Universal phase diagram of dynamics of spin lattice models and strongly correlated bosons

Andrei Maltsev        Landau Institute
Aleksander Prokofiev  Landau Institute
Eugene Demler         Harvard

Supported by Atomtronics MURI, DARPA OLE, Quantum simulation MURI, NSF
Devices classical and quantum: out of equilibrium physics

Emitter and collector currents

\[ I_E = I_{ES} \left( \frac{V_{BE}}{V_T} \right) \]

Collective behavior allows universal description

Is there emergent collective behavior in quantum systems that allows universal description?
Emergent phenomena in dynamics of classical systems

Universality in quantum many-body systems in equilibrium

Solitons in nonlinear wave propagation

Broken symmetries

Bernard cells in the presence of T gradient

Fermi liquid state

Do we have emergent universal phenomena in nonequilibrium dynamics of many-body quantum systems?
Universal phase diagram of dynamics in 2d and 3d anisotropic Heisenberg model

\[ \mathcal{H} = J_\perp \sum_{\langle ij \rangle} \left( S_i^x S_j^x + S_i^y S_j^y \right) + J_z \sum_{\langle ij \rangle} S_i^z S_j^z \]

Hole solitons. Stable to 2d modulation

Both particle and hole solitons allowed

Particle solitons. Stable to 2d modulation

Decay of inhomogeneities to short wavelength oscillations

Particle solitons. Unstable to 2d modulation

2d lamp solutions

Hole solitons. Unstable to 2d modulation

2d lamp solutions
Quantum magnetism of bosons in optical lattices

\[ \mathcal{H} = J_z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + J_\perp \sum_{\langle ij \rangle} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) \]

\[ J_z = \frac{t_{\uparrow}^2 + t_{\downarrow}^2}{2U_{\downarrow\downarrow}} - \frac{t_{\uparrow}^2}{U_{\uparrow\uparrow}} - \frac{t_{\downarrow}^2}{U_{\downarrow\downarrow}} \quad J_\perp = -\frac{t_{\uparrow}t_{\downarrow}}{U_{\downarrow\downarrow}} \]

Duan et al., PRL (2003)
Bose Hubbard model in the hard core limit

\[ \mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1) \]

Hard core limit

\[ U \to \infty \]

- projector of no multiple occupancies

Spin representation of the hard core bosons Hamiltonian

\[ \mathcal{H} = -t \sum_{\langle ij \rangle} \hat{P} b_i^\dagger b_j \hat{P} \]

\[ \hat{P} \]

\[ \mathcal{H} = -t \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) \]
Semiclassical dynamics of anisotropic Heisenberg Hamiltonian

\[ \mathcal{H} = -J_\perp \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) - J_z \sum_{\langle ij \rangle} S_i^z S_j^z \]

Semiclassical equations of motion

Time-dependent variational wavefunction

\[ |\Psi\rangle = \prod_i \left[ \sin \frac{\theta_i}{2} e^{-\varphi_i/2} |\downarrow_i\rangle + \cos \frac{\theta_i}{2} e^{\varphi_i/2} |\uparrow_i\rangle \right] \]

\[ \frac{d}{dt} \langle S_i^x \rangle = \sum_{\langle j \rangle} \left( -J_\perp \langle S_i^z \rangle \langle S_j^y \rangle + J_z \langle S_i^y \rangle \langle S_j^z \rangle \right) \]

Landau-Lifshitz equations

\[ \frac{d}{dt} \langle S_i^y \rangle = \ldots \]
\[ \frac{d}{dt} \langle S_i^z \rangle = \ldots \]

Solitons in the hard core limit of the Hubbard model: Balakrishnan et al., PRL 2009
Equations of Motion
Gradient expansion

Density relative to half filling
\[ \rho = 2\left(n - \frac{1}{2}\right) \]

Phase gradient
\[ k = \varphi_{i+1} - \varphi_i \approx a \nabla_x \phi \]
\approx superfluid velocity

Mass conservation
\[ \dot{\rho} = 4 J_\perp \nabla_x \left[(1 - \rho^2) \sin k\right] \]

Josephson relation
\[ \dot{k} = 8 J_\perp \nabla_x \left[-8J_\perp \rho \cos k + 8J_z \rho\right] \]

Linear hydrodynamics: Bogoliubov mode
Nonlinear hydrodynamics

Keep only the first nonlinear terms.
Separate left and right moving parts

Left-moving part

\[ \partial_t r_1 = (v - C_1 \rho_0 r_1) \nabla_x r_1 \]

Right-moving part

\[ \partial_t r_2 = (-v + C_1 \rho_0 r_2) \nabla_x r_2 \]
Breaking point formation

Regions with different densities move with different velocities

Left-moving part

\[ \partial_t r_1 = (v - C_1 \rho_0 r_1) \nabla_x r_1 \]

Singularity at finite time \( T_0 \)

Right-moving part

\[ \partial_t r_2 = (-v + C_1 \rho_0 r_2) \nabla_x r_2 \]
Dispersion corrections

Left moving part
\[ \partial_t r_1 = (v - C_1 \rho_0 r_1) \nabla_x r_1 + D \nabla_x^3 r_1 \]

Right moving part
\[ \partial_t r_2 = (-v + C_1 \rho_0 r_2) \nabla_x r_2 - D \nabla_x^3 r_2 \]

Competition of nonlinearity and dispersion leads to the formation of soliton structures

Mapping to Kortweg - de Vries equations
In the moving frame and after rescaling
\[ U_T + 6UU_X - U_{XXX} = 0 \quad \text{when} \quad |n - \frac{1}{2}| < \frac{1}{7} \]
\[ U_T + 6UU_X + U_{XXX} = 0 \quad \text{when} \quad |n - \frac{1}{2}| > \frac{1}{7} \]
Stability to transverse fluctuations
Stability to transverse fluctuations

Dispersion

\[ \omega(k) = v \left( k_x^2 + k_\perp^2 \right)^{1/2} \approx v k_x + \frac{v k_\perp^2}{2 k_x} \]

Non-linear waves

\[ \partial_t r_1 = \left( v - C_1 \rho_0 r_1 \right) \nabla_x r_1 + D \nabla_x^3 r_1 \]

Kadomtsev-Petviashvili equation

Planar structures are unstable to transverse modulation if

\[ |n - \frac{1}{2}| > \frac{1}{2\sqrt{7}} \]
Kadomtsev-Petviashvili equation

Stable regime. N-soliton solution.
Plane waves propagating at some angles and interacting

\[ U(x, y, t) = \Phi(\theta_1, \theta_2, \ldots, \theta_n) \]
\[ \theta_i = \omega_i t + \vec{k}_i \vec{r} + c_i \]

Unstable regime.
“Lumps” – solutions localized in all directions. Interactions between solitons do not produce phase shifts.
Universal phase diagram of dynamics in 2d and 3d anisotropic Heisenberg model

\[ \mathcal{H} = J_\perp \sum_{\langle ij \rangle} \left( S^x_i S^x_j + S^y_i S^y_j \right) + J_z \sum_{\langle ij \rangle} S^z_i S^z_j \]

- Hole solitons. Stable to 2d modulation
- Both particle and hole solitons allowed
- Particle solitons. Stable to 2d modulation
- Decay of inhomogeneities to short wavelength oscillations
- Particle solitons. Unstable to 2d modulation
- 2d lamp solutions
- Hole solitons. Unstable to 2d modulation
- 2d lamp solutions

\[ J_z / J_\perp \]
\[ m_z \]
Comparison to solution of lattice model
Full lattice solution of soliton dynamics: stable regime
Full lattice solution of soliton dynamics: unstable regime

Formation of lump solutions
Universal phase diagram of dynamics in 2d and 3d anisotropic Heisenberg model

\[ \mathcal{H} = J_\perp \sum_{\langle ij \rangle} \left( S_i^x S_j^x + S_i^y S_j^y \right) + J_z \sum_{\langle ij \rangle} S_i^z S_j^z \]

Numerical calculations for large amplitude solitons are consistent with the phase diagram based on the analysis of the KdV solitons. **EMERGENT UNIVERSALITY**
Universal dynamical diagram of relaxation in 2d XXZ Heisenberg model

\[ \mathcal{H} = J_\perp \sum_{\langle ij \rangle} \left( S_i^x S_j^x + S_i^y S_j^y \right) + J_z \sum_{\langle ij \rangle} S_i^z S_j^z \]

Hole solitons. Stable to 2d modulation

mKdV regime
Both particle and hole solitons allowed

Particle solitons. 1d solutions unstable to 2d modulation
2d lamp solitons

Hole solitons. 1d solutions unstable to 2d modulation
2d lamp solitons

decay of inhomogeneities to short wavelength oscillations

Numerical calculations for large amplitude solitons are consistent with the phase diagram based on the analysis of the KdV solitons