Learning about order from noise

Quantum noise studies of ultracold atoms

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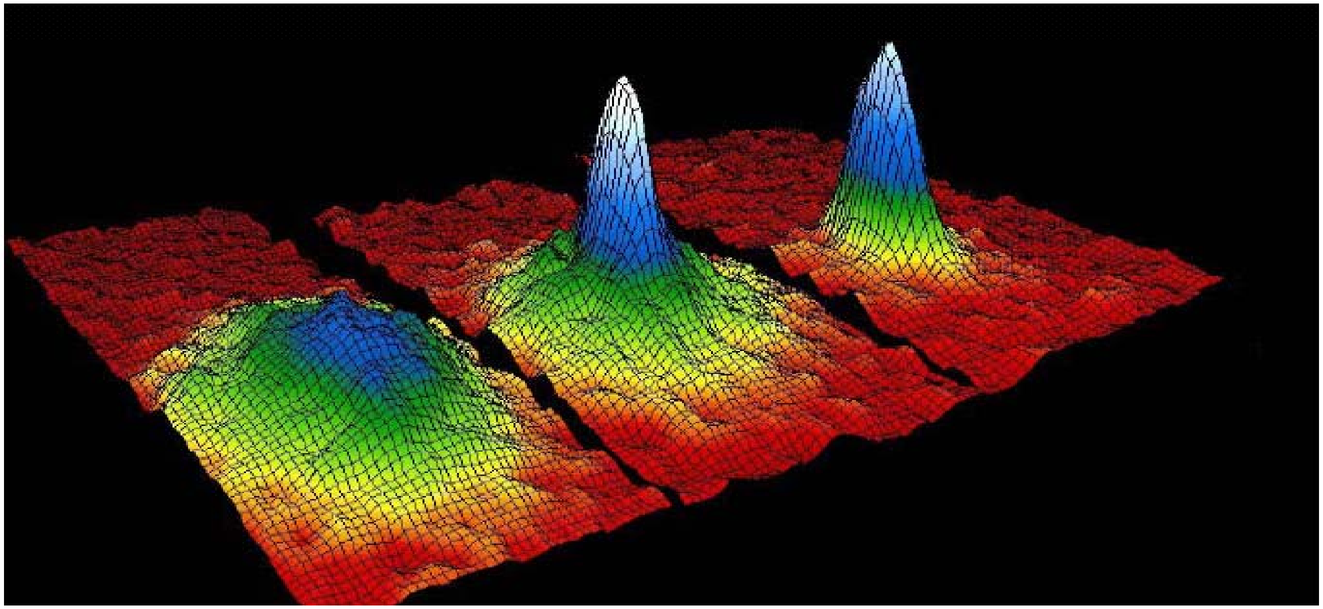
Bose-Einstein condensation of weakly interacting atoms



Density	10^{13} cm^{-1}	
Typical distance between atoms	300 nm	$T_{\mathrm{BEC}} \sim 1\mu\mathrm{K}$
Typical scattering length	10 nm	

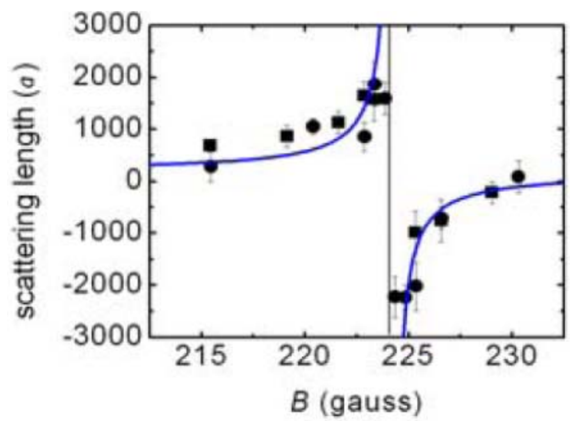
Scattering length is much smaller than characteristic interparticle distances. Interactions are weak

New Era in Cold Atoms Research

Focus on Systems with Strong Interactions

- Feshbach resonances.

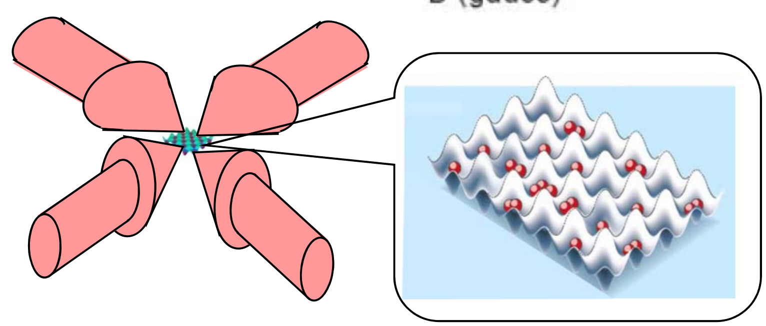
  Scattering length comparable to interparticle distances



E. Tiesinga et al., PRA (1993);
K. O'Hara et al., Science, 2002

- Optical lattices.

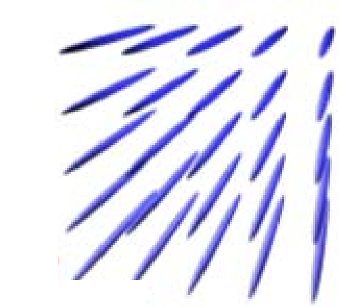
  Suppressed kinetic energy.
  Enhanced role of interactions



D. Jaksch et al., PRL (1998);
M. Greiner et al., Nature, 2002

- Low dimensional systems.

  Strongly interacting regimes at low densities



Kinoshita et al., Science, 2004;
Paredes et al., Nature, 2004

Among major challenges:
detection and characterization of
many-body states of ultracold atoms

This talk:
applications of quantum noise
for studying cold atoms

Outline

Introduction. Historical review

Strongly correlated systems in optical lattices.
Hanbury-Brown-Twiss type experiments

Low dimensional condensates.
Quantum noise in interference experiments

Quantum noise

Classical measurement:

collapse of the wavefunction into eigenstates of x

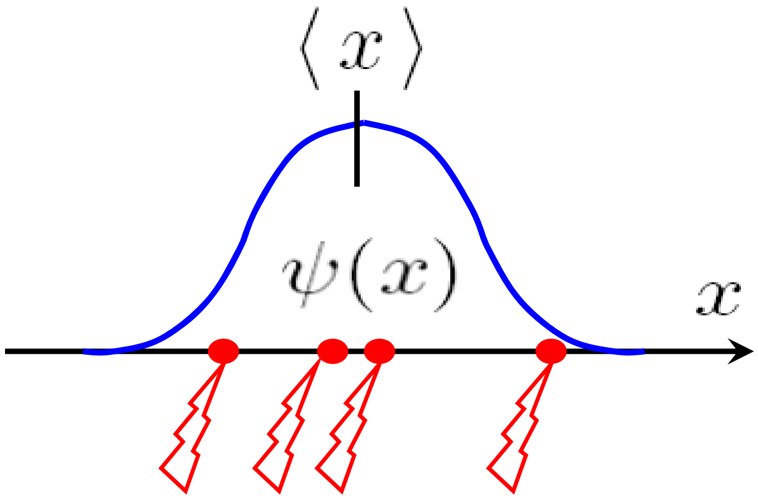
$$\langle x \rangle = \int dx\, x\, |\psi(x)|^2$$

$$\langle x^2 \rangle = \int dx\, x^2\, |\psi(x)|^2$$

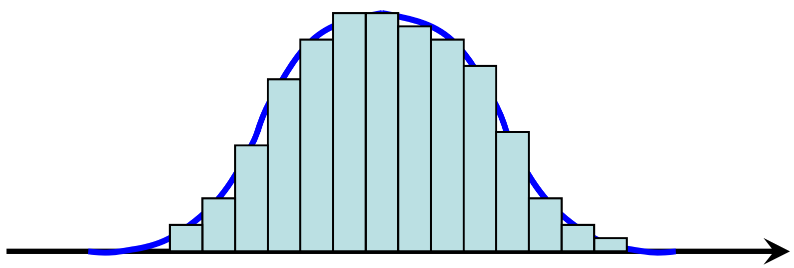
$$\dots$$

$$\langle x^n \rangle = \int dx\, x^n\, |\psi(x)|^2$$

$$\dots$$

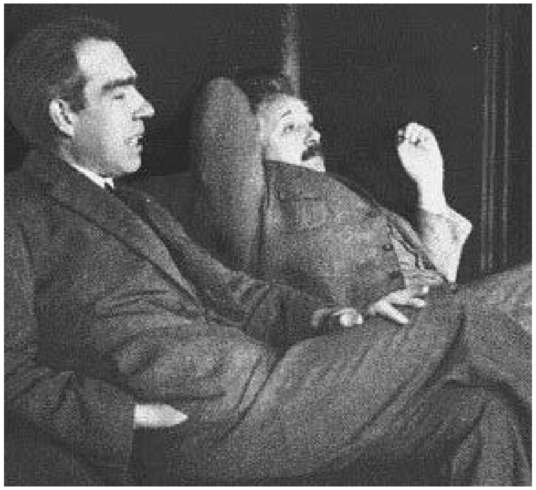


Histogram of measurements of x

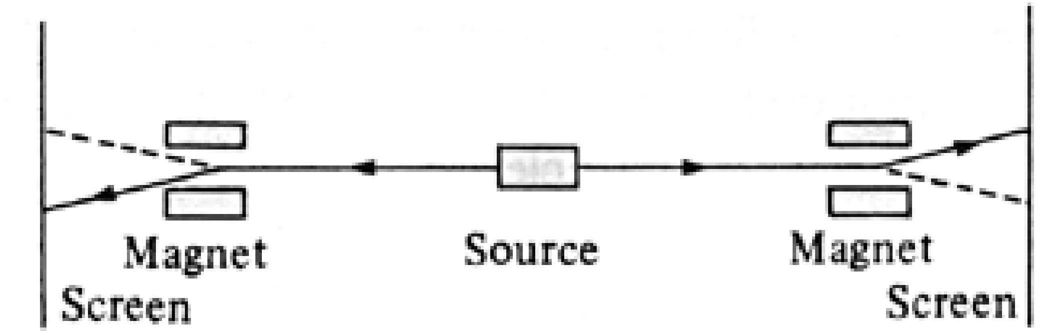


Probabilistic nature of quantum mechanics

Bohr-Einstein debate on spooky action at a distance



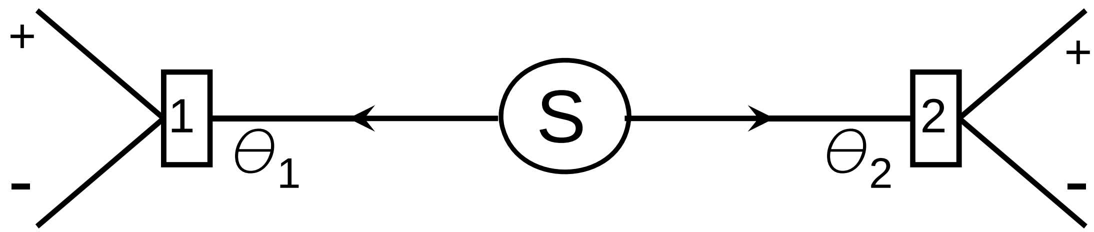
Einstein-Podolsky-Rosen experiment



$$| S = 0 \rangle = | \uparrow \rangle_L | \downarrow \rangle_R - | \downarrow \rangle_L | \uparrow \rangle_R$$

Measuring spin of a particle in the left detector
instantaneously determines its value in the right detector

Aspect's experiments: tests of Bell's inequalities



Correlation function $$E(\theta_1,\theta_2) = \frac{\langle (I_1^+ - I_1^-)(I_2^+ - I_2^-)\rangle}{\langle (I_1^+ + I_1^-)(I_2^+ + I_2^-)\rangle}$$

Classical theories with hidden variable require

$$B = E(\theta_1,\theta_2) - E(\theta_1,\theta_2') + E(\theta_1',\theta_2') - E(\theta_1',\theta_2) \leq 2$$

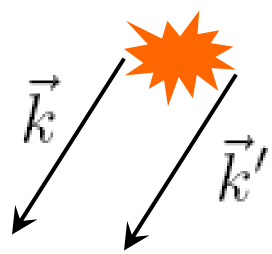
Quantum mechanics predicts B=2.7 for the appropriate choice of Θ's and the state

$$|\psi\rangle = |+\rangle_L |+\rangle_R + |-\rangle_L |-\rangle_R$$

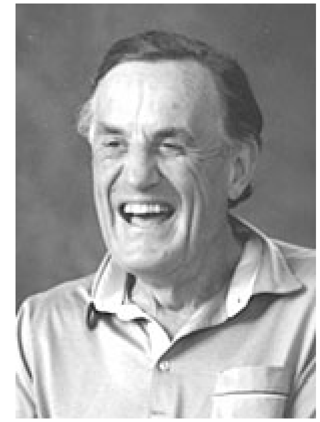
Experimentally measured value B=2.697. Phys. Rev. Let. 49:92 (1982)

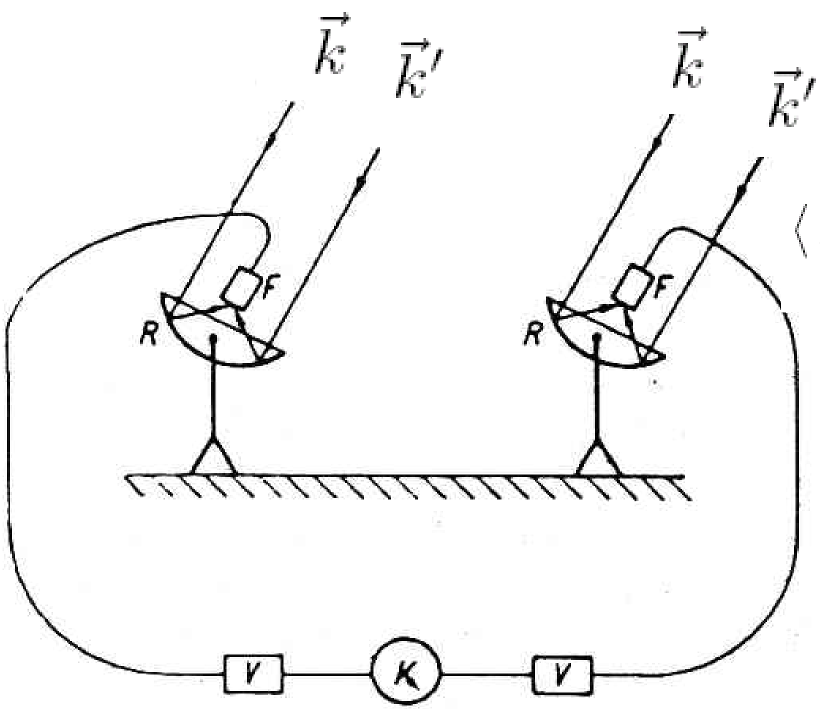
Hanbury-Brown-Twiss experiments

Classical theory of the second order coherence



Hanbury Brown and Twiss,
Proc. Roy. Soc. (*London*),
A, 242, pp. 300-324

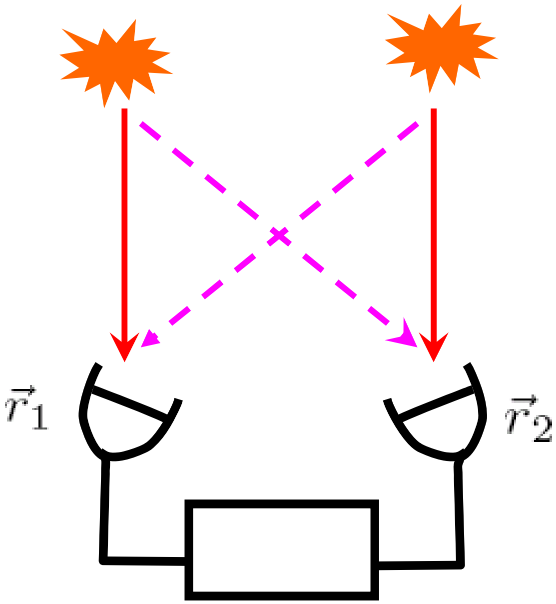




$$\langle I(\vec{r}_1)\ I(\vec{r}_2)\rangle = A + B\ \cos\left((\vec{k}-\vec{k}')\ (\vec{r}_1-\vec{r}_2)\right)$$

Measurements of the angular diameter of Sirius
Proc. Roy. Soc. (*London*), *A*, 248, pp. 222-237

Quantum theory of HBT experiments



Glauber,
Quantum Optics and Electronics (1965)

For bosons

$$A = A_1 + A_2$$

For fermions

$$A = A_1 - A_2$$

HBT experiments with matter

Experiments with neutrons
Ianuzzi et al., Phys Rev Lett (2006)

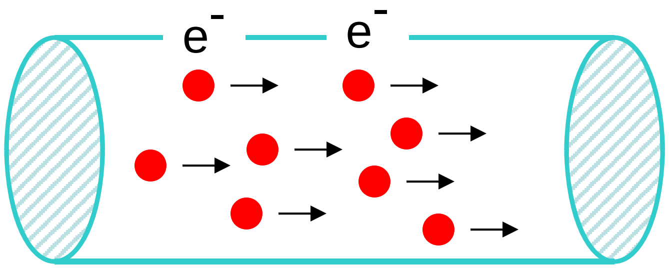
Experiments with electrons
Kiesel et al., Nature (2002)

Experiments with 4He, 3He
Westbrook et al., Nature (2007)

Experiments with ultracold atoms
Bloch et al., Nature (2005,2006)

Shot noise in electron transport

Proposed by Schottky to measure the electron charge in 1918



Spectral density of the current noise

$$S_\omega = \int \langle \{ \delta I(t), \delta I(0) \}_+ \rangle \, e^{i\omega t} \, dt$$

Related to variance of transmitted charge

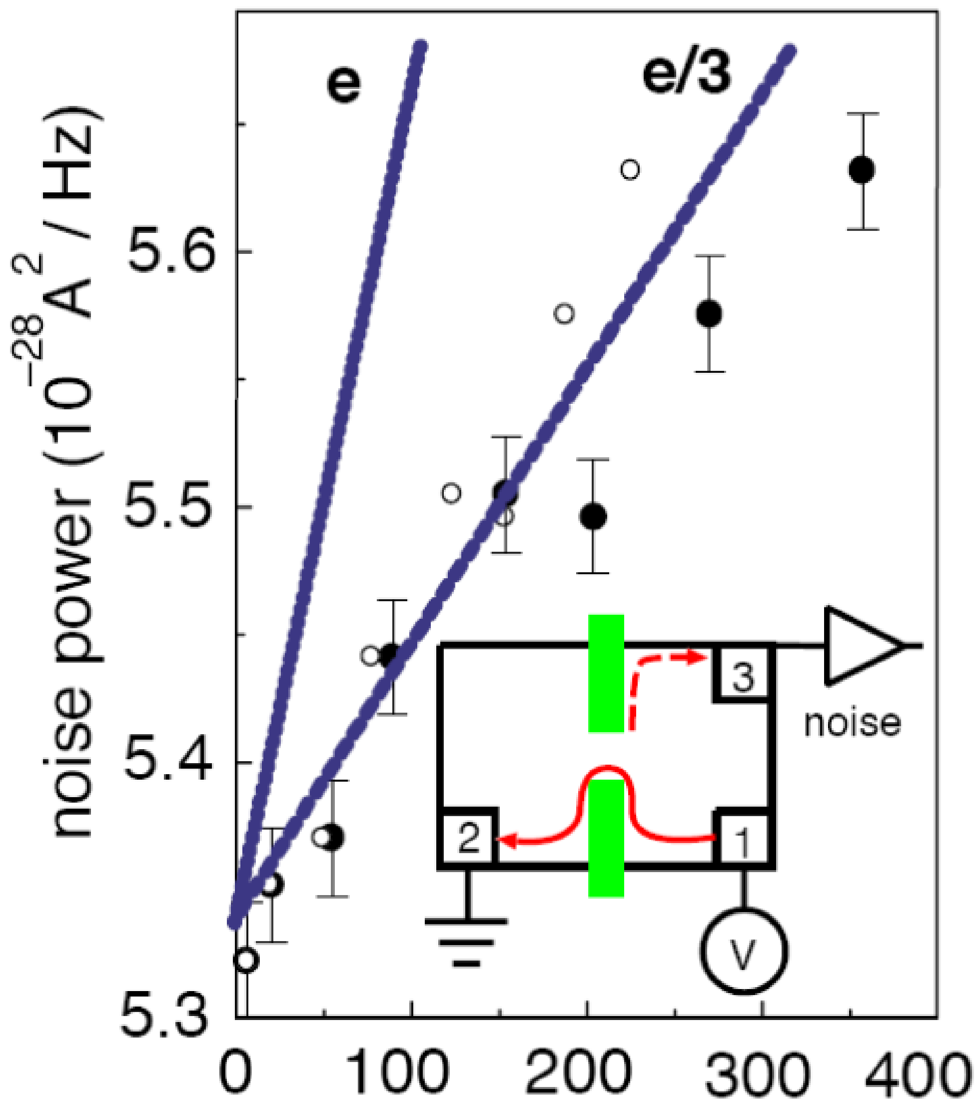
$$S_0 = \frac{2}{\tau} \langle \delta q^2(\tau) \rangle$$

When shot noise dominates over thermal noise

$$\boxed{S_0 = 2 \, e \, I}$$

Poisson process of independent transmission of electrons

Shot noise in electron transport



Current noise for tunneling across a Hall bar on the 1/3 plateau of FQE

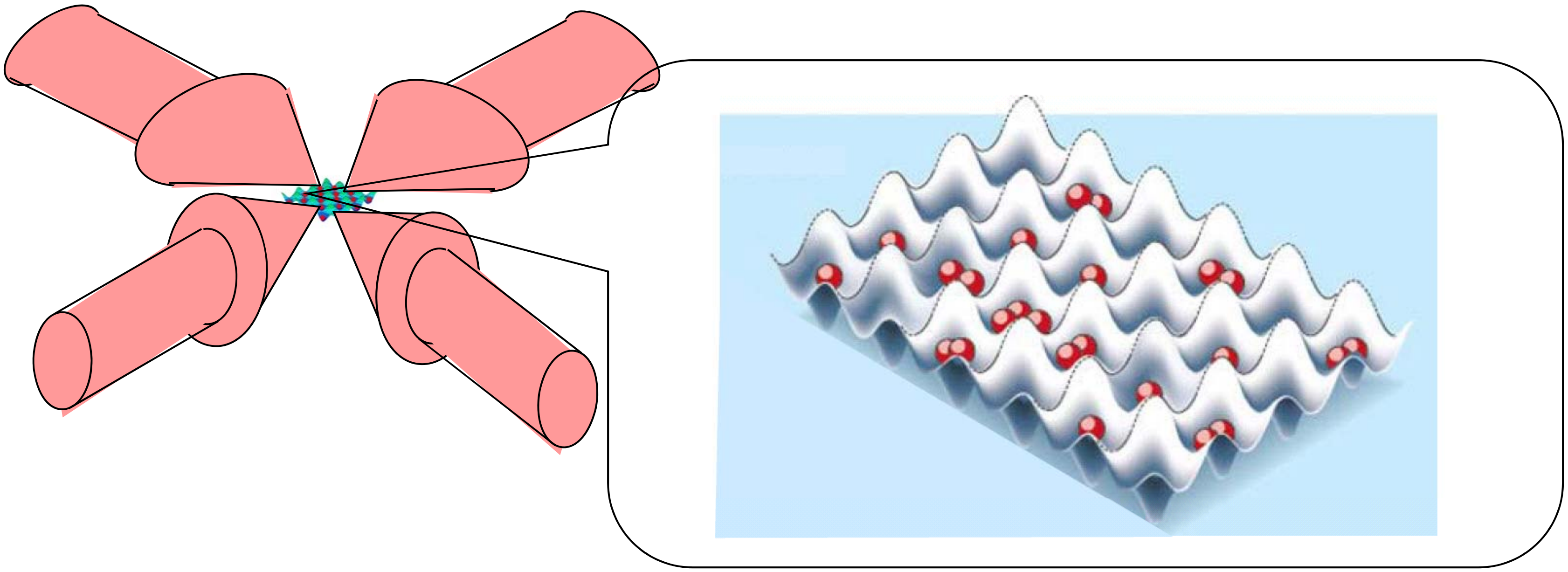
Etien et al. PRL 79:2526 (1997)
see also Heiblum et al. Nature (1997)

Experiments with atoms in optical lattices: Hanburry-Brown-Twiss experiments and beyond

Theory: Altman, Demler, Lukin, PRA 70:13603 (2004)

Experiment: Folling et al., Nature 434:481 (2005);
Spielman et al., PRL 98:80404 (2007);
Tom et al. Nature 444:733 (2006)

Atoms in optical lattices

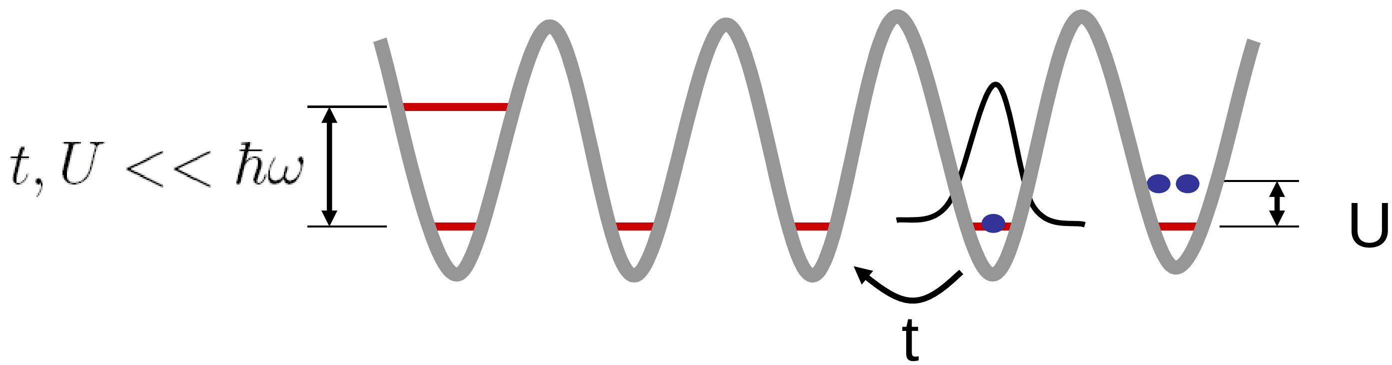


Theory: Jaksch et al. PRL (1998)

Experiment: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Phillips et al., J. Physics B (2002)
Esslinger et al., PRL (2004);
Ketterle et al., PRL (2006)

Bose Hubbard model

M.P.A. Fisher et al., PRB (1989)
D. Jaksch et al. PRL (1998)

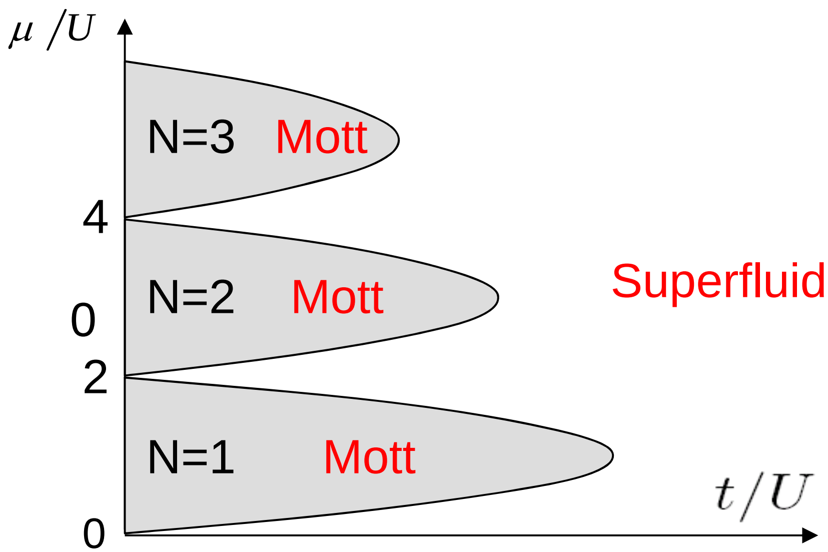


$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i \left(n_i - 1 \right) - \mu \sum_i n_i$$

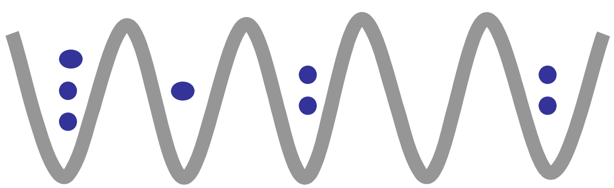
t – tunneling of atoms between neighboring wells

U – repulsion of atoms sitting in the same well

Bose Hubbard model



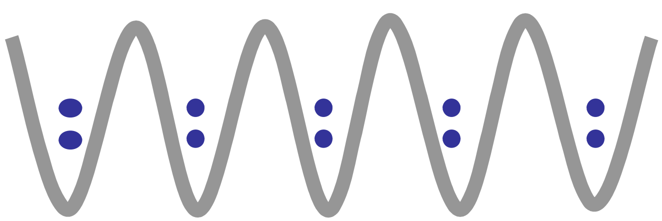
M.P.A. Fisher et al.,
PRB40:546 (1989)



$$U << Nt$$

Superfluid phase

Weak interactions



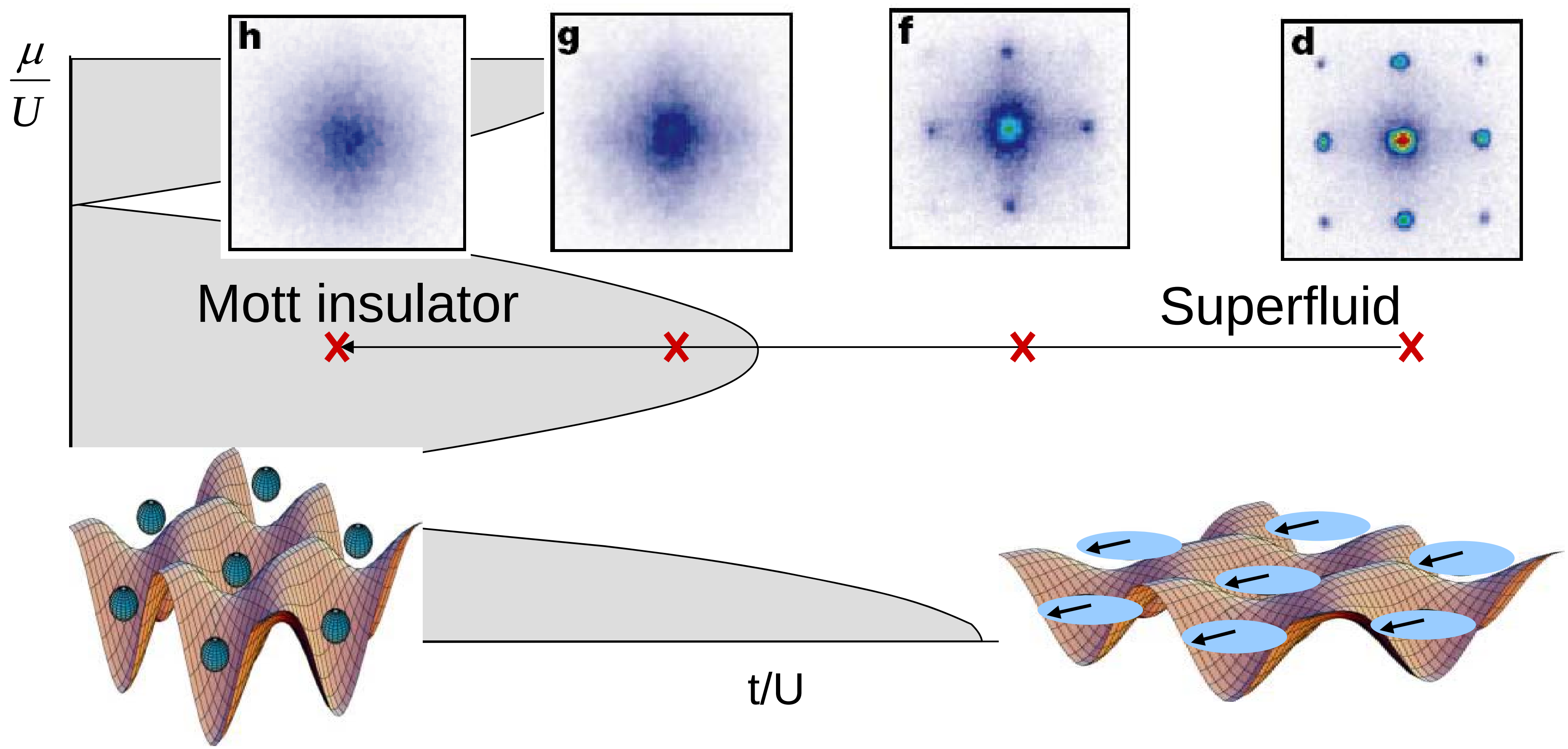
$$U >> Nt$$

Mott insulator phase

Strong interactions

Superfluid to insulator transition in an optical lattice

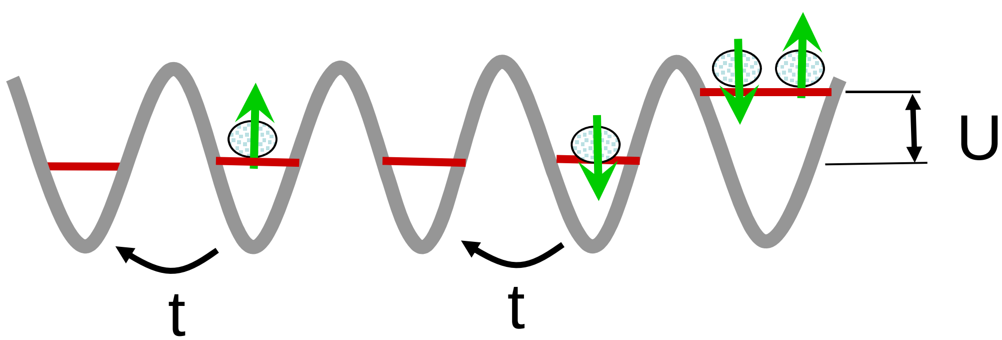
M. Greiner et al., Nature 415 (2002)



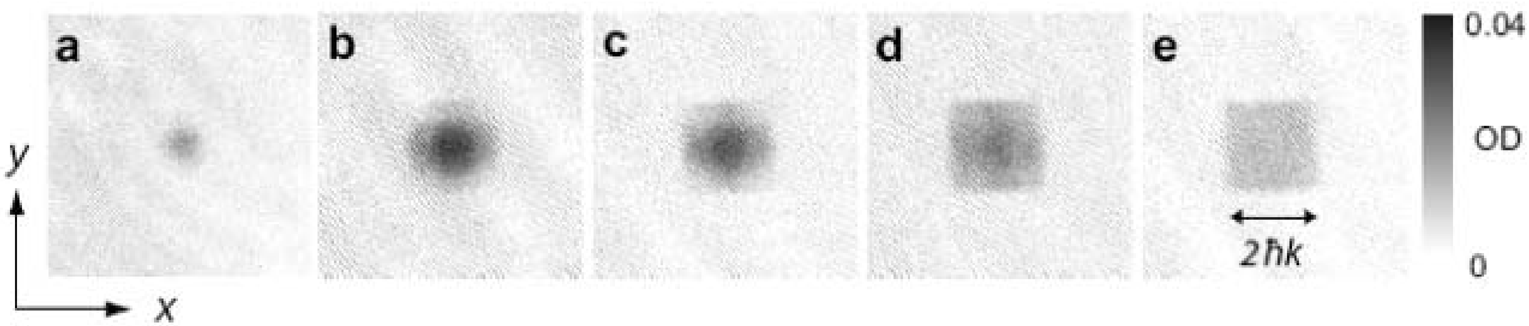
Why study ultracold atoms in optical lattices

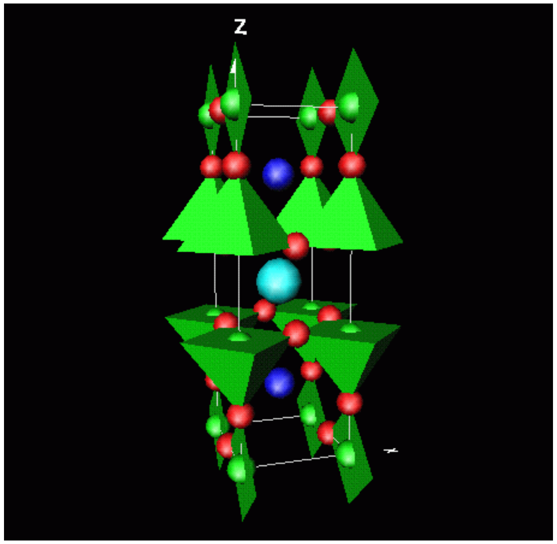
Fermionic atoms in optical lattices

$$\mathcal{H} = -t \sum_{\langle ij \rangle\, \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_{i}$$

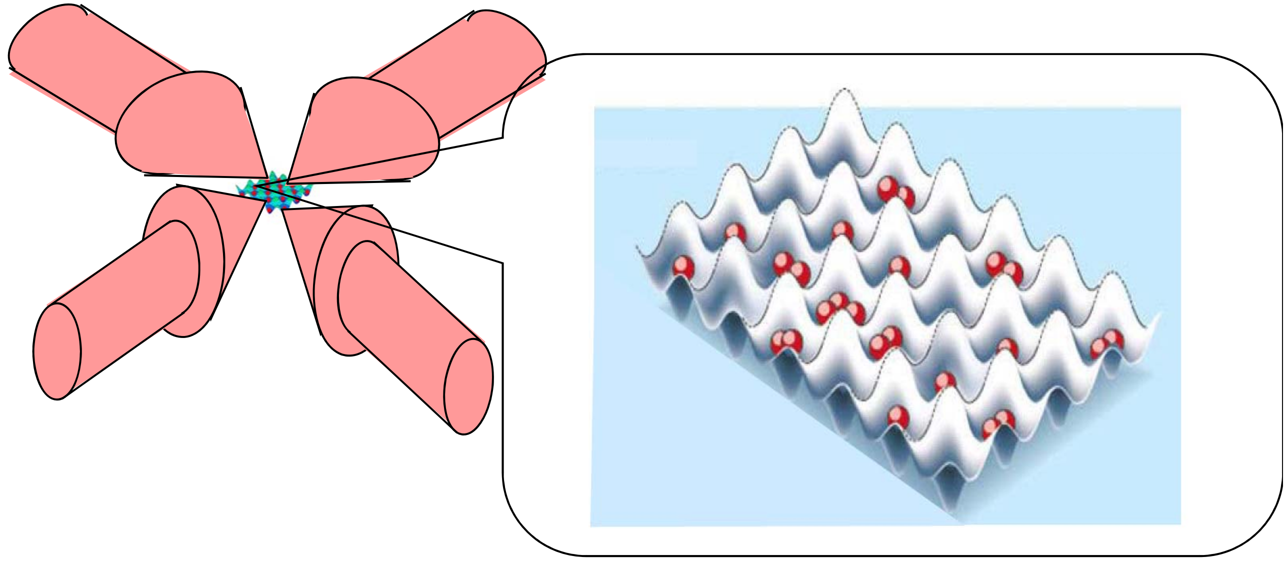


Experiments with fermions in optical lattice, Kohl et al., PRL 2005





$YBa_2Cu_3O_7$



Atoms in optical lattice

Antiferromagnetic and
superconducting Tc
of the order of 100 K

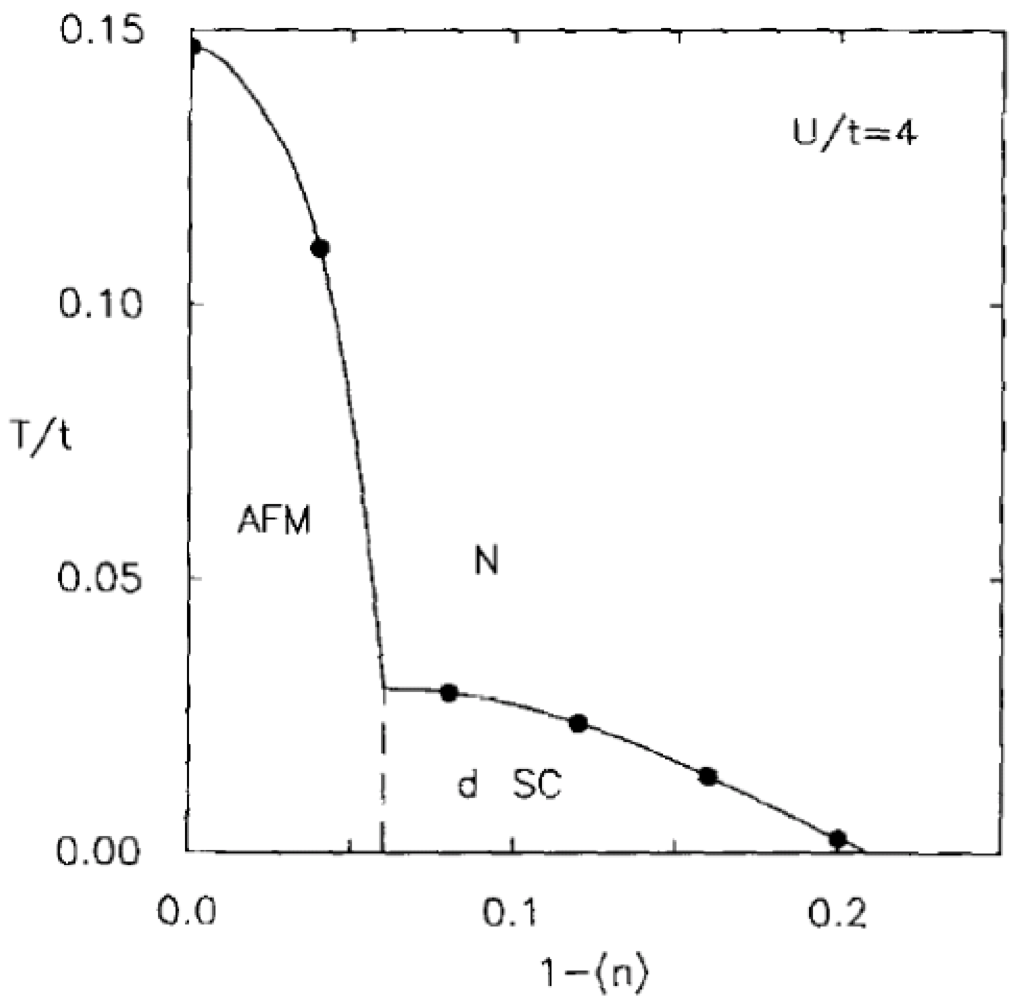
Antiferromagnetism and
pairing at sub-micro Kelvin
temperatures

Same microscopic model

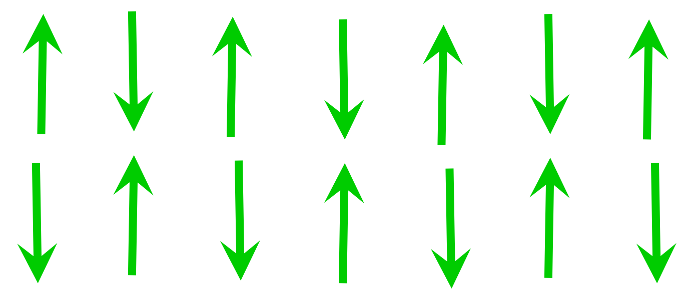
$$\mathcal{H} = -t \sum_{\langle ij \rangle\, \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i$$

Positive U Hubbard model

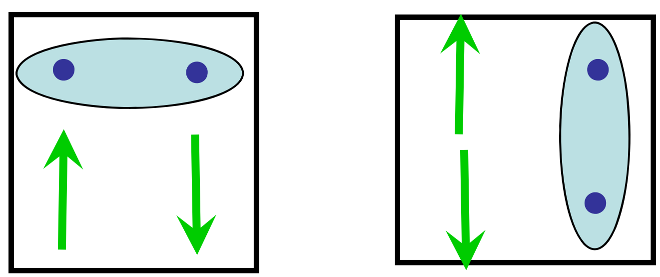
Possible phase diagram. Scalapino, Phys. Rep. 250:329 (1995)

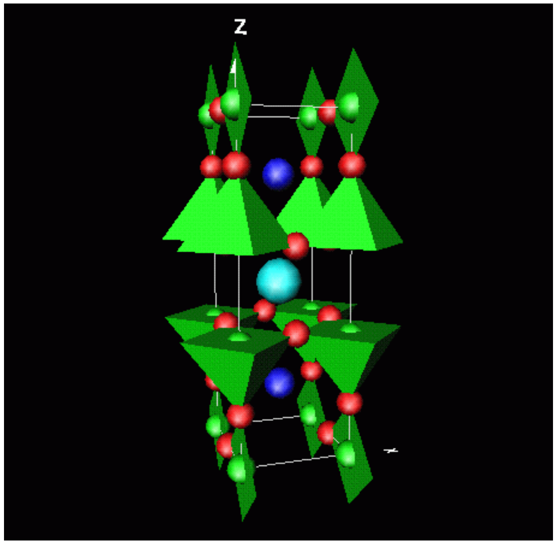


Antiferromagnetic insulator

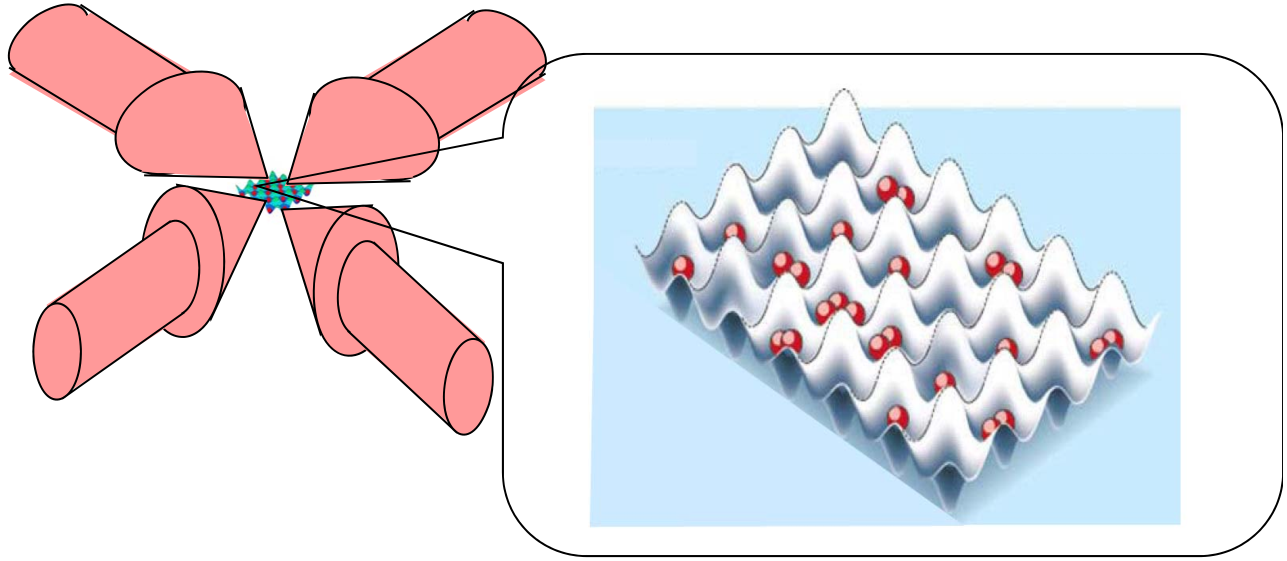


D-wave superconductor





$YBa_2Cu_3O_7$



Atoms in optical lattice

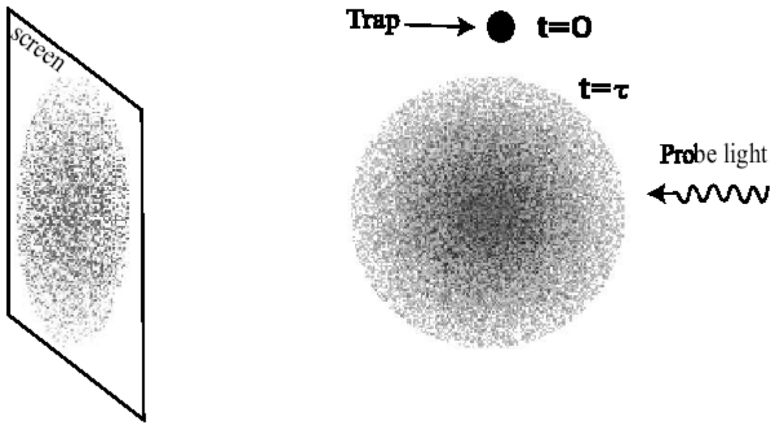
Same microscopic model

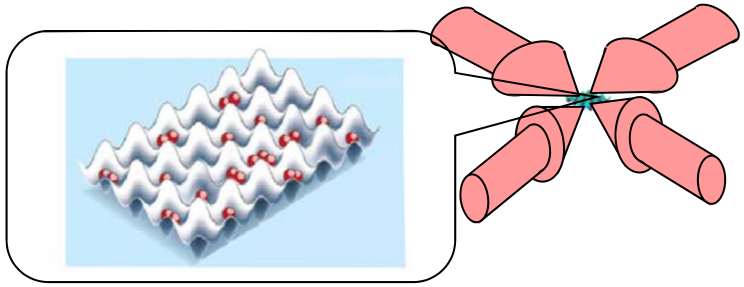
Quantum simulations of strongly correlated electron systems using ultracold atoms

Detection?

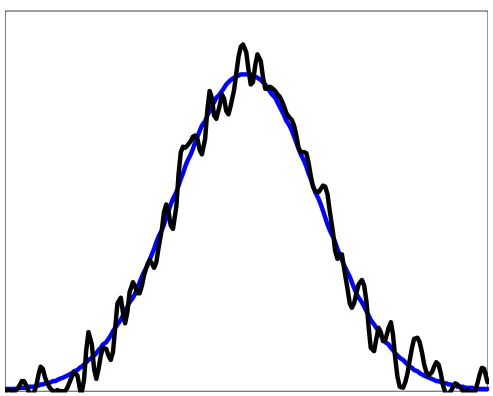
Quantum noise analysis as a probe of many-body states of ultracold atoms

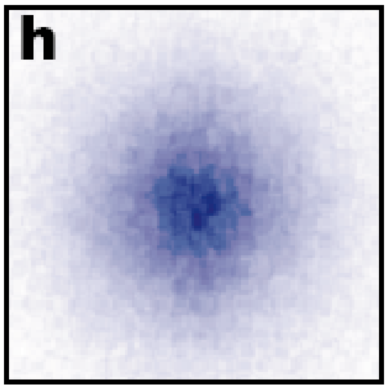
Time of flight experiments





Quantum noise interferometry of atoms in an optical lattice

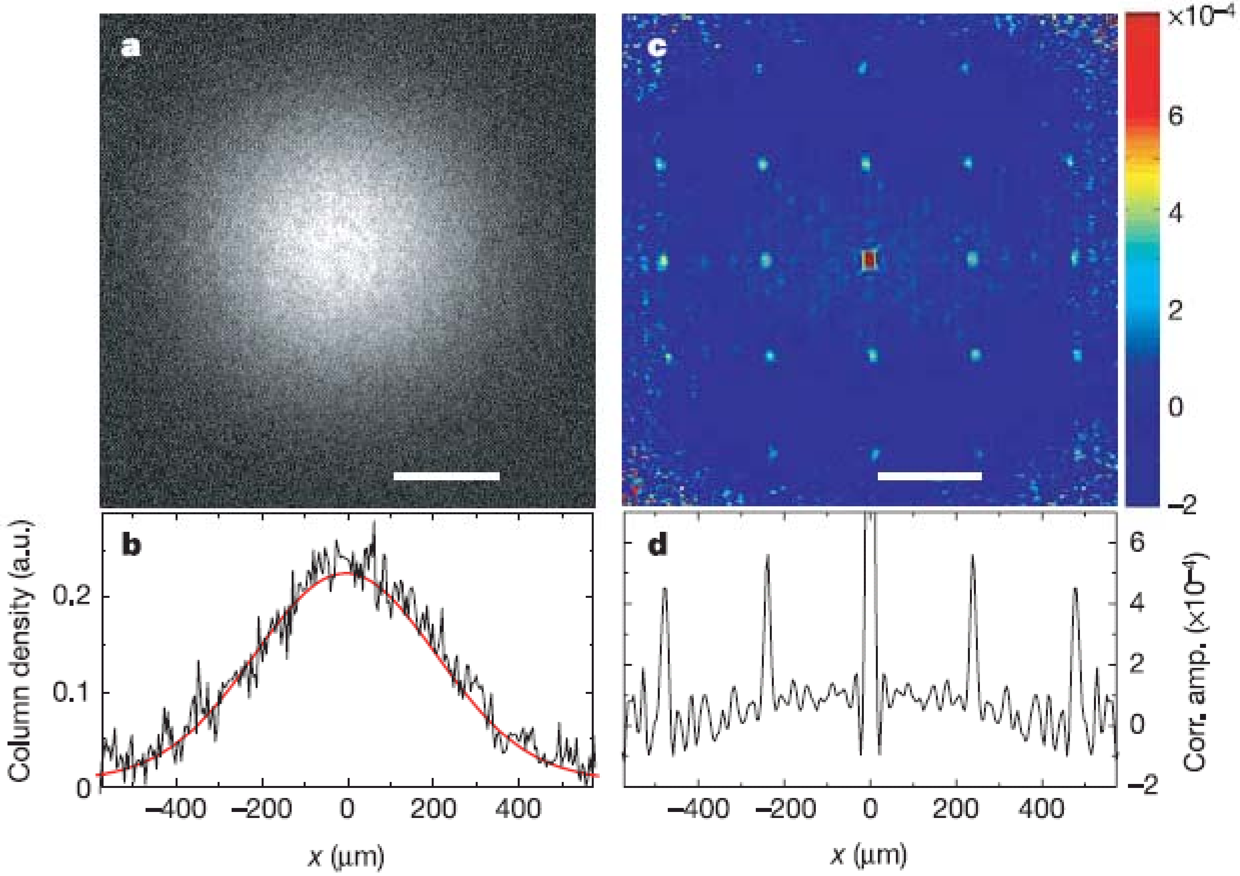




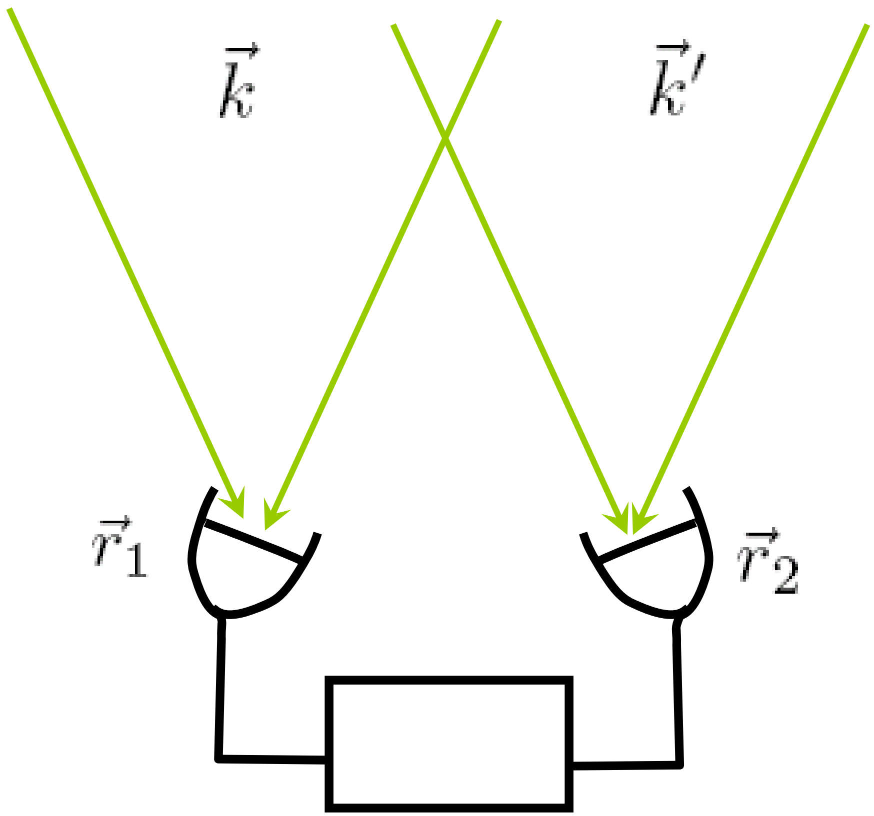
Second order coherence $G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$

Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

Experiment: Folling et al., Nature 434:481 (2005)

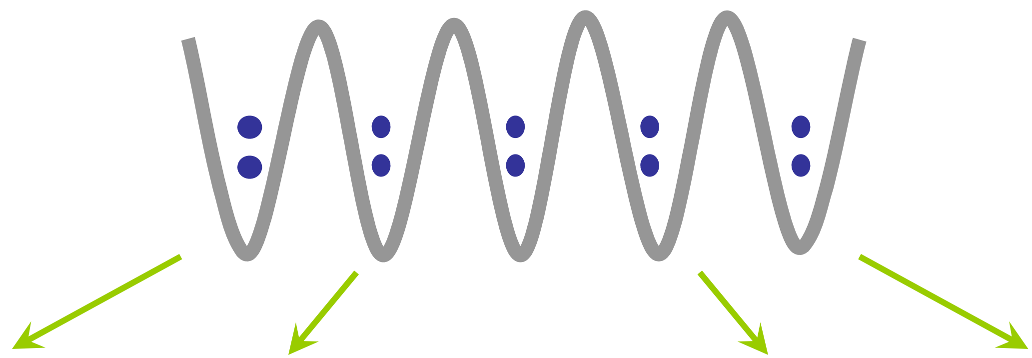


Hanburry-Brown-Twiss stellar interferometer



$$\langle\, I(\vec{r}_1)\ I(\vec{r}_2)\rangle = A + B\ \cos\Big((\vec{k}-\vec{k}')\ (\vec{r}_1-\vec{r}_2)\Big)$$

Second order coherence in the insulating state of bosons



Bosons at quasimomentum $\vec{k}$ expand as plane waves

with wavevectors $\vec{k}, \; \vec{k}+\vec{G}_1, \; \vec{k}+\vec{G}_2$

First order coherence: $\langle \; \rho(\vec{r}) \; \rangle$

Oscillations in density disappear after summing over $\vec{k}$

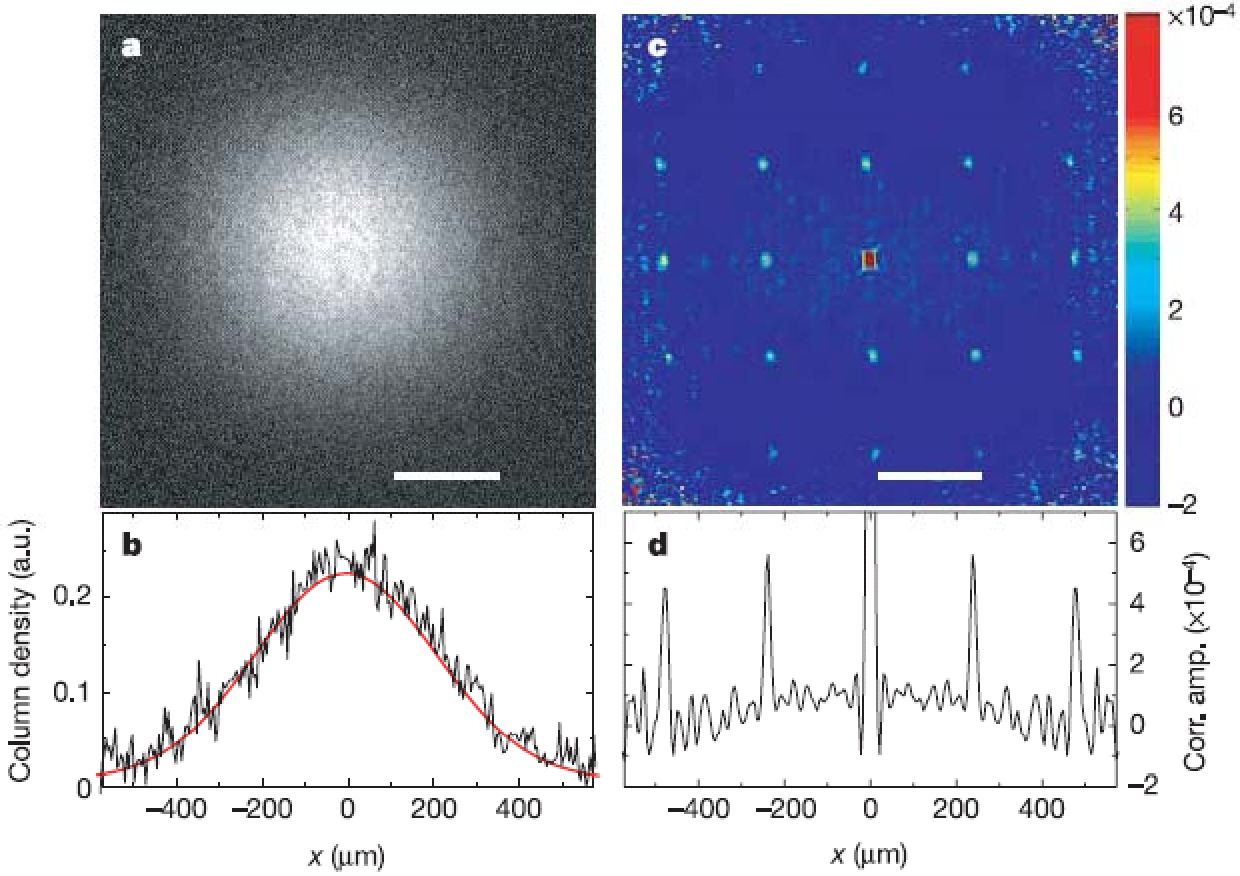
Second order coherence: $\langle \; \rho(\vec{r}_1) \; \rho(\vec{r}_2) \; \rangle$

Correlation function acquires oscillations at reciprocal lattice vectors

$$\langle \; \rho(\vec{r}_1) \; \rho(\vec{r}_2) \; \rangle = A_0 + A_1 \; \cos\Big(\; \vec{G}_1(\vec{r}_1 - \vec{r}_2) \; \Big) + A_2 \; \cos\Big(\; \vec{G}_2(\vec{r}_1 - \vec{r}_2) \; \Big) + \dots$$

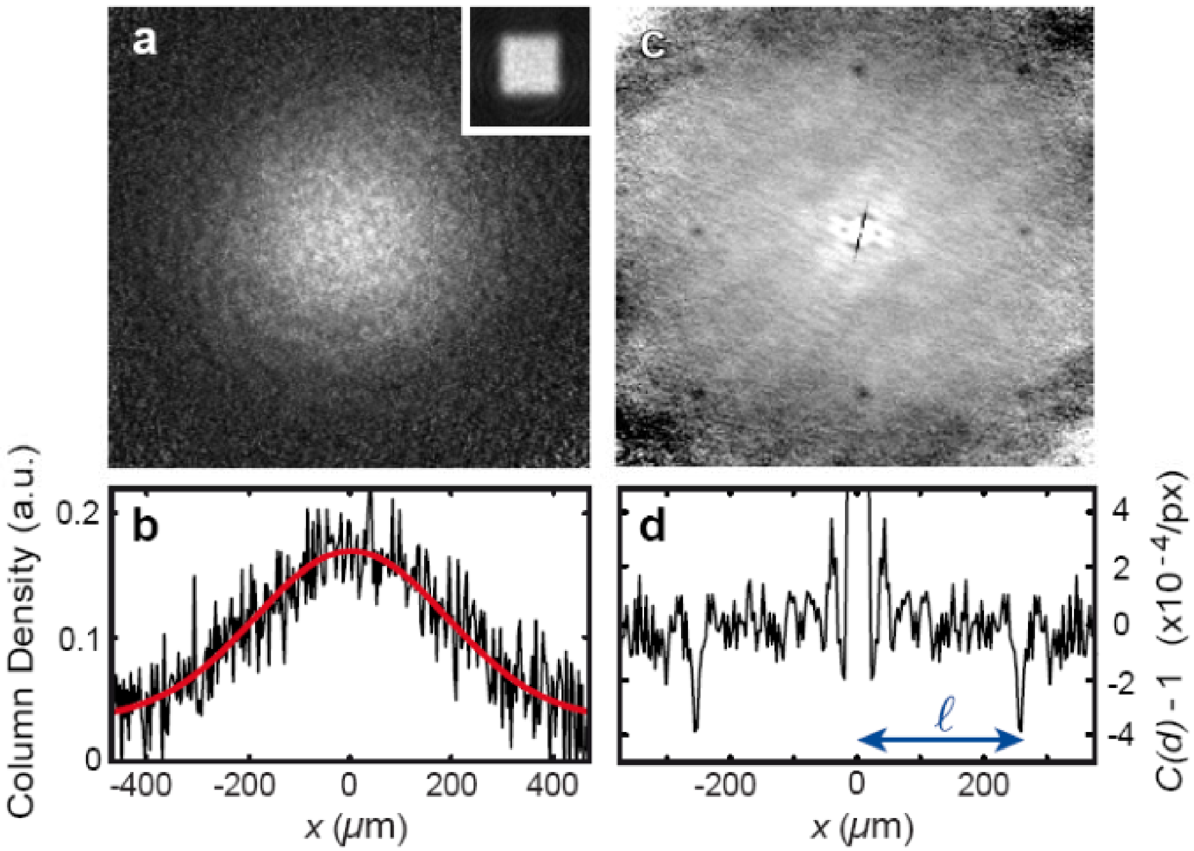
Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

Experiment: Folling et al., Nature 434:481 (2005)



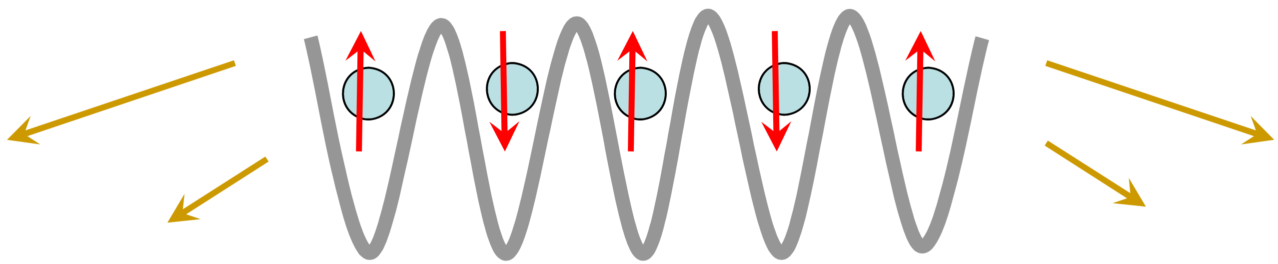
Second order coherence in the insulating state of fermions. Hanburry-Brown-Twiss experiment

Experiment: Tom et al. Nature 444:733 (2006)



How to detect antiferromagnetism

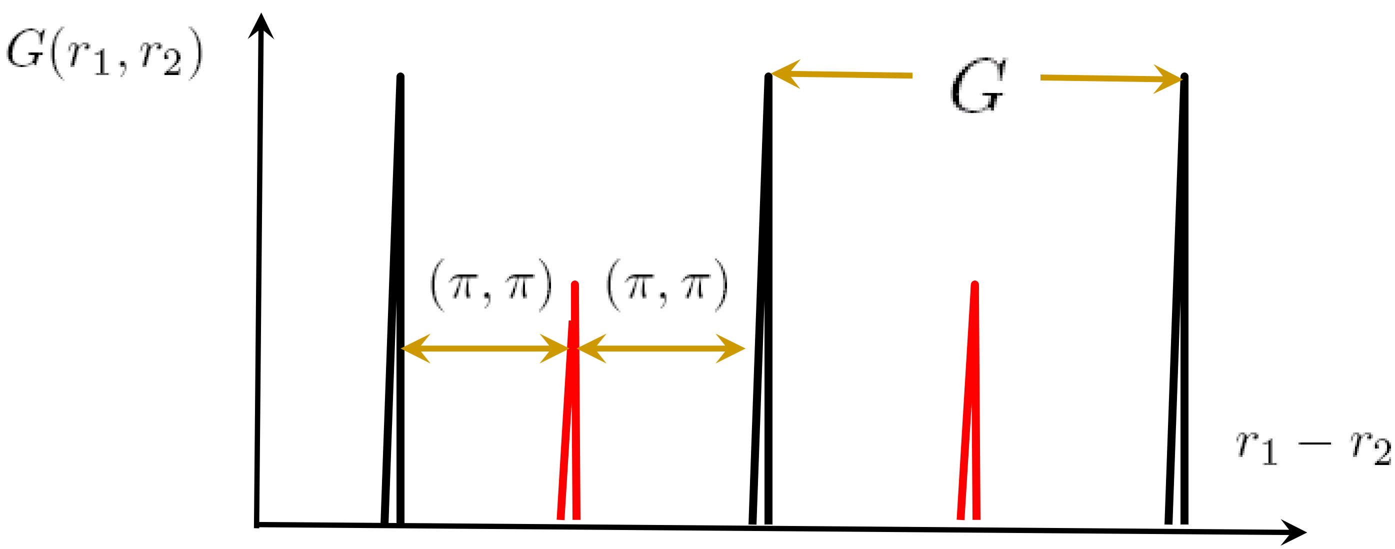
Probing spin order in optical lattices



$$r = \frac{\hbar\, k\, t}{m}$$

Correlation Function Measurements

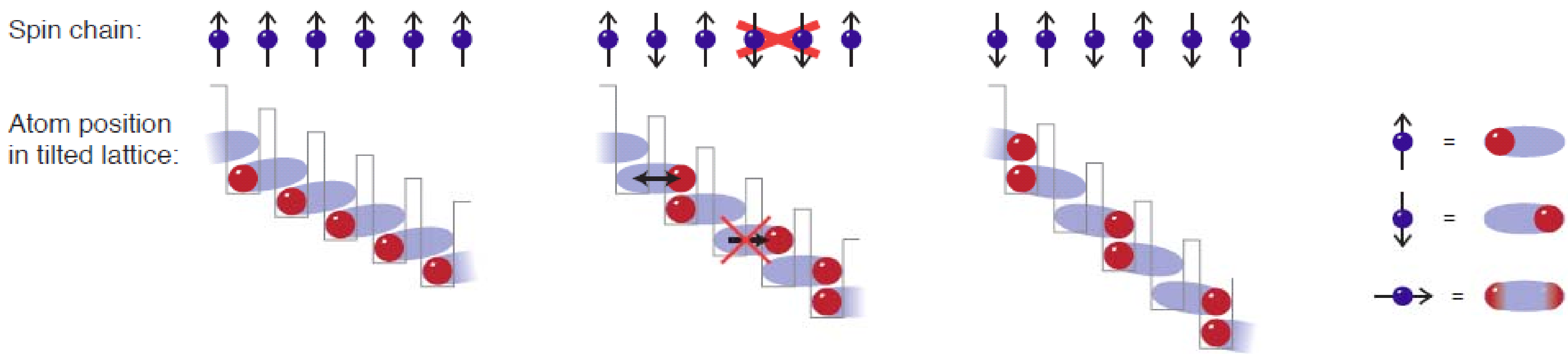
$$\begin{aligned} G(r_1, r_2) &= \langle\, n(r_1)\; n(r_2)\,\rangle_{\mathrm{TOF}} - \langle\, n(r_1)\,\rangle_{\mathrm{TOF}}\,\langle\, n(r_2)\,\rangle_{\mathrm{TOF}} \\ &\sim \langle\, n(k_1)\; n(k_2)\,\rangle_{\mathrm{LAT}} - \langle\, n(k_1)\,\rangle_{\mathrm{LAT}}\,\langle\, n(k_2)\,\rangle_{\mathrm{LAT}} \end{aligned}$$



Extra Bragg peaks appear in the second order correlation function in the AF phase

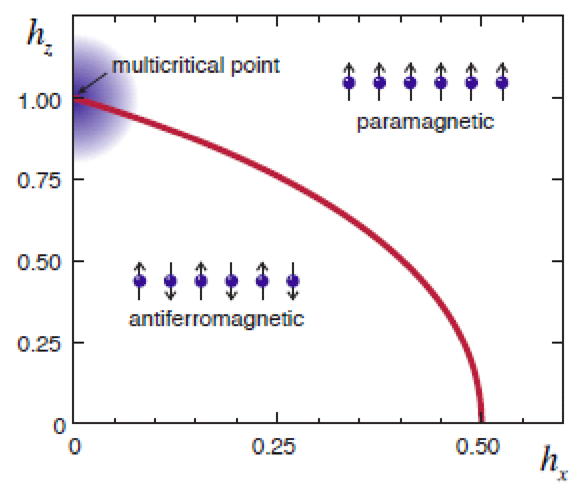
Magnetism in a tilted optical lattice

Theory: S. Sachdev et al., PRB 66:75128 (2002)
Experiment: J. Simon et al., Nature 472, 307(2011)

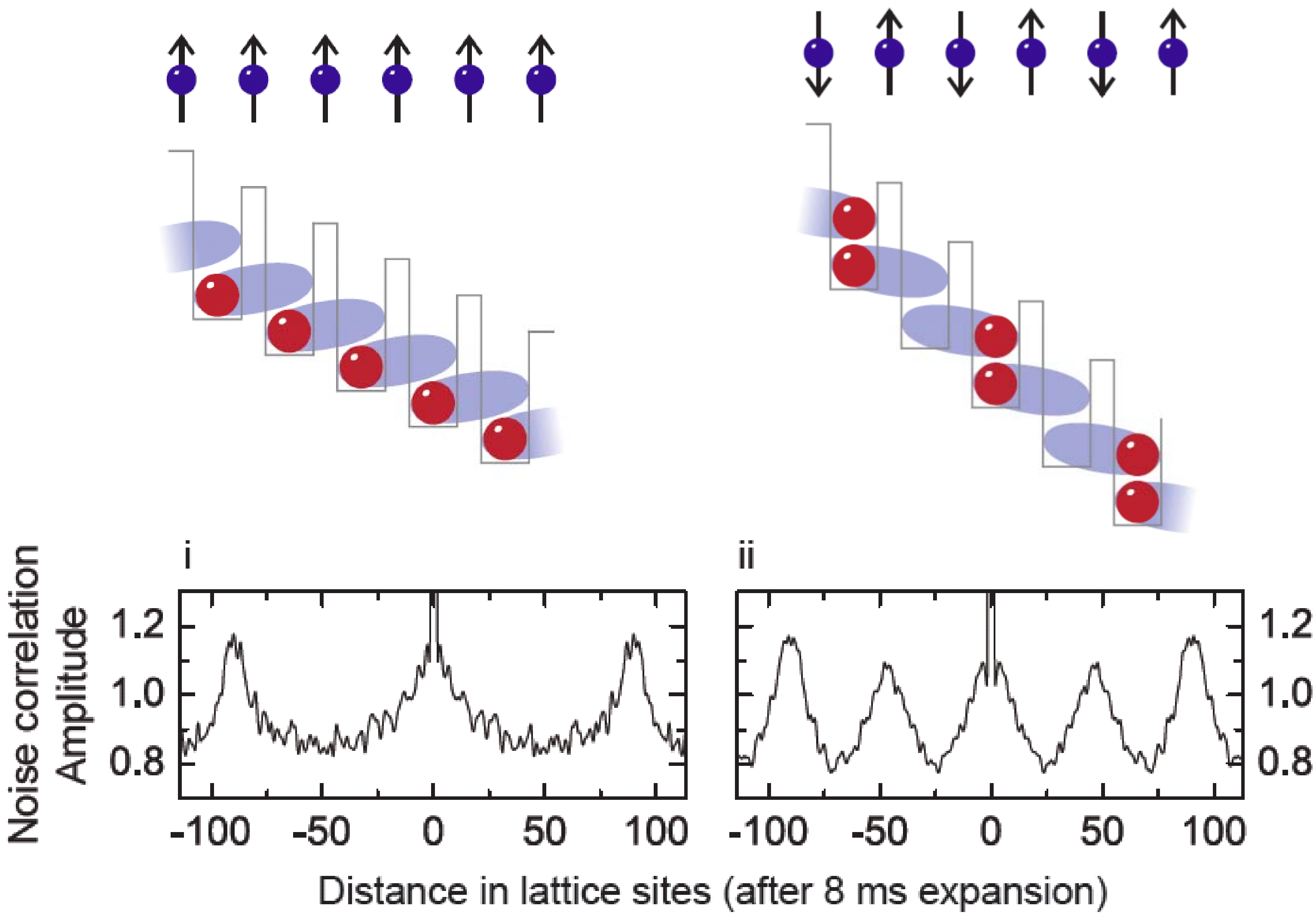


$$H = J\sum_i \overbrace{S_z^i S_z^{i+1}}^{\text{realizes constraint}} \overbrace{\underbrace{-(1-\tilde{\Delta})\,S_z^i}_{h_z} \underbrace{-\,2^{3/2}\,\tilde{t}\,S_x^i}_{h_x}}^{\text{drives quantum phase transition}}$$

magnetic fields: h_z longitudinal, h_x transverse



Magnetism in a tilted optical lattice

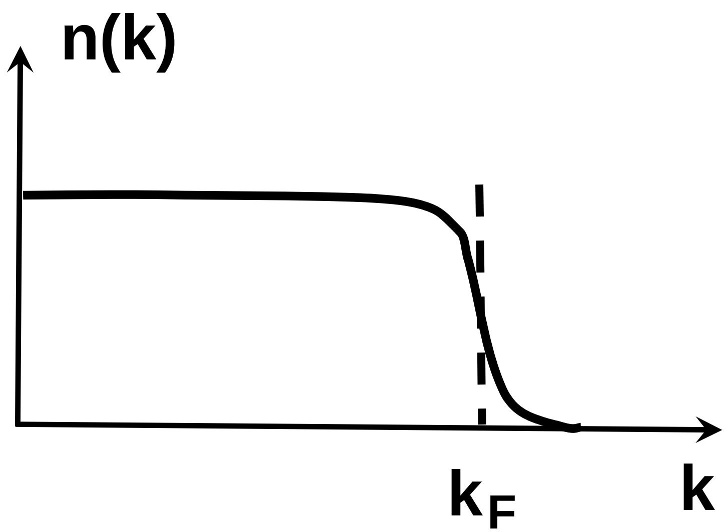


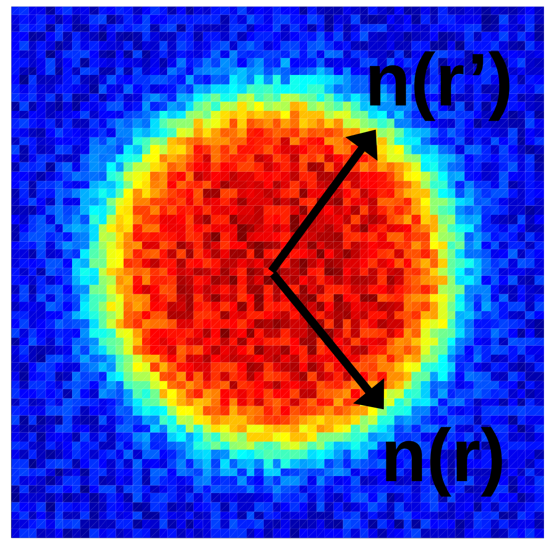
How to detect fermion pairing

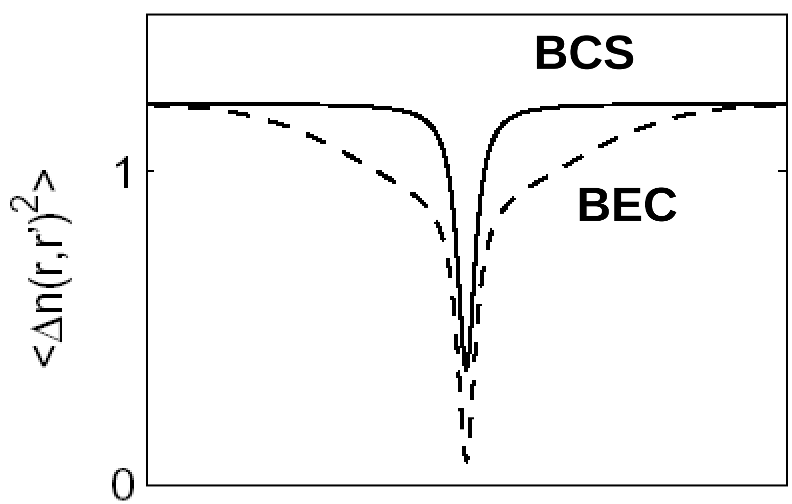
Quantum noise analysis of TOF images
is more than HBT interference

Second order interference from the BCS superfluid

Theory: Altman et al., PRA 70:13603 (2004)





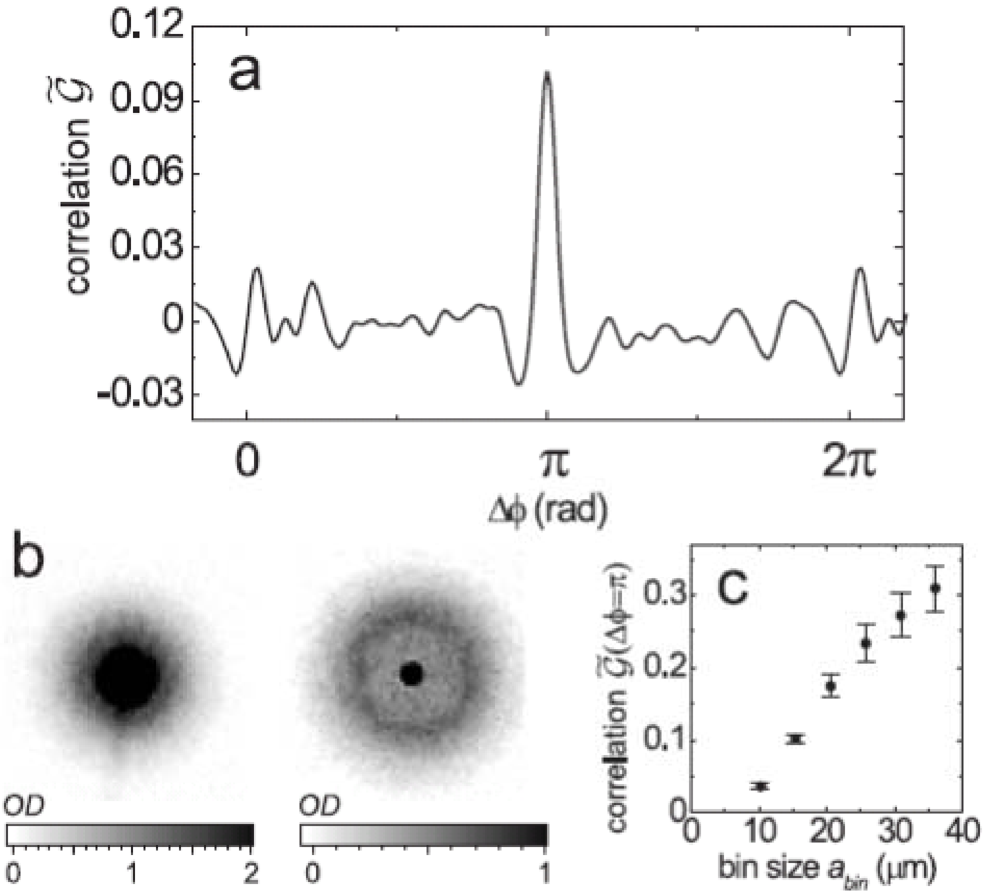


$$\Delta n(\mathbf{r},\mathbf{r}') \equiv n(\mathbf{r}) - n(\mathbf{r}')$$

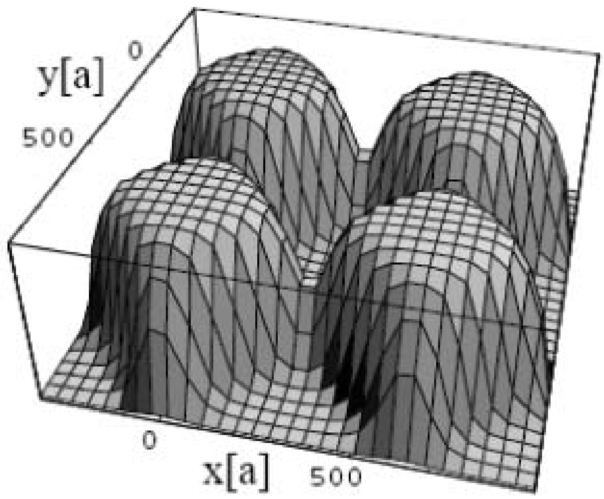
$$\Delta n(\mathbf{r},-\mathbf{r}) \left| \Psi_{BCS} \right\rangle = 0$$

Momentum correlations in paired fermions

Greiner et al., PRL 94:110401 (2005)



Fermion pairing in an optical lattice



Second Order Interference
In the TOF images

$$G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$$

Normal State

$$G_{\mathrm{N}}(r_1, r_2) = \delta(r_1 - r_2)\rho(r_1) - \rho^2(r_1) \sum_G \delta(r_1 - r_2 - \frac{G\hbar t}{m})$$

Superfluid State

$$G_{\mathrm{S}}(r_1, r_2) = G_{\mathrm{N}}(r_1, r_2) + \Psi(r_1) \sum_G \delta(r_1 + r_2 + \frac{G\hbar t}{m})$$

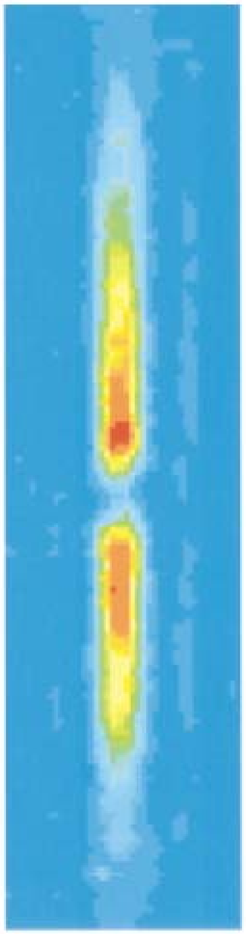
$\Psi(r) = |u(Q(r))v(Q(r))|^2$ **measures the Cooper pair wavefunction**

$$Q(r) = \frac{mr}{\hbar t}$$

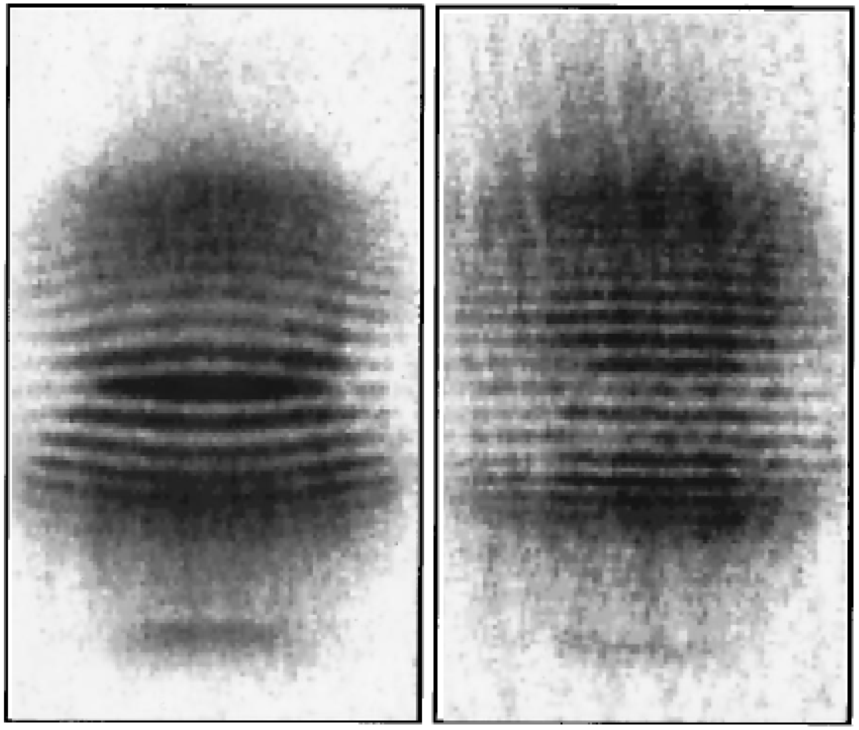
One can identify unconventional pairing

Low dimensional systems. Interference experiments with cold atoms

Interference of independent condensates



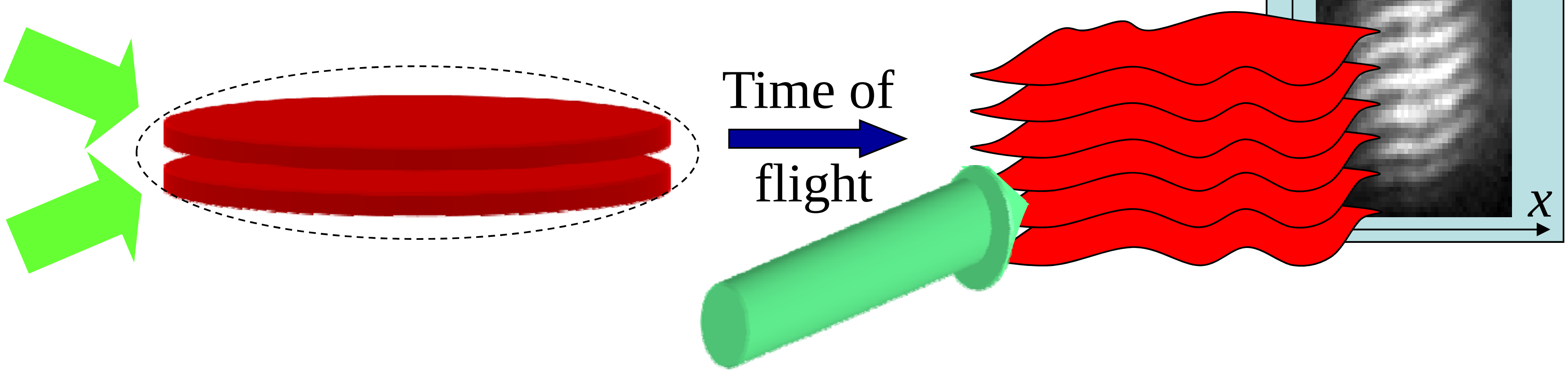
Experiments: Andrews et al., Science 275:637 (1997)



Theory: Javanainen, Yoo, PRL 76:161 (1996)
Cirac, Zoller, et al. PRA 54:R3714 (1996)
Castin, Dalibard, PRA 55:4330 (1997)
and many more

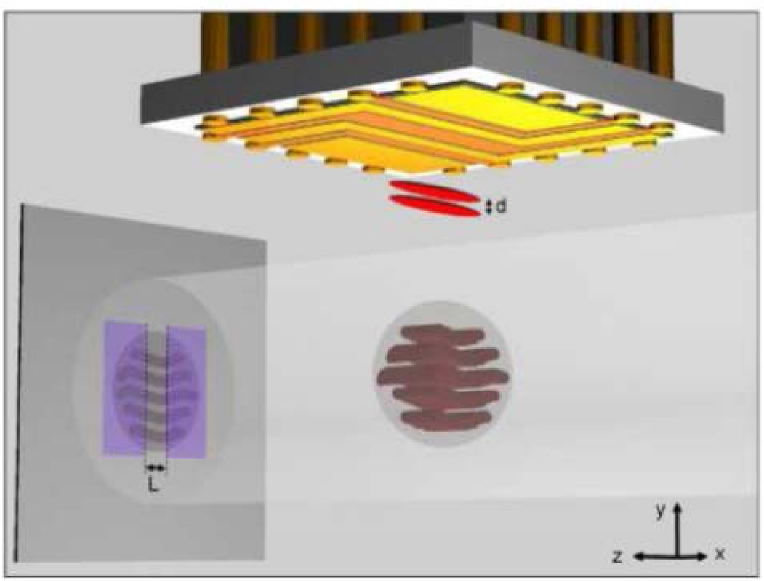
Experiments with 2D Bose gas

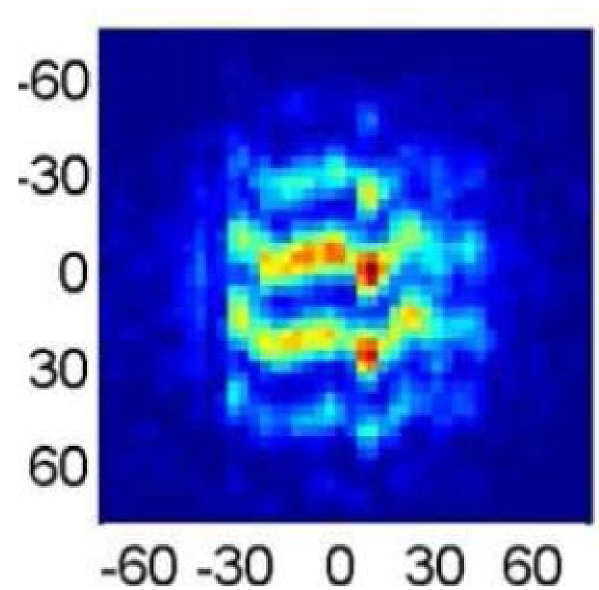
Hadzibabic, Dalibard et al., Nature 2006



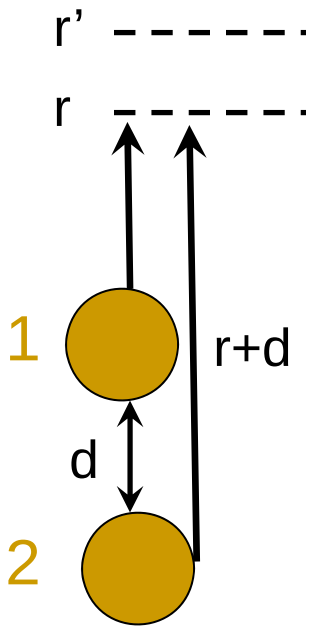
Experiments with 1D Bose gas

Hofferberth et al., Nat. Physics 2008





Interference of two independent condensates



$$\psi(r) = \psi_1(r) + \psi_2(r)$$

$$\rho_{\text{int}}(r) = \psi_1^\dagger(r)\,\psi_2(r) + \text{c.c.}$$

Assuming ballistic expansion

$$\boxed{\rho_{\text{int}}(r) = e^{i\,\frac{m\,d\,r}{\hbar\,t}}\, e^{i\,(\phi_2 - \phi_1)} + \text{c.c.}}$$

Phase difference between clouds 1 and 2
is not well defined

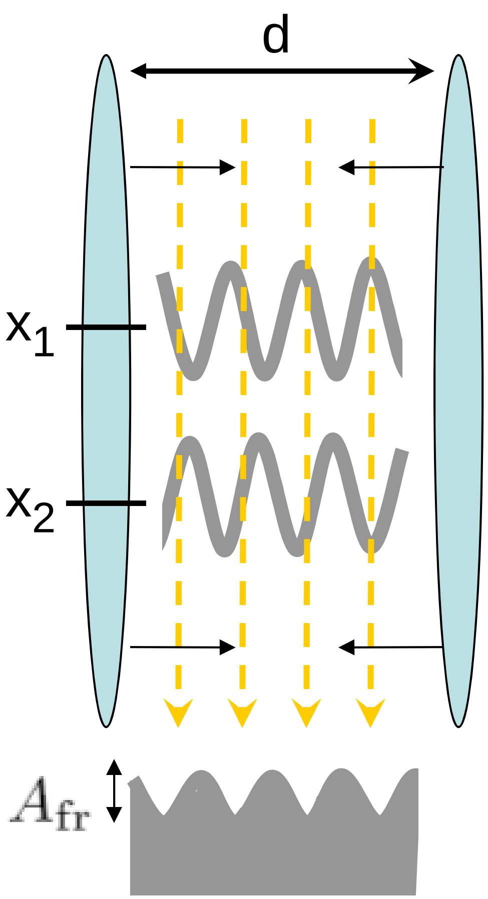
Individual measurements show interference patterns
They disappear after averaging over many shots

$$\langle\, \rho_{\text{int}}(r)\,\rangle = 0$$

$$\langle\, \rho_{\text{int}}(r)\,\rho_{\text{int}}(r')\,\rangle = e^{i\,\frac{m\,d}{\hbar\,t}\,(r-r')} + \text{c.c.}$$

Interference of fluctuating condensates

Polkovnikov, Altman, Demler, PNAS (2006)



Amplitude of interference fringes, A_{fr}

$$|A_{\mathrm{fr}}|\, e^{i\Delta\phi} = \int_0^L dx\, e^{i(\phi_1(x)-\phi_2(x))}$$

For independent condensates A_{fr} is finite but $\Delta\phi$ is random

$$\langle\, |A_{\mathrm{fr}}|^2 \,\rangle = \int_0^L dx_1 \int_0^L dx_2 \,\langle\, e^{i(\phi_1(x_1)-\phi_2(x_1))}\, e^{-i(\phi_1(x_2)-\phi_2(x_2))} \rangle$$

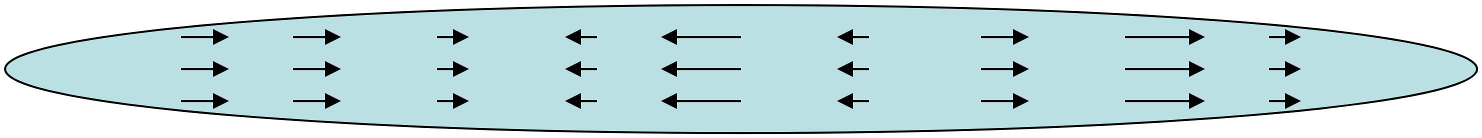
$$\langle\, |A_{\mathrm{fr}}|^2 \,\rangle \approx L \int_0^L dx \,\langle\, e^{i(\phi_1(x)-\phi_1(0))} \rangle \,\langle\, e^{-i(\phi_2(x)-\phi_2(0))} \,\rangle$$

For identical condensates $\langle |A_{\mathrm{fr}}|^2 \rangle = L \int_0^L dx\, (\, G(x)\,)^2$

Instantaneous correlation function $G(x) = \langle\, e^{i\phi(x)}\, e^{-\phi(0)} \,\rangle$

Fluctuations in 1d BEC

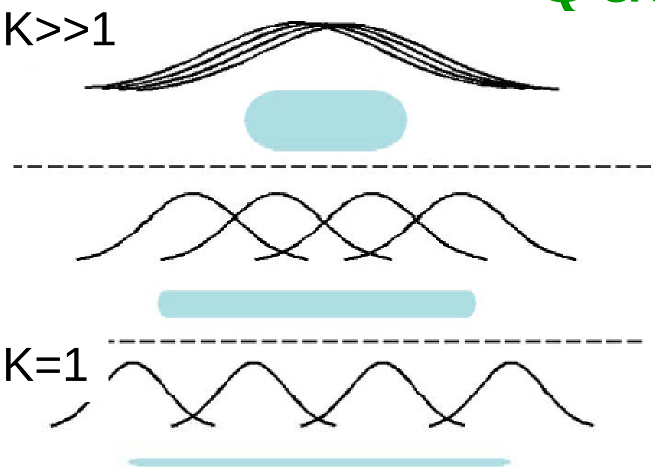
Thermal fluctuations



Thermally energy of the superflow velocity $\mathrm{v}_s = \nabla\phi(x)$

$$\boxed{\langle\, e^{i\phi(x_1)}\, e^{i\phi(x_2)}\, \rangle \sim e^{-|x_1-x_2|/\xi_T}} \qquad \xi_T = \sqrt{\frac{\hbar^2\, m}{T}}$$

Quantum fluctuations



$$\boxed{\langle\, e^{i\phi(x_1)}\, e^{i\phi(x_2)}\, \rangle \sim \left(\frac{\xi_h}{|x_1 - x_2|}\right)^{1/2K}}$$

For weak interactions $K = \sqrt{\frac{n}{g\, m}}$

Experiments with strongly interacting 1d bosonic cold gases:
Weiss et al., (2005, 2006); Bloch et al., (2005), O'Hara et al., (2003-2005)

Interference between Luttinger liquids

Luttinger liquid at T=0 $$G(x) \sim \rho \left(\frac{\xi_h}{x} \right)^{1/2K}$$

$$\langle |A_{\rm fr}|^2 \rangle \sim (\rho \xi_h)^{1/K} (L\rho)^{2-1/K}$$

K – Luttinger parameter

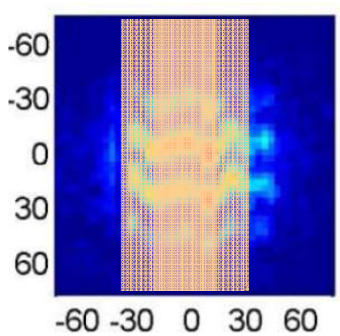
For non-interacting bosons $K = \infty$ and $A_{\rm fr} \sim L$

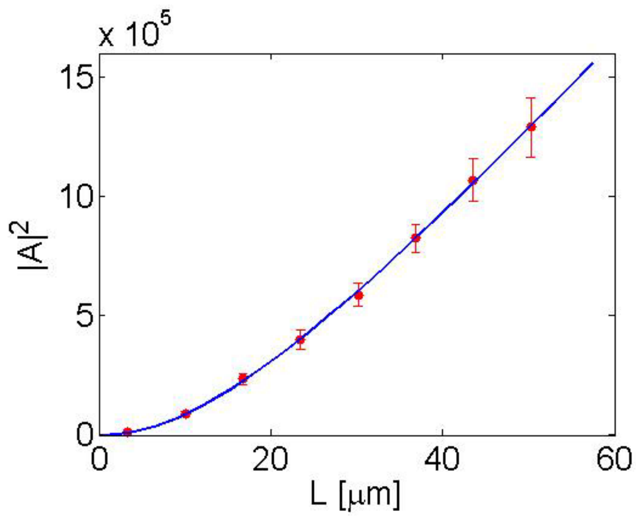
For impenetrable bosons $K = 1$ and $A_{\rm fr} \sim \sqrt{L}$

Finite temperature

$$\langle |A_{\rm fr}|^2 \rangle \sim L \rho^2 \xi_h \left(\frac{\hbar^2}{m \xi_h^2} \frac{1}{T} \right)^{1-1/K}$$

Experiments: Hofferberth, Schumm, Schmiedmayer





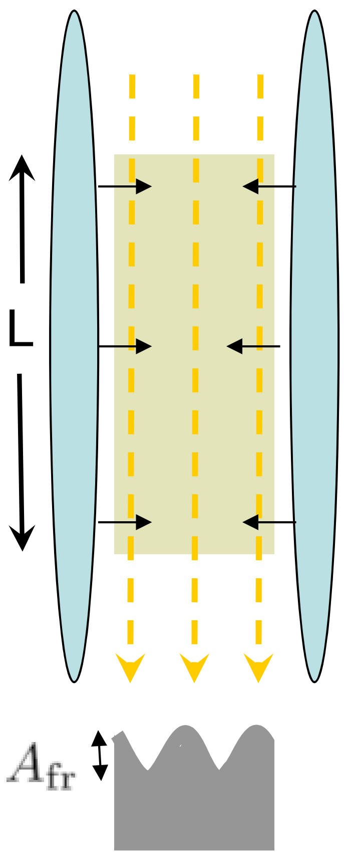
$n_{1d} = 60 \mu\text{m}^{-1}$

$K = 47$

$\boxed{T_{fit} = 84 \pm 22 \text{ nK}}$

Distribution function of fringe amplitudes for interference of fluctuating condensates

Gritsev, Altman, Demler, Polkovnikov, Nature Physics (2006)
Imambekov, Gritsev, Demler, PRA (2007)



A_{fr} is a quantum operator. The measured value of $|A_{\mathrm{fr}}|$ will fluctuate from shot to shot.

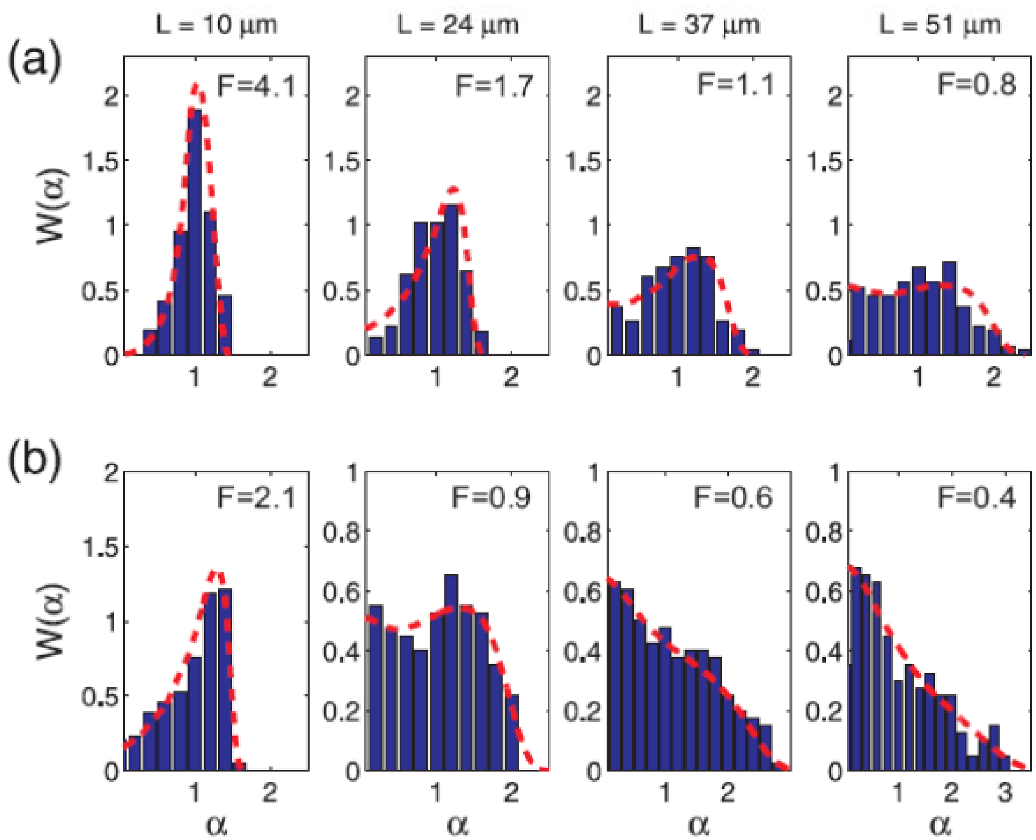
$$\langle\, |A_{\mathrm{fr}}|^{2n} \,\rangle = \int_0^L dz_1 \ldots dz_n' \,|\, \langle\, e^{i\phi(z_1)} \ldots e^{i\phi(z_n)}\, e^{-i\phi(z_1')} \ldots e^{-i\phi(z_n')} \,\rangle |^2$$

Higher moments reflect higher order correlation functions

We need the full distribution function of $|A_{\mathrm{fr}}|$

Distribution function of interference fringe contrast

Hofferberth et al., Nature Physics 2009



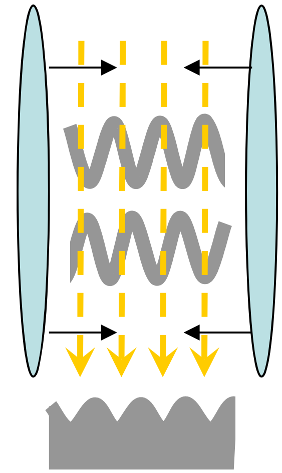
Quantum fluctuations dominate:
asymetric Gumbel distribution
(low temp. T or short length L)

Thermal fluctuations dominate:
broad Poissonian distribution
(high temp. T or long length L)

Intermediate regime:
double peak structure

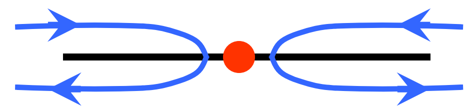
Comparison of theory and experiments: no free parameters
Higher order correlation functions can be obtained

Interference between interacting 1d Bose liquids. Distribution function of the interference amplitude

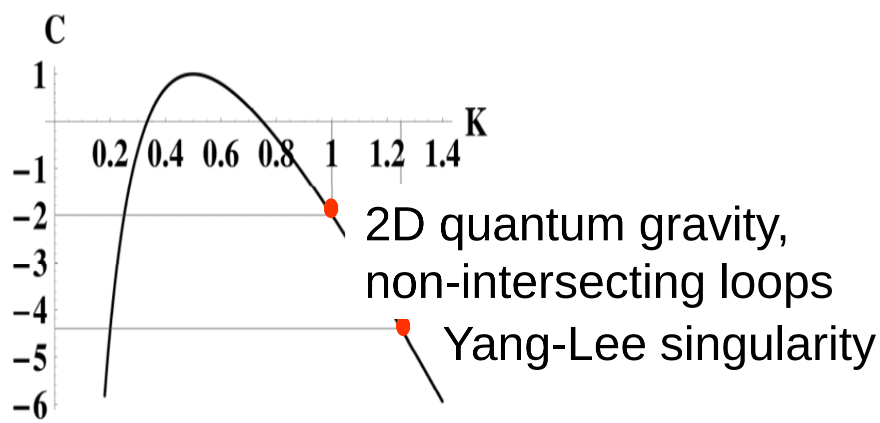


Distribution function of $|A_{\mathrm{fr}}|$

Quantum impurity problem: interacting one dimensional electrons scattered on an impurity

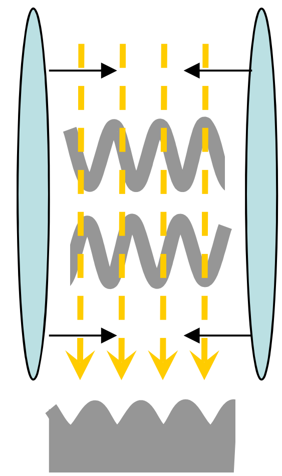


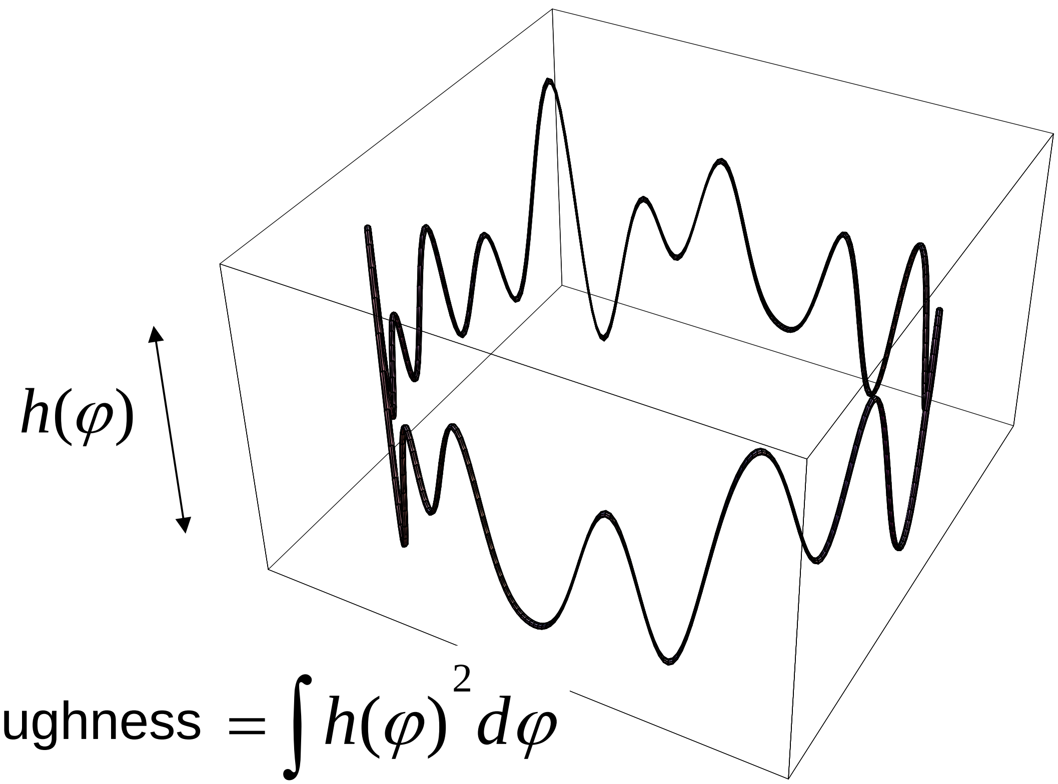
Conformal field theories with negative central charges: 2D quantum gravity, non-intersecting loop model, growth of random fractal stochastic interface, high energy limit of multicolor QCD, …



Fringe visibility and statistics of random surfaces

Distribution function of $|A_{\mathrm{fr}}|$





$$\text{Roughness} = \int h(\varphi)^2 d\varphi$$

Mapping between fringe visibility and the problem of surface roughness for fluctuating random surfaces.
Relation to 1/f Noise and Extreme Value Statistics

Summary

Experiments with ultracold atoms provide a new perspective on the physics of strongly correlated many-body systems. Quantum noise is a powerful tool for analyzing many body states of ultracold atoms

Thanks to:

CUA

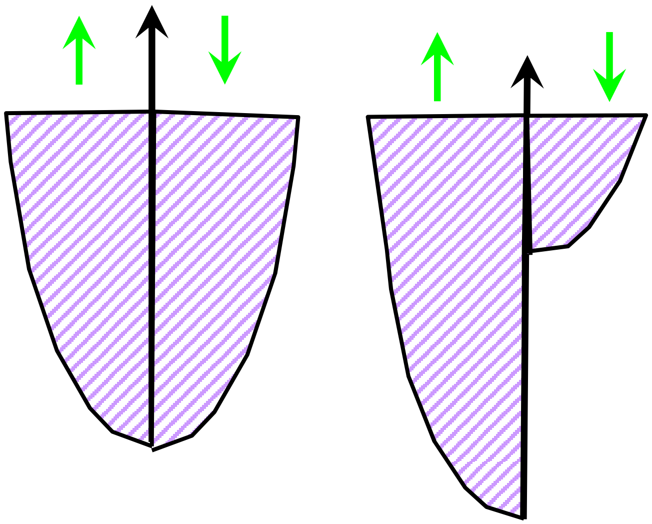
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH · UNITED STATES AIR FORCE

MURI Program in Optical Lattices

DARPA DSO DEFENSE SCIENCES OFFICE

OPTICAL LATTICE EMULATOR OLE

Stoner model of ferromagnetism



Mean-field criterion

$$\boxed{U\,N(0) = 1}$$

U – interaction strength
N(0) – density of states at Fermi level

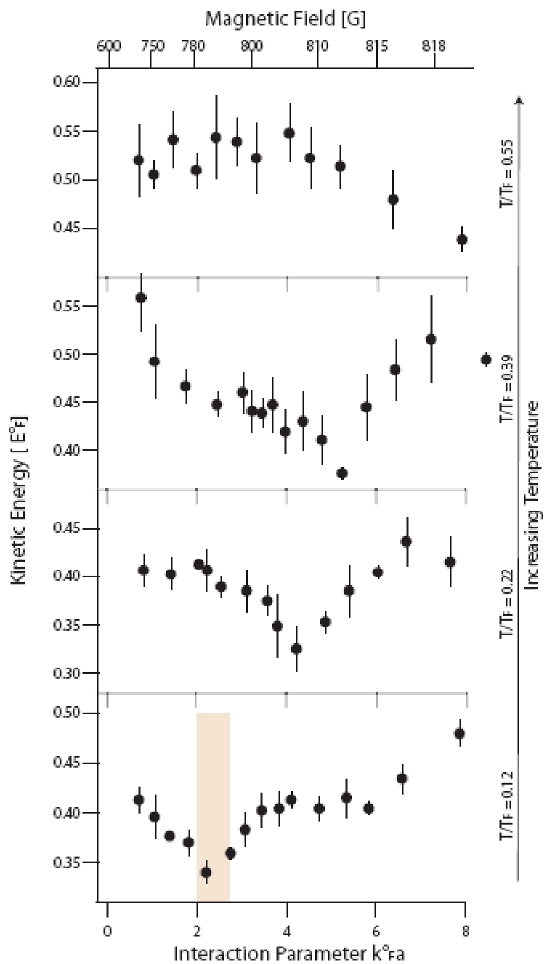
Observation of Stoner transition by G.B. Jo et al., Science (2009)

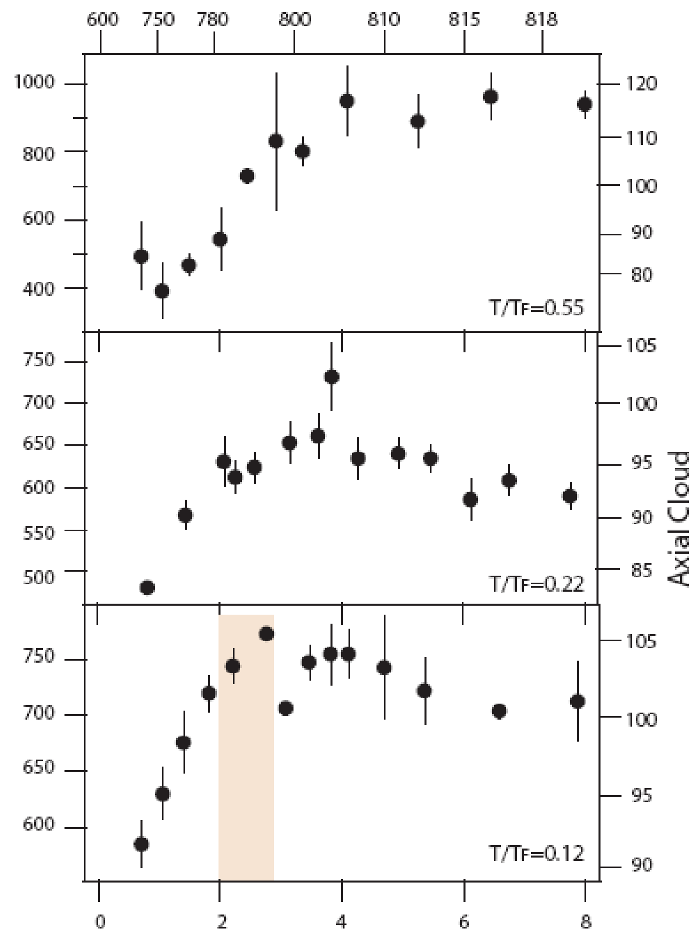
Signatures of ferromagnetic correlations in particles losses, molecule formation, cloud radius

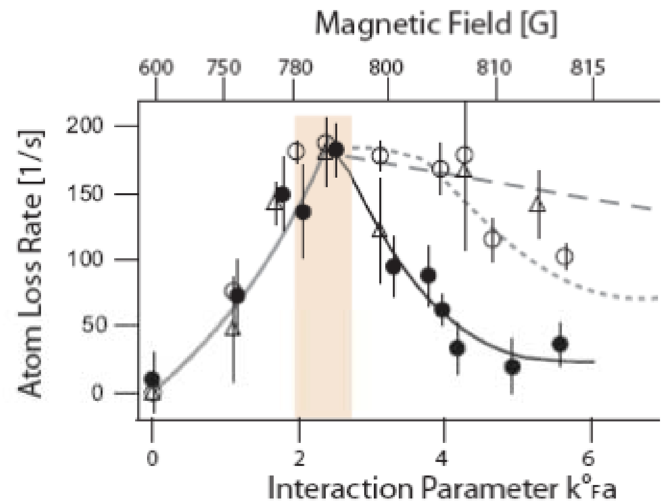
Magnetic domains could not be resolved. Why?

Observation of itinerant ferromagnetism in a strongly interacting Fermi gas of ultracold atoms

Gyu-Boong Jo,[1*] Ye-Ryoung Lee,[1] Jae-Hoon Choi,[1] Caleb A. Christensen,[1]
Tony H. Kim,[1] Joseph H. Thywissen,[2] David E. Pritchard[1], and Wolfgang Ketterle,[1]







Magnetic domains could not be resolved. Why? T.L. Ho (2009)