Understanding quantum dynamics with local probes

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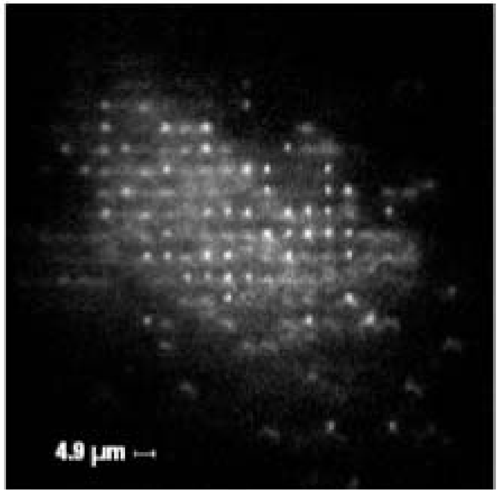
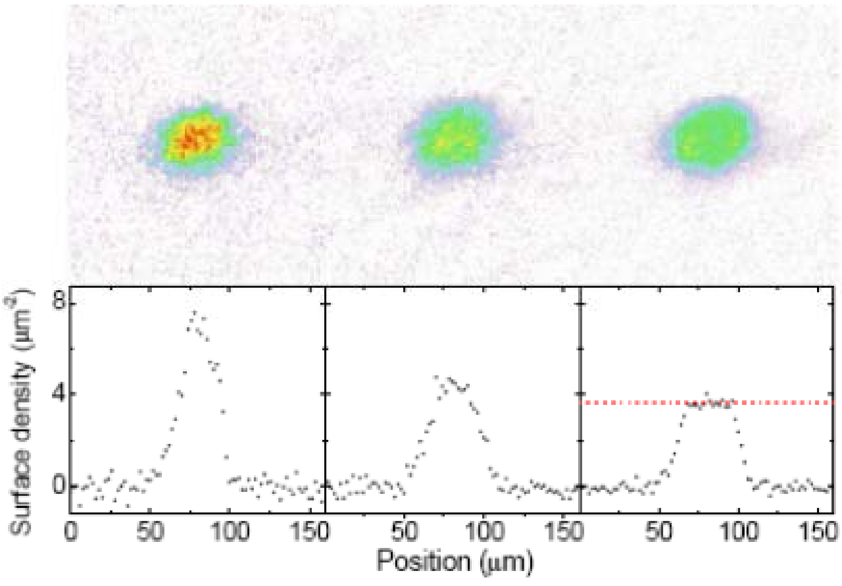
Funded by NSF, Harvard-MIT CUA, AFOSR, DARPA, MURI

Local resolution in optical lattices



Nelson et al., Nature 2007

Imaging single atoms in an optical lattice



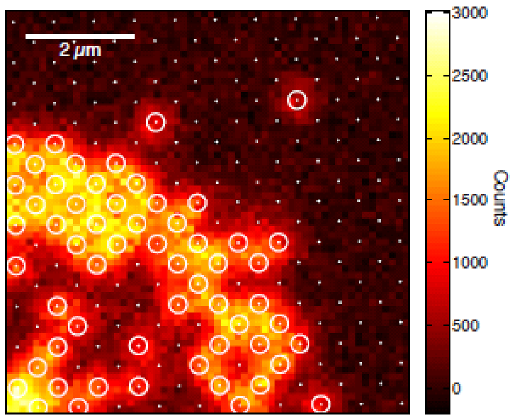
Gemelke et al., Nature 2009

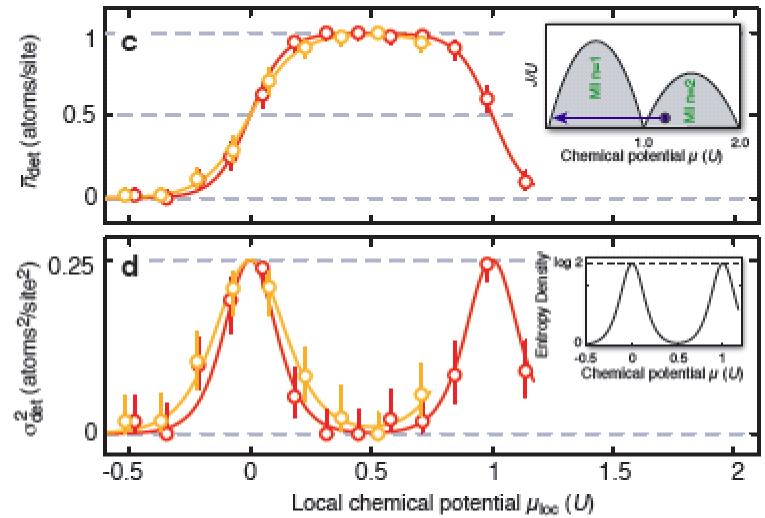
Density profiles in optical lattice: from superfluid to Mott states

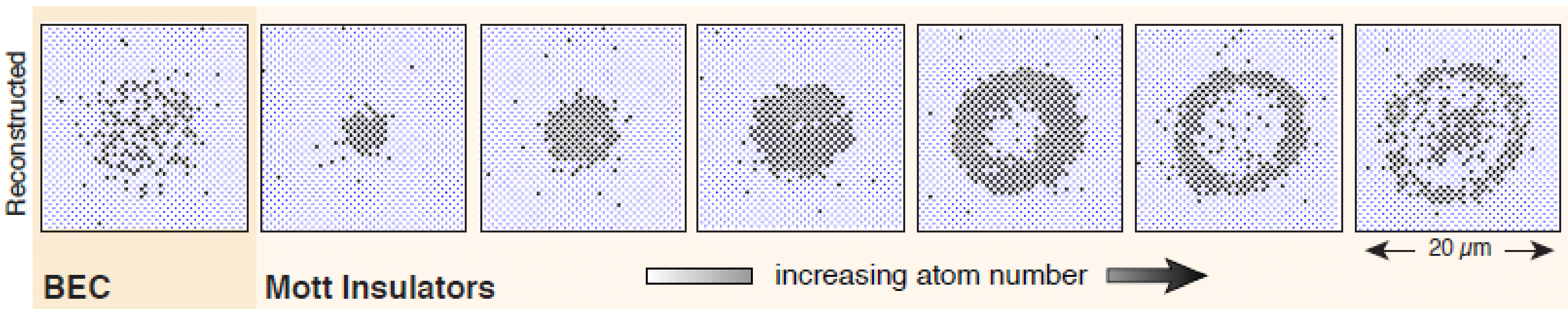
Local picture of SF/Mott shells

J. Sherson et al., Nature (2010)

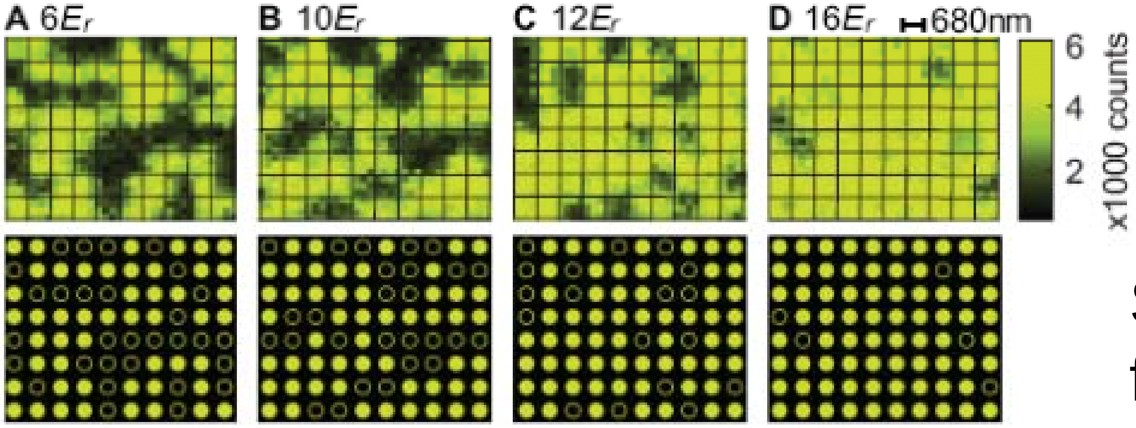
also W. Bakr et. al. Science (2010)





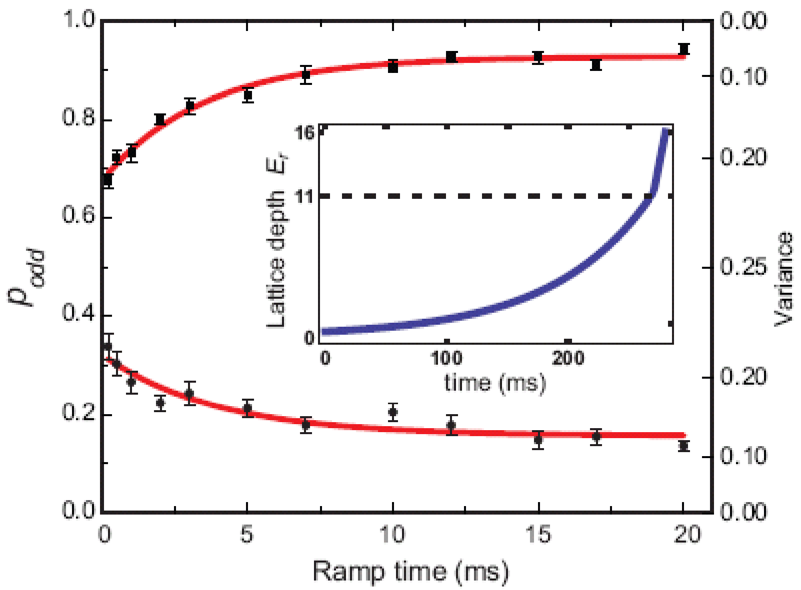


Dynamics and local resolution in systems of ultracold atoms



Bakr et al., Science 2010

Single site imaging from SF to Mott states



Dynamics of on-site number statistics for a rapid SF to Mott ramp

Outline

Observation of prethermalization in quantum dynamics of split one dimensional condensates

Formation of soliton structures in
the dynamics of lattice bosons. Universal
dynamical diagram of relaxation in the
2d XXZ model

Quantum dynamics of split one dimensional condensates

Theory: T. Kitagawa et al., Phys. Rev. Lett. 104:255302 (2010)
Expts: D. Smith et al., arXiv:1104.5631

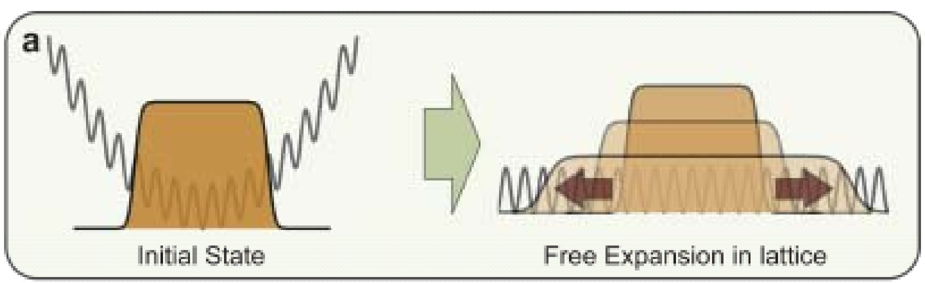
Relaxation to equilibrium

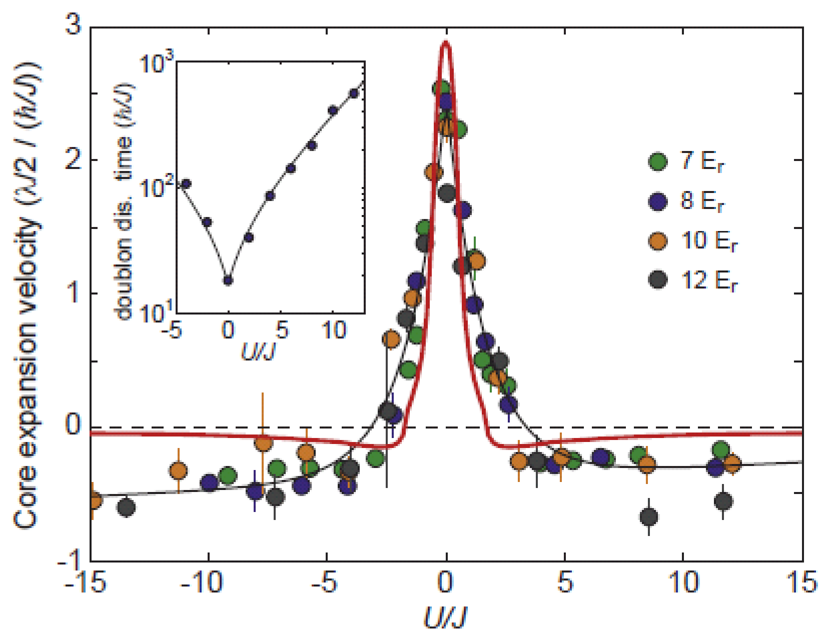
Thermalization: an isolated interacting systems approaches thermal equilibrium (typically at microscopic timescales). All memory about the initial conditions except energy is lost.

Bolzmann equation

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \vec{x}}\vec{v} + \frac{\partial f}{\partial \vec{x}}\vec{F} = -\frac{1}{\tau}(f - f_0)$$

U. Schneider et al., 2010

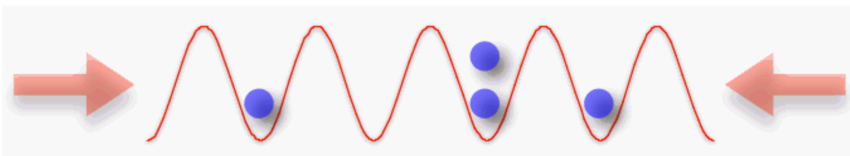


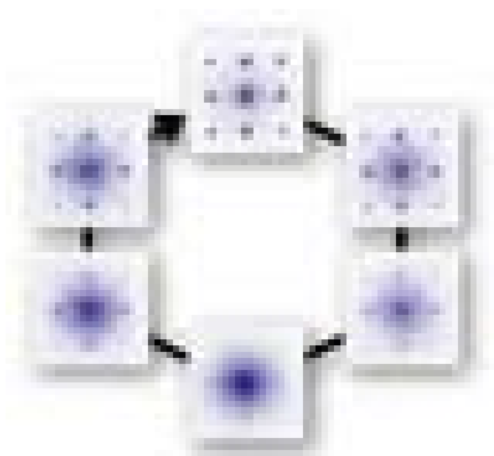


Absence of relaxation

Collapse and revival in Opical Lattice

M. Greiner, I. Bloch, et al.

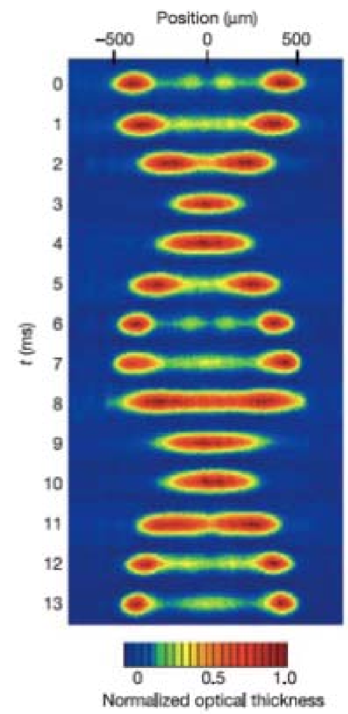




Finite system

Quantum Newton's craddle

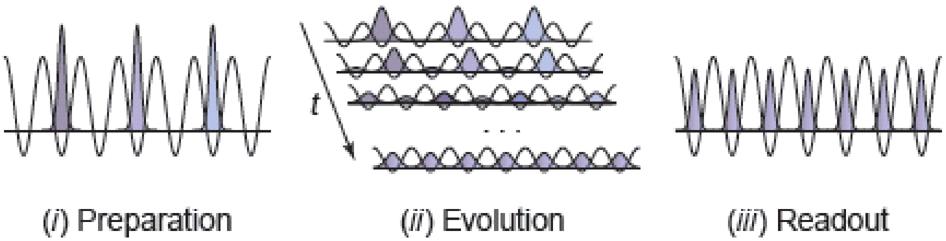
D. Weiss et al.

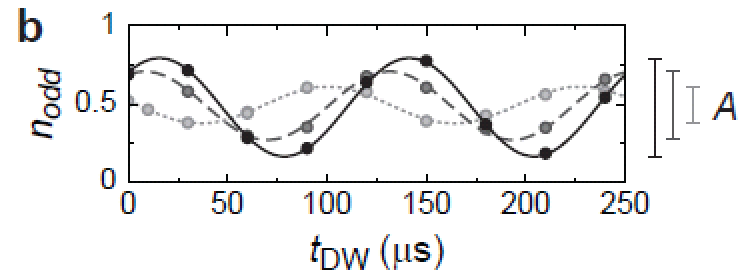


Integrable system

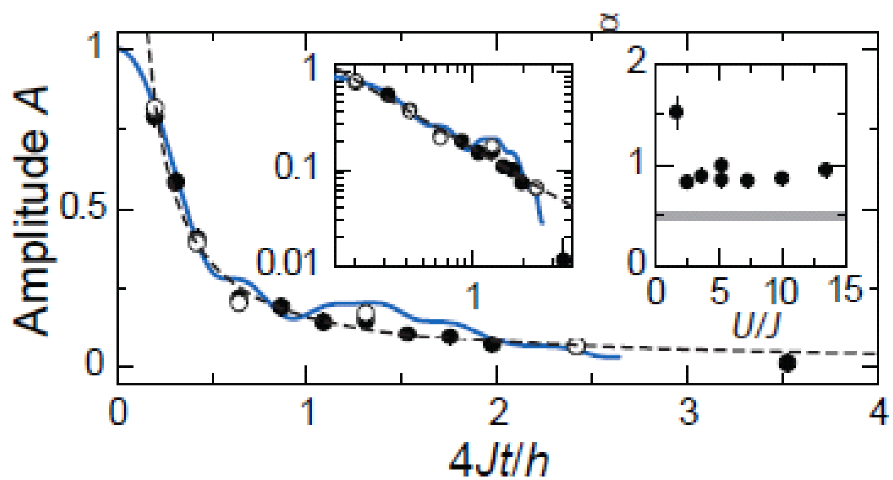
Relaxation at intermediate timescales (?)

S. Trotzky et al, arXiv:1101.2659





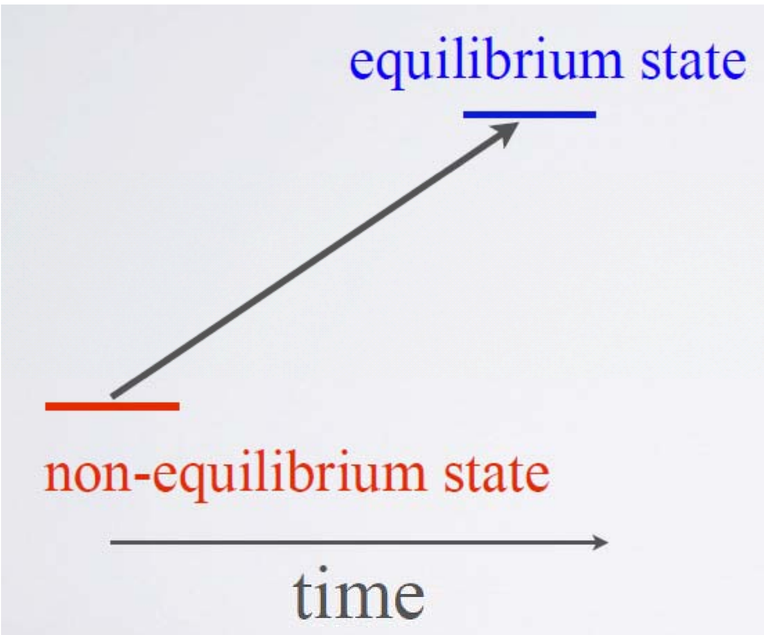
On short timescales, $0 < 4Jt/h < 3$, we find the decay of the amplitude to follow an approximate power-law $\propto t^{-\alpha}$ with $\alpha = 0.86(7)$.



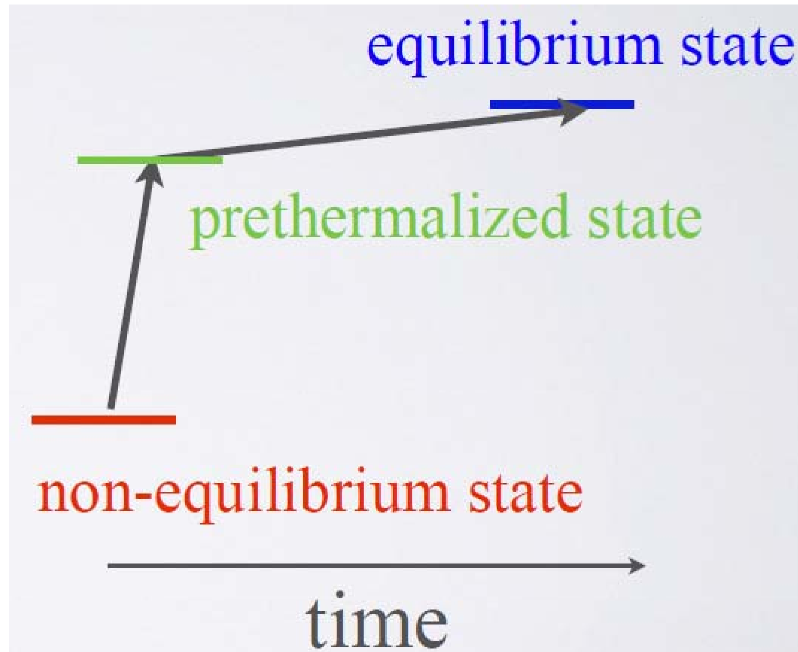
In all cases, the absolute values of the coefficients are larger than the one expected for free particles, where $\alpha = 0.5$, again indicating the faster relaxation in the presence of interactions.

Prethermalization

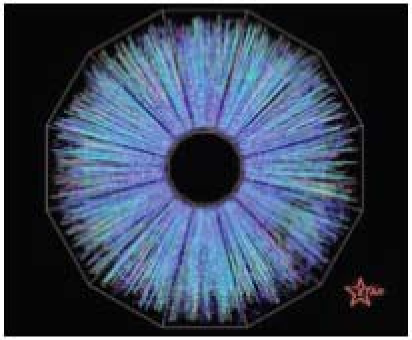
Standard scenario



Prethermalization



Prethermalization



Heavy ions collisions
QCD

PHYSICAL REVIEW D, VOLUME 60, 105026

Time evolution of correlation functions and thermalization

Gian Franco Bonini* and Christof Wetterich†

VOLUME 93, NUMBER 14 PHYSICAL REVIEW LETTERS week ending 1 OCTOBER 2004

Prethermalization

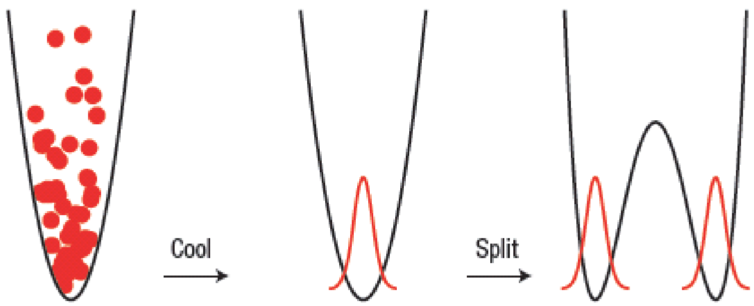
J. Berges, Sz. Borsányi, and C. Wetterich
Institute for Theoretical Physics, Heidelberg University, Philosophenweg 16, 69120 Heidelberg, Germany
(Received 6 April 2004; published 28 September 2004)

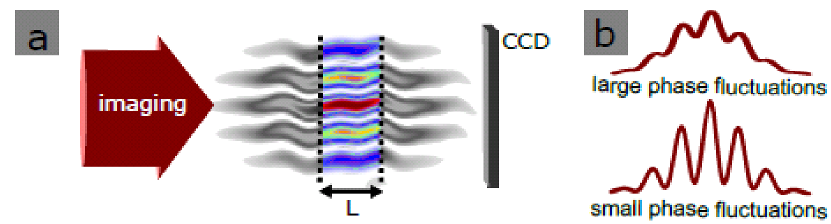
We observe irreversibility and approximate thermalization. At large time the system approaches stationary solution in the vicinity of, but not identical to, thermal equilibrium. The ensemble therefore retains some memory beyond the conserved total energy…This holds for interacting systems and in the large volume limit.

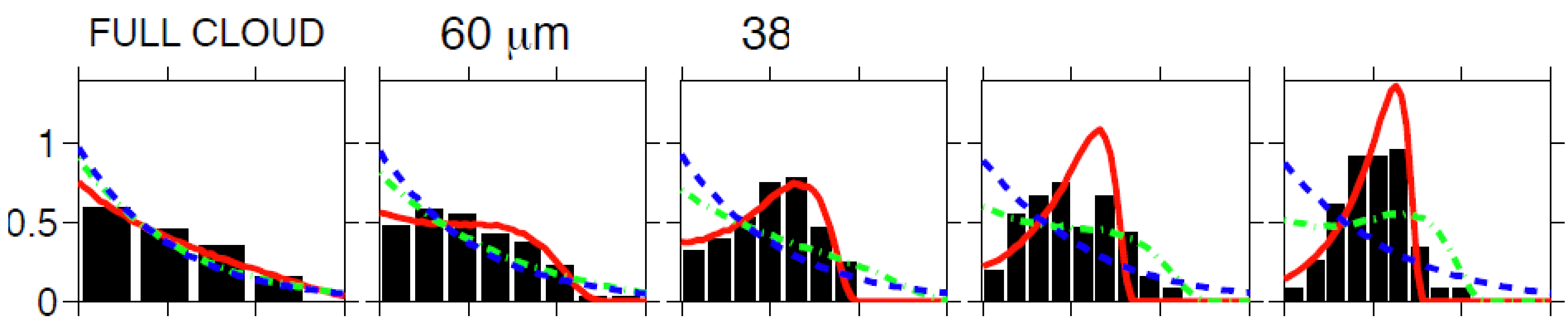
Prethermalization in ultracold atoms, theory: Eckstein et al. (2009); Moeckel et al. (2010), L. Mathey et al. (2010), R. Barnett et al.(2010)

First experimental demonstration of prethermalization

Probing thermolization not only in the average value
but in the complete quantum noise





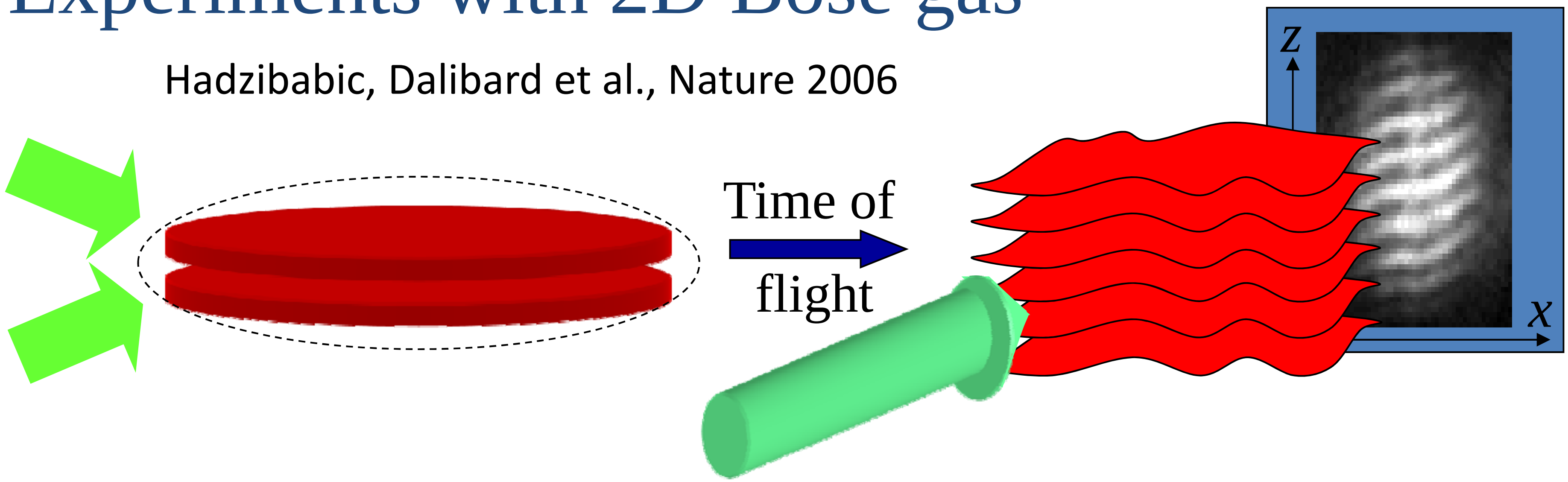


Initial T=120 nK (blue line). After 27.5 ms identical to thermal system at T= 15 nK

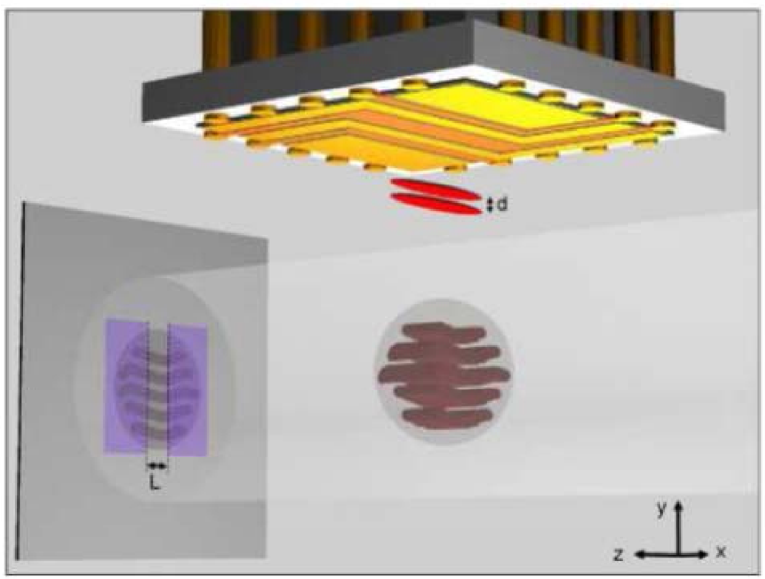
Interference of low dimensional condensates

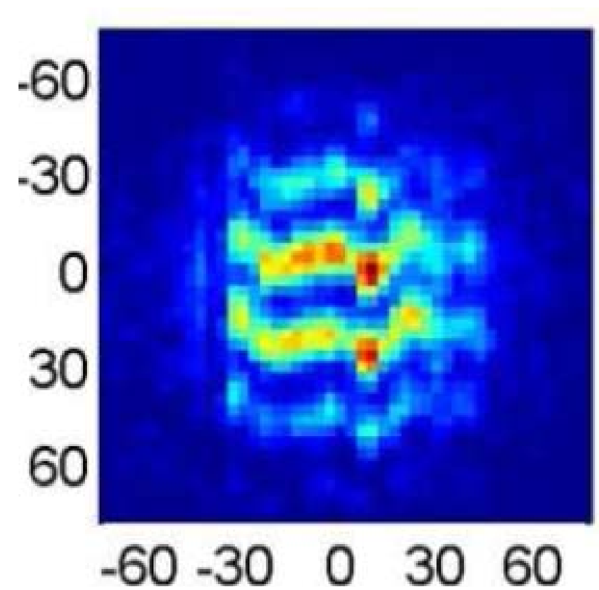
Experiments with 2D Bose gas

Hadzibabic, Dalibard et al., Nature 2006

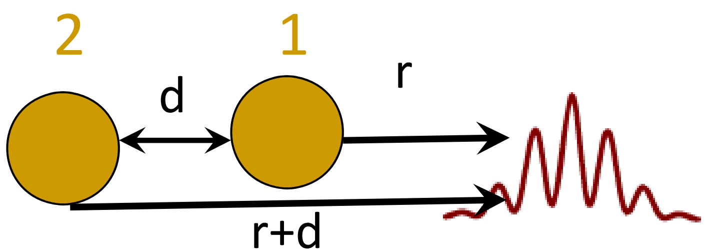


Experiments with 1D Bose gas Hofferberth et al., Nat. Physics 2008





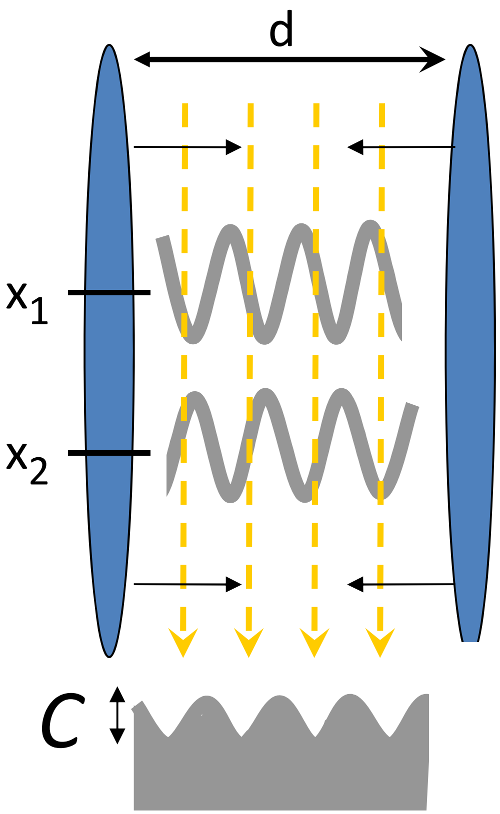
Interference experiments with condensates



$$\psi(r) = \psi_1(r) + \psi_2(r)$$

$$\rho_{\text{int}}(r) = e^{i\,\frac{m\,d\,r}{\hbar\,t}}\, e^{i\,(\phi_2 - \phi_1)} + \text{c.c.}$$

Assuming ballistic expansion



Interference of fluctuating condensates

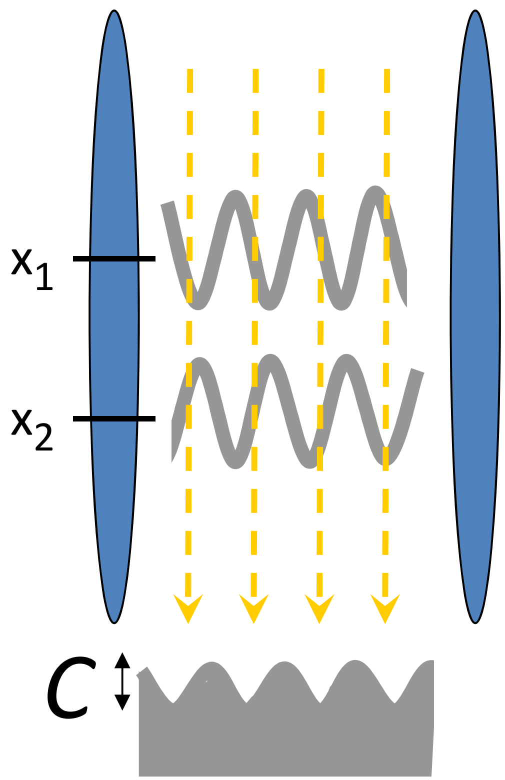
Polkovnikov et al. (2006)

Amplitude of interference fringes

$$C = \int_0^L dx\, e^{i(\phi_1(x) - \phi_2(x))}$$

Distribution function of fringe amplitudes for interference of fluctuating condensates

Polkovnikov et al. (2006), Gritsev et al. (2006), Imambekov et al. (2007)



C is a quantum operator.

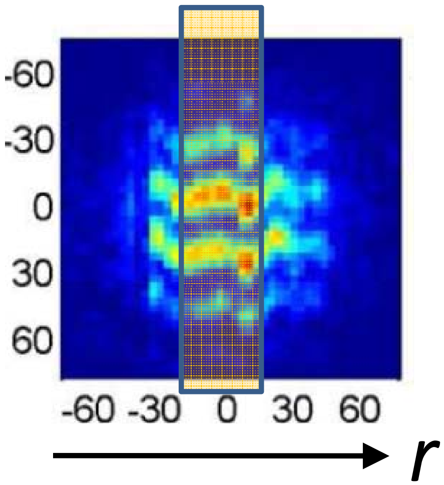
The measured value of C will fluctuate from shot to shot

$$< C^n > = \int dz_1 \cdots dz_n \langle e^{i\phi(z_1)} \ldots e^{i\phi(z_n)} \rangle$$

Higher moments reflect higher order correlation functions

Experiments analyze distribution function of C

Measurements of distribution functions of interference amplitude

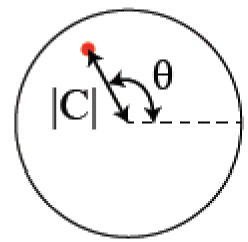


Integrated interference

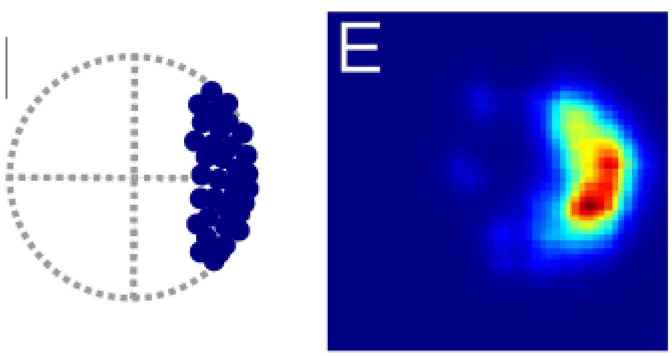
$$C = \int_{-l/2}^{l/2} e^{-i(\phi_1(r)-\phi_2(r))} dr$$

$$C = |C| e^{i\theta}$$

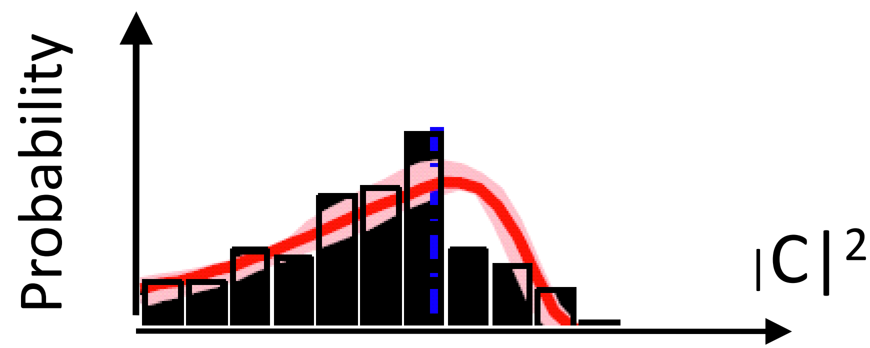
Each experiment measures



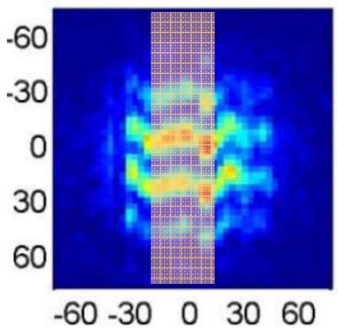
Accumulate statistics



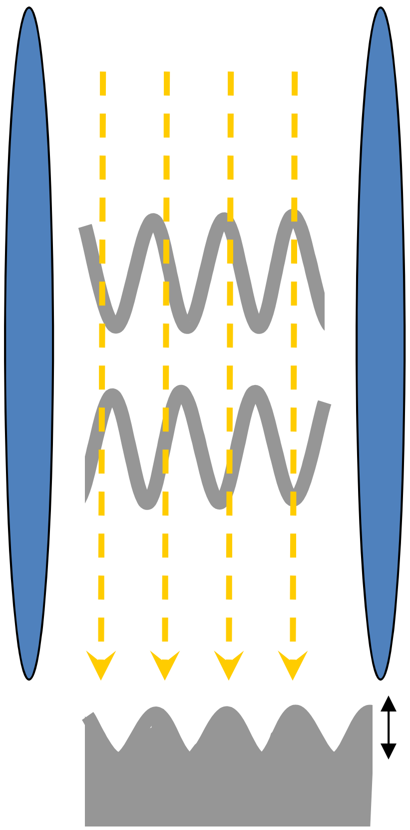
Distribution function of contrast



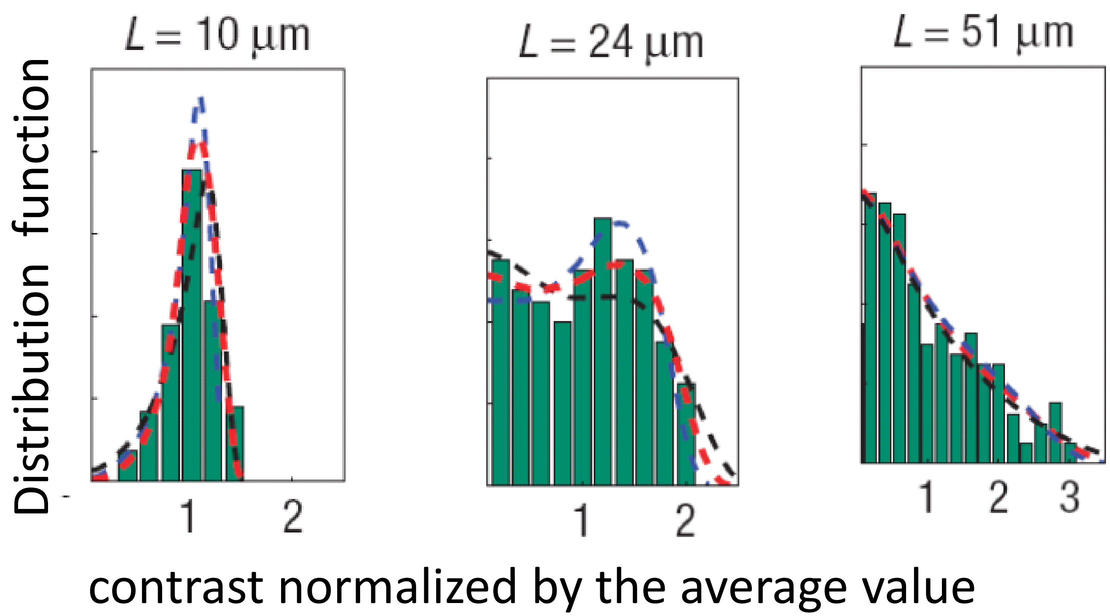
Equilibrium. Interference of independent 1d condensates



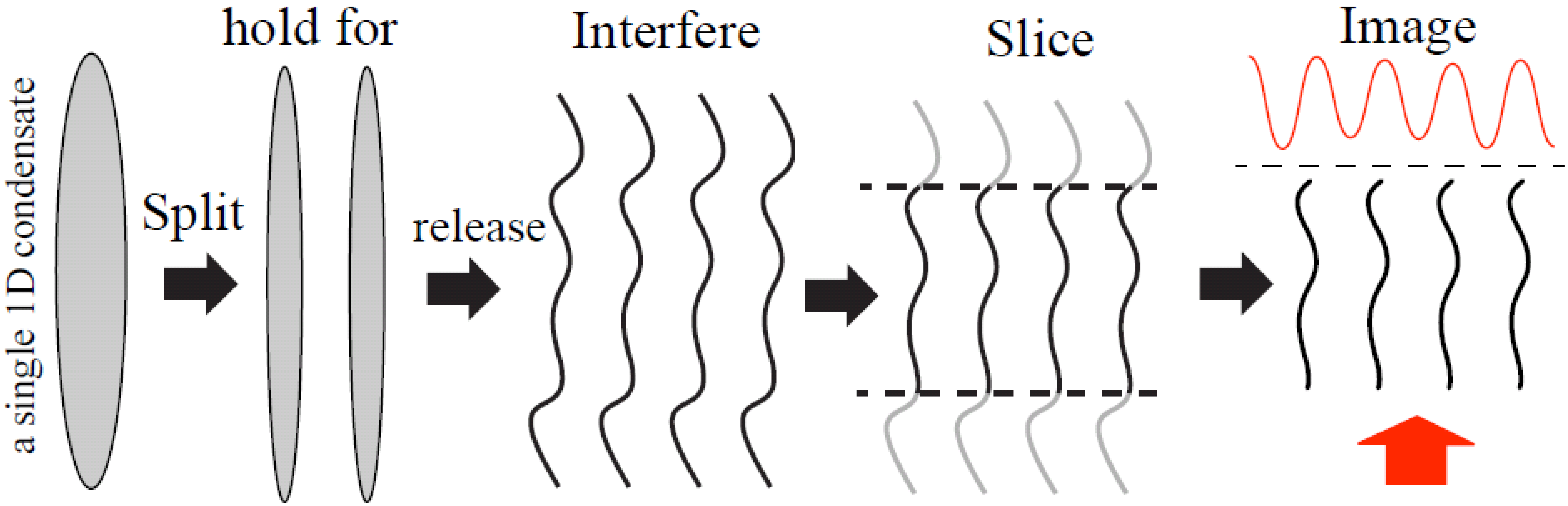
Theory: Altman, Imambekov, Gritsev, Polkovnikov, Demler
Experiments: Hofferberth et al. (2008)

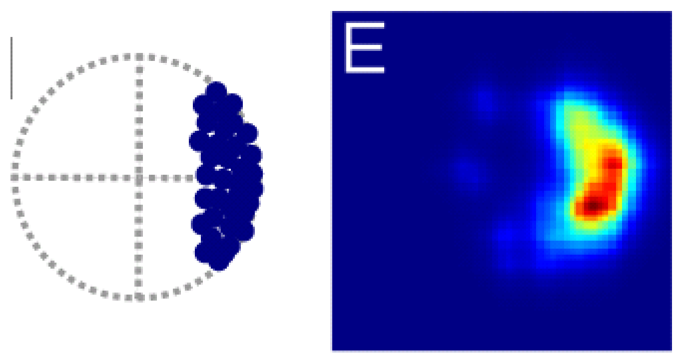


fringe contrast



Measurements of dynamics of split condensate

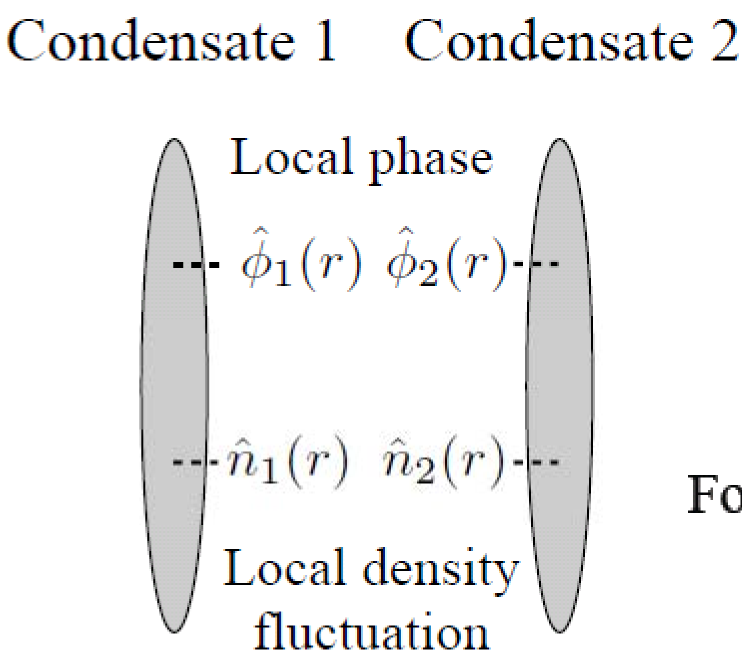




Theoretical analysis of dephasing

Luttinger liquid model

Luttinger liquid model of phase dynamics



$$\hat{\phi}_s(r) = \hat{\phi}_1(r) - \hat{\phi}_2(r)$$

$$2\hat{n}_s(r) = \hat{n}_1(r) - \hat{n}_2(r)$$

$$[\hat{n}_s(r), \hat{\phi}_s(r')] = -i\delta(r - r')$$

For identical average densities, phase difference modes decouple from the phase sum mode

$$H_s = \frac{c_s}{2} \int \left[\frac{K_s}{\pi} (\nabla \hat{\phi}_s(r))^2 + \frac{\pi}{K_s} \hat{n}_s^2(r) \right] dr$$

Luttinger liquid model of phase dynamics

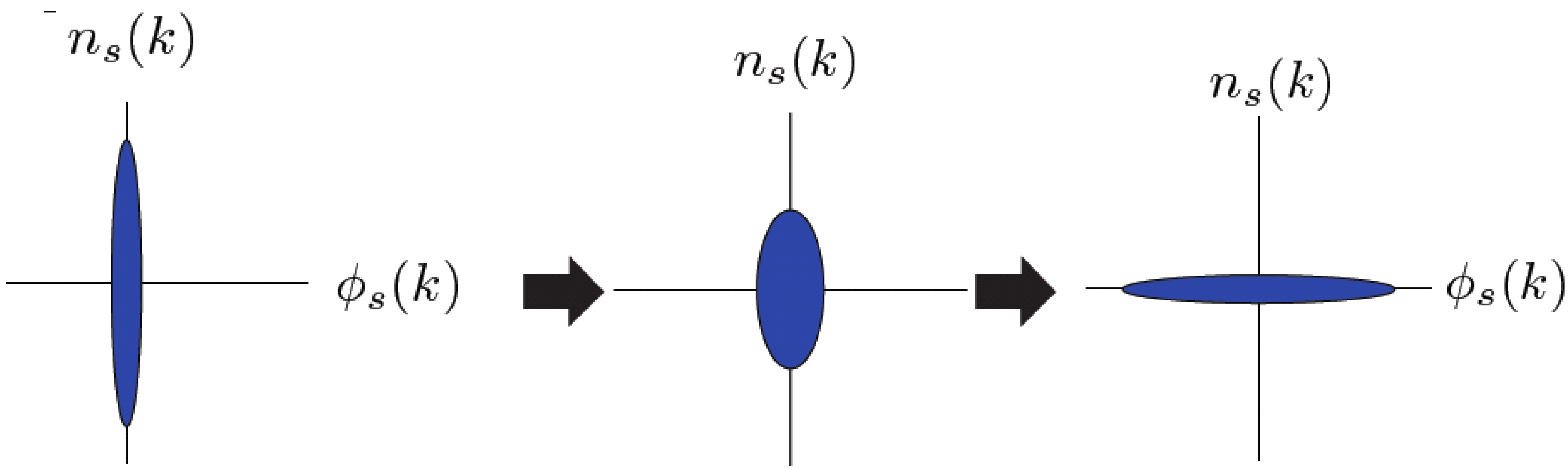
$$H_s = \frac{c_s}{2}\int \left[\frac{K_s}{\pi}(\nabla\hat{\phi}_s(r))^2 + \frac{\pi}{K_s}\hat{n}_s^2(r) \right] dr$$

$$= \frac{c_s}{2}\sum_k \left[\frac{K_s k^2}{\pi}\hat{\phi}_s^\dagger(k)\hat{\phi}_s(k) + \frac{\pi}{K_s}\hat{n}_s^\dagger(k)\hat{n}_s(k) \right]$$

For each k-mode we have simple harmonic oscillators

Fast splitting prepares states with small fluctuations of relative phase

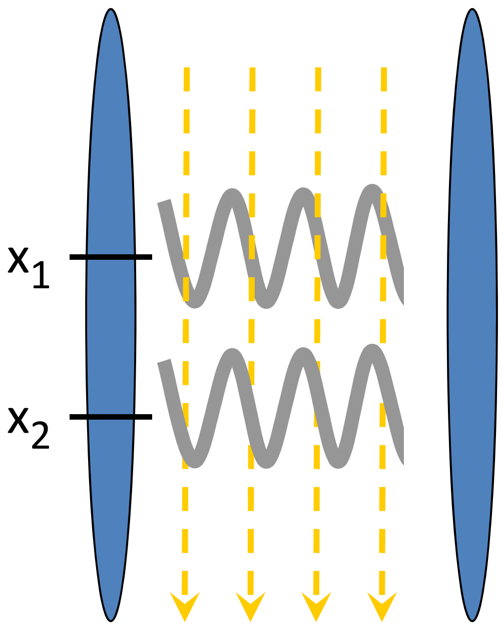
Time evolution



Phase relaxation

Bistrizer, Altman (2007)

Initial state



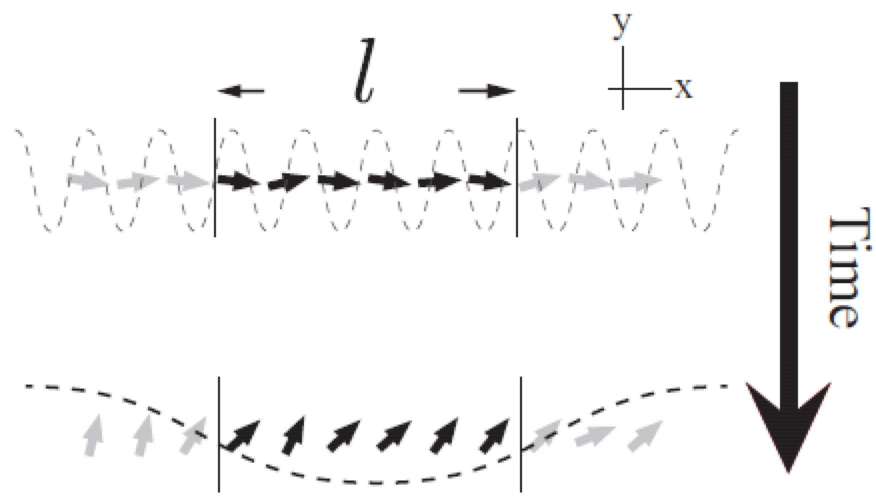
$$\langle \phi_s(x)\, \phi_s(x') \rangle = \phi_0^2\, \delta(x - x')$$

Coarse-graining on the scale of ξ is implied

$$\phi_0^2 = 1/\rho$$

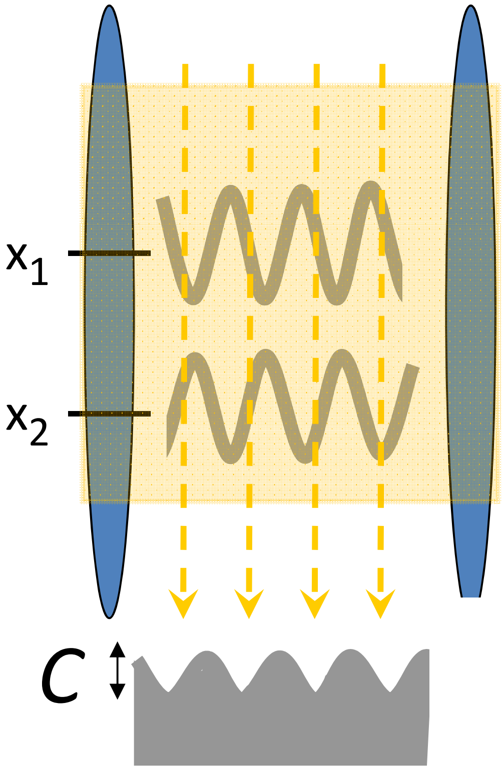
$$\langle (N_1 - N_2)^2 \rangle = N_{tot}$$

Phase dynamics



Short wavelength modes have faster dynamics

Experimental observables



Amplitude of interference fringes

$$C = |C| e^{i\phi}$$

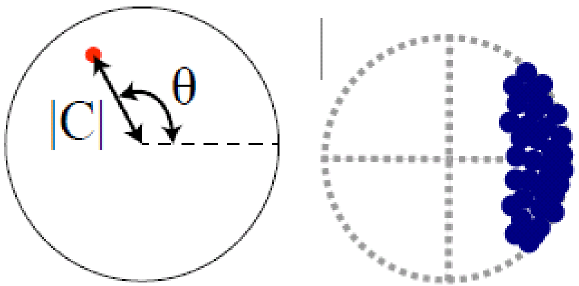
Result of averaging interference amplitudes over many shots

$$\langle C \rangle = e^{-\frac{1}{2} \sum_k \langle \phi_k^2(t) \rangle}$$

Contrast of individual shots $<|C|^2>$ depends on

$$\delta\phi_l = \phi_s(l) - \phi_s(0)$$

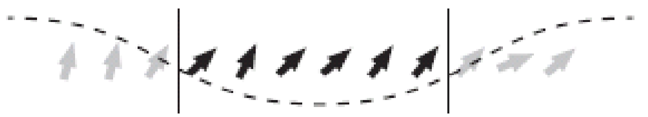
$$\langle \delta\phi_l^2(t) \rangle = \sum_k \langle \phi_k^2(t) \rangle \sin^2 kl$$

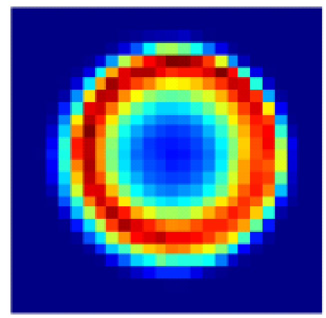


Reduction of average amplitude comes from modes at all wavevectors (uncertainty in both the amplitude and the overall phase). Reduction of contrast comes only from modes with $\lambda > l$

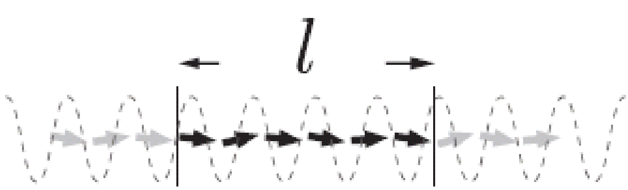
Phase diffusion vs Contrast Decay

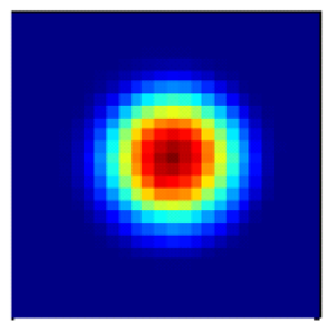
When $\delta\phi_l \leq 2\pi$ we have phase diffusion





When $\delta\phi_l > 2\pi$ we have decay of contrast



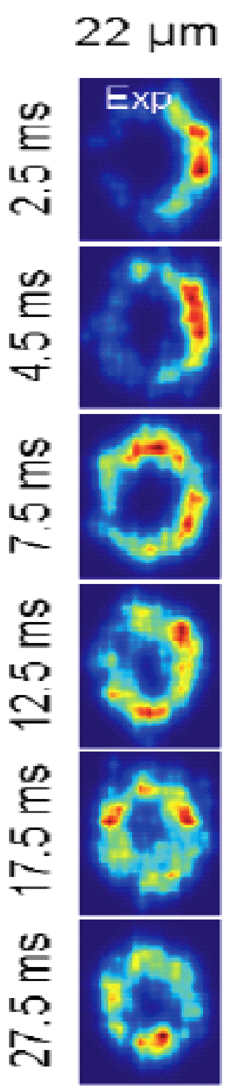


At long times the difference between the two regime occurs for

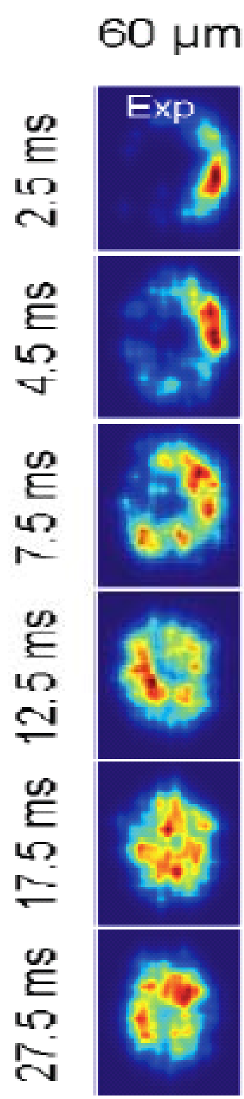
$$l_0 = \frac{8K^2}{\pi^2 \rho}$$

Length dependent phase dynamics

“Short” = phase diffusion

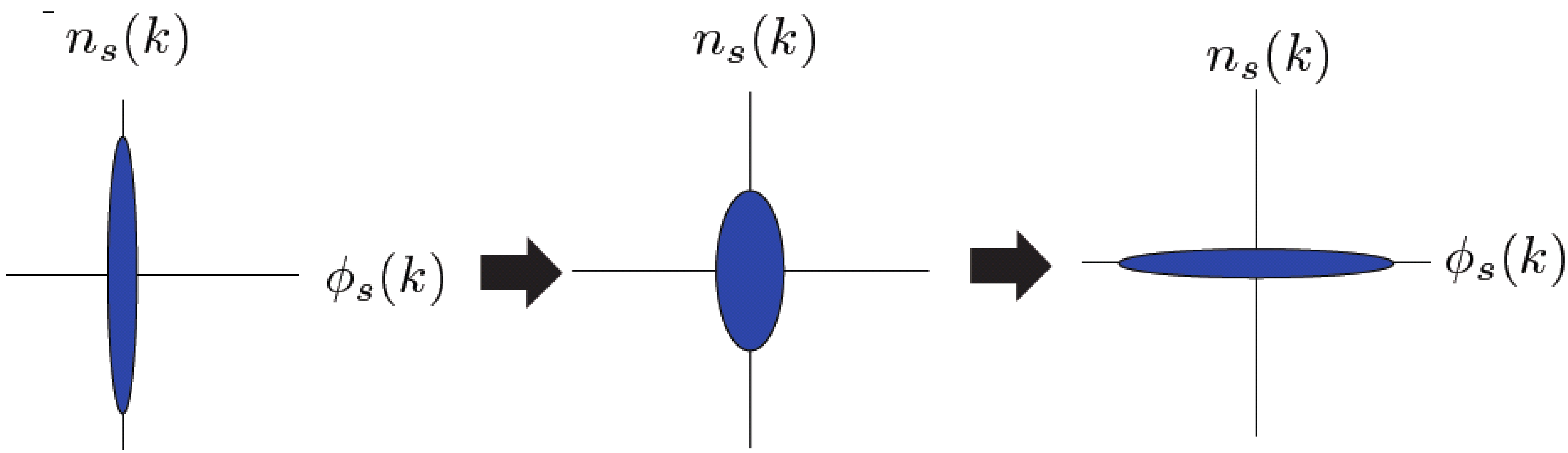


“Long” = contrast decay



Energy distribution

Initially the system is in a squeezed state with large number fluctuations



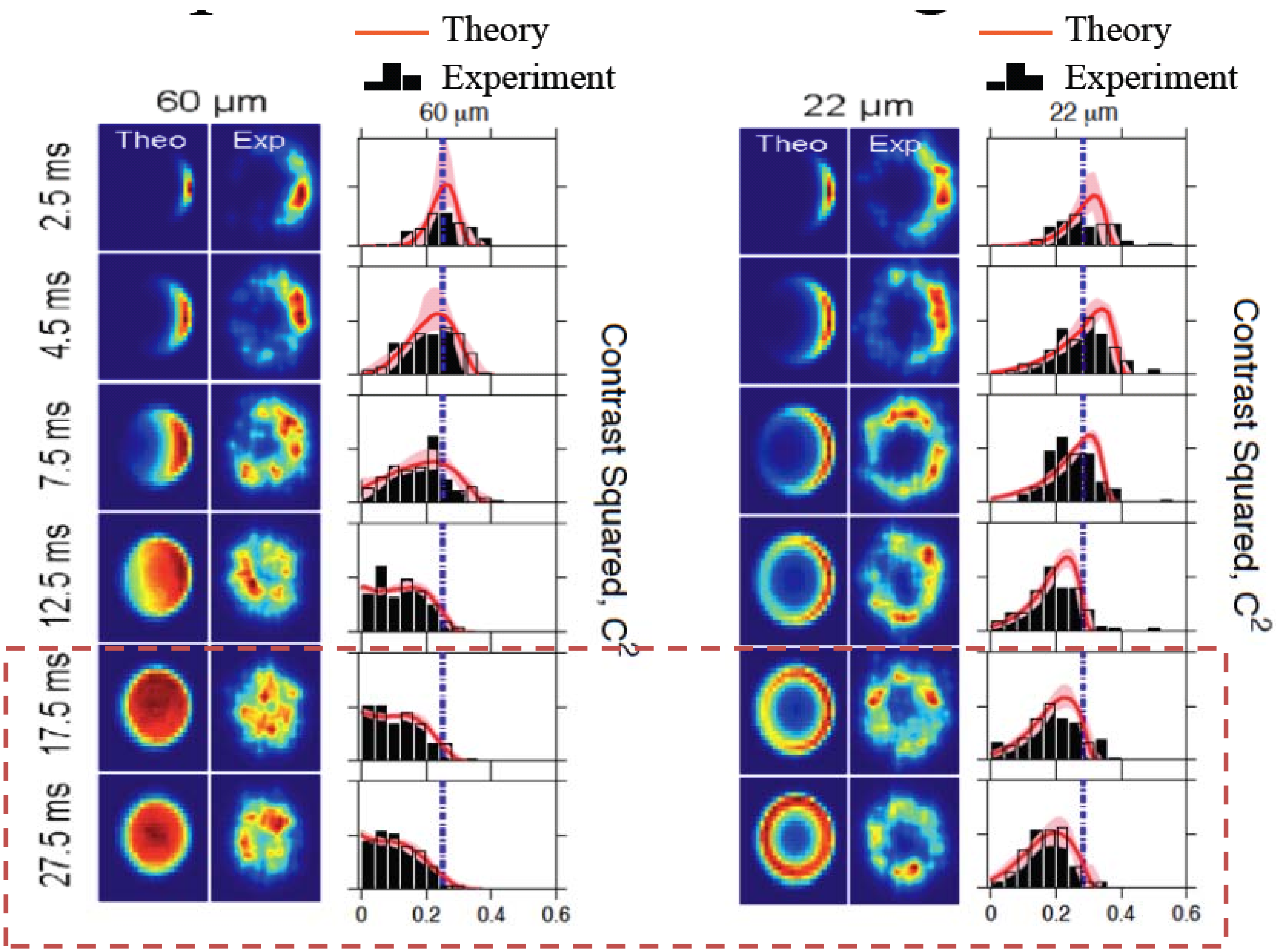
Energy stored in each mode initially

$$E_k = g\, n_s^2(k, t=0) = \frac{g}{\phi_0^2}$$

Equipartition of energy

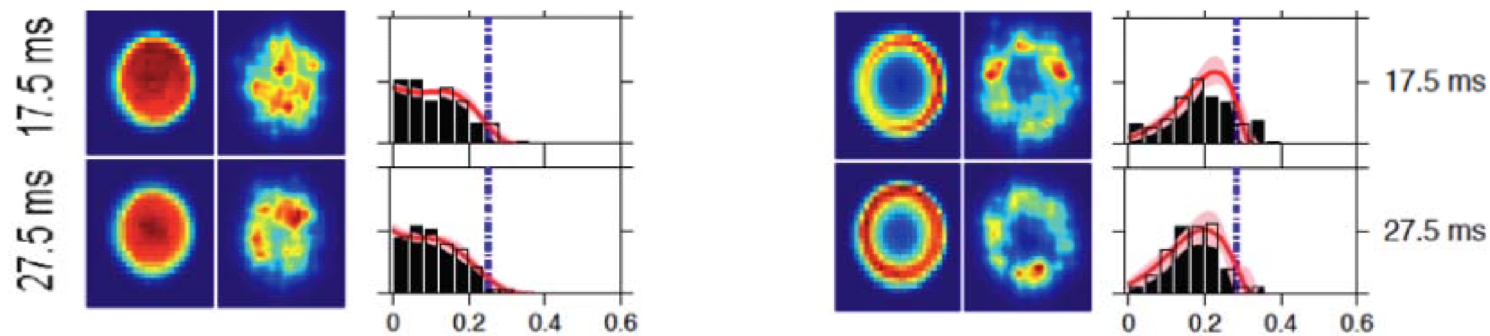
The system should look thermal like after different modes dephase.
Effective temperature is not related to the physical temperature

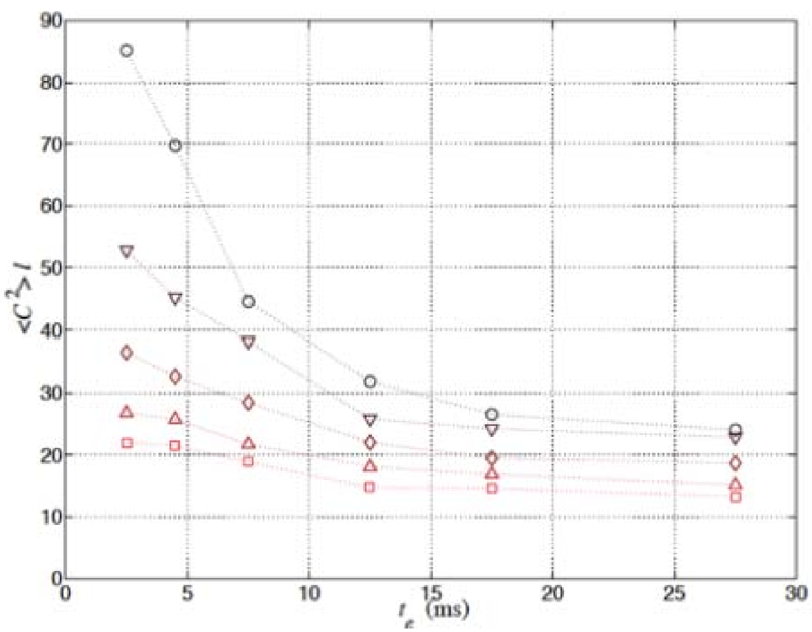
Comparison of experiments and LL analysis



Do we have thermal-like distributions at longer times?

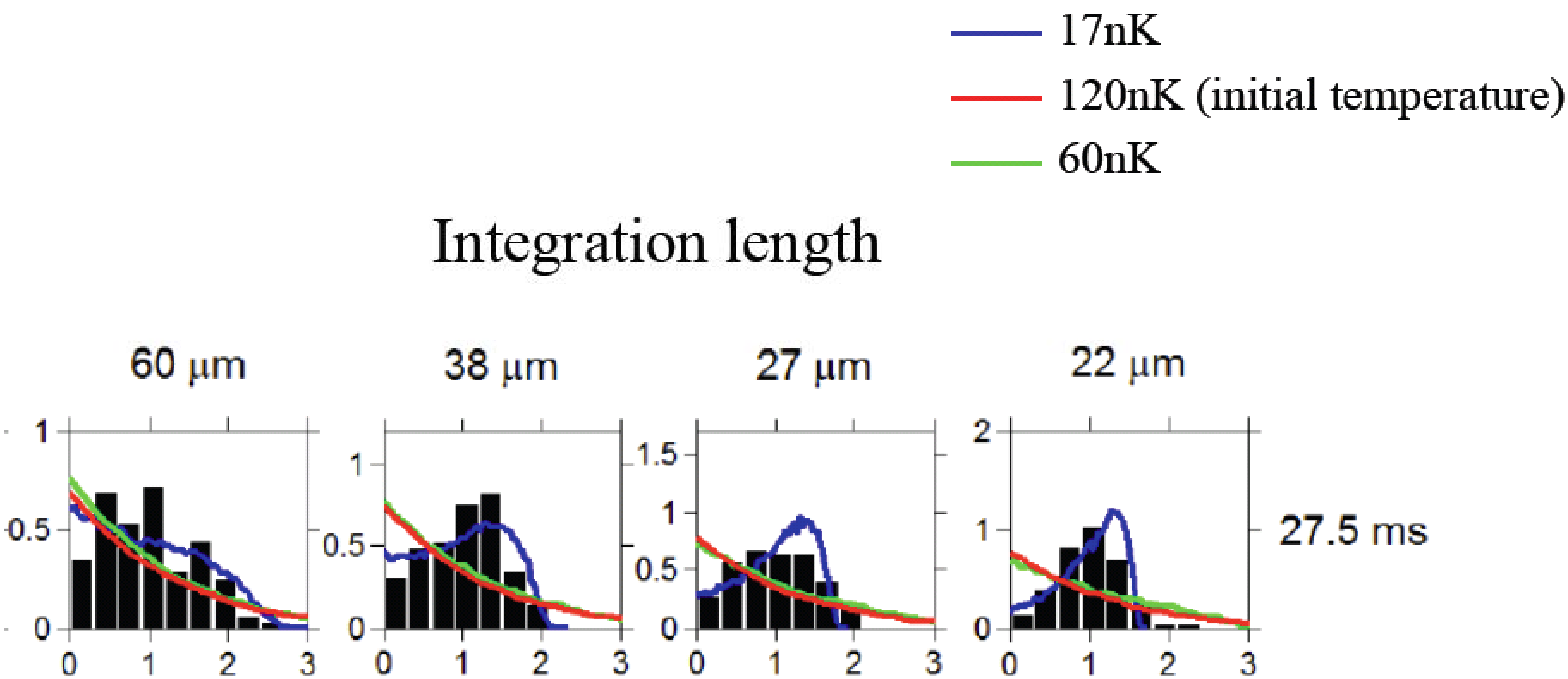
Quasi-steady state





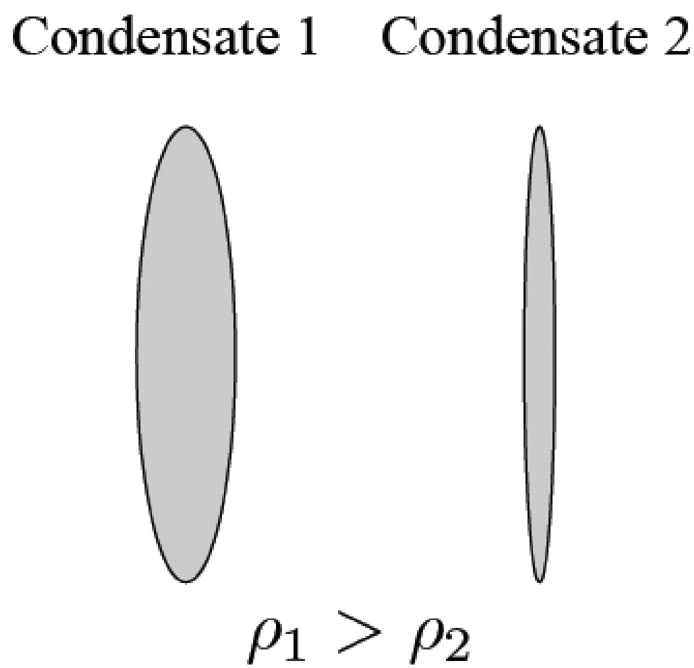
Different lines are different integration length

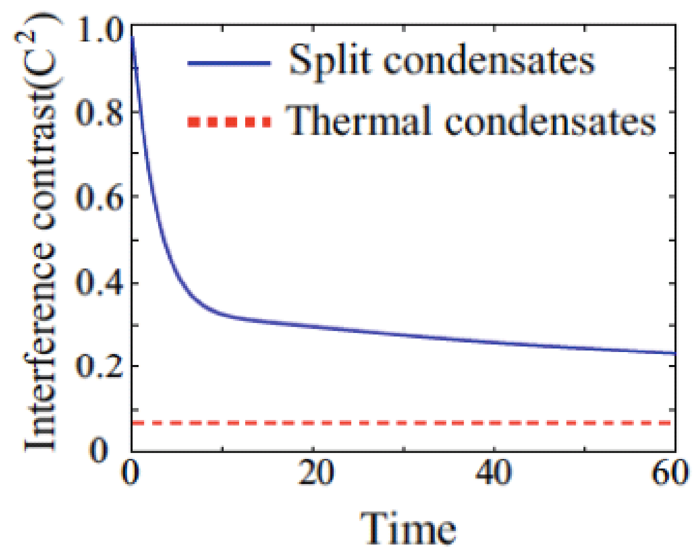
Prethermalization



Interference contrast becomes indistinguishable from the one resulting from thermal condensates but at temperature *much lower* than expected from the initial temperature!

Long time transient





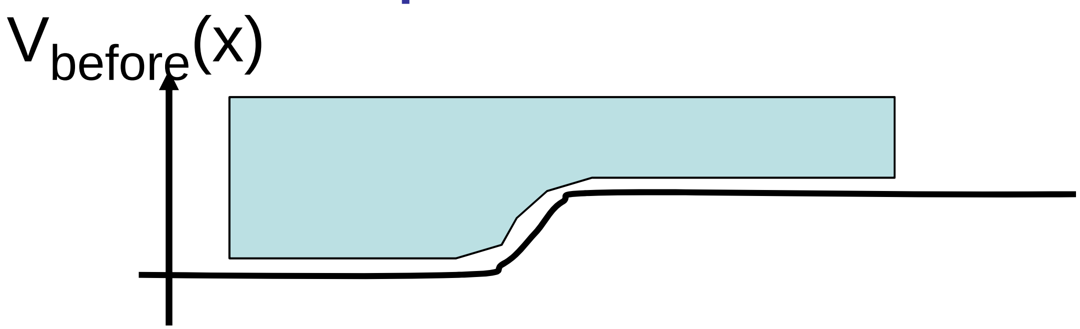
When the average densities of two condensates are different, the interference contrast relaxes to smaller value as time progresses.

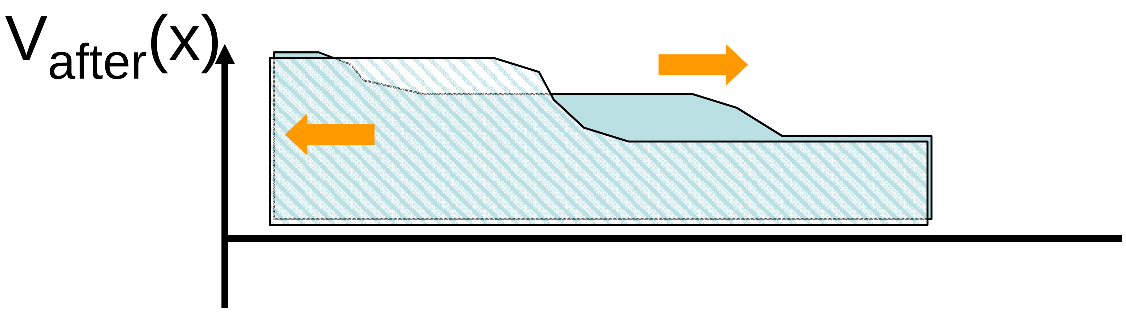
Formation of soliton structures in the dynamics of lattice bosons.

Universal dynamical diagram of relaxation in the 2d XXZ model

Demler, Maltsev, Annals of Physics (2011)
Demler, Maltsev, Prokof'ev, unpublished

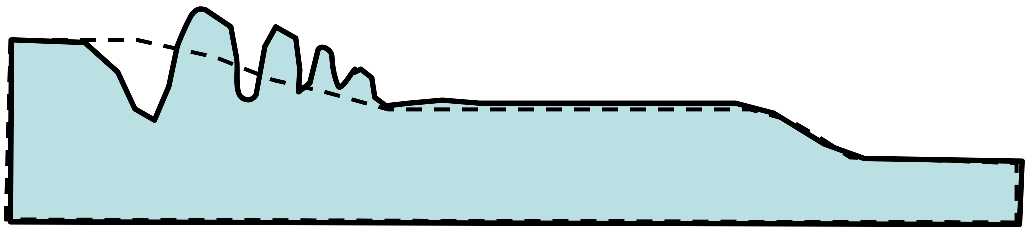
Equilibration of density inhomogeneity





Suddenly change the potential. Observe density redistribution

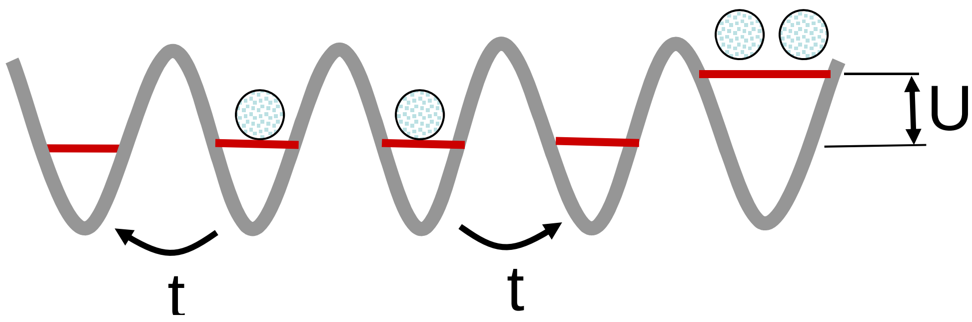
Strongly correlated atoms in an optical lattice: appearance of oscillation zone on one of the edges



Semiclassical dynamics of bosons in optical lattice: Kortweg- de Vries equation

Instabilities to transverse modulation

Bose Hubbard model



$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1)$$

Hard core limit $U \to \infty$

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \hat{P} b_i^\dagger b_j \hat{P}$$

$\hat{P}$ - projector of no multiple occupancies

Spin representation of the hard core bosons Hamiltonian

$$\mathcal{H} = -t \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y)$$

Anisotropic Heisenberg Hamiltonian

$$\mathcal{H} = -J_\perp \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) - J_z \sum_{\langle ij \rangle} S_i^z S_j^z$$

We will be primarily interested in 2d and 3d systems with initial 1d inhomogeneity

Semiclassical equations of motion

Time-dependent variational wavefunction

$$|\Psi\rangle = \prod_i \left[\sin\frac{\theta_i}{2}\, e^{-\varphi_i/2} \,|\downarrow\rangle_i + \cos\frac{\theta_i}{2}\, e^{\varphi_i/2}\,|\uparrow\rangle_i \right]$$

Landau-Lifshitz equations

$$\frac{d}{dt}\langle S_i^x \rangle = \sum_{\langle j \rangle} \left(-J_\perp \langle S_i^z \rangle \langle S_j^y \rangle + J_z \langle S_i^y \rangle \langle S_j^z \rangle \right)$$

$$\frac{d}{dt}\langle S_i^y \rangle = \ldots \qquad \frac{d}{dt}\langle S_i^z \rangle = \ldots$$

Equations of Motion
Gradient expansion

Density relative to half filling $\rho = n - \frac{1}{2}$

Phase gradient
$\approx$ superfluid velocity $q = a \nabla_x \phi$

Mass conservation $\partial_t \rho = 4 J_\perp \nabla_x \left[(1 - \rho^2) \sin q \right]$

Josephson relation $\partial_t q = \nabla_x \left[-8\, J_\perp \, \rho \cos q + 8\, J_z \, \rho \right]$

Expand equations of motion around state with small density modulation and zero superfluid velocity

From wave equation
to solitonic excitations

Equations of Motion

Separate left- and right-moving parts

$$\rho - \rho_0 = \frac{\sqrt{1-\rho_0^2}}{2\sqrt{2}}\,(r_1 + r_2) \qquad q = \frac{1}{2}\sqrt{1 - J_z/J_\perp}\,(r_2 - r_1)$$

First non-linear expansion

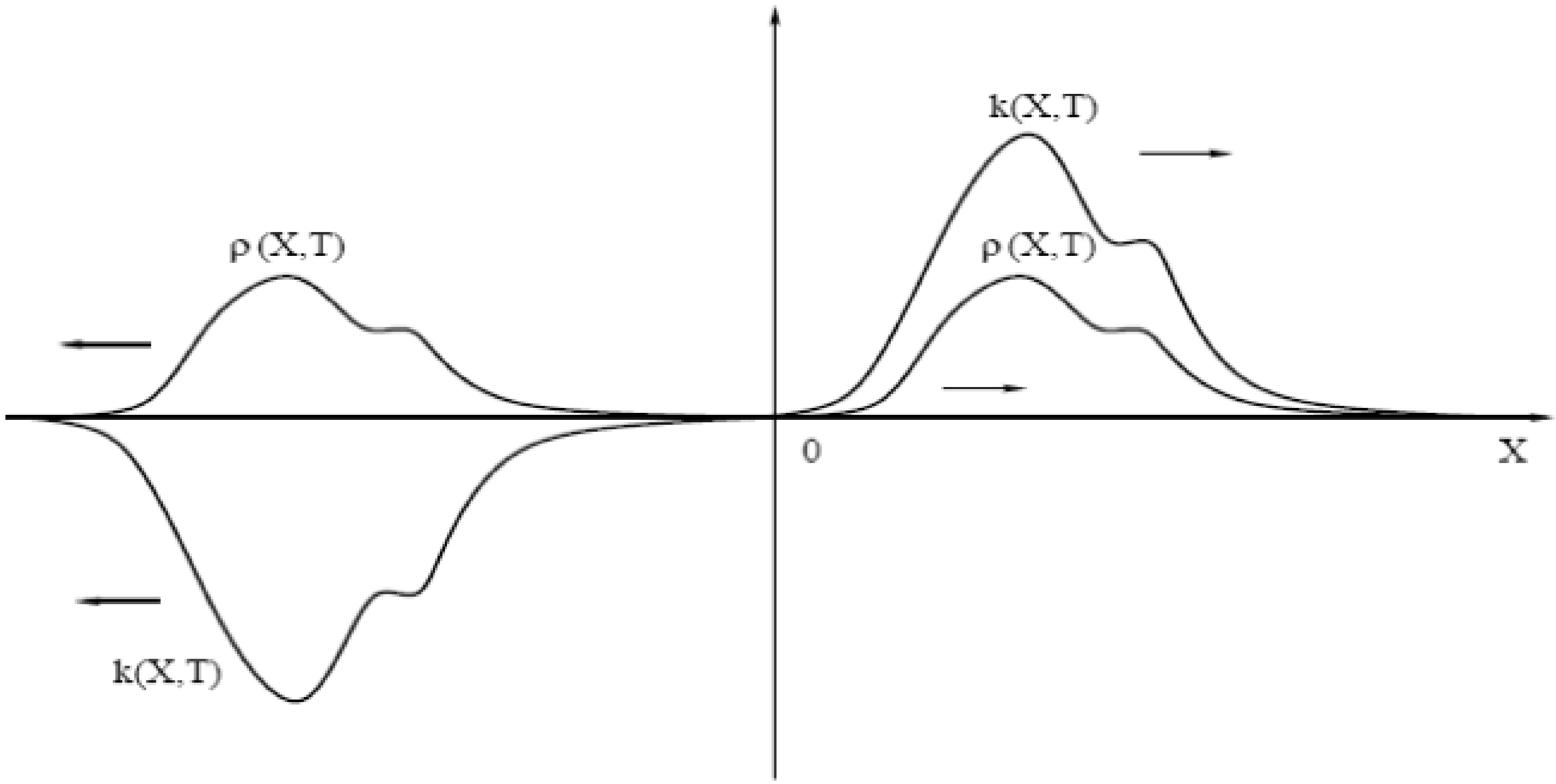
$$\partial_t r_1 = (v - C_1\,\rho_0\,r_1 + C_2\,\rho_0\,r_2)\,\nabla_x r_1$$

Left moving part. Zeroth order solution $r_1(x + v\,t)$

$$\partial_t r_2 = (-v - C_2\,\rho_0\,r_1 + C_1\,\rho_0\,r_2)\,\nabla_x r_1$$

Right moving part. Zeroth order solution $r_2(x - v\,t)$

Assume that left- and right-moving parts separate before nonlinearities become important



Left-moving part $$\partial_t r_1 = (v - C_1 \rho_0 r_1) \nabla_x r_1$$

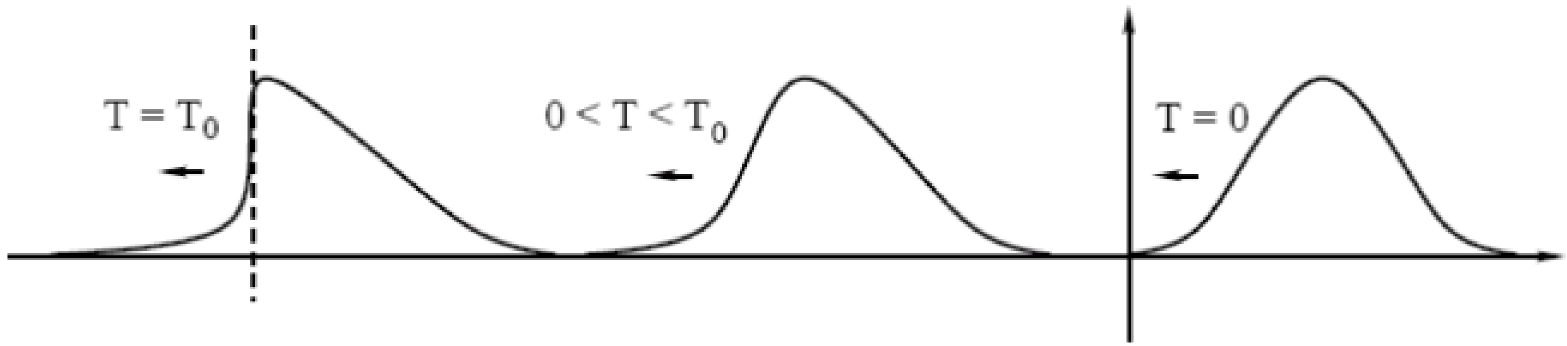
Right-moving part $$\partial_t r_2 = (- v + C_1 \rho_0 r_2) \nabla_x r_2$$

Breaking point formation. Hopf equation

Density below half filling $\rho_0 = n - 1/2 < 0$

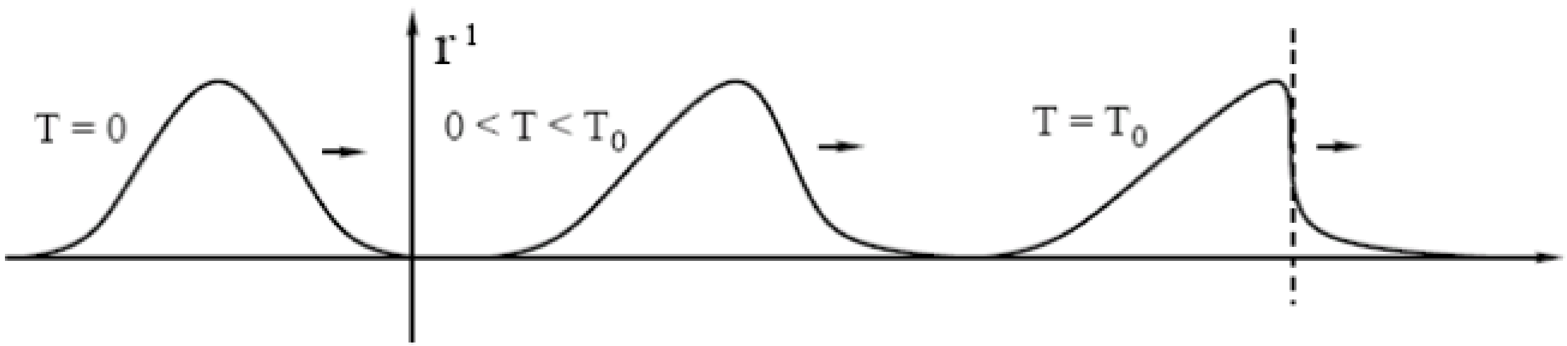
Regions with larger density move faster

Left-moving part $\partial_t r_1 = (v - C_1 \rho_0 r_1) \nabla_x r_1$



Singularity at finite time T_0

Right-moving part $\partial_t r_2 = (-v + C_1 \rho_0 r_2) \nabla_x r_2$



Dispersion corrections

Left moving part

$$\partial_t r_1 = (v - C_1 \rho_0 r_1) \nabla_x r_1 + D \nabla_x^3 r_1$$

Right moving part

$$\partial_t r_2 = (- v + C_1 \rho_0 r_2) \nabla_x r_2 - D \nabla_x^3 r_2$$

Competition of nonlinearity and dispersion leads to the formation of soliton structures

Mapping to Kortweg - de Vries equations

In the moving frame and after rescaling

$$U_T + 6 U U_X - U_{XXX} = 0$$ when $|n - \frac{1}{2}| < 1/7$

$$U_T + 6 U U_X + U_{XXX} = 0$$ when $|n - \frac{1}{2}| > 1/7$

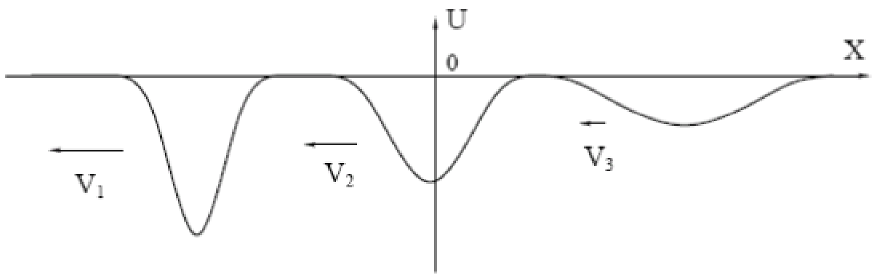
Soliton solutions of Kortweg - de Vries equation

Competition of nonlinearity and dispersion leads to the formation of soliton structures

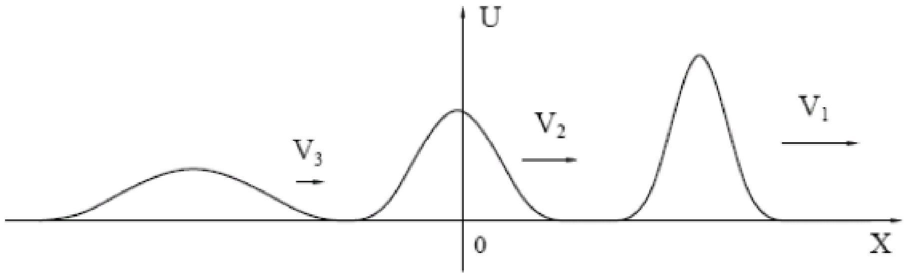
Solitons preserve their form after interactions
Velocity of a soliton is proportional to its amplitude

To solve dynamics: decompose initial state into solitons
Solitons separate at long times

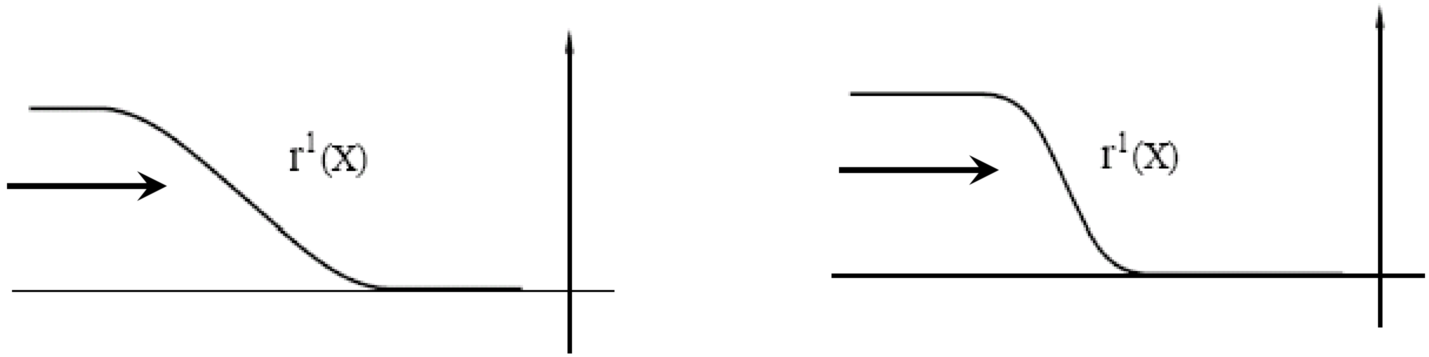
$$U_T + 6\,U\,U_X - U_{XXX} = 0$$



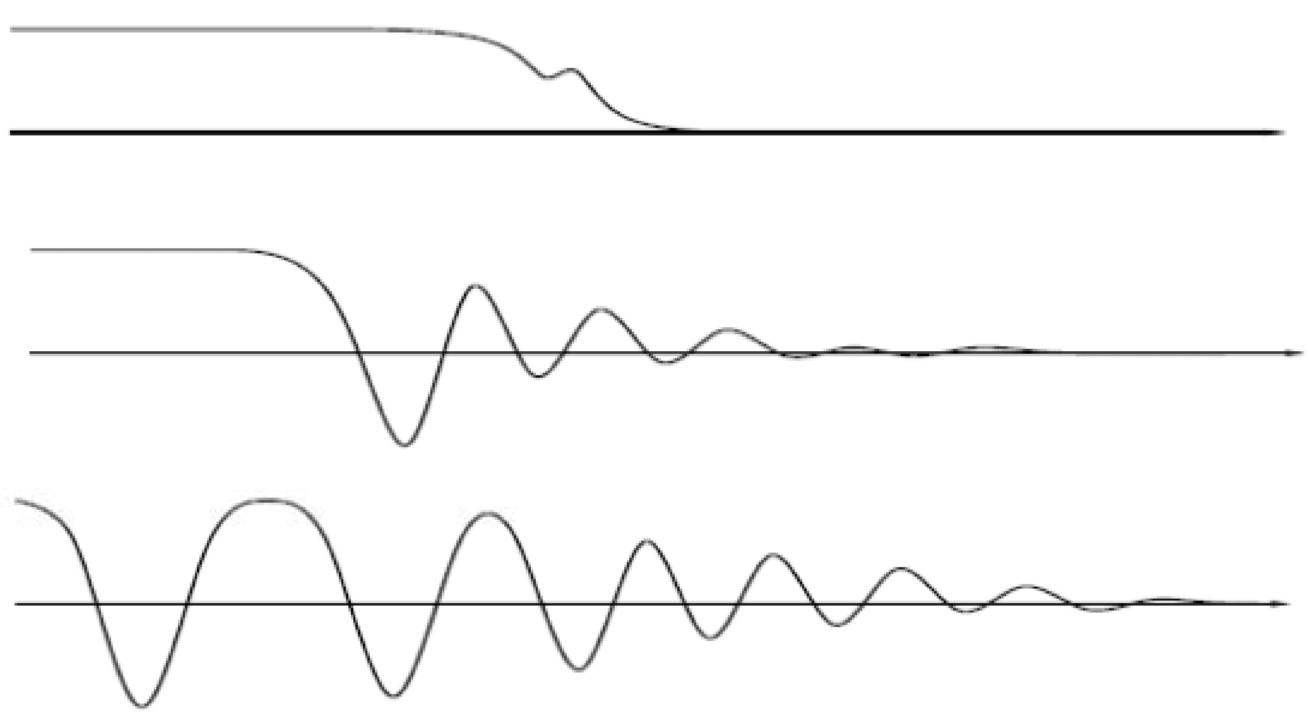
$$U_T + 6\,U\,U_X + U_{XXX} = 0$$



From increase of the steepness

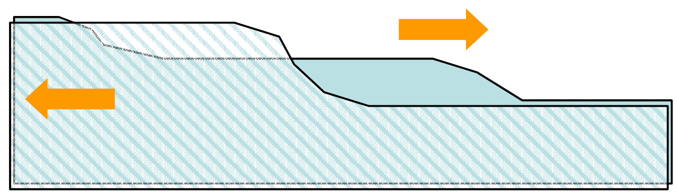


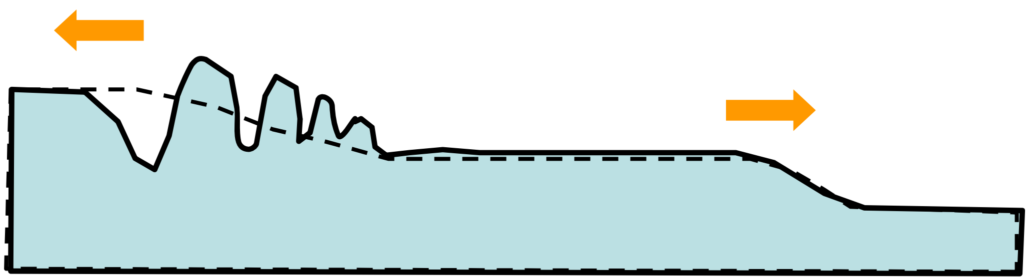
To formation of the oscillation zone



Decay of the step

Above half-filling $\rho_0 = n - 1/2 > 0$

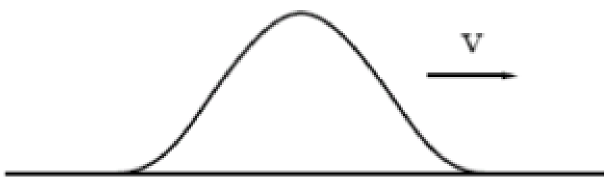




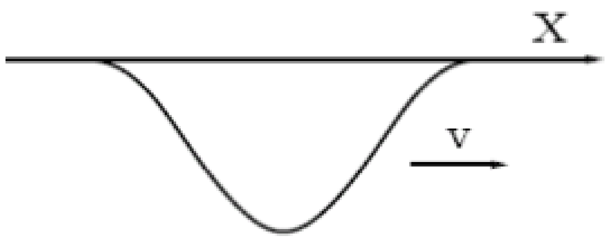
Half filling. Modified KdV equation

$$\partial_t U + (\alpha U + 6U^2)\nabla_x U - \nabla_x^3 U = 0$$

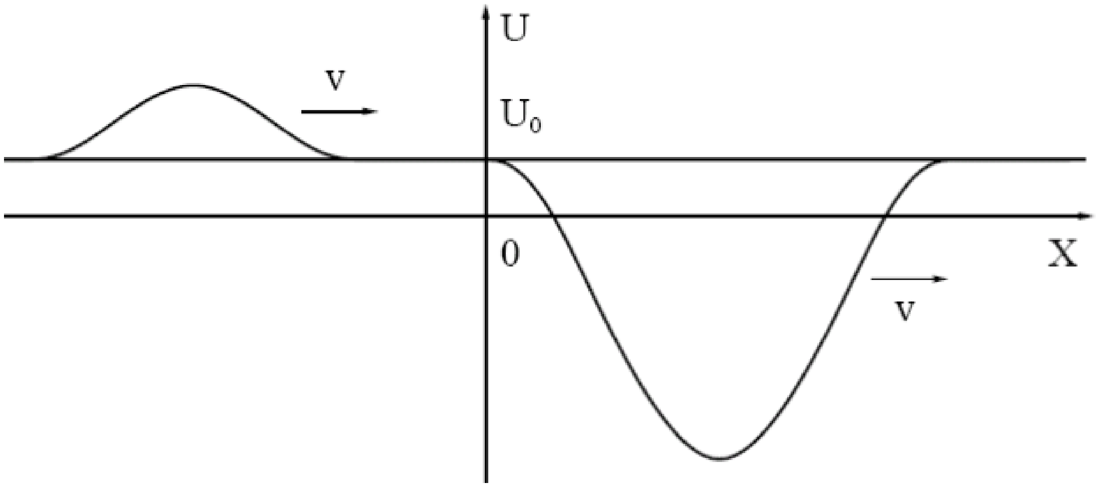
Particle type solitons



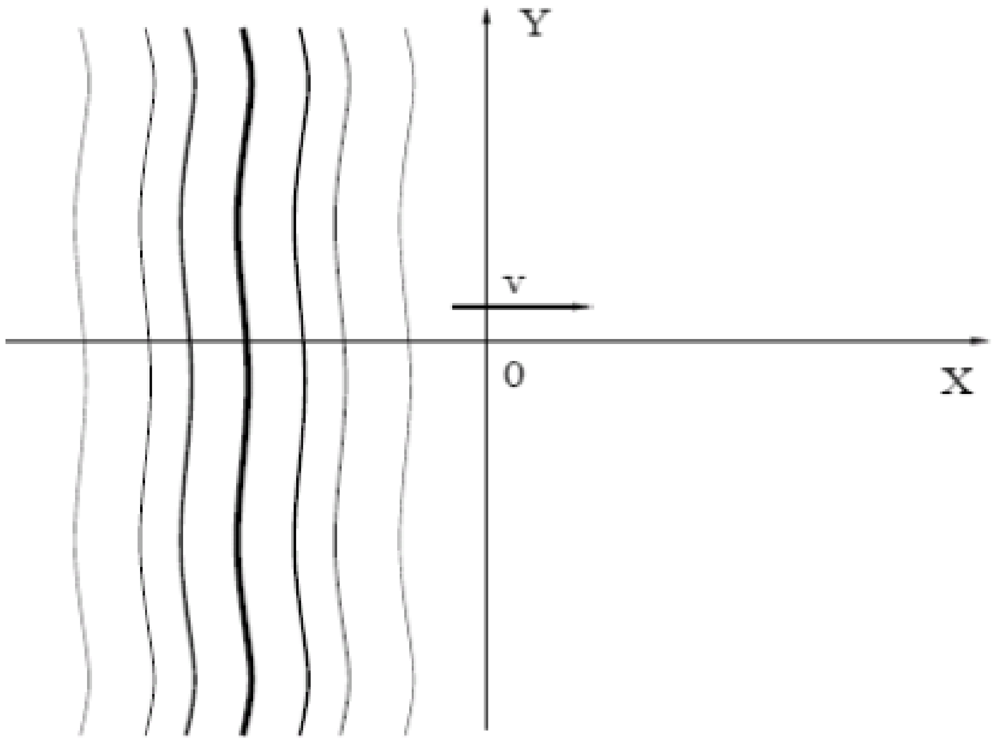
Hole type solitons



Particle-hole solitons



Stability to transverse fluctuations



Stability to transverse fluctuations

Dispersion

$$\omega(\vec{k}) = v\,\left(k_x^2 + k_\perp^2\right)^{1/2} \approx v\,k_x + \frac{v}{2}\,\frac{k_\perp^2}{k_x}$$

Non-linear waves

$$\partial_t\, r_1 = (\,v - C_1\,\rho_0\,r_1\,)\,\nabla_x r_1 + D\,\nabla_x^3 r_1$$

$$\partial_t\,\nabla_x r_1 = (\,v - C_1\,\rho_0\,r_1\,)\,\nabla_x^2 r_1 + D\,\nabla_x^4 r_1 + \frac{v}{2}\nabla_\perp^2 r_1$$

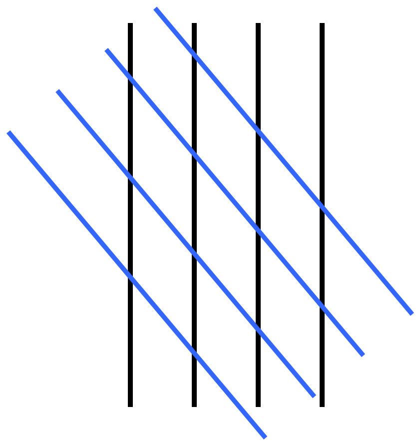
Kadomtsev-Petviashvili equation

Planar structures are unstable to transverse modulation if

$$|\,n - \frac{1}{2}\,| > \frac{1}{2\sqrt{7}}$$

Kadomtsev-Petviashvili equation

Stable regime. N-soliton solution.
Plane waves propagating
at some angles and interacting



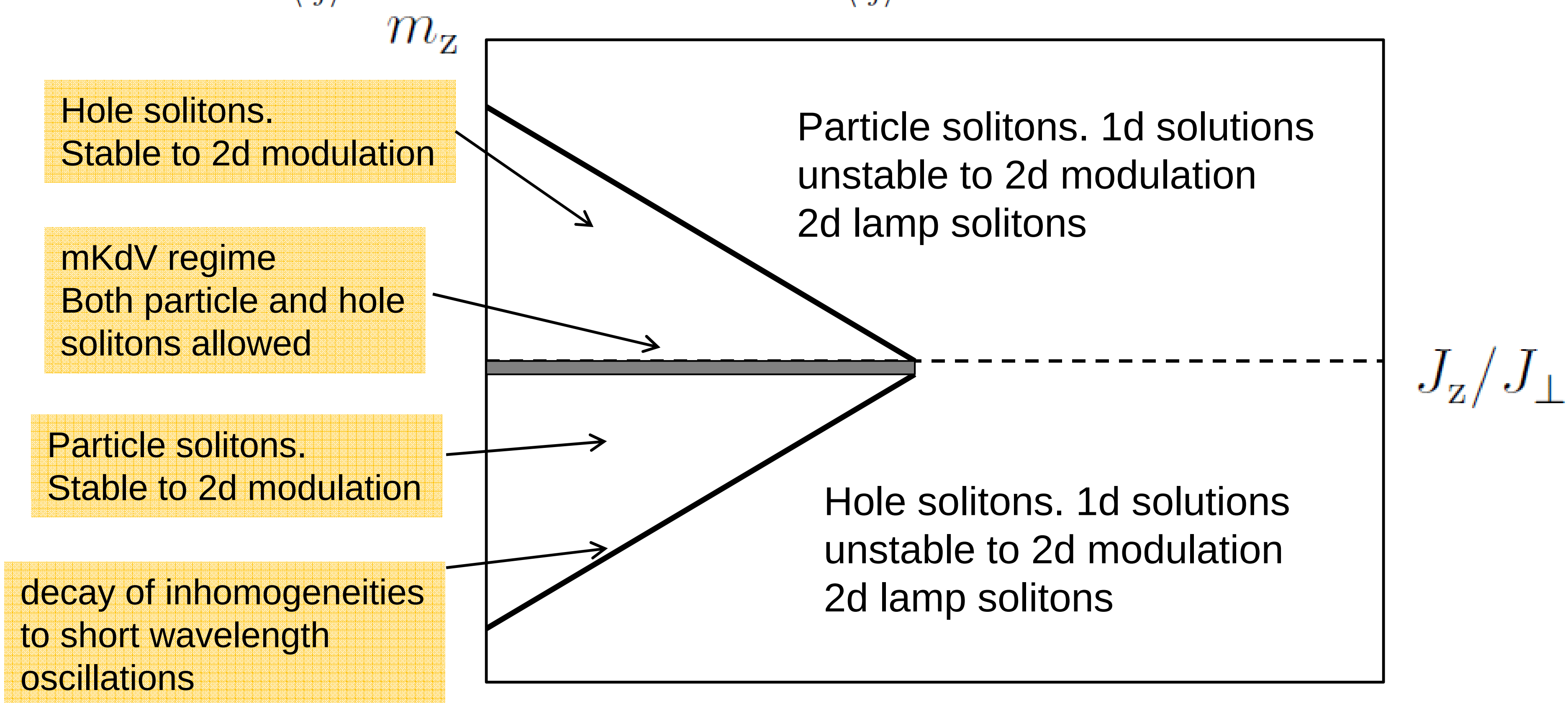
$$U(x, y, t) = \Phi(\theta_1, \theta_2, \ldots, \theta_n)$$

$$\theta_i = \omega_i t + \vec{k}_i \vec{r} + c_i$$

Unstable regime.
"Lumps" – solutions localized in all directions.
Interactions between solitons do not produce
phase shits.

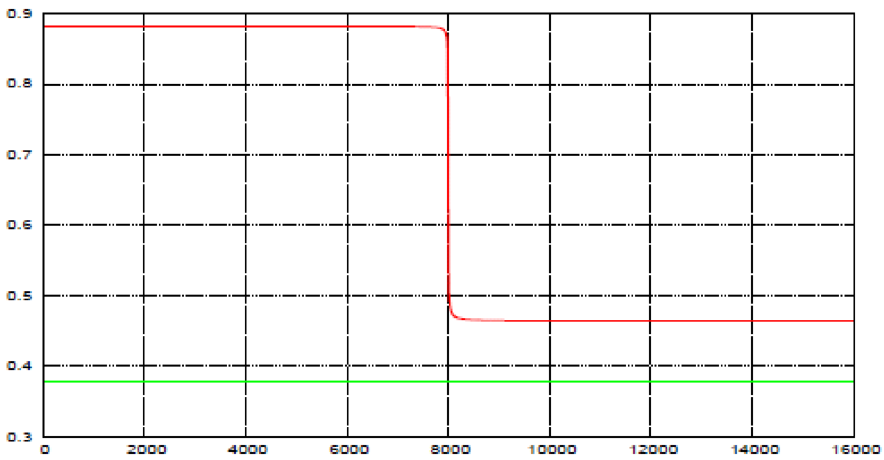
KdV dynamical diagram of relaxation in 2d XXZ Heisenberg model

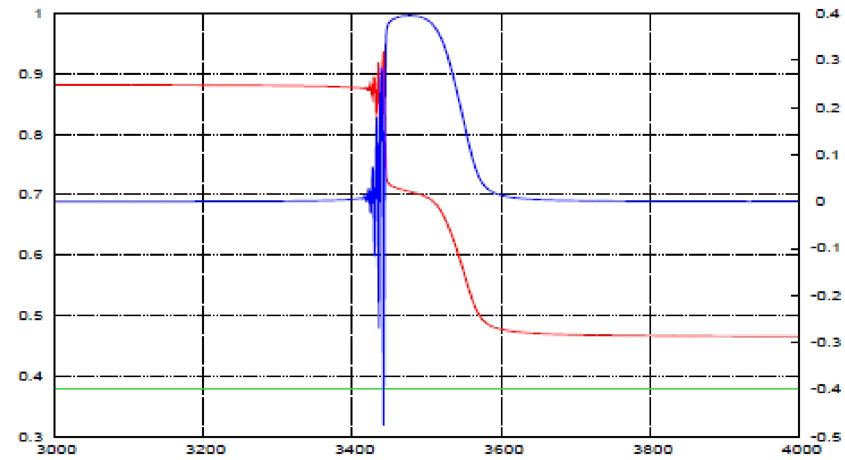
$$\mathcal{H} = J_\perp \sum_{\langle ij \rangle} \left(S_i^x\, S_j^x + S_i^y\, S_j^y \right) + J_z \sum_{\langle ij \rangle} S_i^z\, S_j^z$$



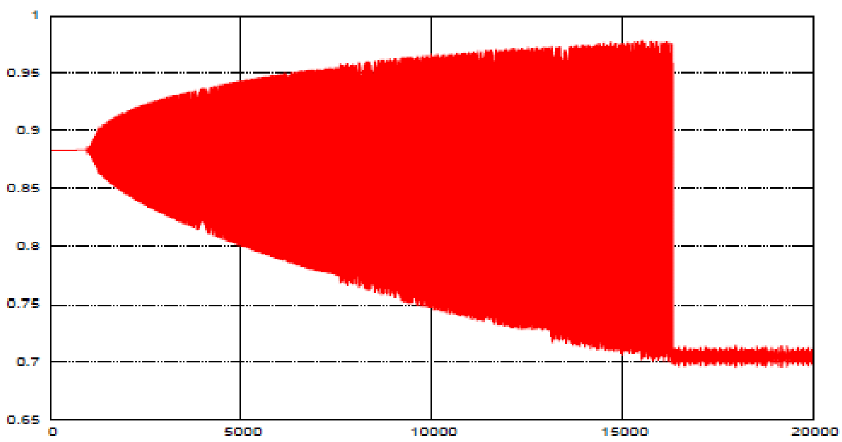
Dynamics of density step decay. Regime stable to 2d modulation

t=50

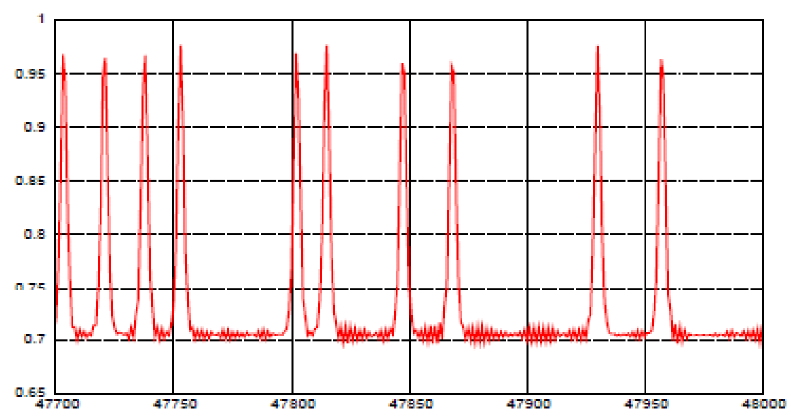




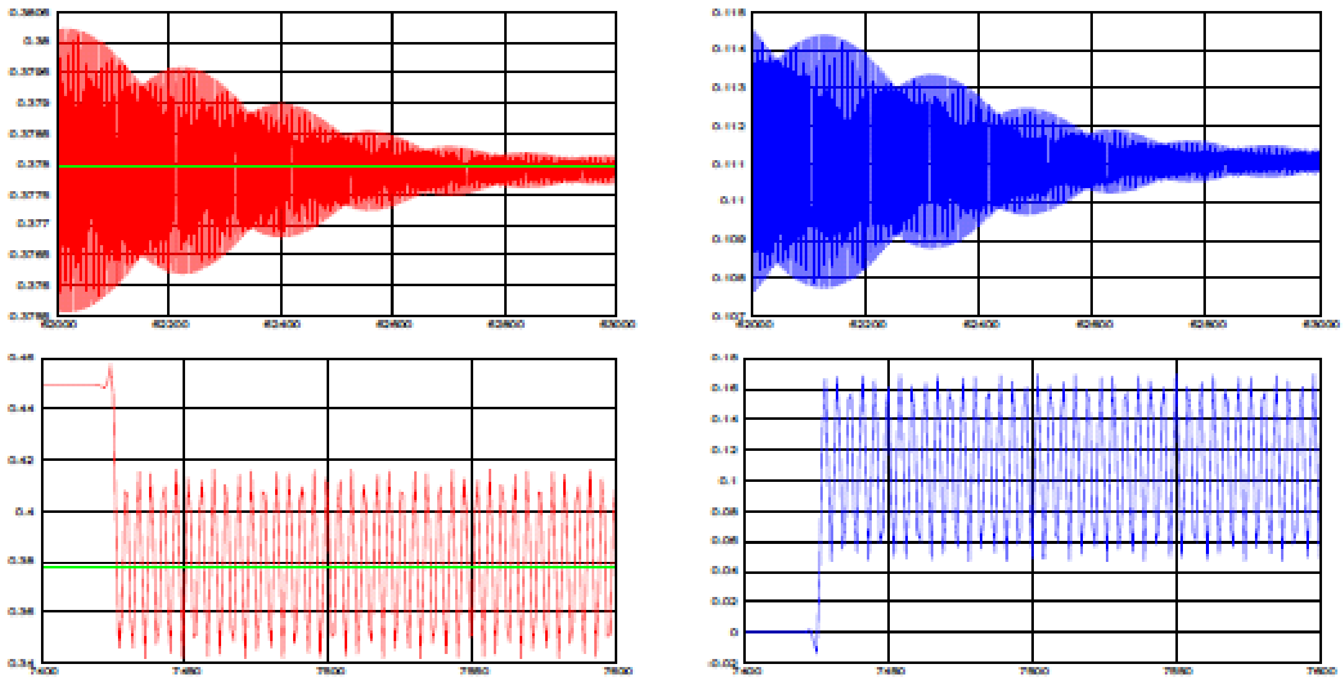
t=5000



t=5000 expanded

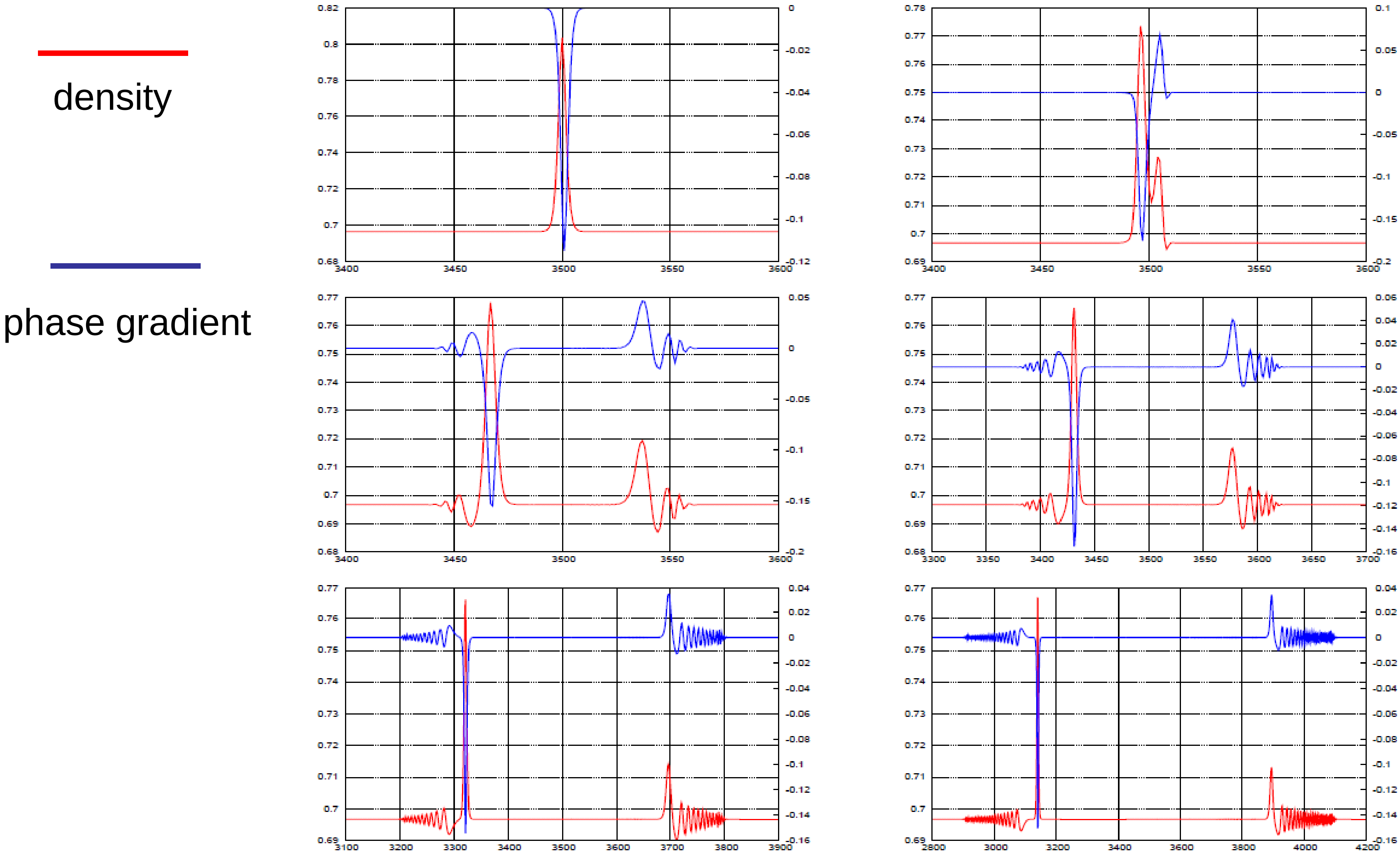


Dynamics of density step decay. Near the instability line



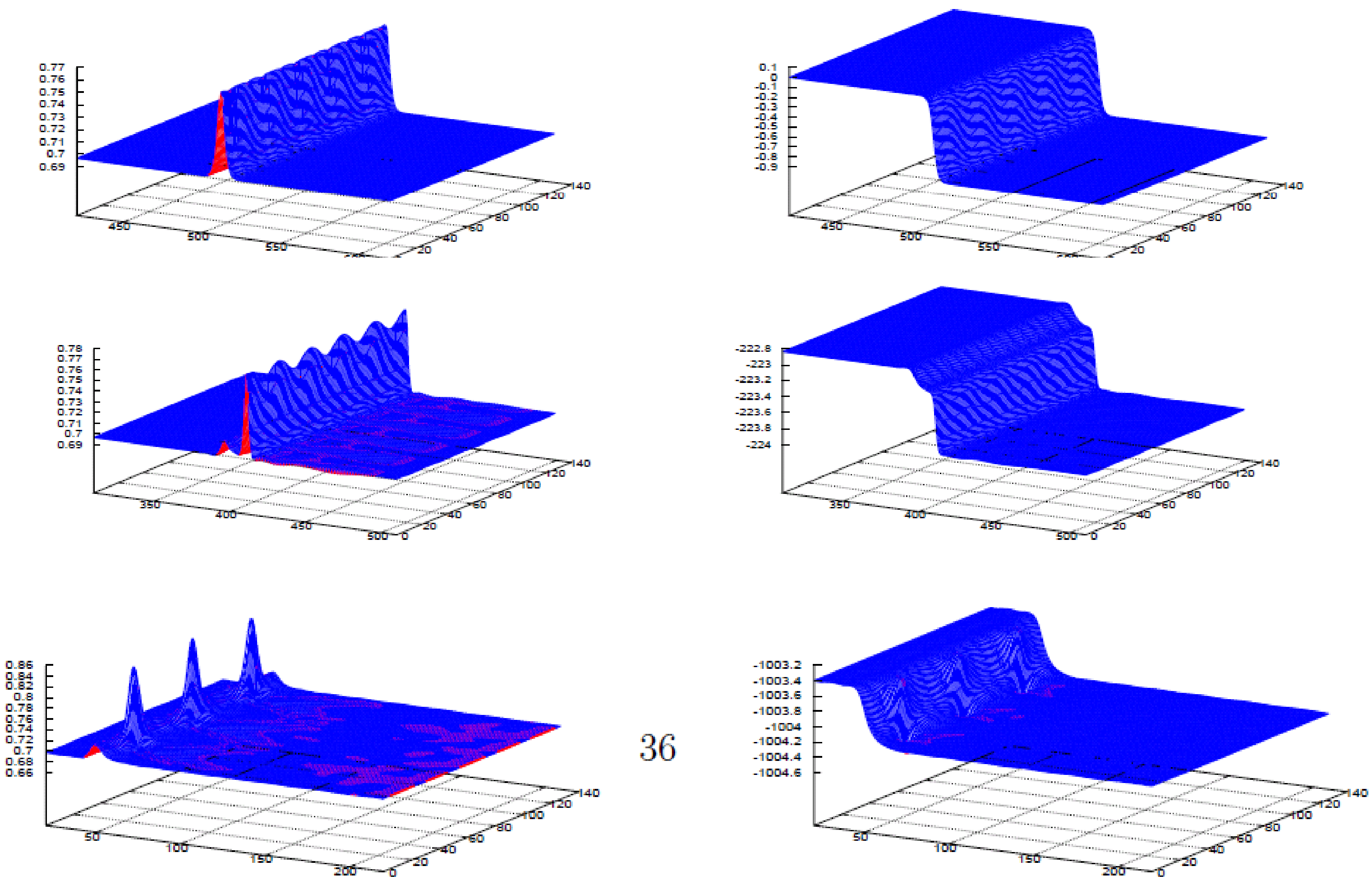
Dynamical separation of solitons and wavetrains

t=0, 2, 10, 20



Instability of soliton to 2d modulation

density phase



t=0

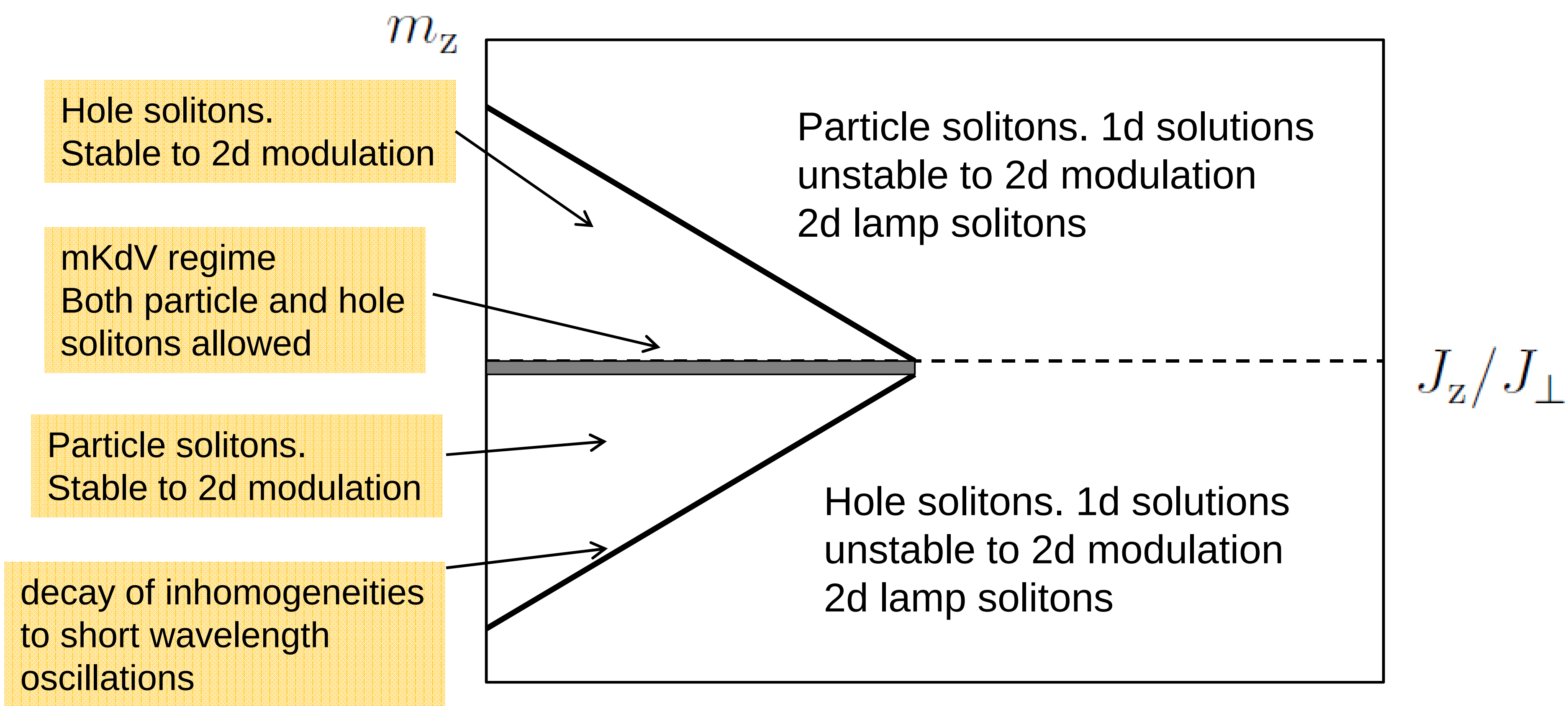
t=20

t=60

36

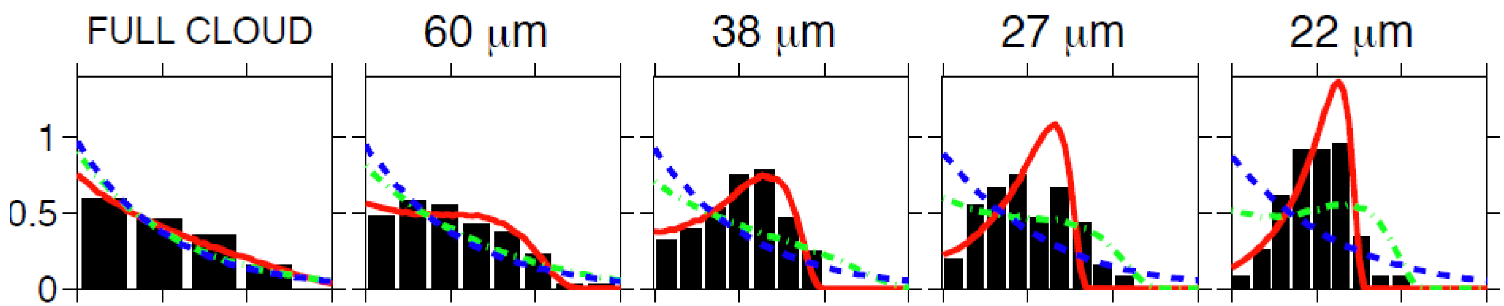
Universal dynamical diagram of relaxation in 2d XXZ Heisenberg model

Numerical semiclassical calculations are consistent withthe dynamical diagram based on the analysis of KdV equations

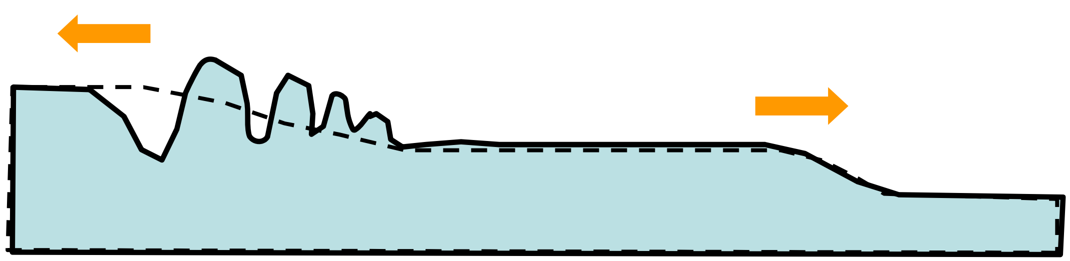


Summary

Observation of prethermalization in quantum dynamics of split one dimensional condensates



Formation of soliton structures in the dynamics of lattice bosons. Universal dynamical diagram of relaxation in 2d XXZ model

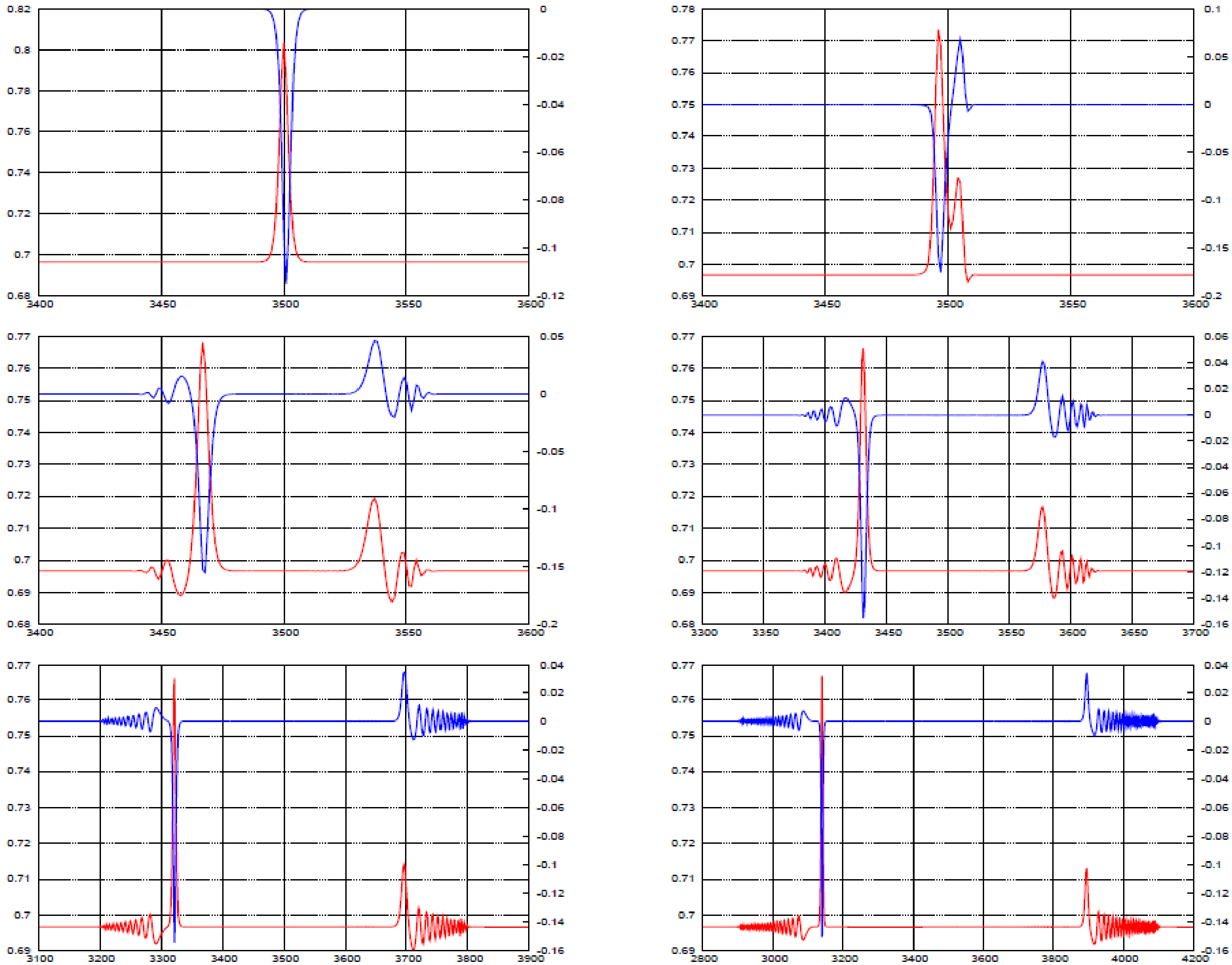


Dynamical separation of solitons and wavetrains

t=0, 2, 10, 20

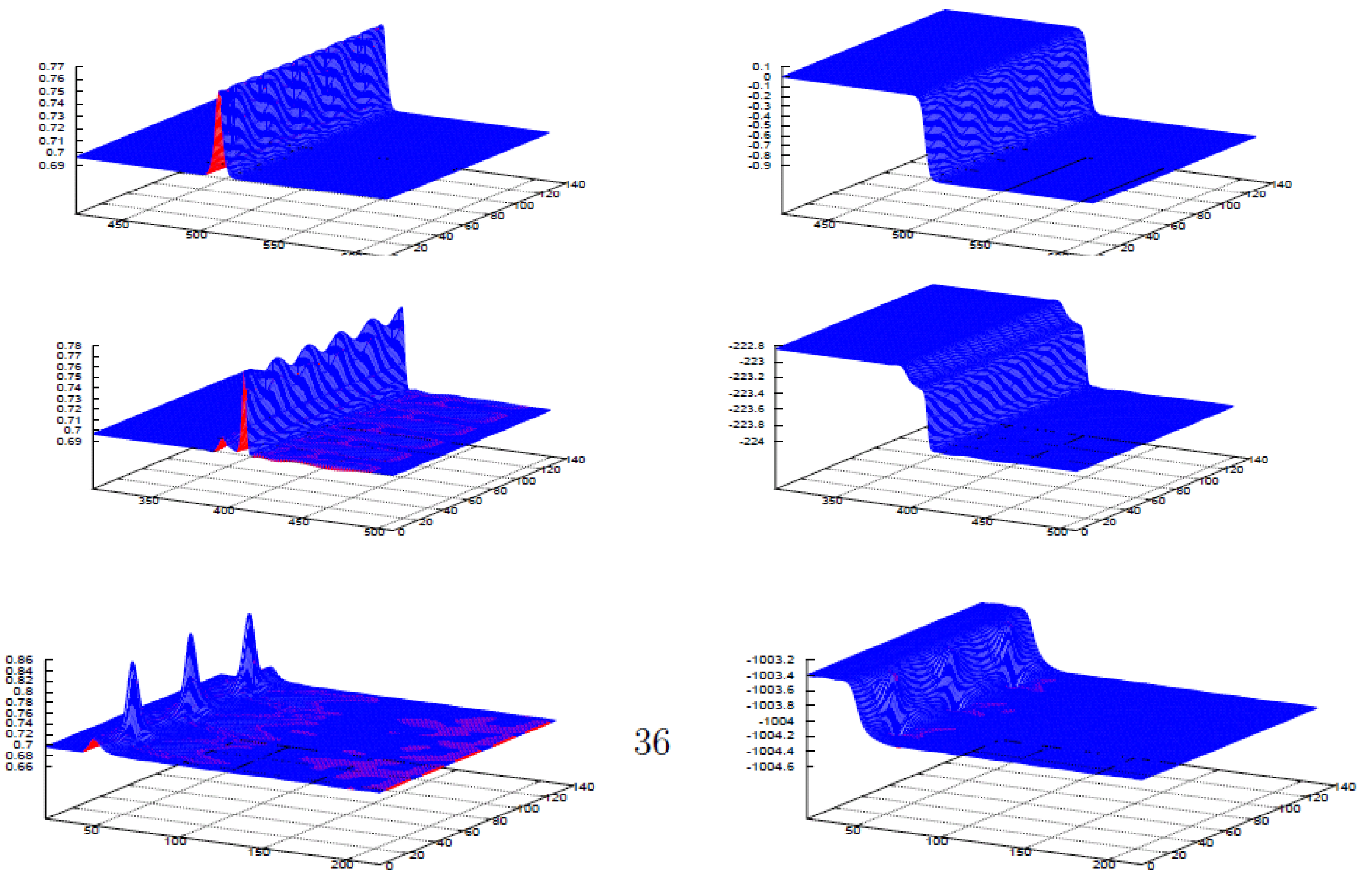
density

phase gradient



Instability of soliton to 2d modulation

density phase

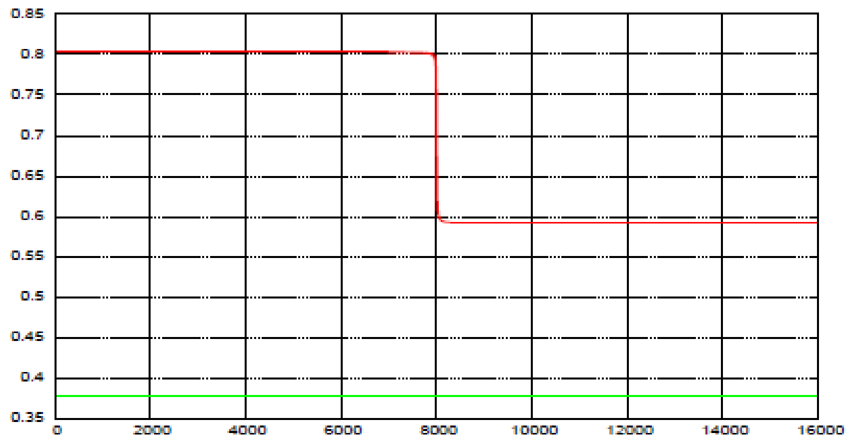


t=0

t=20

t=60

36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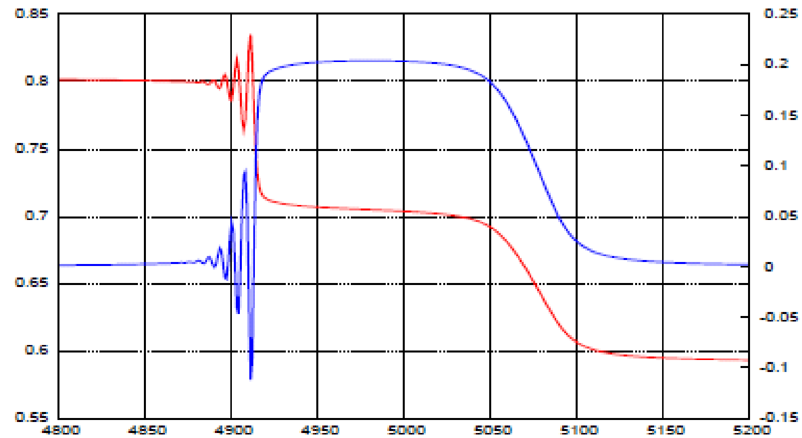
16000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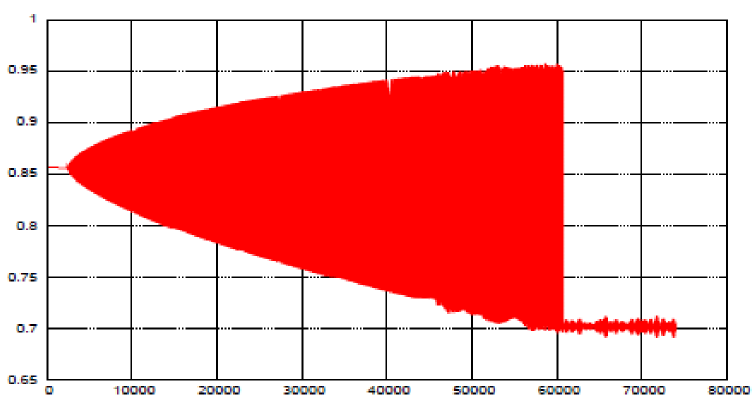
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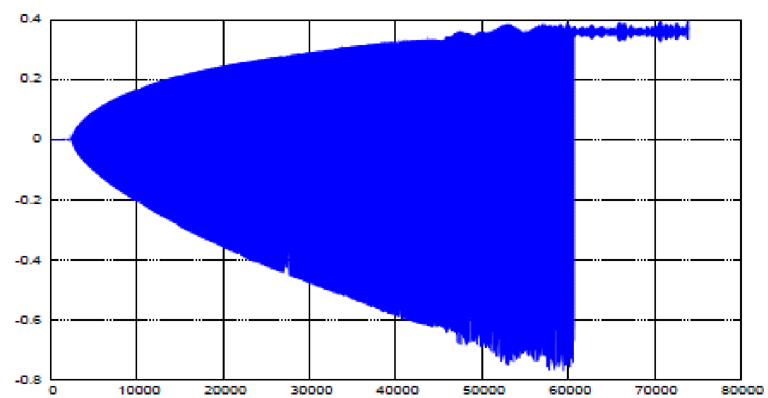
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80000

Summary and outlook

Formation of soliton structures in the dynamics of lattice bosons within semiclassical approximation.

Solitons beyond longwavelength approximation.
Quantum solitons

Beyond semiclassical approximation. Emission on Bogoliubov modes. Dissipation.

Transverse instabilities. Dynamics of lump formation

Multicomponent generalizations. Matrix KdV

Harvard-MIT

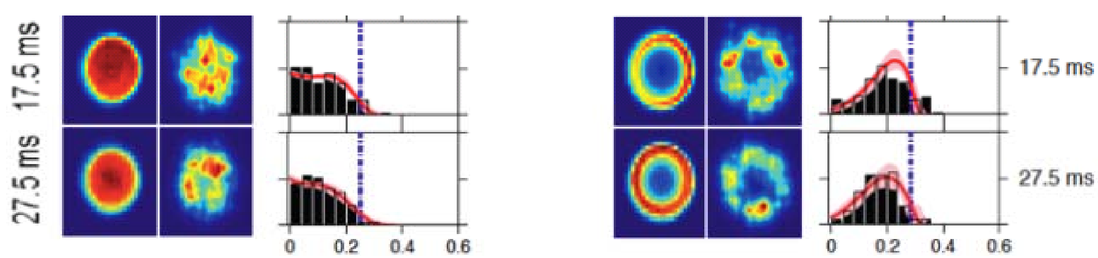
$$ RFBR, NSF, AFOSR MURI, DARPA

Conclusions

• 1D nature of the dynamics in split condensates is well characterized by quantum noise

• Using the theoretical and experimental access of distributions, we observed the phenomena of prethermalization in 1D quantum system.

• The ability to quantitatively analyze 1D systems allows the tomography of many-body 1D system in the future work.



Relaxation to equilibrium

Thermalization: an isolated interacting systems approaches thermal equilibrium at large times (typically at microscopic timescales). The asymptotic values of the correlation functions can then be computed from the resulting microcanonical equilibrium ensemble. All memory about the initial conditions except energy is lost.

Bolzmann equation

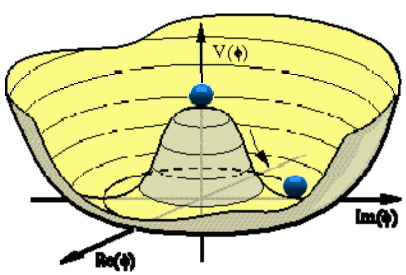
$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \vec{x}} \vec{v} + \frac{\partial f}{\partial \vec{x}} \vec{F} = - \frac{1}{\tau} (f - f_0)$$

Bloch equations for nuclear spin relaxation

$$\frac{d M_z}{dt} = (\vec{M} \times \vec{B})_z - \frac{1}{\tau} (M_z - M_0)$$

Time dependent Ginzburg-Landau

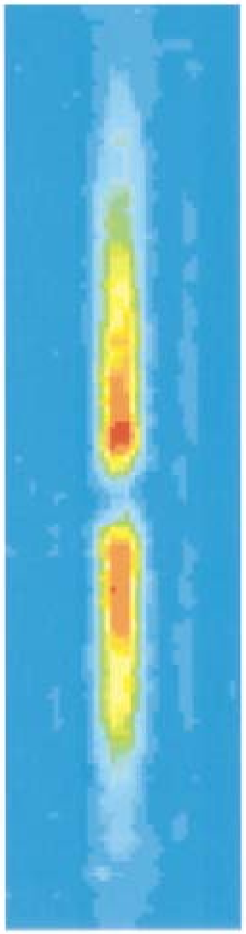
Halperin, Hohenberg (1977)



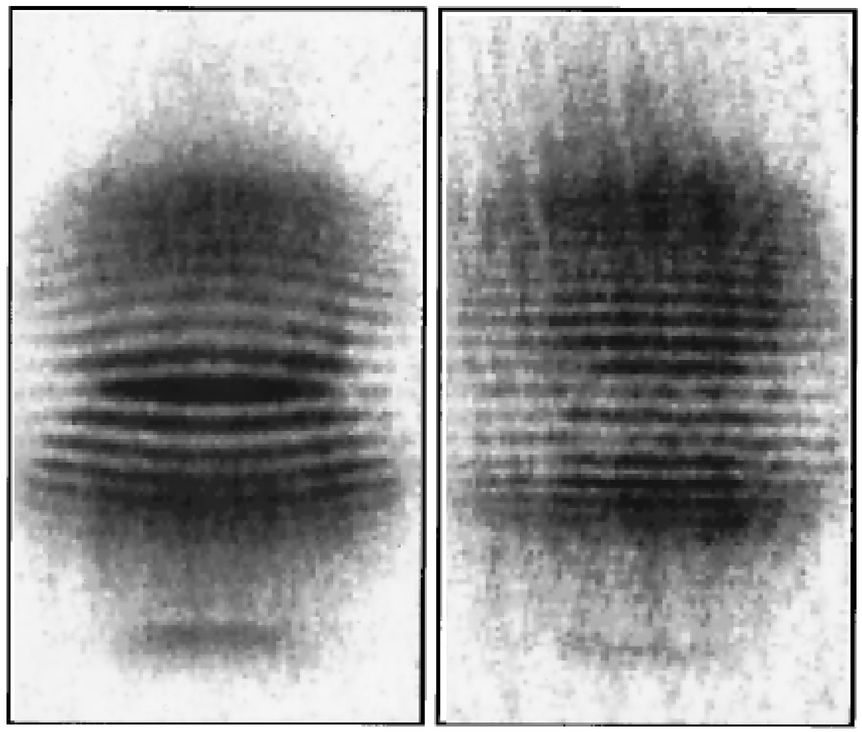
$$\frac{\partial \psi_\alpha(\mathbf{x}, t)}{\partial t} = -\Gamma_0 \frac{\delta F_0}{\delta \psi_\alpha(\mathbf{x}, t)}$$

$$F_0 = \int d^d x \{ \tfrac{1}{2} r_0 \psi^2 + \tfrac{1}{2} |\nabla \psi|^2 + u_0 \psi^4 \}$$

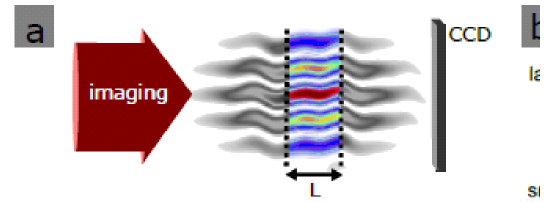
Interference of macroscopic condensates



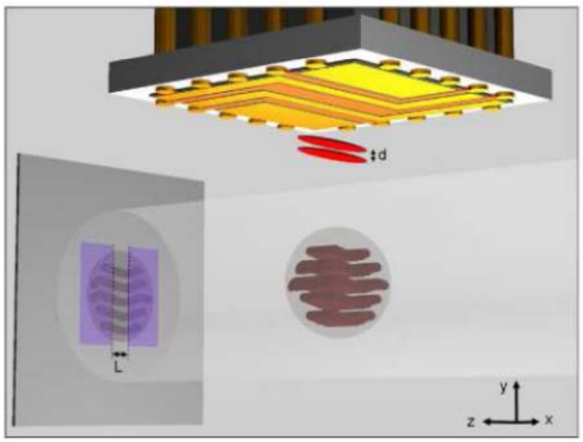
Experiments: Andrews et al., Science 275:637 (1997)

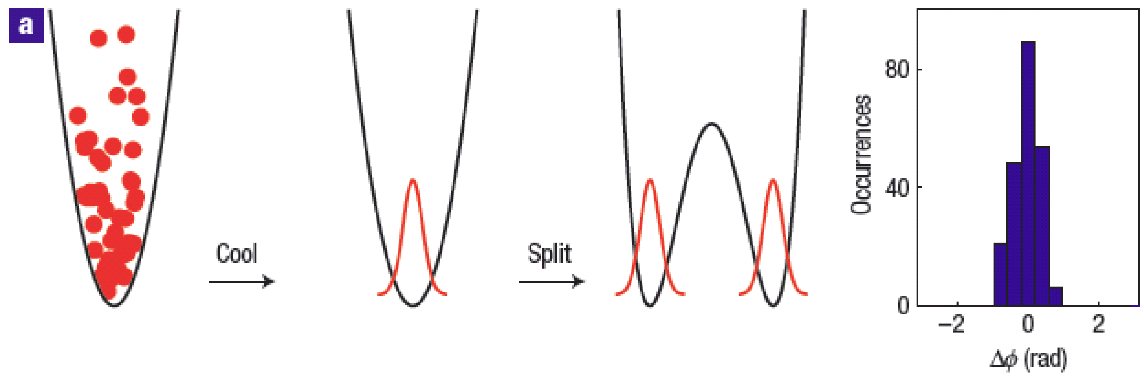


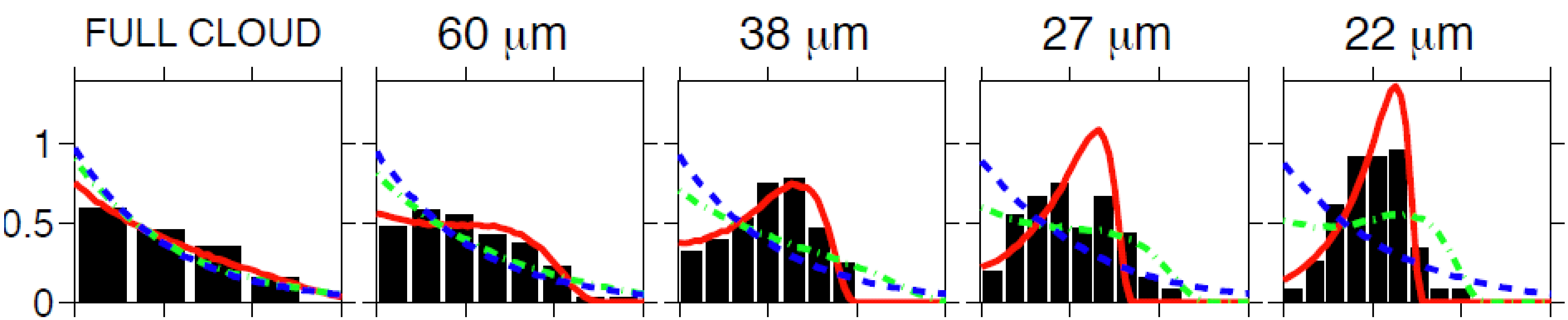
First experimental demonstration of pre



Probing thermolization not only in the avera
but in the complete quantum noise







Initial T=120 nK (blue line). After 27.5 ms identical to thermal system at T= 15 nK