Ramsey interference as a probe of “synthetic” condensed matter systems

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Ramsey interference

\[ |\Psi\rangle = e^{-iE_1t} |\uparrow\rangle + e^{-iE_2t} |\downarrow\rangle \]
Ramsey interference in atomic clocks

Atomic clocks and Ramsey interference:
Working with $N$ atoms improves the precision by $\sqrt{N}$

Using interactions to improve the clock precision
Kitagawa, Ueda, PRA 47:5138 (1993)

Classical BEC
Single mode approximation

$$\mathcal{H}_{SMA} = \frac{g_s}{V} (N_1 - N_2)^2$$
Ramsey interference as a probe of 1d dynamics

Interaction induced collapse of Ramsey fringes.

Only weak spin echo was observed in experiments.


Decoherence of Ramsey fringes due to many-body dynamics of low dimensional BECs

New probe of dynamics of 1d many-body systems
This talk:
Exploring “synthetic” condensed matter systems with Ramsey interference

1. Measuring topological properties of band structures. Berry and Zak phases

2. Orthogonality catastrophe
Measuring topological properties of band structures. Berry and Zak phases
Topological aspects of band structures

All evidence is indirect: (transport, edge states)
Can there be a direct measurement of Berry phase of Bloch states?
Example of band structure with Berry phase.

Hexagonal (graphene) lattice

Tight binding model on a hexagonal lattice

\[ H = -J \sum_{\langle i,j \rangle} c_i^\dagger c_j \]

Dirac fermions in optical lattices, Tarruell et al., Nature 2012
Berry phase in hexagonal lattice

- Eigenvectors lie in the XY plane
- Around each Dirac point eigenvector makes $2\pi$ rotation
- Integral of the Berry phase is $\pi$

$$H(k) = -J \left( \begin{array}{cc} 0 & A(k) \\ A^*(k) & 0 \end{array} \right)$$

$$A(k) = e^{i k \cdot a_1} + e^{i k \cdot a_2} + e^{-i k \cdot (a_1 + a_2)}$$

$$\int_C \langle \psi_k | \frac{\partial}{\partial \vec{k}} | \psi_k \rangle \, d\vec{k} = \pi$$
How to measure Berry phase in hexagonal lattice

Naïve approach:

Move atom on a closed trajectory around Dirac point
Measure accumulated phase

\[
\Delta \phi = \Delta \phi_{\text{dyn}} + \Delta \phi_{\text{Berry}}
\]

\[
\Delta \phi_{\text{dyn}} = \int E(t) \, dt
\]

\[
\Delta \phi_{\text{Berry}} = \int_C \langle \psi_k | \partial_{\vec{k}} | \psi_k \rangle \, d\vec{k}
\]

Problems with this approach:
Need to move atom on a complicated curved trajectory
Need to separate dynamical phase
From Berry phase to Zak phase

Integral of the Berry phase is only well defined on a closed trajectory.

\[ \vec{A} = \langle \psi_k | \frac{\partial}{\partial k} | \psi_k \rangle \]

\[ \int_A^{B} \vec{A} \, d\vec{l} \]

is not gauge invariant

\[ \int_C \vec{A} \, d\vec{l} \]

gauge invariant integral of Berry curvature
How to measure Zak phase using Ramsey interference sequence

Two hyperfine spin states experience the same optical potential

Advantages
Requires only straight trajectory
Dynamical phase cancels between two spin states
How to measure Zak phase using Ramsey interference sequence

This scheme measures the phase accumulation under Bloch oscillation

Berry phase

\[ \Delta \phi = \pi \]

\[ \text{Dirac point} \]

Zak phase

\[ \Delta \phi = \frac{\pi}{3} \]

Zak phase is related to Berry phase through a geometrical symmetry of the lattice!
Dynamically induced topological phases in a hexagonal lattice

T. Kitagawa et al.,
Dynamically induced topological phases in a hexagonal lattice

Floquet spectrum on a strip \( J/\omega = \frac{3}{32} \)

Edge states indicate the appearance of topologically non-trivial phases

Equivalent to Haldane model
Detection of topological band structure

Gap opening leads to trivial/non-trivial topology of bands

\[ \Delta \phi = \pi / 3 \]

Small band gap does not affect the Zak phase in this trajectory

Detection of topological band structure

Consider the trajectory that directly goes over the Dirac points.

In Haldane model, topologically non-trivial $\rightarrow \pi$
topologically trivial $\rightarrow 0$

It detects topological phase transition!

Gap is crucial to stay in the lower band!
Zak phase as a probe of band topology
1d version
Su-Schrieffer-Heeger model of polyacetylene

\[ H(u) = - \sum_{n,s} [t_0 + (-1)^n 2\alpha u] (c_{n+1,s}^{\dagger} c_{n,s} + c_{n,s}^{\dagger} c_{n+1,s}) \]

Analogous to bichromatic optical lattice potential

LMU/MPQ
Dimerized model

Topology of the band shows up in the winding of the eigenstate. Zak phase is equal to $\pi$.

$$\int_{-\pi/a}^{\pi/a} \langle \psi_k | \partial_k | \psi_k \rangle d k = \pi$$
Characterizing SSH model using Zak phase

Two hyperfine spin states experience the same optical potential

Zak phase is equal to $\pi$

$$\int_{-\pi/a}^{\pi/a} \langle \psi_k | \partial_k | \psi_k \rangle \, dk = \pi$$
Orthogonality catastrophe
Single impurity problems in condensed matter physics

Rich many-body physics

- **Edge singularities** in the X-ray absorption spectra
  (exact solution of non-equilibrium many-body problem)

- **Kondo effect**: entangled state of impurity spin and fermions

Influential area, both for methods (renormalization group) and for strongly correlated materials
Probing impurity physics in solids is limited

-Many unknowns;
Simple models hard to test
(complicated band structure, unknown impurity parameters, coupling to phonons)
Probing impurity physics in solids is limited

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X-ray absorption in Na
Probing impurity physics in solids is limited

- Many unknowns;
- Simple models hard to test
  (complicated band structure, unknown impurity parameters, coupling to phonons)
- Limited probes
  (usually only absorption spectra)
- Dynamics beyond linear response
  out of reach
  (relevant time scales GHz-THz, experimentally difficult)
Cold atoms: new opportunities for studying impurity physics

- Parameters known allow precise tests of theory
- Exact solutions available
- Tunable by the Feshbach resonance
  \[ f(k) = \frac{a}{1 + ika} \]
- Fast control of microscopic parameters (compared to many-body scales)
- Rich toolbox for probing many-body states

Ketterle group '98
Cold atoms: new opportunities for studying impurity physics

- Parameters known allow precise tests of theory
- Exact solutions available
- Tunable by the Feshbach resonance
- Describe dynamics
- Characterize complicated transient many-body states
- Explore new many-body phenomena
- Fast control of microscopic parameters (compared to many-body scales)
- Rich toolbox for probing many-body states

New challenges for theory:
Introduction to Anderson orthogonality catastrophe (OC)

- Overlap
  \[ S = \langle FS | FS' \rangle \]

- \( S \to 0 \) as system size \( L \to \infty \), "orthogonality catastrophe"

- Infinitely many low-energy electron-hole pairs produced

Fundamental property of the Fermi gas
Orthogonality catastrophe in X-ray absorption spectra

-Relevant overlap: \[ S(t) = \langle FS | e^{iH_0 t} e^{-iH_f t} | FS \rangle \propto t^{-\delta^2/\pi^2} \]

\[ \delta -- \text{scattering phase shift at Fermi energy, } \tan \delta = -k_F a \]

-Manifests in a power-law singularity in the absorption spectrum

\[ A(\omega) \propto \int \exp(i\omega t) S(t) dt \propto \frac{1}{\omega^{1-\delta^2/\pi^2}} \]
Orthogonality catastrophe in X-ray absorption spectra

- Relevant overlap: \( S(t) = \left\langle FS \left| e^{iH_0t} e^{-iH_f t} \right| FS \right\rangle \propto t^{-\delta^2 / \pi^2} \)

\( \delta \) -- scattering phase shift at Fermi energy, \( \tan \delta = -k_F a \)

- Manifests in a power-law singularity in the absorption spectrum

\[ A(\omega) \propto \int \exp(i\omega t)S(t)dt \propto \frac{1}{\omega^{1-\delta^2 / \pi^2}} \]
Orthogonality catastrophe with cold atoms: Setup

- Fermi gas + single impurity
- Two pseudospin states of impurity, $|\uparrow\rangle$ and $|\downarrow\rangle$
- $|\uparrow\rangle$ - state scatters fermions
- $|\downarrow\rangle$ - state does not
- Scattering length $a$
- Fermion Hamiltonian for pseudospin $|\uparrow\rangle, |\downarrow\rangle$ -- $H_0, H_f$

Earlier theoretical work on Kondo in cold atoms: Zwerger, Lamacraft, …
Ramsey fringes – new manifestation of OC

- Utilize control over spin
- Access coherent coupled dynamics of spin and Fermi gas
- Ramsey interferometry

1) $\pi/2$ pulse
\[ |\downarrow\rangle|FS\rangle \rightarrow \frac{1}{\sqrt{2}} |\downarrow\rangle|FS\rangle + \frac{1}{\sqrt{2}} |\uparrow\rangle|FS\rangle \]

2) Evolution
\[ \frac{1}{\sqrt{2}} |\downarrow\rangle e^{-iH_0 t} |FS\rangle + \frac{1}{\sqrt{2}} |\uparrow\rangle e^{-iH_f t} |FS\rangle \]

3) Use $\pi/2$ pulse to measure
\[ <S_x> = \text{Re}[S(t)] \]
\[ S(t) = \langle FS | e^{iH_0 t} e^{-iH_f t} |FS\rangle \]

Direct measurement of OC in the time domain
Ramsey fringes as a probe of OC
First principle calculations
Spin echo: probing non-trivial dynamics of the Fermi gas

-Response of Fermi gas to process in which impurity switches between different states several times

\[ S_1(t) = \left\langle FS | e^{iH_0t} e^{iH_{ft}} e^{-iH_0t} e^{-iH_{ft}} | FS \right\rangle \]

-Such responses play central role in analysis of Kondo problem but could not be studied in solid state systems

-Advantage: insensitive to slowly fluctuating magnetic fields (unlike Ramsey)
Spin echo response: features

- Power-law decay at long times with an enhanced exponent

\[ S_1(t) \propto t^{-3\delta^2/\pi^2} \]

- Unlike the usual situation (spin-echo decays slower than Ramsey)
- Cancels magnetic field fluctuations
- Universal
- Generalize to \( n \) pi-pulses to study even more complex response functions
RF spectroscopy of impurity atom

Free atom

Atom in a Fermi sea – many-body effects completely change absorption function

RF spectra can be calculated exactly
Edge singularities in the RF spectra: Intuitive picture

Photon → pseudospin flipped + massive production of excitations
Exact RF spectra

\(a < 0\); no impurity bound state

**Single threshold**

in absorption

\(a > 0\); bound state

**Two thresholds**

- bound state filled after absorption
- bound state empty
**Dynamic Friedel oscillations**

- Time-of-flight experiment following impurity spin flip;
- Access real-time development of Friedel oscillations

![Graphs showing dynamic Friedel oscillations](image)

**Excess density $\delta \rho(r)$**

- For $t=777$: 
  
- For $t=1555$: 
  
- For $t=2333$: 
  
- For $t=3111$: 

**Friedel oscillations of density**

$$\delta \rho(r) \propto \frac{\cos(2k_Fr + \delta)}{r}$$

**Excess charge pushed out by impurity**
Generalizations: non-equilibrium OC, non-abelian Riemann-Hilbert problem

- Impurity coupled to several Fermi seas at different chemical potentials
- Theoretical works in the context of quantum transport
- Mathematically, reduces to non-abelian Riemann-Hilbert problem (challenging)
- Experiments lacking

Muzykantskii et al’03
Abanin, Levitov ‘05
Generalizations: non-equilibrium OC, non-abelian Riemann-Hilbert problem

- Multi-component Fermi gas coupled to impurity
- Imbalance different species
- Mix them by pi/2 pulses
- Realization of non-equilibrium OC problem
- "Simulator" of quantum transport and non-abelian Riemann-Hilbert problem
- Charge full counting statistics can be probed

\[ \text{Hyperfine state 1} \quad \text{Hyperfine state 2} \]
Other directions

-Multi-component Fermi gas: non-equilibrium orthogonality catastrophe, non-abelian Riemann-Hilbert problem

-Spinful fermions → dynamics in the Kondo problem

-Dynamics: many-body effects in Rabi oscillations of impurity spin

-Very different physics for bosons
Exploring “synthetic” condensed matter systems with Ramsey interference

Measuring topological properties of band structures. Berry and Zak phases

Orthogonality catastrophe
Interaction induced collapse of Ramsey fringes

Two component BEC. Single mode approximation

\[ \mathcal{H} = \chi_s (N_\uparrow - N_\downarrow)^2 \]

Ramsey fringe visibility

Experiments in 1d tubes:
A. Widera et al. PRL 100:140401 (2008)
Spin echo. Time reversal experiments

Single mode approximation

\[ \mathcal{H} = x_s (S^z_{\text{tot}})^2 \]
\[ g_s = \frac{g_{11} - g_{12}}{2} \]

The Hamiltonian can be reversed by changing \( a_{12} \)

\[ a_s \rightarrow -a_s \]
\[ \mathcal{H}_{\text{SMA}} \rightarrow -\mathcal{H}_{\text{SMA}} \]

\[ e^{i \int_{-T}^{T} \mathcal{H}_{\text{SMA}}(t) dt} \times e^{i \int_0^T \mathcal{H}_{\text{SMA}}(t) dt} = 1 \]

Predicts perfect spin echo
Spin echo. Time reversal experiments

Expts: A. Widera, I. Bloch et al.

No revival?

Experiments done in array of tubes. Strong fluctuations in 1d systems. Single mode approximation does not apply. Need to analyze the full model.
Interaction induced collapse of Ramsey fringes in one dimensional systems

Only $q=0$ mode shows complete spin echo. Finite $q$ modes continue decay.

The net visibility is a result of competition between $q=0$ and other modes.

Decoherence due to many-body dynamics of low dimensional systems.

New probe of dynamics of 1d many-body systems.
Berry phase in Hexagonal lattice (graphene)

\[ H(k) = -J \begin{pmatrix} 0 & A(k) \\ A^*(k) & 0 \end{pmatrix} \]

\[ A(k) = e^{ik \cdot a_1} + e^{ik \cdot a_2} + e^{-ik \cdot (a_1 + a_2)} \]

This non-trivial geometrical phase is responsible for

1. Anomalous quantum Hall effect
2. Integer quantum Hall effect without Landau Levels
Manifestations of topological character of band structures

Integer quantum Hall effect

Chern number

All evidence is indirect: (transport, edge states)

Can there be a direct measurement of Berry phase of Bloch states?
Topologically different dimerizations

Dimerized tunneling

\[ \circ -- \circ -- \circ -- \circ -- \circ \]

topological

Zak phase is equal to \( \pi \)

Dimerized on-site potential

\[ \bullet -- \circ -- \bullet -- \circ -- \circ -- \bullet \]

non-topological

Zak phase is equal to 0

Dimerized without inversion symmetry

\[ \circ -- \bullet -- \circ -- \bullet -- \circ \]

Zak phase can be arbitrary