Quantum many-body dynamics of ultracold atoms

**Near equilibrium dynamics: collective modes**
Observation of the amplitude Higgs mode in the superfluid state of bosons in optical lattices

Experiment: Manuel Endres, Immanuel Bloch and MPQ team
Theory: David Pekker (Caltech), Eugene Demler

**Far from equilibrium dynamics: collective modes**
Quantum dynamics of split one dimensional condensates. Prethermalization

Experiment: David Smith, Joerg Schmiedmayer and Vienna team
Theory: Takuya Kitagawa et al.,

Supported by NSF, DARPA OLE, AFOSR MURI, ARO MURI
Near equilibrium dynamics: Observation of the amplitude Higgs mode in the superfluid state of bosons in an optical lattice

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Collective modes of strongly interacting superfluid bosons

Order parameter

\[ \langle b_i \rangle = \Phi = |\Phi| e^{i\theta} \]

Breaks U(1) symmetry

Figure from Bissbort et al. (2010)

Phase (Goldstone) mode = gapless Bogoliubov mode

\[ \omega = c |\vec{q}| \]

Gapped amplitude mode (Higgs mode)

\[ \omega = \sqrt{\Delta^2 + c^2 q^2} \]
Excitations of the Bose Hubbard model

Softening of the amplitude mode is the defining characteristic of the second order Quantum Phase Transition
Is there a Higgs resonance 2d?

D. Podolsky et al., arXiv:1108.5207

Why it is difficult to observe the amplitude mode

Stoferle et al., PRL(2004)

Peak at $U$ dominates and does not change as the system goes through the SF/Mott transition
Exciting the amplitude mode

Absorbed energy

\[ E = 2\pi (\delta J)^2 S(\omega) \omega T_{\text{mod}} \]
Exciting the amplitude mode

Manuel Endres, Immanuel Bloch and MPQ team
Experiments: full spectrum
Manuel Endres, Immanuel Bloch and MPQ team
Theory. Time dependent mean-field

Semiclassical equations of motion

\[ |\Psi\rangle = \prod_i \left[ \sin \frac{\theta_i}{2} e^{-\varphi_i/2} |\downarrow_i\rangle + \cos \frac{\theta_i}{2} e^{\varphi_i/2} |\uparrow_i\rangle \right] \]

\[ \frac{d}{dt} \langle S^x_i \rangle = \sum_{\langle j \rangle} \left( -J_{\perp} \langle S^z_i \rangle \langle S^y_j \rangle + J_z \langle S^y_i \rangle \langle S^z_j \rangle \right) \]

\[ \frac{d}{dt} \langle S^y_i \rangle = \ldots \]

\[ \frac{d}{dt} \langle S^z_i \rangle = \ldots \]

Threshold for absorption is captured very well
Time dependent cluster mean-field

Lattice height 9.5 Er: (1x1 vs 2x2)

2x2 captures width of spectral feature
Absorption spectra. Theory (1x1 calculations)

- Disappearing amplitude mode
- Breathing mode
- Spectrum remains gapped due to trap
- Details at the QCP
Far from equilibrium dynamics: Quantum dynamics of split one dimensional condensates Prethermalization

Theory: Takuya Kitagawa et al.

arXiv:1112.0013
Relaxation to equilibrium

**Thermalization:** an isolated interacting systems approaches thermal equilibrium at long times (typically at microscopic timescales). All memory about the initial conditions except energy is lost.

**Bolzmann equation**

\[
\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} \vec{v} + \frac{\partial f}{\partial x} \vec{F} = -\frac{1}{\tau} (f - f_0)
\]

U. Schneider et al., arXiv:1005.3545
We observe irreversibility and approximate thermalization. At large time the system approaches stationary solution in the vicinity of, but not identical to, thermal equilibrium. The ensemble therefore retains some memory beyond the conserved total energy...This holds for interacting systems and in the large volume limit.

Prethermalization in ultracold atoms, theory: Eckstein et al. (2009); Moeckel et al. (2010), L. Mathey et al. (2010), R. Barnett et al. (2010)
Experimental demonstration of prethermalization

Probing thermolization using local resolution and complete characterization of quantum noise

Initial $T=120$ nK (blue line).
After 27.5 ms identical to thermal system at $T=15$ nK
At all lengthscales
In all correlation functions
Experiments with 2D Bose gas

Experiments with 1D Bose gas
Hofferberth et al., Nat. Physics 2008
Interference experiments with condensates

Assuming ballistic expansion

Interference of fluctuating condensates

Polkovnikov et al. (2006)

Amplitude of interference fringes

\[ C = \int_0^L dx \, e^{i(\phi_1(x) - \phi_2(x))} \]
Distribution function of fringe amplitudes for interference of fluctuating condensates

Polkovnikov et al. (2006), Gritsev et al. (2006), Imambekov et al. (2007)

\[ \mathcal{C} \text{ is a quantum operator.} \]

The measured value of \( \mathcal{C} \) will fluctuate from shot to shot

\[ < \mathcal{C}^n > = \int dz_1 \cdots dz_n \left< e^{i\phi(z_1)} \cdots e^{i\phi(z_n)} \right> \]

Higher moments reflect higher order correlation functions

Experiments analyze distribution function of \( \mathcal{C} \)
FDF of phase and contrast

- Matter-wave interferometry
FDF of phase and contrast

- Matter-wave interferometry

- Plot as circular statistics

contrast

phase
FDF of phase and contrast

- Matter-wave interferometry: repeat $FDF$ of phase and contrast many times

$\text{contrast}_i$ accumulate statistics

$\text{phase}$
Equilibrium. Interference of independent 1d condensates

Theory: Altman, Imambekov, Gritsev, Polkovnikov, Demler
Measurements of dynamics of split condensate
Theoretical analysis of dephasing Luttinger liquid model
Luttinger liquid model of phase dynamics

Condensate 1  Condensate 2

\[ \hat{\phi}_s(r) = \hat{\phi}_1(r) - \hat{\phi}_2(r) \]

\[ 2\hat{n}_s(r) = \hat{n}_1(r) - \hat{n}_2(r) \]

\[ [\hat{n}_s(r), \hat{\phi}_s(r')] = -i\delta(r - r') \]

For identical average densities, phase difference modes decouple from the phase sum mode

\[ H_s = \frac{c_s}{2} \int \left[ \frac{K_s}{\pi} (\nabla \hat{\phi}_s(r))^2 + \frac{\pi}{K_s} \hat{n}_s^2(r) \right] \, dr \]
Luttinger liquid model of phase dynamics

\[ H_s = \frac{c_s}{2} \int \left[ \frac{K_s}{\pi} (\nabla \hat{\phi}_s(r))^2 + \frac{\pi}{K_s} \hat{n}_s^2(r) \right] dr \]

\[ = \frac{c_s}{2} \sum_k \left[ \frac{K_s k^2}{\pi} \hat{\phi}_s^\dagger(k)\hat{\phi}_s(k) + \frac{\pi}{K_s} \hat{n}_s^\dagger(k)\hat{n}_s(k) \right] \]

For each k-mode we have simple harmonic oscillators

\[ \hat{n}_s(k) \quad \hat{\phi}_s(k) \quad \hat{n}_s(k) \quad \hat{\phi}_s(k) \]
Phase diffusion vs Contrast Decay

Segment size is smaller than the fluctuation lengthscale

Segment size is longer than the fluctuation lengthscale

At long times the difference between the two regime occurs for

\[ l_0 = \frac{8 K^2}{\pi^2 \rho} \]
Length dependent phase dynamics

“Short segments” = phase diffusion

“Long segments” = contrast decay
Energy distribution

Initially the system is in a squeezed state with large number fluctuations.

Energy stored in each mode initially

\[ E_k = g n_s^2(k, t = 0) = \frac{g}{\phi_0^2} \]

 Equipartition of energy

For 2d also pointed out by Mathey, Polkovnikov in PRA (2010)

The system should look thermal like after different modes dephase. Effective temperature is not related to the physical temperature.
Comparison of experiments and LL analysis

Do we have thermal-like distributions at longer times?
Prethermalization

Interference contrast is described by thermal distributions but at temperature much lower than the initial temperature.
Long time transient

Condensate 1  Condensate 2

\[ \rho_1 > \rho_2 \]

When the average densities of two condensates are different, the interference contrast relaxes to smaller value as time progresses.
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