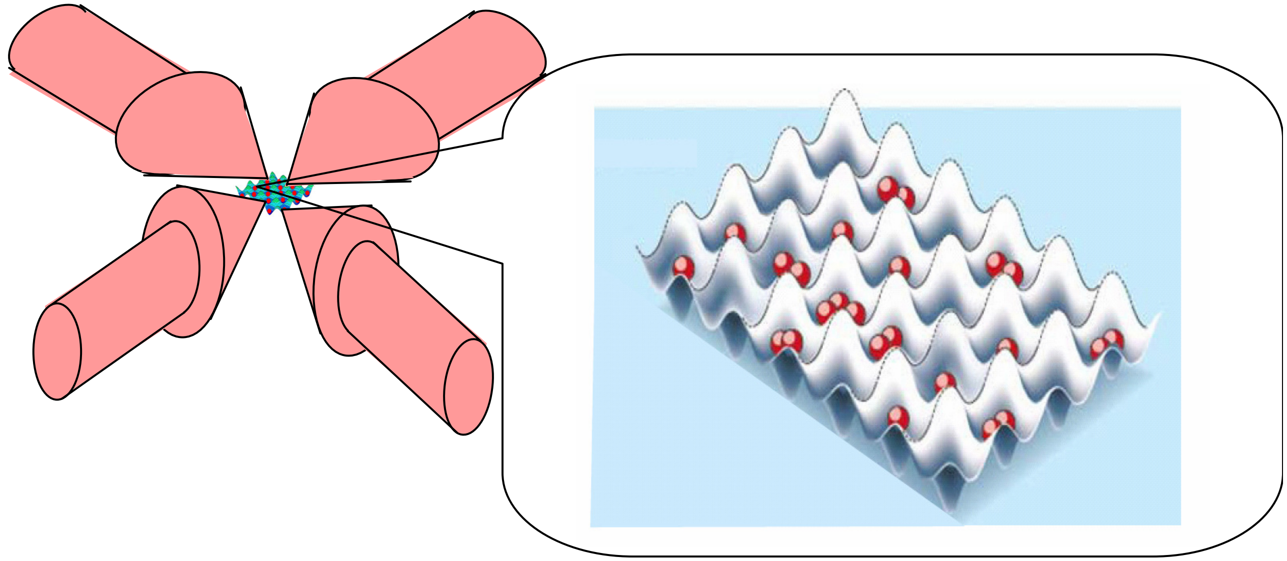
Collaboration with experimental group of I. Bloch

Harvard-MIT CUA

$$ NSF, MURI, DARPA, AFOSR



$YBa_2Cu_3O_7$



Atoms in optical lattice

Antiferromagnetic and superconducting Tc of the order of 100 K

Antiferromagnetism and pairing at sub-micro Kelvin temperatures

Same microscopic model

$$\mathcal{H} = -t \sum_{\langle ij \rangle\, \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_i$$

New Phenomena in quantum many-body systems of ultracold atoms

Long intrinsic time scales

- Interaction energy and bandwidth ~ 1kHz
- System parameters can be changed over this time scale

Decoupling from external environment

- Long coherence times

Can achieve highly non equilibrium quantum many-body states

$$H_i \rightarrow H_f \qquad\qquad | \Psi(t) \rangle = e^{-iH_f t} | \Psi_i \rangle$$

Other theoretical work on many-body nonequilibrium dynamics of ultracold atoms: E. Altman, J.S. Caux, A. Cazalilla, K. Collath, A.J. Daley, T. Giamarchi, V. Gritsev, T.L. Ho, L. Levitov, A. Muramatsu, A. Polkovnikov, S. Sachdev, P. Zoller and many more

Paradigms for equilibrium states of many-body systems

- Broken symmetry phases (magnetism, pairing, etc.)
- Order parameters
- RG flows and fixed points (e.g. Landau Fermi liquids)
- Effective low energy theories
- Classical and quantum critical points
- Scaling

Do we get any collective (universal?) phenomena in the case of nonequilibrium dynamics?

Outline

Emergence of collective phenomena
in non-equilibrium dynamics

Quench dynamics of spin chains.
Emergent time scales

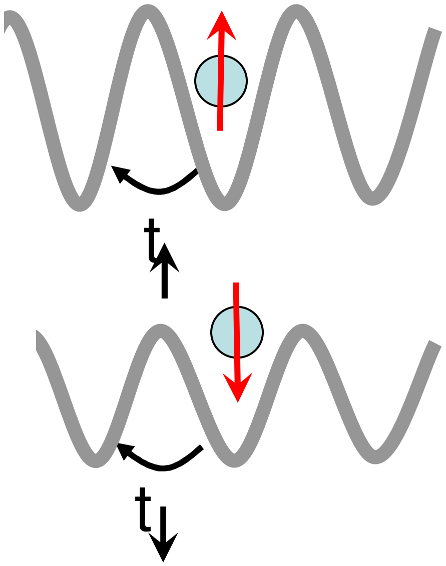
Scaling solution for dynamics with time changing parameters. Dynamic "Fermionization"

Many-body systems in the presence of external noise. Nonequilibrium critical state

Emegent timescales in quench dynamics of spin chains

Two component Bose mixture in optical lattice

Example: $^{87}\mathrm{Rb}$. Mandel et al., Nature 425:937 (2003)



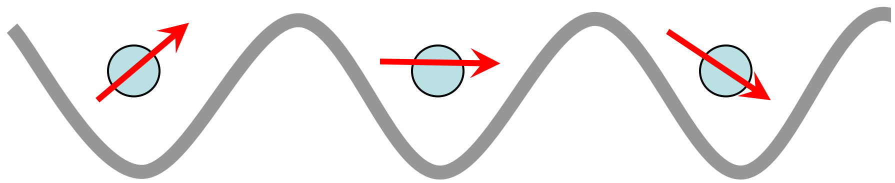
$$|\uparrow\rangle = |F=1, m_F=-1\rangle$$

$$|\downarrow\rangle = |F=2, m_F=-2\rangle$$

Two component Bose Hubbard model

$$\begin{aligned}\mathcal{H} = & -t_\uparrow \sum_{\langle ij\rangle} b^\dagger_{i\uparrow} b_{j\uparrow} - t_\downarrow \sum_{\langle ij\rangle} b^\dagger_{i\downarrow} b_{j\downarrow} + U_{\uparrow\uparrow} \sum_i n_{i\uparrow}(n_\uparrow - 1) \\ & + U_{\downarrow\downarrow} \sum_i n_{i\downarrow}(n_\downarrow - 1) + U_{\uparrow\downarrow} \sum_i n_{i\uparrow} n_\downarrow\end{aligned}$$

Quantum magnetism of bosons in optical lattices



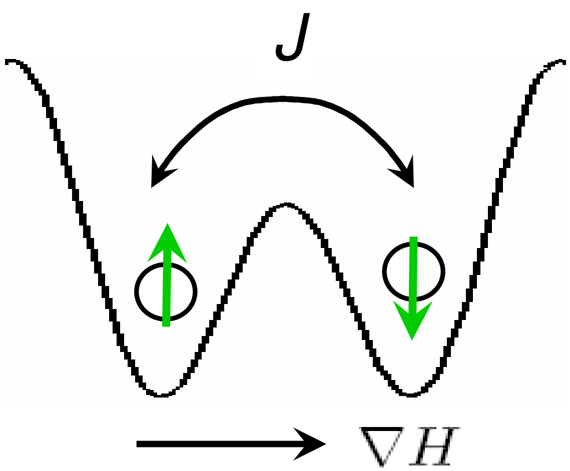
$$\mathcal{H} = J_z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + J_\perp \sum_{\langle ij \rangle} \left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right)$$

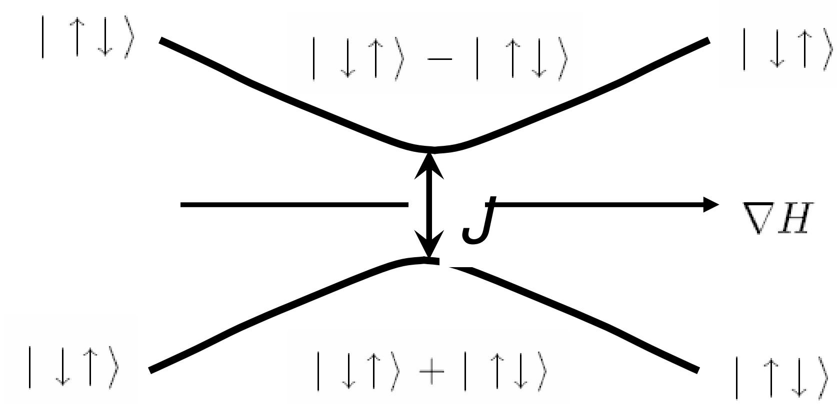
$$J_z = \frac{t_\uparrow^2 + t_\downarrow^2}{2U_{\uparrow\downarrow}} - \frac{t_\uparrow^2}{U_{\uparrow\uparrow}} - \frac{t_\downarrow^2}{U_{\downarrow\downarrow}} \qquad J_\perp = -\frac{t_\uparrow t_\downarrow}{U_{\uparrow\downarrow}}$$

- Ferromagnetic $\quad U_{\uparrow\downarrow} >> U_{\uparrow\uparrow},\ U_{\downarrow\downarrow}$
- Antiferromagnetic $\quad U_{\uparrow\downarrow} << U_{\uparrow\uparrow},\ U_{\downarrow\downarrow}$

Observation of superexchange in a double well potential

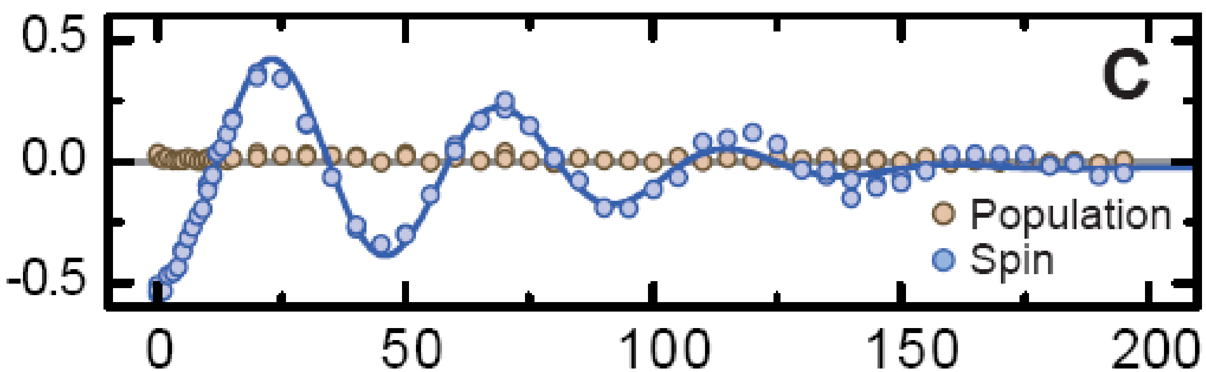
Theory: A.M. Rey et al., PRL 2008





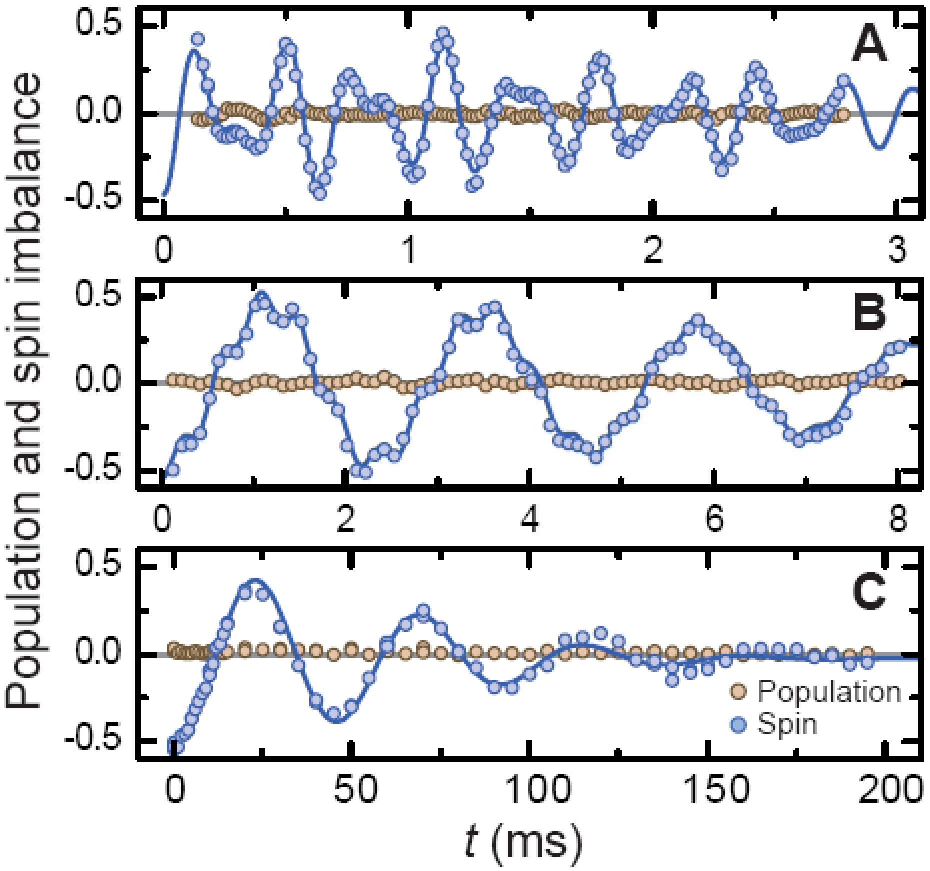
Use magnetic field gradient to prepare a state $|\downarrow\uparrow\rangle$

Observe oscillations between $|\downarrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$ states

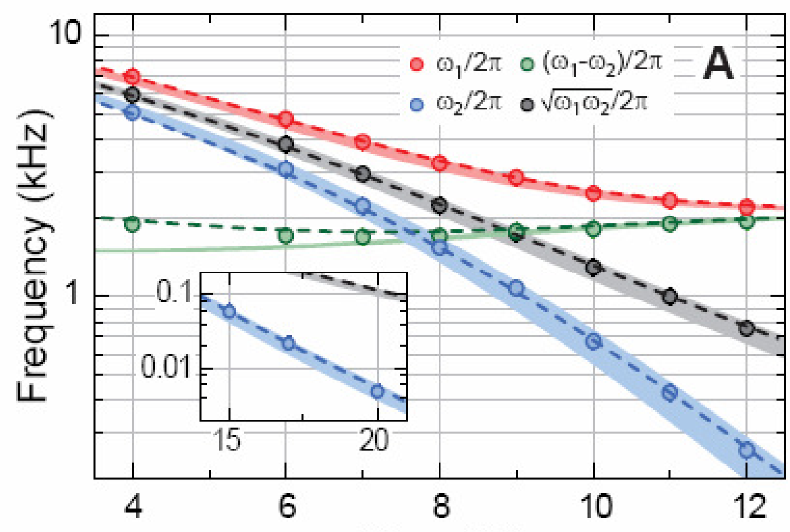


Experiments:
S. Trotzky et al.
Science 2008

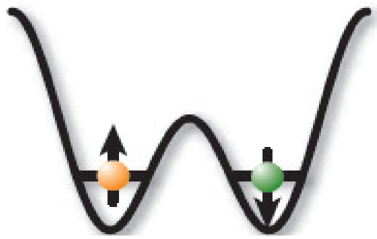
Comparison to the Hubbard model

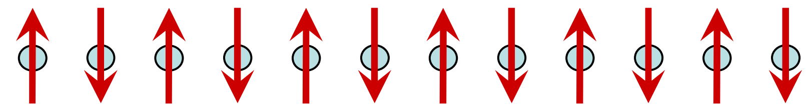


$$\hbar\omega_{1,2} = \frac{U}{2}\left(\sqrt{\left(\frac{4J}{U}\right)^2 + 1} \pm 1\right)$$



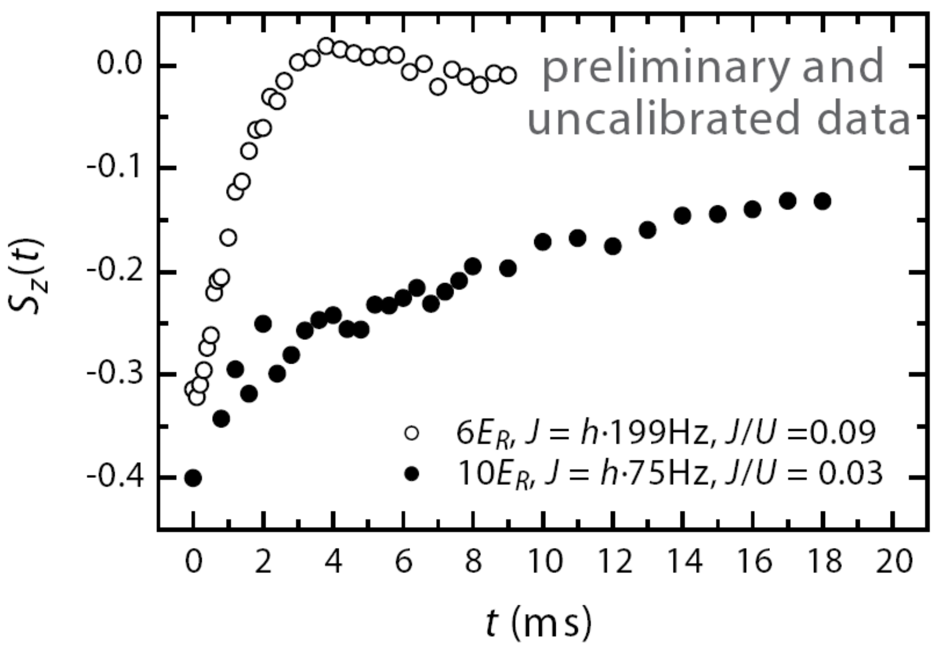
From two spins to a spin chain





Spin oscillations

?



Data courtesy of
S. Trotzky
(group of I. Bloch)

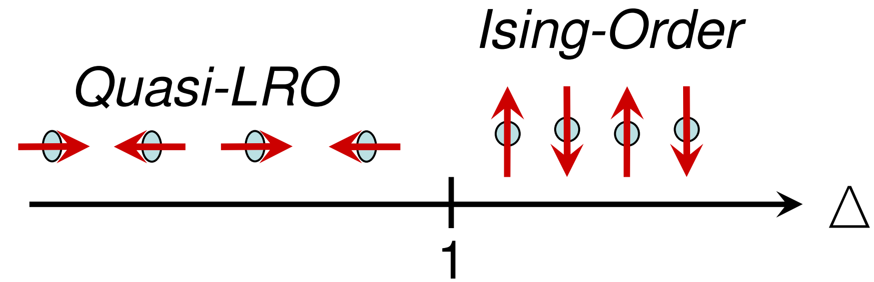
1D: XXZ dynamics starting from the classical Neel state

P. Barmettler et al, PRL 2009

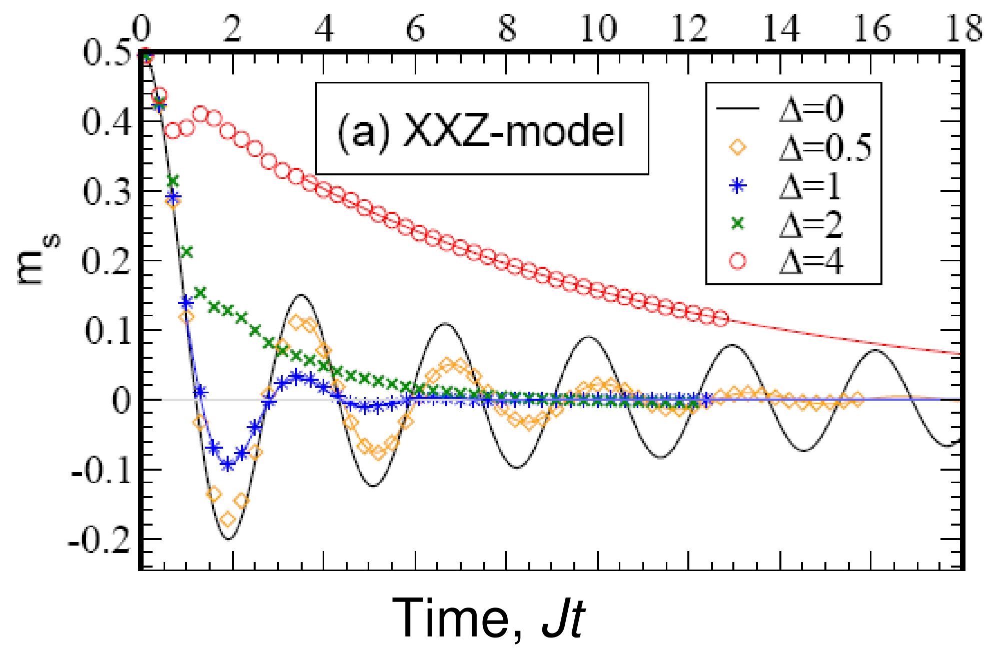
$$|\Psi(t{=}0)\rangle = |\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\rangle$$

$$H_{\mathrm{XXZ}} = J\sum_j \{S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z\}$$

Equilibrium phase diagram:

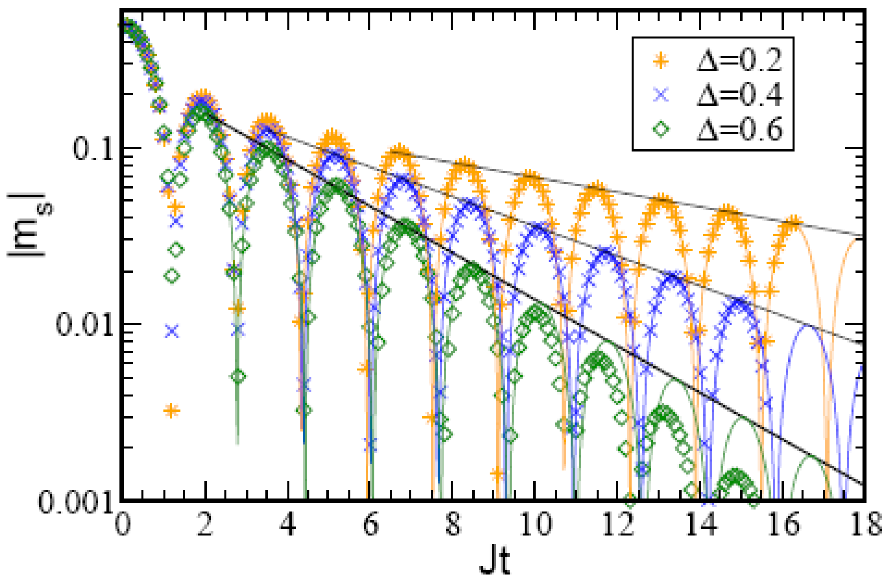


- DMRG
- Bethe ansatz
- XZ model: exact solution



XXZ dynamics starting from the classical Neel state

$$H_{XXZ} = J\sum_j \{S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z\}$$

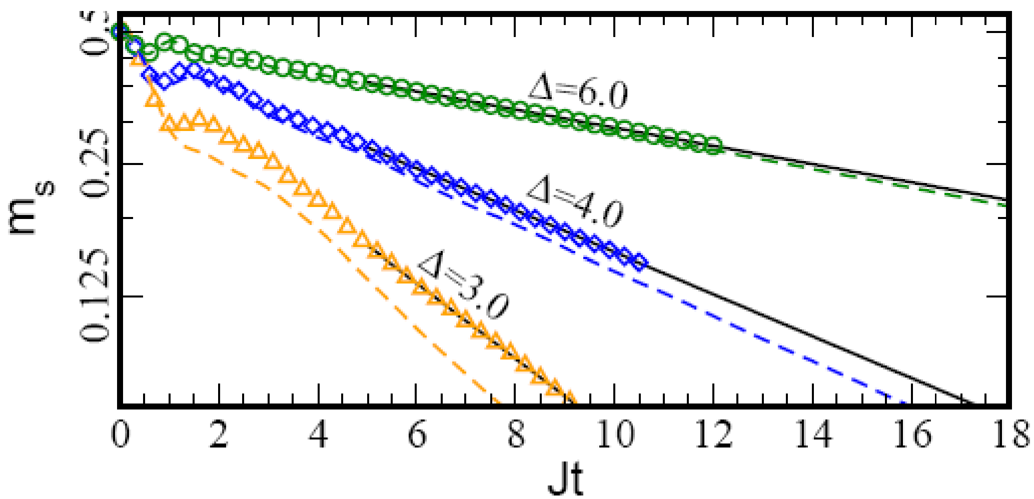


△<1, XY easy plane anisotropy

Oscillations of staggered moment,
Exponential decay of envelope

Except at solvable xx point where:

$$m_s(t) \sim \frac{1}{\sqrt{4\pi t}}\cos(2Jt - \frac{\pi}{4})$$

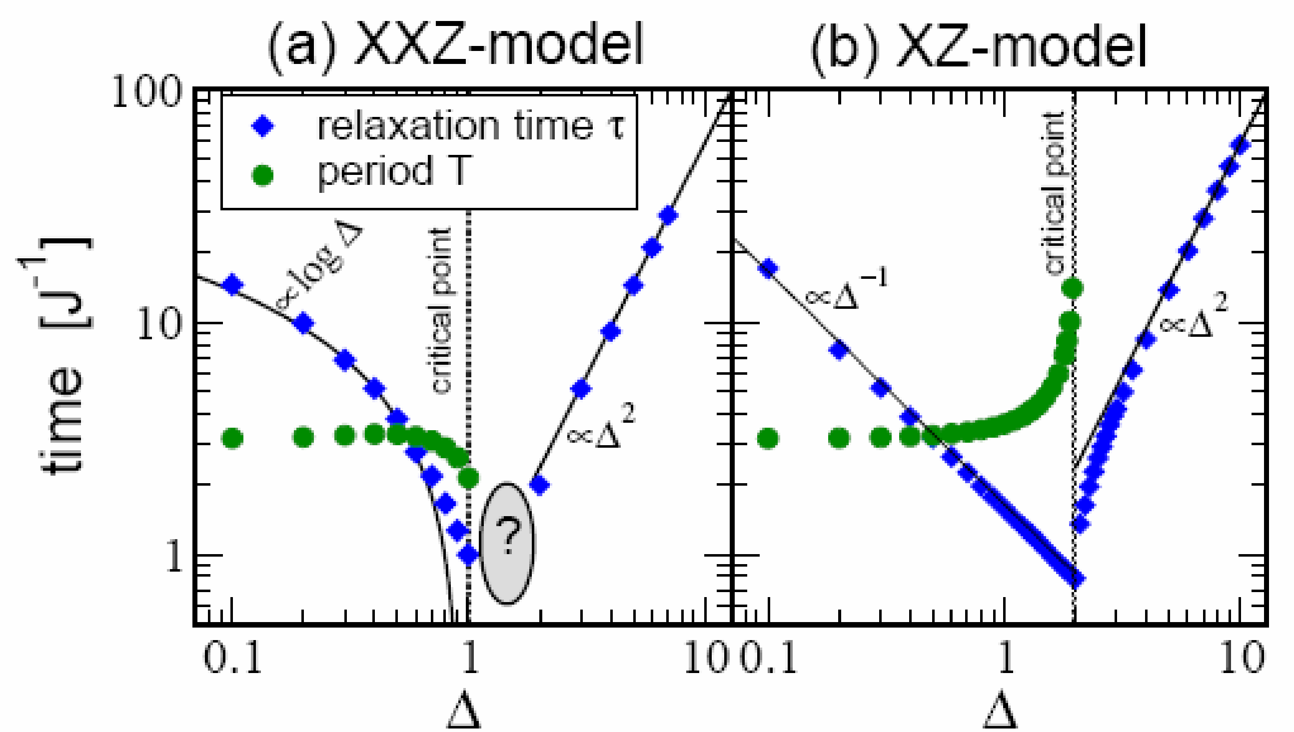


△>1, Z axis anisotropy

Exponential decay of
staggered moment

Behavior of the relaxation time with anisotropy

$$H_{\mathrm{XXZ}} = J\sum_{i}\{S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z\}$$



See also: Sengupta, Powell & Sachdev (2004)

- Moment always decays to zero. Even for high easy axis anisotropy
- Minimum of relaxation time at the QCP. Opposite of classical critical slowing.
- Divergent relaxation time at the XX point.

Scaling solution of dynamics with time changing parameters.
Dynamic “Fermionization”

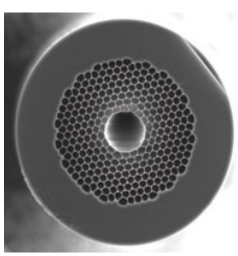
Dynamics with time changing parameters

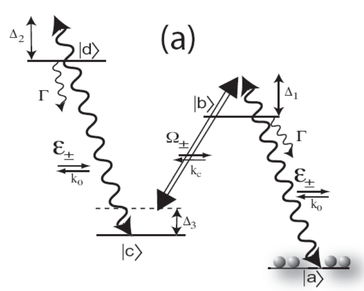
$$\mathcal{H}(t) = \frac{1}{2m(t)} \sum_i p_i^2 + \sum_{ij} V(r_i - r_j, t) + \frac{m(t)\omega^2(t)}{2} \sum_i r_i^2$$

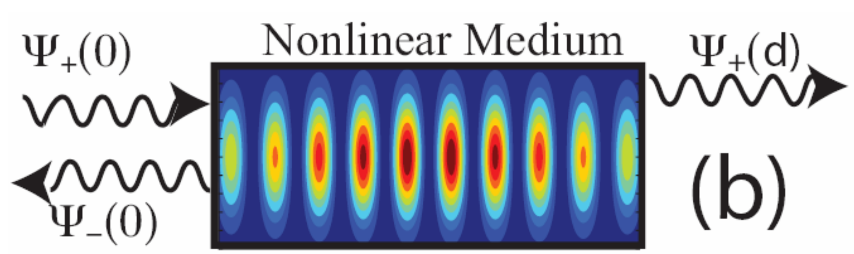
Examples of ultracold atoms and molecules:

- tuning interaction with Feshbah resonance for atoms
- tuning interaction with electric field for molecules
- tuning mass with optical lattice
- tuning external confinement

Other relevant systems in nonlinear quantum optics.
Tuning nonlinearities using EIT







Changing interaction and confining potential. Many-body scaling transformation

– Time-dependent quantum many-body nonequilibrium problem

$$i\frac{\partial\Psi(\mathbf{x}_1,\ldots,\mathbf{x}_N;t)}{\partial t} = -\frac{1}{2m(t)}\sum_{i=1}^{N}\Delta_{x_i}^{(D)}\Psi(\mathbf{x}_1,\ldots,\mathbf{x}_N;t) + [\mu(t)N + \mathbf{g}(t)\sum_{i=1}^{N}\mathbf{x}_i + \frac{m(t)\omega(t)}{2}\sum_{i=1}^{N}\mathbf{x}_i^2 + \sum_{i\neq j=1}^{N}V(\mathbf{x}_i-\mathbf{x}_j;t)]\Psi(\mathbf{x}_1,\ldots,\mathbf{x}_N;t).$$

– Is mapped to the equilibrium problem

$$i\frac{\partial\Phi(y_1,\ldots,y_N;\tau)}{\partial\tau} = -\frac{1}{2}\sum_{i=1}^{N}\Delta_{y_i}^{(D)}\Phi(y_1,\ldots,y_N;\tau) + \frac{\omega_0}{2}\sum_{i}\mathbf{y}_i^2 + \sum_{i\neq j=1}^{N}V(y_i-y_j;0)\Phi(y_1,\ldots,y_N;\tau)$$

– scaling transformation

$$\Psi(\mathbf{x}_1,\ldots,\mathbf{x}_N;t) = \frac{1}{R^N(t)}\exp(i[F(t)\sum_{i=1}^{N}\mathbf{x}_i^2 + \mathbf{G}(t)\sum_{i=1}^{N}\mathbf{x}_i + M(t)N])\Phi(\frac{\mathbf{x}_i}{L(t)} + \mathbf{S}(t);\tau(t))$$

Extends scaling transformation for mean-field GP equation, Yu. Kagan et al. PRA 96

Changing interaction and confining potential. Many-body scaling transformation

$$V(\lambda \mathrm{x};\, t) = \lambda^{\alpha} V(\mathrm{x};\, t) \qquad V(\mathrm{x};\, t) \equiv V(\mathrm{x}) v(t) \qquad V(\mathrm{x};\, 0) \equiv V(\mathrm{x}) v_0$$

– Examples of interaction potentials: contact, Coulomb, centrifugal, dipole, van der Waals, etc.

Time-dependent parameters of the transformation satisfy a set of coupled differential equations. They have a unique solution if

$$L(t) = \left(\frac{v_0}{v(t) m(t)} \right)^{\frac{1}{\alpha+2}}$$

satisfies Ermakov equation:

$$\ddot{L}(t) + \frac{\dot{m}(t)}{m(t)} \dot{L}(t) + \omega(t) L(t) = \frac{\omega_0}{m^2(t) L^3(t)}$$

Nonlinear superposition principle

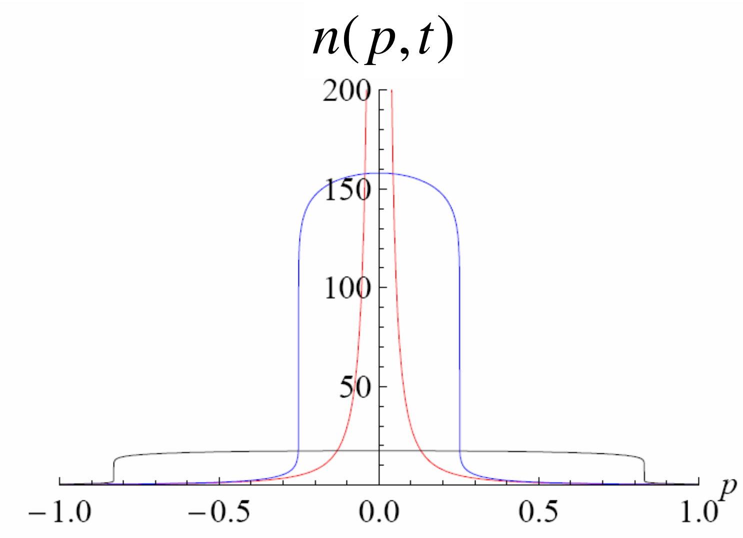
$\ddot{y}(t) + \Omega(t) y(t) = \dfrac{\omega_0}{y^3(t)}$ can be reduced to solving linear problem $\ddot{x}(t) + \Omega(t) x(t) = 0$

Changing interaction and confining potential. Many-body scaling transformation

Emergent collective behavior: “fermionization” of momentum distribution for a wide range of time dependent problems

In the case of free expansion of hard core bosons in 1d
discussed by Muramatsu et al (2004), Minguzzi and Gangardt (2005)

1D, 2D, finite T



Example: 1D Bose gas

$$\omega(t) = t$$

$$m(t) = \exp(2t)$$

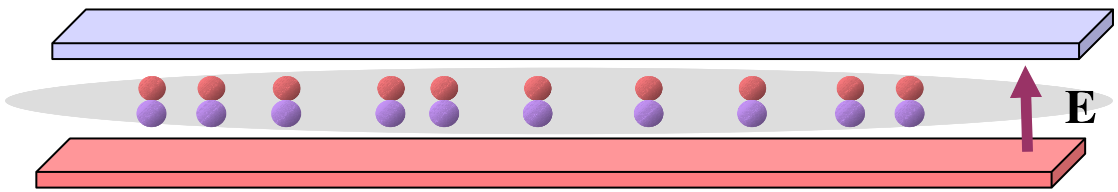
Many-body systems in the presence of external noise.

Nonequilibrium critical state

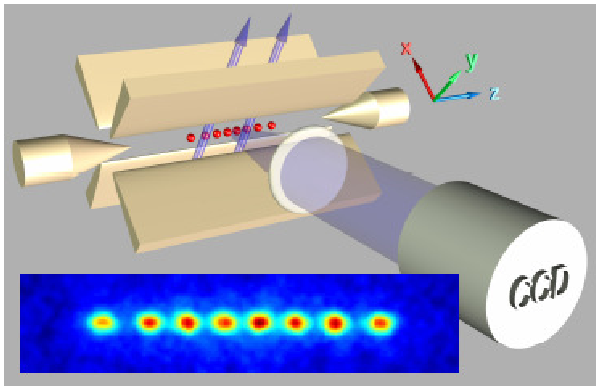
Question:

What happens to low dimensional quantum systems when they are subjected to external non-equilibrium noise?

Ultracold polar molecules



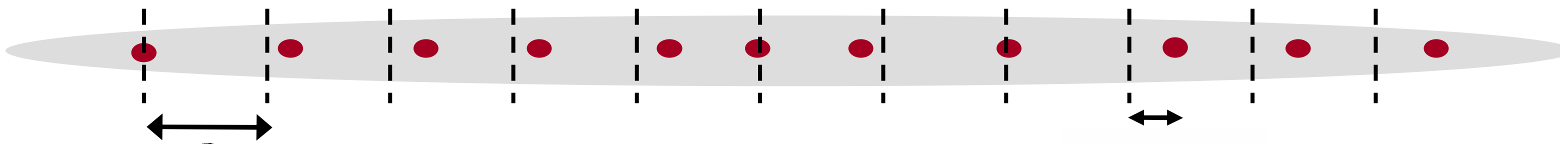
Trapped ions



One dimensional Luttinger state can evolve into a new critical state. This new state has intriguing interplay of quantum critical and external noise driven fluctuations

A brief review:
Universal long-wavelength theory of 1D systems

Haldane (81)



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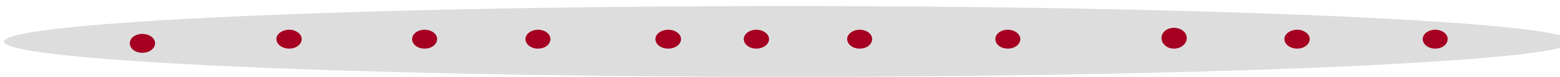
Displacement field: $l\pi\phi(x)$

Long wavelength density fluctuations (phonons): $\delta\rho_0(x) = \frac{1}{\pi}\partial_x\phi(x)$

$$\boxed{S_0 = \frac{K}{2}\int dx d\tau \left[(\partial_\tau\phi)^2 + (\partial_x\phi)^2\right]}$$

Weak interactions: $K >> 1$
Hard core bosons: $K = 1$
Strong long range interactions: $K < 1$

1D review cont'd: Wigner crystal correlations



$$S_0 = \frac{K}{2}\int dx d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2\right]$$

Wigner crystal order parameter:

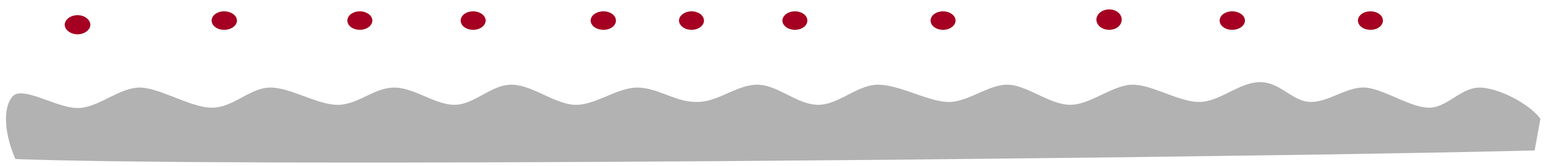
$$\langle \delta\rho_{2\pi/l}\rangle = \langle \cos(2\phi(x))\rangle = 0$$

No crystalline order !

$$\langle \cos(2\phi(x))\cos(2\phi(0))\rangle \sim \frac{1}{x^{2K}}$$

Scale invariant critical state (Luttinger liquid)

1D review cont'd:
Effect of a weak commensurate lattice potential



$$S = \frac{K}{2}\int dx d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2\right] - V\int dx d\tau \cos(2\phi)$$

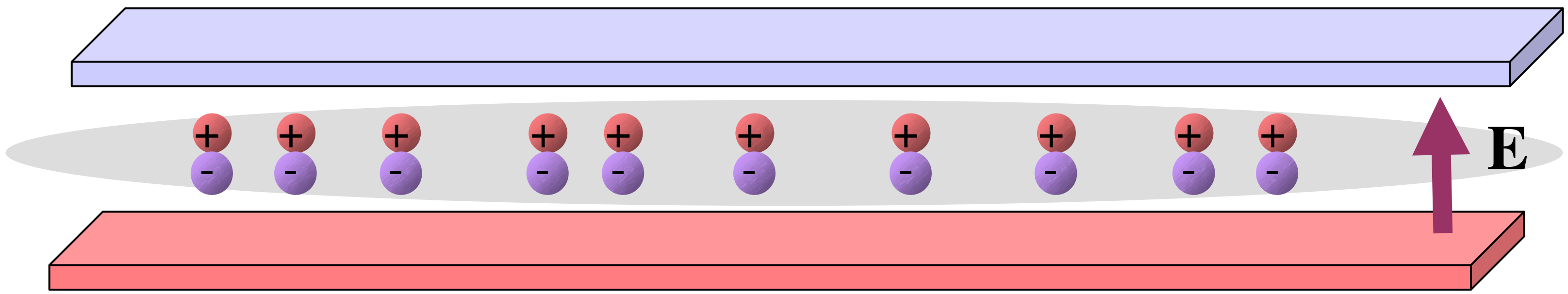
How does the lattice potential change under rescaling ?

$$\langle \cos\left(2\phi(x)\right)\cos\left(2\phi(0)\right)\rangle \sim \frac{1}{x^{2K}} \quad \Rightarrow \quad [dx d\tau \cos(2\phi)] \sim [x]^{2-K}$$

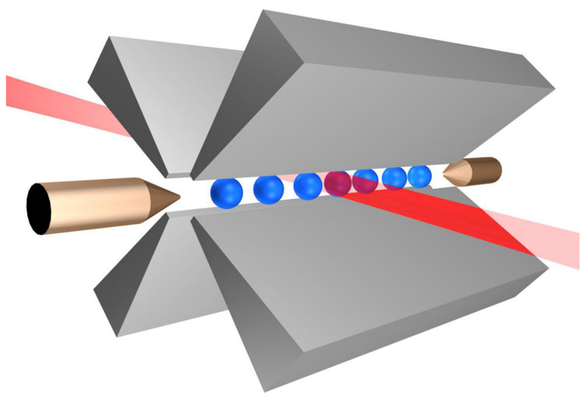
Quantum phase transition: K<2 – Pinning by the lattice ("Mott insulator")
K>2 – Critical phase (Luttinger liquid)

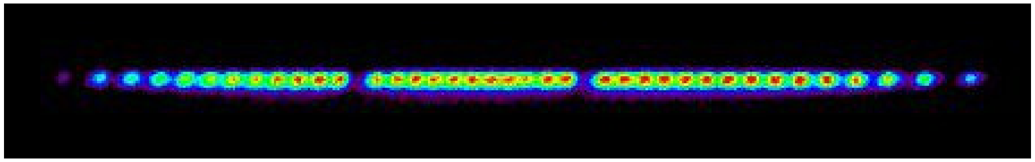
New systems more prone to external disturbance

Ultracold polar molecules



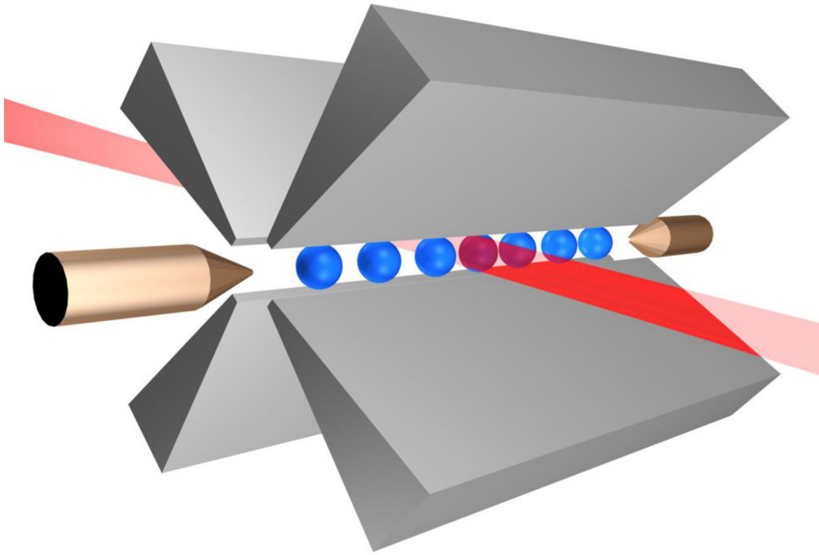
Trapped ions

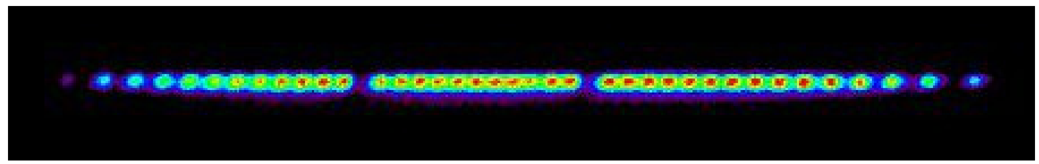




(from NIST group)

Linear ion trap





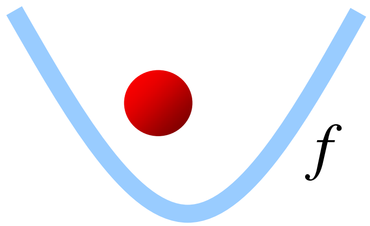
Linear coupling to the noise:

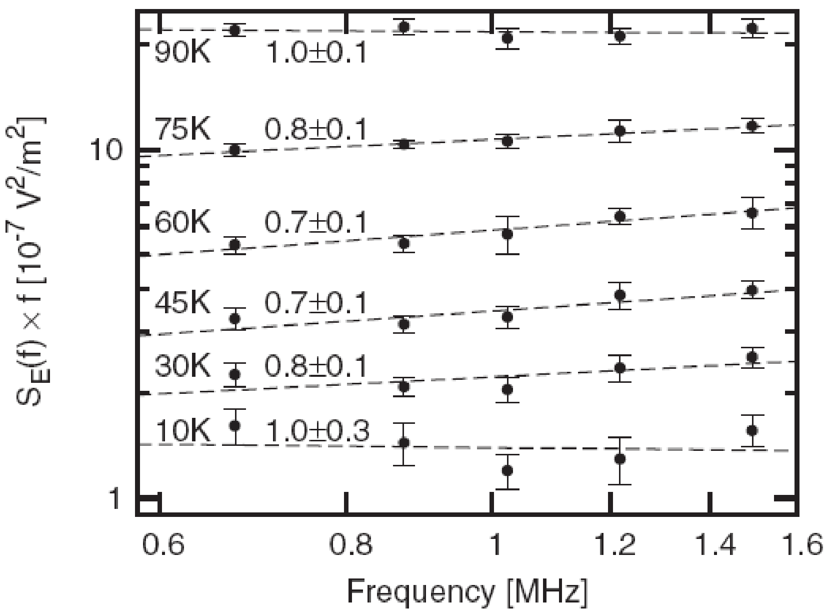
$$H = H_{Coul} - Q \int dx \; \delta V(x,t) \hat{\rho}(x,t)$$

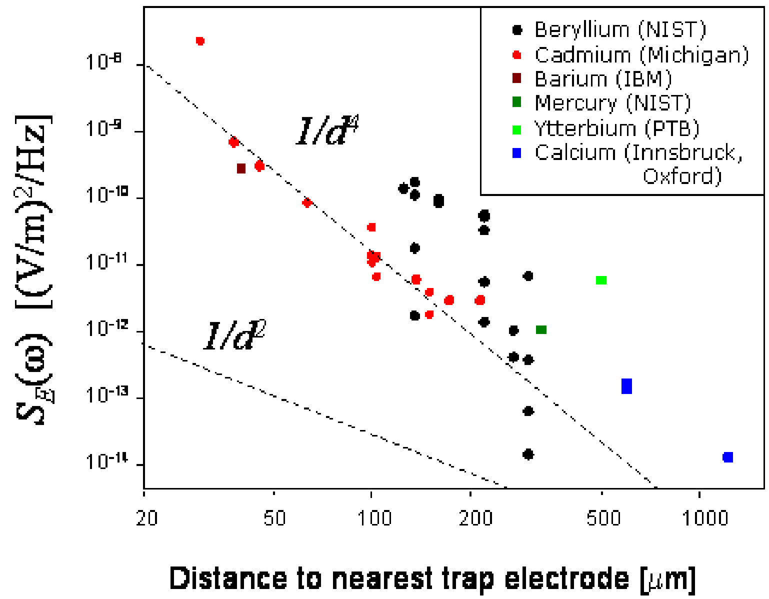
Measured noise spectrum in ion trap

From dependence of heating rate on trap frequency.

Monroe group, PRL (06), Chuang group, PRL (08)





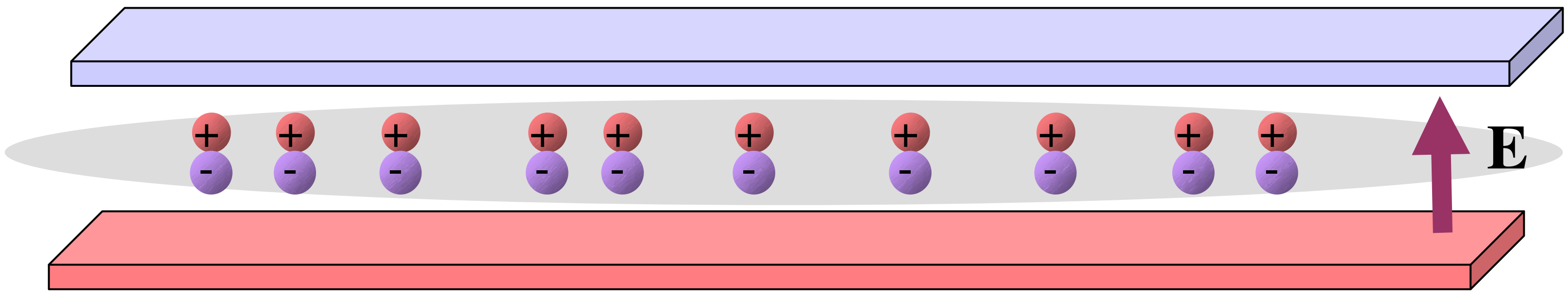


- Direct evidence that noise spectrum is *1/f*

- Short range spatial correlations (~ distance from electrodes)

$$\langle \delta E^{\star}_{q\omega} \delta E_{q\omega} \rangle \approx F_0/\omega$$

Ultra cold polar molecules



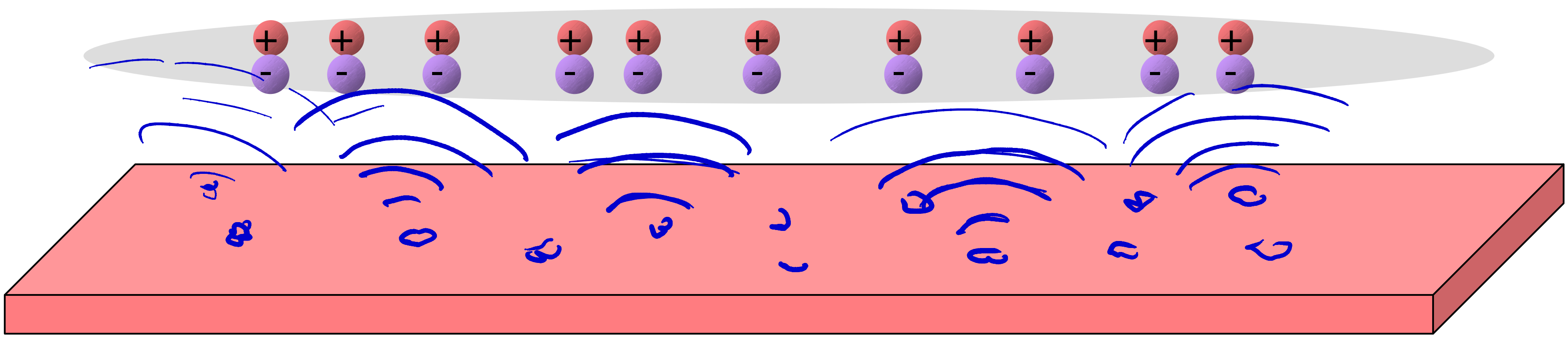
Polarizing electric field: $\mathbf{E(x,t) = E_0 + \delta E(x,t)}$

$$H = H_0 - \alpha_m \int dx \; \delta E(x,t) \hat{\rho}(x,t)$$

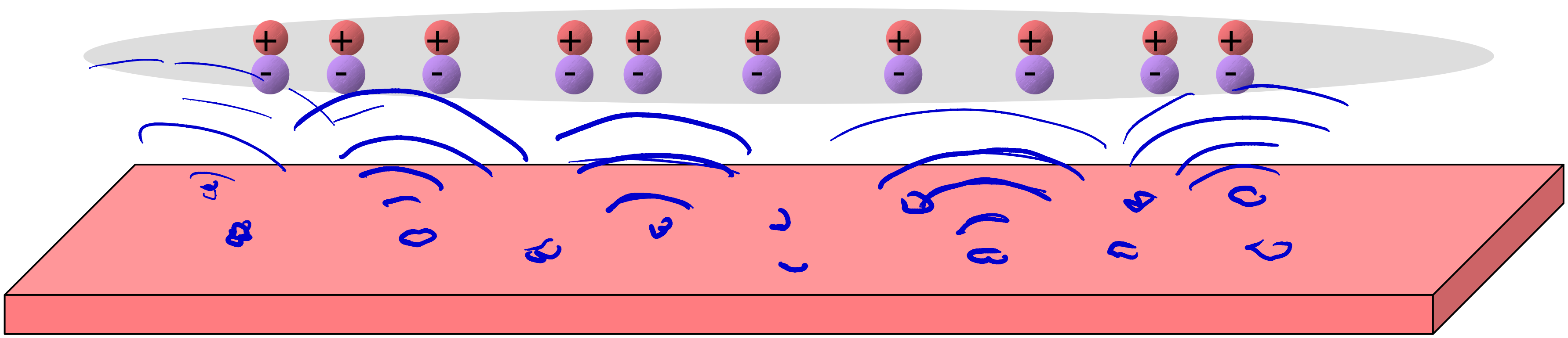
Molecule polarizability

System is subject to electric field noise from the electrodes !

Long wavelength description of noisy low D systems



Effective coupling to external noise

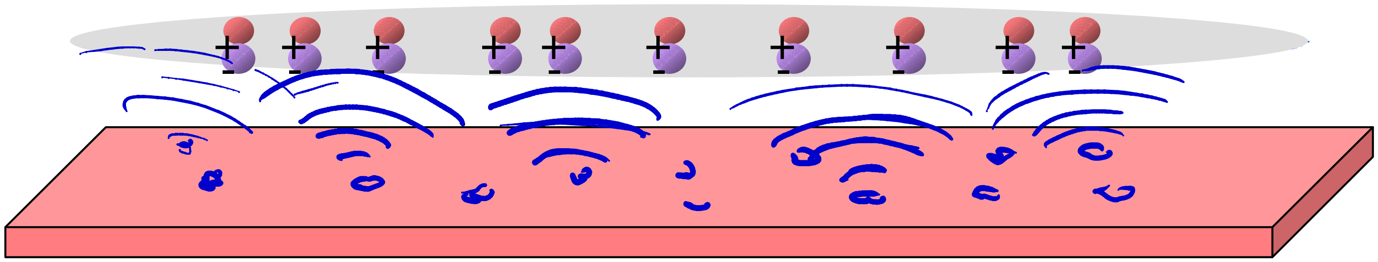


$$g \int dx \; \delta E(x,t)\hat{\rho}(x,t) \;\rightarrow\; \int dx \, f(x,t)\partial_x \phi \;+ \int dx \;\zeta(x,t)\cos\left[2\phi(x)\right]$$

Long wavelength component of noise >> Component of noise at wavelengths near the inter-particle spacing

The "backscattering" ζ can be neglected if the distance to the noisy electrode is much larger than the inter-particle spacing.

Effective harmonic theory of the noisy system



(Quantum) Langevin dynamics:

$$K^{-1}\left(\partial_t^2\phi - \partial_x^2\phi\right) + \eta\partial_t\phi = \xi(x,t) + \partial_x f(x,t)$$

$$\langle \xi^*_{q\omega}\xi_{q\omega}\rangle = \eta\omega\coth\left(\frac{\omega}{2T}\right)$$ Thermal bath

$$\langle f^*_{q\omega} f_{q,\omega}\rangle = F(q,\omega)$$ External noise

Dissipative coupling to bath needed to ensure steady state
(removes the energy pumped in by the external noise)

Implementation of bath: continuous cooling

Wigner crystal correlations

$$\langle \phi^*_{cl}(q,\omega)\phi_{cl}(q,\omega)\rangle = \frac{K}{2|\omega|}\delta(|\omega|-|q|)\left(1+\frac{|\omega|F(q,\omega)}{\eta}\right)$$

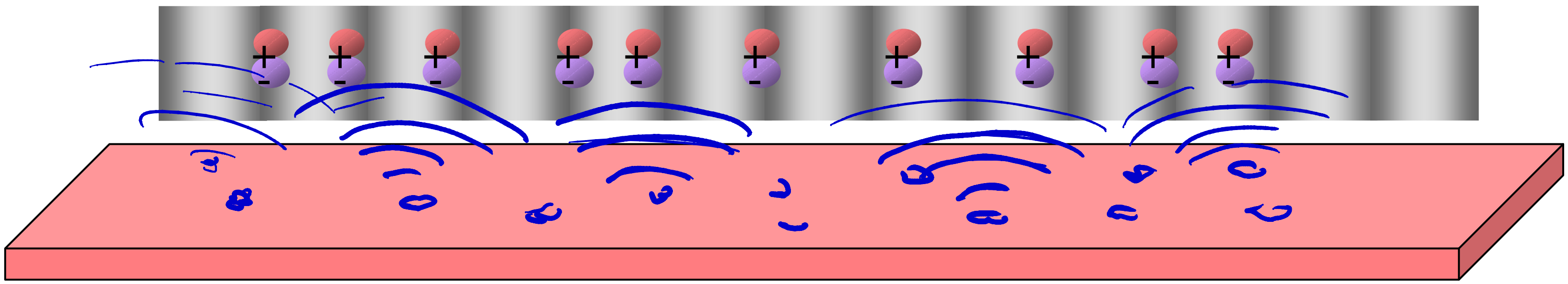
Case of local 1/f noise: $F(q,\omega) = F_0/\omega$

$$\Rightarrow \quad \boxed{\langle \cos(2\phi(x))\cos(2\phi(0))\rangle \sim \left(\frac{1}{x}\right)^{2K(1+F_0/\eta)}}$$

- Decay of crystal correlations remains power-law.
- Decay exponent tuned by the 1/f noise power.

1/f noise is a marginal perturbation ! $\Rightarrow$ Critical steady state

Effect of a weak commensurate lattice potential



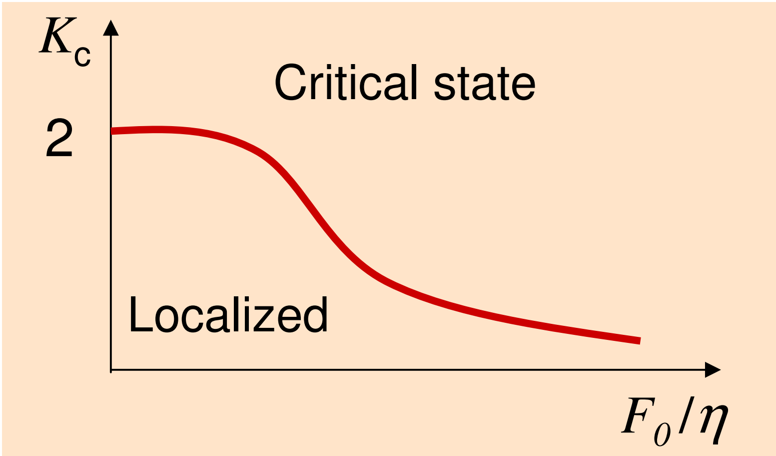
Without lattice: Scale invariant steady state.

How does the lattice change under a scale transformation?

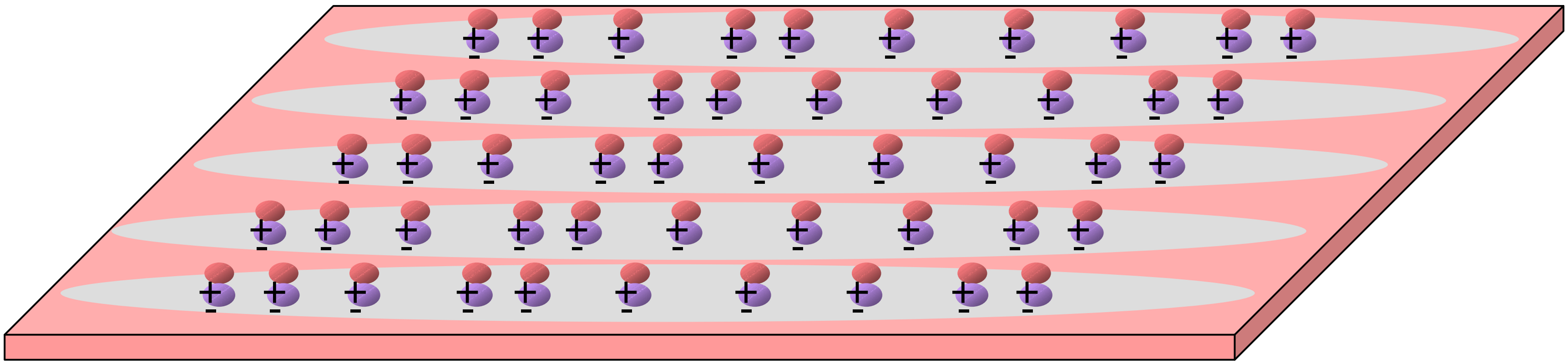
$$\langle \cos(2\phi(x)) \cos(2\phi(0)) \rangle \sim x^{-2K(1+F_0/\eta)} \quad \Rightarrow \quad [dx dt \cos(2\phi)] \sim [x]^{2-K(1+F_0/\eta)}$$

Phase transition tuned by noise power

(Supported also by a full RG analysis within the Keldysh formalism)



1D-2D transition of coupled tubes

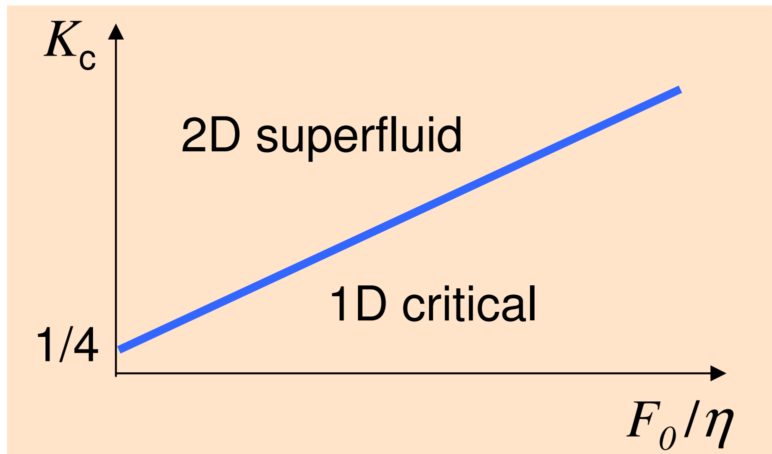


Scaling of the inter-tube hopping:

$$\langle \cos\left[\theta_{cl}(x) - \theta_{cl}(0)\right]\rangle \sim x^{-(1+F_0/\eta)/2K}$$

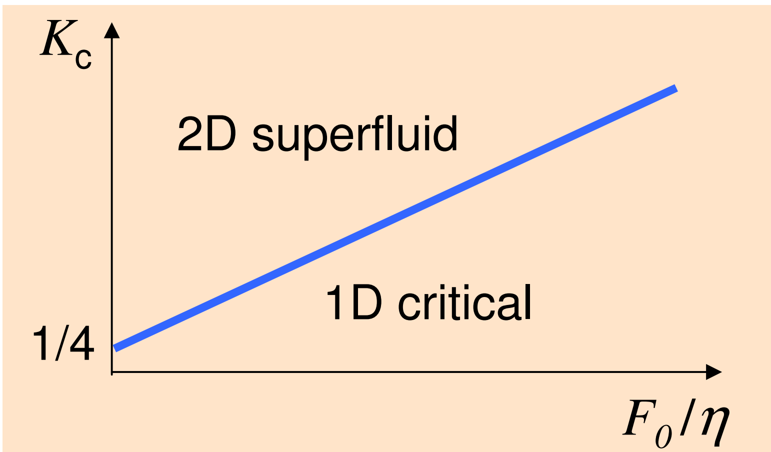
$$\Downarrow$$

$$\left[dx dt \cos\left(\theta_i(x) - \theta_j(x)\right)\right] = x^{2-(1+F_0/\eta)/2K}$$

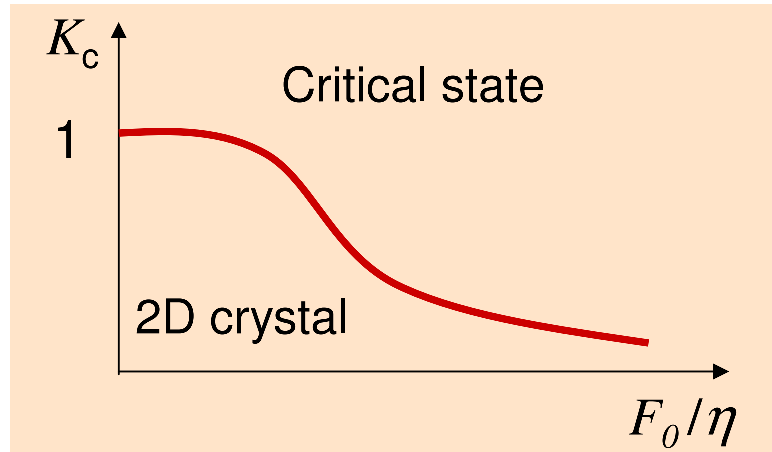


Global phase diagram

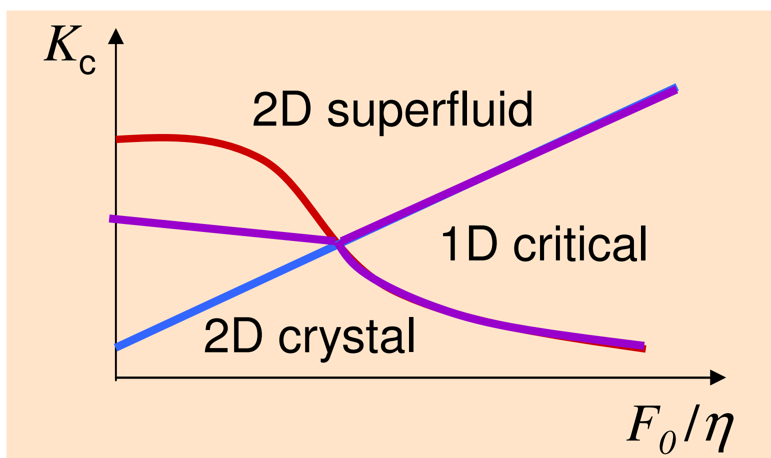
Inter-tube tunneling



Inter-tube interactions



Both perturbations



Summary

Scaling solution for dynamics with time changing parameters. Dynamic “Fermionization”

Quench dynamics of spin chains.
Emergent time scales

Many-body systems in the presence of external noise. Nonequilibrium critical state