Strongly correlated many-body systems: from electronic materials to ultracold atoms

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Collaborations with expt. groups of I. Bloch, J. Schmiedmayer, T. Esslinger, W. Ketterle
“Conventional” solid state materials

Bloch theorem for non-interacting electrons in a periodic potential
Consequences of the Bloch theorem

Metals

Insulators and Semiconductors

First semiconductor transistor

\[ V_H = \frac{I B}{n e^* d} \]
“Conventional” solid state materials

Electron-phonon and electron-electron interactions are irrelevant at low temperatures

\[ \frac{1}{\tau_{e-e}} \sim \epsilon^2 \]
\[ \frac{1}{\tau_{e-ph}} \sim \epsilon^3 \]

Landau Fermi liquid theory: when frequency and temperature are smaller than \( E_F \) electron systems are equivalent to systems of non-interacting fermions

\[ \rho = \rho_0 + aT^2 \]
\[ c/T = \text{const} \]
\[ \kappa/T = \text{const} \]
Strongly correlated electron systems

Quantum Hall systems
kinetic energy suppressed by magnetic field

Heavy fermion materials
many puzzling non-Fermi liquid properties

High temperature superconductors
Unusual “normal” state,
Controversial mechanism of superconductivity,
Several competing orders
What is the connection between strongly correlated electron systems and ultracold atoms?
Bose-Einstein condensation of weakly interacting atoms

Scattering length is much smaller than characteristic interparticle distances. Interactions are weak.

\[ n \sim 10^{14} \text{ cm}^{-3} \quad \quad T_{\text{BEC}} \sim 1 \mu \text{K} \]
New Era in Cold Atoms Research
Focus on Systems with Strong Interactions

• Feshbach resonances
• Rotating systems
• Low dimensional systems
• Atoms in optical lattices
• Systems with long range dipolar interactions
Feshbach resonance and fermionic condensates


Ketterle et al.,
One dimensional systems

1D confinement in optical potential
Weiss et al., Science (05);
Bloch et al.,
Esslinger et al.,

\[ E_{\text{kin}} \sim \frac{\hbar^2}{md^2} \sim \frac{\hbar^2 n^2}{m} \]

\[ E_{\text{int}} \sim gn \]

\[ \gamma = \frac{E_{\text{int}}}{E_{\text{kin}}} \sim \frac{gm}{\hbar^2 n} \]

Strongly interacting regime can be reached for low densities

One dimensional systems in microtraps.
Thywissen et al., Eur. J. Phys. D. (99);
Hansel et al., Nature (01);
Atoms in optical lattices

Theory: Jaksch et al. PRL (1998)

Experiment: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Phillips et al., J. Physics B (2002);
Esslinger et al., PRL (2004);
and many more …
Strongly correlated systems

Electrons in Solids

\[ E_{\text{int}} \sim 1 \div 4 \text{ eV} \sim 10^4 \text{ K} \]
\[ E_{\text{kin}} \sim 1 \div 10 \text{ eV} \sim 10^5 \text{ K} \]

Atoms in optical lattices

\[ E_{\text{int}} \sim E_{\text{kin}} \sim 10 \text{ kHz} \sim 10^{-6} \text{ K} \]

Simple metals \( E_{\text{int}} < E_{\text{kin}} \)

Perturbation theory in Coulomb interaction applies.
Band structure methods work.

Strongly Correlated Electron Systems \( E_{\text{int}} \geq E_{\text{kin}} \)

Band structure methods fail.

Novel phenomena in strongly correlated electron systems:

Quantum magnetism, phase separation, unconventional superconductivity, high temperature superconductivity, fractionalization of electrons …
Strongly correlated systems of ultracold atoms should also be useful for applications in quantum information, high precision spectroscopy, metrology.

By studying strongly interacting systems of cold atoms we expect to get insights into the mysterious properties of novel quantum materials: Quantum Simulators.

BUT

Strongly interacting systems of ultracold atoms and photons: are NOT direct analogues of condensed matter systems. These are independent physical systems with their own “personalities”, physical properties, and theoretical challenges.

Strongly correlated systems of ultracold atoms should also be useful for applications in quantum information, high precision spectroscopy, metrology.
New Phenomena in quantum many-body systems of ultracold atoms

Long intrinsic time scales
- Interaction energy and bandwidth $\sim 1\text{kHz}$
- System parameters can be changed over this time scale

Decoupling from external environment
- Long coherence times

Can achieve highly non equilibrium quantum many-body states

$$H_i \rightarrow H_f \quad \quad \quad \quad \quad |\Psi(t)\rangle = e^{-iH_f t} |\Psi_i\rangle$$

New detection methods

Interference, higher order correlations
Strongly correlated many-body systems of photons
Linear geometrical optics

Newton’s experiment for splitting white light into a spectrum
Strongly correlated systems of photons

Strongly interacting polaritons in coupled arrays of cavities
M. Hartmann et al., Nature Physics (2006)

Strong optical nonlinearities in nanoscale surface plasmons

Crystallization (fermionization) of photons in one dimensional optical waveguides
D. Chang et al., Nature Physics (2008)
Outline of these lectures

• Introduction. Cold atoms in optical lattices. Bose Hubbard model and extensions
• Bose mixtures in optical lattices
  Quantum magnetism of ultracold atoms.
  Current experiments: observation of superexchange
• Fermions in optical lattices
  Magnetism and pairing in systems with repulsive interactions.
  Current experiments: Mott state
• Detection of many-body phases using noise correlations
• Experiments with low dimensional systems
  Interference experiments. Analysis of high order correlations

Emphasis of these lectures:
• Detection of many-body phases
• Dynamics
Atoms in optical lattices.
Bose Hubbard model
Bose Hubbard model

\[ \mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1) - \mu \sum_i n_i \]

- \( t \) — tunneling of atoms between neighboring wells
- \( U \) — repulsion of atoms sitting in the same well
Bose Hubbard model. Mean-field phase diagram

\[ \mu / U \]

\[ t / U \]

- $N=1$: Mott
- $N=2$: Mott
- $N=3$: Mott

Superfluid

Mott

\[ U \ll Nt \]
Superfluid phase
Weak interactions

\[ U \gg Nt \]
Mott insulator phase
Strong interactions

M.P.A. Fisher et al., PRB (1989)
Bose Hubbard model

\[
\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1) - \mu \sum_i n_i
\]

Set \( t = 0 \) Hamiltonian eigenstates are Fock states \( |n\rangle = \frac{1}{\sqrt{n!}} (b_i^\dagger)^n |0\rangle \)

\[
|0\rangle \quad \epsilon = 0
\]

\[
|1\rangle \quad \epsilon = -\mu
\]

\[
|2\rangle \quad \epsilon = 2U - 2\mu
\]

\[
|3\rangle \quad \epsilon = 6U - 3\mu
\]
Bose Hubbard Model. Mean-field phase diagram

\[ \mu / U \]

\begin{align*}
N=3 & \quad \text{Mott} \\
N=2 & \quad \text{Mott} \\
N=1 & \quad \text{Mott} \\
\end{align*}

\( t / U \)

- Superfluid
- Mott insulator phase
- Particle-hole excitation

\[ \Delta E \sim U - N \ t \]

\[ U \sim N \ t \]
Gutzwiller variational wavefunction

\[ |\Psi\rangle = \prod_i ( f_0 |0\rangle + f_1 |1\rangle + f_2 |2\rangle + \ldots )_i \]

\[ = \prod_i ( f_0 + f_1 b^\dagger_i + \frac{f_2}{\sqrt{2}} (b^\dagger_i)^2 + \ldots ) |0\rangle_i \]

Normalization

\[ |f_0|^2 + |f_1|^2 + |f_2|^2 + \ldots = 1 \]

Interaction energy

\[ \epsilon_U = 2 \ U |f_2|^2 + 6 \ U |f_3|^2 + \ldots \]

Kinetic energy

\[ \epsilon_t = -zt \left| f_0^* f_1 + \sqrt{2} f_1^* f_2 + \sqrt{3} f_2^* f_3 + \ldots \right|^2 \]

\( z \) – number of nearest neighbors
Phase diagram of the 1D Bose Hubbard model. Quantum Monte-Carlo study

Batrouni and Scaletter, PRB (1992)
Superfluid to insulator transition in an optical lattice

Optical lattice and parabolic potential

Jaksch et al.,
Shell structure in optical lattice

S. Foelling et al., PRL (2006)

Observation of spatial distribution of lattice sites using spatially selective microwave transitions and spin changing collisions
Extended Hubbard Model

\[ \mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \sum_{ij} n_i \ U_{ij} \ n_j - \mu \sum_i n_i \]

- on site repulsion \( U_0 \)
- nearest neighbor repulsion \( U_1 \)

Checkerboard phase:

Crystal phase of bosons.
Breaks translational symmetry
Extended Hubbard model. Mean field phase diagram

van Otterlo et al., PRB (1995)

\[
\frac{U_1}{U_0} = \frac{1}{5}
\]

Hard core bosons. \[
\frac{U_2}{U_1} = \frac{1}{10}
\]

Supersolid – superfluid phase with broken translational symmetry
Extended Hubbard model. Quantum Monte Carlo study

Hebert et al., PRB (2002)  
Sengupta et al., PRL (2005)
Dipolar bosons in optical lattices

\[ V_{\text{int}} = d^2 \frac{1 - 3 \cos^2\theta}{|\mathbf{r} - \mathbf{r}'|^3} + \frac{4 \pi \hbar^2 a}{m} \delta(\mathbf{r} - \mathbf{r}') \]

Goral et al., PRL (2002)
Bose Hubbard model away from equilibrium. Dynamical Instability of strongly interacting bosons in optical lattices
Moving condensate in an optical lattice. Dynamical instability

Theory: Niu et al. PRA (01), Smerzi et al. PRL (02)
Experiment: Fallani et al. PRL (04)
Dynamical instability

Classical limit of the Hubbard model. $N t \gg U$

Discreet Gross-Pitaevskii equation

$$i \frac{d \Psi_j}{dt} = -t \sum_{\langle k \rangle} \Psi_k + U |\Psi_j|^2 \Psi_j$$

Current carrying states $\Psi_j \sim e^{ipx_j}$

Linear stability analysis: States with $p > \pi/2$ are unstable

Amplification of density fluctuations
Dynamical instability. Gutzwiller approximation

Wavefunction

\[ |\Psi(t)\rangle = \prod_j \left[ \sum_{n=0}^{\infty} f_{jn}(t) |n\rangle_j \right] \]

Time evolution

\[ -i \frac{df_{jn}}{dt} = -t (f_{jn-1} \phi_j + f_{jn+1} \phi_j^*) + \frac{U}{2} n (n-1) f_{jn} \]

\[ \phi_j(t) = \sum_{\langle i \rangle} \langle \Psi(t) | a_i | \Psi(t) \rangle \]

We look for stability against small fluctuations

Phase diagram. Integer filling

Altman et al., PRL 95:20402 (2005)
The first instability develops near the edges, where $N=1$. Optical lattice and parabolic trap. Gutzwiller approximation.

Gutzwiller ansatz simulations (2D)

$U=0.01 \, t$

$J=1/4$
Phase diagram for a Bose-Einstein condensate moving in an optical lattice

Jongchul Mun, Patrick Medley, Gretchen K. Campbell,* Luis G. Marcassa,† David E. Pritchard, and Wolfgang Ketterle
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, and Department of Physics, MIT, Cambridge, Massachusetts 02139, USA.

PRL (2007)
Beyond semiclassical equations. Current decay by tunneling

Current carrying states are metastable. They can decay by thermal or quantum tunneling

Thermal activation

Quantum tunneling
Current decay by thermal phase slips

Theory: Polkovnikov et al., PRA (2005)
Current decay by quantum phase slips

Experiment: Ketterle et al., PRL (2007)
Engineering magnetic systems using cold atoms in an optical lattice
Two component Bose mixture in optical lattice


$|\uparrow\rangle = |F = 1, m_F = -1\rangle$

$|\downarrow\rangle = |F = 2, m_F = -2\rangle$

Two component Bose Hubbard model

\[
\mathcal{H} = -t_\uparrow \sum_{\langle ij \rangle} b_{i\uparrow}^\dagger b_{j\uparrow} - t_\downarrow \sum_{\langle ij \rangle} b_{i\downarrow}^\dagger b_{j\downarrow} + U_{\uparrow\uparrow} \sum_i n_{i\uparrow}(n_{\uparrow} - 1) \\
+ U_{\downarrow\downarrow} \sum_i n_{i\downarrow}(n_{\downarrow} - 1) + U_{\uparrow\downarrow} \sum_i n_{i\uparrow}n_{\downarrow}
\]
Quantum magnetism of bosons in optical lattices

Duan, Demler, Lukin, PRL (2003)

\[ \mathcal{H} = J_z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + J_\perp \sum_{\langle ij \rangle} \left( \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right) \]

\[ J_z = \frac{t^2_{\parallel} + t^2_{\perp}}{2U_{\downarrow\downarrow}} - \frac{t^2_{\parallel}}{U_{\uparrow\uparrow}} - \frac{t^2_{\perp}}{U_{\downarrow\downarrow}} \]

\[ J_\perp = -\frac{t_{\parallel} t_{\perp}}{U_{\uparrow\downarrow}} \]

- Ferromagnetic
- Antiferromagnetic

\[ U_{\downarrow\downarrow} \gg U_{\uparrow\uparrow}, U_{\downarrow\downarrow} \]

\[ U_{\downarrow\downarrow} \ll U_{\uparrow\uparrow}, U_{\downarrow\downarrow} \]
Exchange Interactions in Solids

Kinetic energy dominates: antiferromagnetic state

Coulomb energy dominates: ferromagnetic state
Two component Bose mixture in optical lattice. Mean field theory + Quantum fluctuations

Altman et al., NJP (2003)
Realization of spin liquid using cold atoms in an optical lattice

Theory: Duan, Demler, Lukin PRL (03)


\[ H = - J_x \sum \sigma_i^x \sigma_j^x - J_y \sum \sigma_i^y \sigma_j^y - J_z \sum \sigma_i^z \sigma_j^z \]

Questions:
Detection of topological order
Creation and manipulation of spin liquid states
Detection of fractionalization, Abelian and non-Abelian anyons
Melting spin liquids. Nature of the superfluid state
Superexchange interaction in experiments with double wells

Theory: A.M. Rey et al., PRL 2008
Experiments: S. Trotzky et al., Science 2008
Observation of superexchange in a double well potential

Theory: A.M. Rey et al., PRL 2008

Use magnetic field gradient to prepare a state \(| \downarrow \uparrow \rangle\)

Observe oscillations between \(| \downarrow \uparrow \rangle\) and \(| \uparrow \downarrow \rangle\) states

Experiments: S. Trotzky et al. Science 2008
Preparation and detection of Mott states of atoms in a double well potential

Reversing the sign of exchange interaction
Comparison to the Hubbard model

\[ \hbar \omega_{1,2} = \frac{U}{2} \left( \sqrt{\left( \frac{4J}{U} \right)^2 + 1 \pm 1} \right) \]
Beyond the basic Hubbard model

Basic Hubbard model includes only local interaction

Extended Hubbard model takes into account non-local interaction

\[
\hat{H}^{EHM} = \hat{H}^{HM} - \Delta J \sum_{\sigma \neq \sigma'} (\hat{n}_{\sigma L} + \hat{n}_{\sigma R}) \left( \hat{a}_{\sigma' L}^{\dagger} \hat{a}_{\sigma' R} + \hat{a}_{\sigma' R}^{\dagger} \hat{a}_{\sigma' L} \right) \\
+ U_{LR} \sum_{\sigma \neq \sigma'} \left( \hat{n}_{\sigma L} \hat{n}_{\sigma' R} + \hat{a}_{\sigma L}^{\dagger} \hat{a}_{\sigma' R}^{\dagger} \hat{a}_{\sigma' L} \hat{a}_{\sigma R} + \frac{1}{2} \hat{a}_{\sigma L}^{\dagger} \hat{a}_{\sigma' L} \hat{a}_{\sigma' R} \hat{a}_{\sigma R} + \frac{1}{2} \hat{a}_{\sigma R}^{\dagger} \hat{a}_{\sigma' R} \hat{a}_{\sigma' L} \hat{a}_{\sigma L} \right),
\]
Beyond the basic Hubbard model
From two spins to a spin chain

Spin oscillations

Data courtesy of S. Trotzky (group of I. Bloch)
1D: XXZ dynamics starting from the classical Neel state

\[ \Psi(t=0) = \uparrow \downarrow \downarrow \downarrow \downarrow \downarrow \]

Equilibrium phase diagram:

- DMRG
- Bethe ansatz
- XZ model: exact solution

\[ H_{XXZ} = J \sum_j \{ S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \} \]

P. Barmettler et al, PRL 2009
XXZ dynamics starting from the classical Neel state

\[ H_{XXZ} = J \sum_j \{ S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \} \]

\( \Delta < 1, \) XY easy plane anisotropy

Oscillations of staggered moment, Exponential decay of envelope

Except at solvable xx point where:

\[ m_s(t) \sim \frac{1}{\sqrt{4\pi t}} \cos(2Jt - \frac{\pi}{4}) \]

\( \Delta > 1, \) Z axis anisotropy

Exponential decay of staggered moment
Behavior of the relaxation time with anisotropy

- Moment always decays to zero. Even for high easy axis anisotropy
- **Minimum** of relaxation time at the QCP. Opposite of classical critical slowing.
- Divergent relaxation time at the XX point.

See also: Sengupta, Powell & Sachdev (2004)
Magnetism in optical lattices

Higher spins and higher symmetries
F=1 spinor condensates

Spin symmetric interaction of F=1 atoms

\[ U(r_1 - r_2) = \delta(r_1 - r_2) \left( W_0 + W_2 \vec{S}_1 \cdot \vec{S}_2 \right) \]

\[ W_2 = \frac{4\pi\hbar^2}{3m} \left( a_2 - a_0 \right) \]

Ferromagnetic Interactions for \( W_2 < 0 \)

87Rb

\[ a_0 = 110 \pm 4 a_B \]

\[ a_2 = 107 \pm 4 a_B \]

Antiferromagnetic Interactions for \( W_2 > 0 \)

23Na

\[ a_0 = 46 \pm 5a_B \]

\[ a_2 = 52 \pm 5a_B \]
Antiferromagnetic spin F=1 atoms in optical lattices

**Hubbard Hamiltonian**  

\[
\mathcal{H} = -t \sum_{\langle ij \rangle} a_{im}^\dagger a_{jm} + U_0 \sum_i n_i^2 + U_2 \sum_i \vec{S}_i^2 - \mu \sum_i n_i
\]

**Symmetry constraints**  
\[n_i + S_i = \text{even}\]

---

**Nematic Mott Insulator**

\[
|\Psi\rangle = \prod_i (n_x a_{ix}^\dagger + n_y a_{iy}^\dagger + n_z a_{iz}^\dagger)^N |0\rangle
\]

**Spin Singlet Mott Insulator**

\[
|\Psi\rangle = \prod_i (a_{ix}^\dagger + a_{iy}^\dagger + a_{iz}^\dagger)^{N/2} |0\rangle
\]
Nematic insulating phase for $N=1$

**Effective $S=1$ spin model**  
Imambekov et al., PRA (2003)

$$\mathcal{H} = -J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - J_2 \sum_{\langle ij \rangle} \left( \vec{S}_i \cdot \vec{S}_j \right)^2$$

$$J_1 = \frac{2t^2}{U_0 + U_2} \quad J_2 = \frac{2t^2}{3(U_0 + U_2)} + \frac{4t^2}{3(U_0 - U_2)}$$

When $J_2 > J_1$ the ground state is nematic in $d=2,3$.

$$\langle S_a \rangle = 0 \quad \langle S_a S_b \rangle \neq 0$$

One dimensional systems are dimerized: Rizzi et al., PRL (2005)
SU(N) Magnetism with Ultracold Alkaline-Earth Atoms

A. Gorshkov et al., arXiv:0905.2610

Example: $^{87}\text{Sr}$ ($I = 9/2$)

nuclear spin decoupled from electrons \quad SU(N=2I+1) symmetry

SU(N) spin models

Example: Mott state with $n_A$ atoms in sublattice A and $n_B$ atoms in sublattice B

\[ \mathcal{H} = J \sum_{\langle ij \rangle} S^m_m(i) S^m_n(j) \quad S^m_n = c^\dagger_n c_m \]

Phase diagram for $n_A + n_B = N$

There are also extensions to models with additional orbital degeneracy
Ultracold fermions in optical lattices
Fermionic atoms in optical lattices

\[ \mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i \]

Experiments with fermions in optical lattice, Kohl et al., PRL 2005
Antiferromagnetic and superconducting $T_c$ of the order of 100 K

Same microscopic model

$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c_{i \sigma}^{\dagger} c_{j \sigma} + U \sum_i n_{i \uparrow} n_{i \downarrow} - \mu \sum_i n_i$$
Fermionic Hubbard model
Phenomena predicted

Superexchange and antiferromagnetism (P.W. Anderson)

Itinerant ferromagnetism. Stoner instability (J. Hubbard)

Incommensurate spin order. Stripes (Schulz, Zaannen, Emery, Kivelson, White, Scalapino, Sachdev, …)

Mott state without spin order. Dynamical Mean Field Theory (Kotliar, Georges, …)

d-wave pairing (Scalapino, Pines, …)

d-density wave (Affleck, Marston, Chakravarty, Laughlin, …)
Superexchange and antiferromagnetism in the Hubbard model. Large U limit

Singlet state allows virtual tunneling and regains some kinetic energy

\[ E_S = -\frac{4t^2}{U} \]

Triplet state: virtual tunneling forbidden by Pauli principle

\[ E_T = 0 \]

Effective Hamiltonian: Heisenberg model

\[ \mathcal{H}_{\text{eff}} = J \vec{S}_i \cdot \vec{S}_j \]

\[ J = \frac{4t^2}{U} \]
Hubbard model for small U.
Antiferromagnetic instability at half filling

Fermi surface for $n=1$

Analysis of spin instabilities. Random Phase Approximation

$$\chi_{\text{RPA}}(q, \omega) = \frac{\chi_0(q, \omega)}{1 - U \chi_0(q, \omega)}$$

$$\chi_0(q, \omega) = \sum_p \frac{n_p - n_{p+q}}{\omega - \epsilon_p + \epsilon_{p+q}}$$

$$\chi_0(q, \omega = 0) \sim \frac{1}{t} \log\left(\frac{t}{T}\right)$$

Nesting of the Fermi surface leads to singularity

BCS-type instability for weak interaction

$$T_N \sim t e^{-\frac{t}{\sigma}}$$
Hubbard model at half filling

Paramagnetic Mott phase:
- one fermion per site
- charge fluctuations suppressed
- no spin order

BCS-type theory applies

Heisenberg model applies

$T \sim U$

$T_N \sim t e^{-\frac{t}{\sigma}}$

$T_N \sim \frac{t^2}{U}$
Doped Hubbard model
Attraction between holes in the Hubbard model

Loss of superexchange energy from 8 bonds

Loss of superexchange energy from 7 bonds
Pairing of holes in the Hubbard model

Leading instability: d-wave
Scalapino et al, PRB (1986)
Pairing of holes in the Hubbard model

BCS equation for pairing amplitude

\[ \Delta_k = - \sum_{k'} V_{kk'} \Delta_{k'} \]

\[ V_{kk'} \sim \chi_S(k - k') \]

Systems close to AF instability:

\( \chi(Q) \) is large and positive

\( \Delta_k \) should change sign for \( k' = k + Q \)
Stripe phases in the Hubbard model

Stripes:
Antiferromagnetic domains separated by hole rich regions

Antiphase AF domains stabilized by stripe fluctuations


\[ \mathcal{H} = -t \sum_{\langle ij \rangle} c_{i\sigma}^{\dagger} c_{j\sigma} - \frac{U}{4} \sum_{i} \langle \mathbf{S}_{i} \rangle c_{i\alpha}^{\dagger} \sigma_{\alpha\beta} c_{i\beta} \]
Stripe phases in ladders

t-J model

\[ H_{tJ} = -t \sum_{\langle ij \rangle} P c_{i\sigma}^\dagger c_{j\sigma} P + J \sum_i \vec{S}_i \cdot \vec{S}_j \]

DMRG study of t-J model on ladders
Scalapino, White, PRL 2003
After several decades we do not yet know the phase diagram.

AF – antiferromagnetic
SDW - Spin Density Wave (Incommens. Spin Order, Stripes)
D-SC – d-wave paired
Fermionic Hubbard model

From high temperature superconductors to ultracold atoms

\[ \mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i \]

YBa$_2$Cu$_3$O$_7$

Antiferromagnetic and superconducting Tc of the order of 100 K

Atoms in optical lattice

Antiferromagnetism and pairing at sub-micro Kelvin temperatures
Signatures of incompressible Mott state of fermions in optical lattice

Suppression of double occupancies

Compressibility measurements
Fermions in optical lattice. Next challenge: antiferromagnetic state

\[ T \sim U \]

current experiments

\[ T_N \sim \frac{t^2}{U} \]

\[ T_N \sim t e^{-\frac{t}{\Delta}} \]
Antiferromagnetism beyond Hubbard model

Mathy, Huse 2009

Correction to superexchange from higher bands

Optimizing $T_N$ by changing the lattice height
Lattice modulation experiments with fermions in optical lattice.

Probing the Mott state of fermions
Sensarma, Pekker, Lukin, Demler, PRL 2009

Related theory work: Kollath et al., PRA (2006)
Huber, Ruegg, PRB (2009)
Lattice modulation experiments
Probing dynamics of the Hubbard model

Modulate lattice potential $V_0$

Measure number of doubly occupied sites

$t \sim \exp(-\sqrt{V_0/E_R})$

$U \sim \left(\frac{V_0}{E_R}\right)^{3/4}$

Main effect of shaking: modulation of tunneling

$\mathcal{H}_{\text{pert}}(\tau) = \lambda t \cos \omega \tau \sum_{\langle i,j \rangle \sigma} c_{i \sigma}^+ c_{j \sigma}$

Doubly occupied sites created when frequency $\omega$ matches Hubbard $U$
Lattice modulation experiments
Probing dynamics of the Hubbard model

Mott state

Regime of strong interactions $U \gg t$.

Mott gap for the charge forms at $T \sim U$

Antiferromagnetic ordering at $T_N \sim J = \frac{4t^2}{U}$

“High” temperature regime $T_N << T << U$

All spin configurations are equally likely. Can neglect spin dynamics.

“Low” temperature regime $T \leq T_N$

Spins are antiferromagnetically ordered or have strong correlations
Schwinger bosons and Slave Fermions

\[ c_{i\sigma}^\dagger = a_{i\sigma}^\dagger h_i + \sigma a_{i-\sigma}^\dagger d_i^\dagger \]

Constraint:
\[ a_{i\sigma}^\dagger a_{i\sigma} + d_i^\dagger d_i + h_i^\dagger h_i = 1 \]

Singlet Creation

\[ A_{ij}^\dagger = a_{i\uparrow}^\dagger a_{j\downarrow}^\dagger - a_{i\downarrow}^\dagger a_{j\uparrow}^\dagger \]

Boson Hopping

\[ F_{ij}^\dagger = a_{i\uparrow}^\dagger a_{j\uparrow}^\dagger + a_{i\downarrow}^\dagger a_{j\downarrow}^\dagger \]
Schwinger bosons and slave fermions

Fermion hopping

\[ c_{i\uparrow}^\dagger c_{j\uparrow} + c_{i\downarrow}^\dagger c_{j\downarrow} + \text{h.c.} = (d_{i\uparrow}^\dagger d_{j\downarrow} - h_{i\uparrow}^\dagger h_{j\downarrow}) F_{ij} + d_{i\uparrow}^\dagger h_{j\downarrow}^\dagger A_{ij} + \text{h.c.} \]

Propagation of holes and doublons is coupled to spin excitations. Neglect spontaneous doublon production and relaxation.

Doublon production due to lattice modulation perturbation

\[ \mathcal{H}(\tau) = \lambda t \sin \omega \tau \sum_{\langle ij \rangle} (d_{i\uparrow}^\dagger h_{j\downarrow}^\dagger A_{ij} + \text{h.c.}) \]

Second order perturbation theory. Number of doublons

\[ N_d(\tau) = t^2 \lambda^2 \int_0^T dt' \int_0^T dt'' \sin[\omega t'] \sin[\omega t''] \sum_{\langle ij \rangle \langle lm \rangle} \langle A_{ij}^\dagger(t') d_i(t') h_j(t') h_m^\dagger(t'') d_l^\dagger(t'') A_{lm}(t'') \rangle \]
“Low” Temperature \( T \ll T_N \)

Schwinger bosons Bose condensed

Propagation of holes and doublons strongly affected by interaction with spin waves

Assume independent propagation of hole and doublon (neglect vertex corrections)

Self-consistent Born approximation

Spectral function for hole or doublon

Sharp coherent part: dispersion set by \( J \), weight by \( J/t \)

Incoherent part: dispersion \( 4t \times \text{dimension} \)
Propogation of doublons and holes

Spectral function:
Oscillations reflect shake-off processes of spin waves

Comparison of Born approximation and exact diagonalization: Dagotto et al.

Hopping creates string of altered spins: bound states
“Low” Temperature $T << T_N$

Rate of doublon production

- Sharp absorption edge due to coherent quasiparticles
- Broad continuum due to incoherent part
- Spin wave shake-off peaks
"High" Temperature

Atomic limit. Neglect spin dynamics. All spin configurations are equally likely.

$A_{ij}(t')$ replaced by probability of having a singlet

$$N_d(\tau) = \frac{1}{4} t^2 \lambda^2 \int_0^\tau dt' \int_0^\tau dt'' \sin[\omega t'] \sin[\omega t''] \sum_{\langle ij \rangle \langle lm \rangle} \langle d_i(t') h_j(t') h_m^\dagger(t'') d_l^\dagger(t'') \rangle$$

Assume independent propagation of doublons and holes. Rate of doublon production

$$P_d(\omega) = \frac{\pi^3}{2} t^2 \lambda^2 \sum_{r \delta \delta'} \int d\omega' A^d(r + \delta, \omega') A^h(r + \delta', \omega - U - \omega')$$

$A^{d(h)}$ is the spectral function of a single doublon (holon)
Propogation of doublons and holes

Hopping creates string of altered spins

Retraceable Path Approximation  *Brinkmann & Rice, 1970*

Consider the paths with no closed loops

Spectral Fn. of single hole  Doublon Production Rate  Experiments
Lattice modulation experiments. Sum rule

\[ P_d(\omega) = \frac{\pi^3}{2} t^2 \lambda^2 \sum_{r,\delta,\delta'} \int d\omega' A^d(r + \delta, \omega') A^h(r + \delta', \omega - U - \omega') \]

\( A^{d(h)} \) is the spectral function of a single doublon (holon)

**Sum Rule:**

\[ \int d\omega P_d(\omega) = \frac{\pi^3}{2} t^2 \lambda^2 z \]

**Experiments:**

Most likely reason for sum rule violation: nonlinearity

The total weight does not scale quadratically with \( t \)
Fermions in optical lattice.
Decay of repulsively bound pairs

Ref: N. Strohmaier et al., arXiv:0905.2963
Experiment: T. Esslinger’s group at ETH
Theory: Sensarma, Pekker, Altman, Demler
Fermions in optical lattice.
Decay of repulsively bound pairs

Experiments: N. Strohmaier et. al.
Relaxation of doublon-hole pairs in the Mott state

Energy U needs to be absorbed by spin excitations

- Energy carried by spin excitations
  \[ \sim J = 4t^2/U \]

- Relaxation requires creation of \( \sim U^2/t^2 \) spin excitations

Relaxation rate

\[ W \sim t(t/U)^{U^2/t^2} \]

Very slow, not relevant for ETH experiments
Doublon decay in a compressible state

Excess energy $U$ is converted to kinetic energy of single atoms

Compressible state: Fermi liquid description

Doublon can decay into a pair of quasiparticles with many particle-hole pairs
Doublon decay in a compressible state

Perturbation theory to order \( n = \frac{U}{6t} \)

Decay probability

\[
P \sim \left( \frac{t}{U} \right)^{\text{const} \cdot \frac{U}{6t}} \sim e^{-\text{const} \cdot \frac{U}{6t} \cdot \log\left( \frac{U}{t} \right)}
\]

To calculate the rate: consider processes which maximize the number of particle-hole excitations
Doublon decay in a compressible state

Comparison of approximations

Changes of density around 30%
Why understanding doublon decay rate is important

Prototype of decay processes with emission of many interacting particles.
Example: resonance in nuclear physics: (i.e. delta-isobar)

Analogy to pump and probe experiments in condensed matter systems

Response functions of strongly correlated systems at high frequencies. Important for numerical analysis.

Important for adiabatic preparation of strongly correlated systems in optical lattices
Learning about order from noise

Quantum noise studies of ultracold atoms
Quantum noise

Classical measurement:
collapse of the wavefunction into eigenstates of $x$

$$\langle x \rangle = \int dx \, x \, |\psi(x)|^2$$

$$\langle x^2 \rangle = \int dx \, x^2 \, |\psi(x)|^2$$

$$\ldots$$

$$\langle x^n \rangle = \int dx \, x^n \, |\psi(x)|^2$$

$$\ldots$$

Histogram of measurements of $x$
Probabilistic nature of quantum mechanics

Bohr-Einstein debate on spooky action at a distance

Einstein-Podolsky-Rosen experiment

\[ |S = 0\rangle = |\uparrow\rangle_L |\downarrow\rangle_R - |\downarrow\rangle_L |\uparrow\rangle_R \]

Measuring spin of a particle in the left detector instantaneously determines its value in the right detector.
Aspect’s experiments: tests of Bell’s inequalities

Correlation function

\[ E(\theta_1, \theta_2) = \frac{\langle (I_1^+ - I_1^-)(I_2^+ - I_2^-) \rangle}{\langle (I_1^+ + I_1^-)(I_2^+ + I_2^-) \rangle} \]

Classical theories with hidden variable require

\[ B = E(\theta_1, \theta_2) - E(\theta_1, \theta'_2) + E(\theta'_1, \theta'_2) - E(\theta'_1, \theta_2) \leq 2 \]

Quantum mechanics predicts \( B = 2.7 \) for the appropriate choice of \( \theta \)'s and the state

\[ |\psi\rangle = |+\rangle_L |+\rangle_R + |-\rangle_L |-\rangle_R \]

Hanbury-Brown-Twiss experiments

Classical theory of the second order coherence

\[ \langle I(\vec{r}_1) I(\vec{r}_2) \rangle = A + B \cos \left( (\vec{k} - \vec{k}') (\vec{r}_1 - \vec{r}_2) \right) \]


Measurements of the angular diameter of Sirius
Quantum theory of HBT experiments

For bosons

\[ A = A_1 + A_2 \]

For fermions

\[ A = A_1 - A_2 \]

Glauber, *Quantum Optics and Electronics* (1965)

HBT experiments with matter

Experiments with neutrons

Experiments with electrons

Experiments with 4He, 3He
Westbrook et al., Nature (2007)

Experiments with ultracold atoms
Shot noise in electron transport

Proposed by Schottky to measure the electron charge in 1918

\[ S_\omega = \int \langle \{ \delta I(t), \delta I(0) \} \rangle_+ e^{i\omega t} dt \]

Spectral density of the current noise

Related to variance of transmitted charge

\[ S_0 = \frac{2}{\tau} \langle \delta q^2(\tau) \rangle \]

When shot noise dominates over thermal noise

\[ S_0 = 2eI \]

Poisson process of independent transmission of electrons
Shot noise in electron transport

Current noise for tunneling across a Hall bar on the 1/3 plateau of FQE

Etien et al. PRL 79:2526 (1997)
see also Heiblum et al. Nature (1997)
Quantum noise analysis of time-of-flight experiments with atoms in optical lattices: Hanbury-Brown-Twiss experiments and beyond


Experiment: Folling et al., Nature (2005);
Spielman et al., PRL (2007);
Time of flight experiments

Quantum noise interferometry of atoms in an optical lattice

Second order coherence

\[ G(r_1, r_2) = \langle n(r_1)n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle \]
Second order coherence in the insulating state of bosons. Hanbury-Brown-Twiss experiment

Hanbury-Brown-Twiss stellar interferometer

\[ \langle I(\vec{r}_1) I(\vec{r}_2) \rangle = A + B \cos \left( (\vec{k} - \vec{k}') (\vec{r}_1 - \vec{r}_2) \right) \]
Second order coherence in the insulating state of bosons

Bosons at quasimomentum $\vec{k}$ expand as plane waves with wavevectors $\vec{k}, \vec{k} + \vec{G}_1, \vec{k} + \vec{G}_2$

First order coherence: $\langle \rho(\vec{r}) \rangle$
Oscillations in density disappear after summing over $\vec{k}$

Second order coherence: $\langle \rho(\vec{r}_1) \rho(\vec{r}_2) \rangle$
Correlation function acquires oscillations at reciprocal lattice vectors

$$\langle \rho(\vec{r}_1) \rho(\vec{r}_2) \rangle = A_0 + A_1 \cos \left( \vec{G}_1 (\vec{r}_1 - \vec{r}_2) \right) + A_2 \cos \left( \vec{G}_2 (\vec{r}_1 - \vec{r}_2) \right) + \ldots$$
Second order coherence in the insulating state of bosons. Hanbury-Brown-Twiss experiment

Second order coherence in the insulating state of fermions. Hanbury-Brown-Twiss experiment

How to detect antiferromagnetism
Probing spin order in optical lattices

Correlation Function Measurements

\[ G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle_{TOF} - \langle n(r_1) \rangle_{TOF} \langle n(r_2) \rangle_{TOF} \]
\[ \sim \langle n(k_1) n(k_2) \rangle_{LAT} - \langle n(k_1) \rangle_{LAT} \langle n(k_2) \rangle_{LAT} \]

Extra Bragg peaks appear in the second order correlation function in the AF phase
How to detect fermion pairing

Quantum noise analysis of TOF images is more than HBT interference
Second order interference from the BCS superfluid

Theory: Altman et al., PRA (2004)

\[ \Delta n(r, r') \equiv n(r) - n(r') \]

\[ \Delta n(r, -r) \left| \Psi_{BCS} \right> = 0 \]
Momentum correlations in paired fermions

Greiner et al., PRL (2005)
Fermion pairing in an optical lattice

Second Order Interference
In the TOF images

\[ G(r_1, r_2) = \langle n(r_1)n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle \]

Normal State

\[ G_N(r_1, r_2) = \delta(r_1 - r_2)\rho(r_1) - \rho^2(r_1) \sum_G \delta(r_1 - r_2 - \frac{G \hbar t}{m}) \]

Superfluid State

\[ G_S(r_1, r_2) = G_N(r_1, r_2) + \Psi(r_1) \sum_G \delta(r_1 + r_2 + \frac{G \hbar t}{m}) \]

\[ \Psi(r) = |u(Q(r))v(Q(r))|^2 \] measures the Cooper pair wavefunction

\[ Q(r) = \frac{mr}{\hbar t} \] One can identify unconventional pairing
Interference experiments with cold atoms
Interference of independent condensates


Theory: Javanainen, Yoo, PRL (1996)
Castin, Dalibard, PRA (1997)
and many more
Interference of two independent condensates

\[ \psi(r) = \psi_1(r) + \psi_2(r) \]

\[ \rho_{\text{int}}(r) = \psi_1^\dagger(r) \psi_2(r) + \text{c.c.} \]

\[ \psi_1(r) = e^{i\phi_1 + i k_1 r} \quad k_1 = \frac{m v_1}{\hbar} = \frac{m r}{\hbar t} \]

\[ \psi_2(r) = e^{i\phi_2 + i k_2 r} \quad k_2 = \frac{m v_2}{\hbar} = \frac{m (r + d)}{\hbar t} \]

\[ \rho_{\text{int}}(r) = e^{i(k_2 - k_1)r} e^{i(\phi_2 - \phi_1)} + \text{c.c.} \]

\[ \rho_{\text{int}}(r) = e^{i \frac{m d r}{\hbar t}} e^{i(\phi_2 - \phi_1)} + \text{c.c.} \]

Clouds 1 and 2 do not have a well defined phase difference. However each individual measurement shows an interference pattern

\[ \langle \rho_{\text{int}}(r) \rangle = 0 \]

\[ \langle \rho_{\text{int}}(r) \rho_{\text{int}}(r') \rangle = e^{i \frac{m d}{\hbar t} (r - r')} + \text{c.c.} \]
Experiments with 2D Bose gas  

Experiments with 1D Bose gas  
Interference of fluctuating condensates

Polkovnikov, Altman, Demler, PNAS (2006)

Amplitude of interference fringes,

\[ |A_{fr}| e^{i\Delta \phi} = \int_0^L dx \ e^{i(\phi_1(x)-\phi_2(x))} \]

For independent condensates \( A_{fr} \) is finite but \( \Delta \phi \) is random

\[ \langle |A_{fr}|^2 \rangle = \int_0^L dx_1 \int_0^L dx_2 \ \langle e^{i(\phi_1(x_1)-\phi_2(x_1))} e^{-i(\phi_1(x_2)-\phi_2(x_2))} \rangle \]

\[ \langle |A_{fr}|^2 \rangle \approx L \int_0^L dx \ \langle e^{i(\phi(x)-\phi(0))} \rangle \langle e^{-i(\phi(x)-\phi(0))} \rangle \]

For identical condensates

\[ \langle |A_{fr}|^2 \rangle = L \int_0^L dx \ (G(x))^2 \]

Instantaneous correlation function

\[ G(x) = \langle e^{i\phi(x)} e^{-\phi(0)} \rangle \]
Fluctuations in 1d BEC

Thermal fluctuations

Thermally energy of the superflow velocity

\[
\langle e^{i\phi(x_1)} e^{i\phi(x_2)} \rangle \sim e^{-|x_1 - x_2|/\xi_T}
\]

\[
\xi_T = \sqrt{\frac{\hbar^2 m}{T}}
\]

Quantum fluctuations

\[
\langle e^{i\phi(x_1)} e^{i\phi(x_2)} \rangle \sim \left(\frac{\xi_h}{|x_1 - x_2|}\right)^{1/2K}
\]

\[
K = \sqrt{\frac{n}{g m}}
\]
Interference between Luttinger liquids

Luttinger liquid at $T=0$

$$\langle |A_{fr}|^2 \rangle \sim (\rho \xi_h)^{1/K} (L\rho)^{2-1/K}$$

$K$ – Luttinger parameter

For non-interacting bosons

$K = \infty$ and $A_{fr} \sim L$

For impenetrable bosons

$K = 1$ and $A_{fr} \sim \sqrt{L}$

Finite temperature

Experiments: Hofferberth, Schumm, Schmiedmayer

$$\langle |A_{fr}|^2 \rangle \sim L \rho^2 \xi_h \left( \frac{\hbar^2}{m \xi_h^2} \frac{1}{T} \right)^{1-1/K}$$

$n_{1d} = 60 \mu m^{-1}$

$K = 47$

$T_{fit} = 84 \pm 22$ nK
Distribution function of fringe amplitudes for interference of fluctuating condensates

\[ A_{fr} \] is a quantum operator. The measured value of \[ |A_{fr}| \] will fluctuate from shot to shot.

\[
\langle |A_{fr}|^{2n} \rangle = \int_0^L dz_1 \ldots dz_n \left| \langle e^{i\phi(z_1)} \ldots e^{i\phi(z_n)} e^{-i\phi(z_1')} \ldots e^{-i\phi(z_n')} \rangle \right|^2
\]

Higher moments reflect higher order correlation functions.

We need the full distribution function of \[ |A_{fr}| \]
Distribution function of interference fringe contrast

Theory: Imambekov et al., PRA 2008
Experiments: Hofferberth et al., Nature Physics 2008

Comparison of theory and experiments: no free parameters
Higher order correlation functions can be obtained

Quantum fluctuations dominate:
asymmetric Gumbel distribution
(low temp. $T$ or short length $L$)

Thermal fluctuations dominate:
broad Poissonian distribution
(high temp. $T$ or long length $L$)

Intermediate regime:
double peak structure
Interference between interacting 1d Bose liquids. Distribution function of the interference amplitude

Distribution function of $|A_{fr}|$

Quantum impurity problem: interacting one dimensional electrons scattered on an impurity

Conformal field theories with negative central charges: 2D quantum gravity, non-intersecting loop model, growth of random fractal stochastic interface, high energy limit of multicolor QCD, …
Fringe visibility and statistics of random surfaces

Distribution function of $|A_{fr}|$

Mapping between fringe visibility and the problem of surface roughness for fluctuating random surfaces. Relation to 1/f Noise and Extreme Value Statistics

Roughness $= \int h(\varphi)^2 d\varphi$
Many-body systems in the presence of external noise

Nonequilibrium critical state

E. Dalla Torre, E. Altman, T. Giamarchi, E. Demler
New systems more prone to external disturbance

Ultracold polar molecules

Trapped ions

(from NIST group)
Linear ion trap

Linear coupling to the noise:

\[ H = H_{Coul} - Q \int dx \, \delta V(x, t) \hat{\rho}(x, t) \]
Ultra cold polar molecules

Polarizing electric field: \[ E(x, t) = E_0 + \delta E(x, t) \]

\[ H = H_0 - \alpha_m \int dx \, \delta E(x, t) \hat{\rho}(x, t) \]

Molecule polarizability

System is subject to electric field noise from the electrodes!
Measured noise spectrum in ion trap

From dependence of heating rate on trap frequency.

Monroe group, PRL (06), Chuang group, PRL (08)

- Direct evidence that noise spectrum is $1/f$
- Short range spatial correlations ($\sim$ distance from electrodes)

$$\langle \delta E_{q\omega}^* \delta E_{q\omega} \rangle \approx F_0/\omega$$
Question:

What happens to low dimensional quantum systems when they are subjected to external non-equilibrium noise?

One dimensional Luttinger state can evolve into a **new critical state**. This new state has intriguing interplay of quantum critical and external noise driven fluctuations.
A brief review:
Universal long-wavelength theory of 1D systems
Haldane (81)

Long wavelength density fluctuations (phonons):
\[ \delta \rho_0(x) = \frac{1}{\pi} \partial_x \phi(x) \]

\[ S_0 = \frac{K}{2} \int dx d\tau \left [ (\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right ] \]

Weak interactions: \( K >> 1 \)
Hard core bosons: \( K = 1 \)
Strong long range interactions: \( K < 1 \)
1D review cont’d: Wigner crystal correlations

\[ S_0 = \frac{K}{2} \int dx d\tau \left[ (\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right] \]

Wigner crystal order parameter:

\[ \langle \delta \rho_{2\pi/l} \rangle = \langle \cos(2\phi(x)) \rangle = 0 \]

\[ \langle \cos(2\phi(x)) \cos(2\phi(0)) \rangle \sim \frac{1}{x^{2K}} \]

No crystalline order!

Scale invariant critical state (Luttinger liquid)
Long wavelength description of noisy low D systems
Effective coupling to external noise

The “backscattering” $\zeta$ can be neglected if the distance to the noisy electrode is much larger than the inter-particle spacing.
Effective harmonic theory of the noisy system

(Quantum) Langevin dynamics:

\[ K^{-1} \left( \partial_t^2 \phi - \partial_x^2 \phi \right) + \eta \partial_t \phi = \xi(x, t) + \partial_x f(x, t) \]

\[ \langle \xi^{*}_{q,\omega} \xi_{q,\omega} \rangle = \eta \omega \coth \left( \frac{\omega}{2T} \right) \]

\[ \langle f^{*}_{q,\omega} f_{q,\omega} \rangle = F(q, \omega) \]

Dissipative coupling to bath needed to ensure steady state (removes the energy pumped in by the external noise)

Implementation of bath: continuous cooling

Thermal bath

External noise
Effective coupling to external noise

\[ \langle \cos(2\phi(x)) \cos(2\phi(0)) \rangle \sim \left( \frac{1}{x} \right)^{2K(1+F_0/\eta)} \]

- Decay of crystal correlations remains power-law.
- Decay exponent tuned by the $1/f$ noise power.

Novel phase transitions tuned by a competition of noise and quantum fluctuations
Global phase diagram

Inter-tube tunneling

Inter-tube interactions

Both perturbations

2D superfluid

1D critical

2D crystal

Critical state
Summary

Experiments with ultracold atoms provide a new perspective on the physics of strongly correlated many-body systems. They pose new questions about new strongly correlated states, their detection, and nonequilibrium many-body dynamics.