Learning about order from noise

Quantum noise studies of ultracold atoms

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Bose-Einstein condensation of weakly interacting atoms

Density                                           $10^{13}$ cm$^{-1}$
Typical distance between atoms                    300 nm
Typical scattering length                         10 nm

$T_{\text{BEC}} \sim 1 \mu K$

Scattering length is much smaller than characteristic interparticle distances. Interactions are weak.
New Era in Cold Atoms Research

Focus on Systems with Strong Interactions

- **Feshbach resonances.**
  Scattering length comparable to interparticle distances
  E. Tiesinga et al., PRA (1993);
  K. O’Hara et al., Science, 2002

- **Optical lattices.**
  Suppressed kinetic energy.
  Enhanced role of interactions
  D. Jaksch et al., PRL (1998);
  M. Greiner et al., Nature, 2002

- **Low dimensional systems.**
  Strongly interacting regimes at low densities
  Kinoshita et al., Science, 2004;
  Paredes et al., Nature, 2004
Quantum simulations

Among major challenges: detection and characterization of many-body states of ultracold atoms

This talk:
applications of quantum noise for studying cold atoms
Outline

Introduction. Historical review

Strongly correlated systems in optical lattices. Hanbury-Brown-Twiss type experiments

Interferometric measurements of topological order parameter in optical lattices
Quantum noise

Classical measurement:
collapse of the wavefunction into eigenstates of $x$

$$
\langle x \rangle = \int dx \ x \ |\psi(x)|^2 \\
\langle x^2 \rangle = \int dx \ x^2 \ |\psi(x)|^2 \\
\ldots \\
\langle x^n \rangle = \int dx \ x^n \ |\psi(x)|^2 \\
\ldots \\
$$

Histogram of measurements of $x$
Hanbury-Brown-Twiss experiments

Classical theory of the second order coherence

\[ \langle I(\vec{r}_1) I(\vec{r}_2) \rangle = A + B \cos \left( (\vec{k} - \vec{k}') (\vec{r}_1 - \vec{r}_2) \right) \]

Measurements of the angular diameter of Sirius


Quantum theory of HBT experiments

Glauber, *Quantum Optics and Electronics* (1965)

HBT experiments with matter

Experiments with neutrons

Experiments with electrons

Experiments with 4He, 3He
Westbrook et al., Nature (2007)

Experiments with ultracold atoms

For bosons

\[ A = A_1 + A_2 \]

For fermions

\[ A = A_1 - A_2 \]
Experiments with atoms in optical lattices: Hanbury-Brown-Twiss experiments and beyond


Experiment: Folling et al., Nature 434:481 (2005);
Spielman et al., PRL 98:80404 (2007);
Quantum noise analysis as a probe of many-body states of ultracold atoms
Time of flight experiments

Quantum noise interferometry of atoms in an optical lattice

Second order coherence $G(r_1, r_2) = \langle n(r_1)n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$
Second order coherence in the insulating state of bosons. Hanbury-Brown-Twiss experiment

Hanbury-Brown-Twiss stellar interferometer

\[ \langle I(\vec{r}_1) \ I(\vec{r}_2) \rangle = A + B \ \cos \left( (\vec{k} - \vec{k}') \ (\vec{r}_1 - \vec{r}_2) \right) \]
Second order coherence in the insulating state of bosons

Bosons at quasimomentum $\vec{k}$ expand as plane waves with wavevectors $\vec{k}, \vec{k} + \vec{G}_1, \vec{k} + \vec{G}_2$

First order coherence: $\langle \rho(\vec{r}) \rangle$
Oscillations in density disappear after summing over $\vec{k}$

Second order coherence: $\langle \rho(\vec{r}_1) \rho(\vec{r}_2) \rangle$
Correlation function acquires oscillations at reciprocal lattice vectors

$$\langle \rho(\vec{r}_1) \rho(\vec{r}_2) \rangle = A_0 + A_1 \cos \left( \vec{G}_1(\vec{r}_1 - \vec{r}_2) \right) + A_2 \cos \left( \vec{G}_2(\vec{r}_1 - \vec{r}_2) \right) + \ldots$$
Second order coherence in the insulating state of bosons.

Hanbury-Brown-Twiss experiment

Second order coherence in the insulating state of fermions. Hanburry-Brown-Twiss experiment

How to detect antiferromagnetism
Probing spin order in optical lattices

Correlation Function Measurements

\[ G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle_{\text{TOF}} - \langle n(r_1) \rangle_{\text{TOF}} \langle n(r_2) \rangle_{\text{TOF}} \]

\[ \sim \langle n(k_1) n(k_2) \rangle_{\text{LAT}} - \langle n(k_1) \rangle_{\text{LAT}} \langle n(k_2) \rangle_{\text{LAT}} \]

Extra Bragg peaks appear in the second order correlation function in the AF phase
Magnetism in a tilted optical lattice


\[ H = J \sum_i S^i_z S^{i+1}_z - (1 - \tilde{\Delta}) S^i_z - 2^{3/2} \tilde{\eta} S^i_x \]

magnetic fields:
- \( h_z \) longitudinal
- \( h_x \) transverse
Magnetism in a tilted optical lattice

![Diagram showing magnetism in a tilted optical lattice with two sets of data representing different distances in lattice sites (after 8 ms expansion).]
How to detect fermion pairing

Quantum noise analysis of TOF images is more than HBT interference
Second order interference from the BCS superfluid


\[ n(r') \]

\[ n(k) \]

\[ k_F \]

\[ \Delta n(r, r') \equiv n(r) - n(r') \]

\[ \Delta n(r, -r) \left| \Psi_{BCS} \right>= 0 \]
Momentum correlations in paired fermions

Greiner et al., PRL 94:110401 (2005)
Fermion pairing in an optical lattice

Second Order Interference
In the TOF images

\[ G(r_1, r_2) = \langle n(r_1)n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle \]

Normal State

\[ G_N(r_1, r_2) = \delta(r_1 - r_2)\rho(r_1) - \rho^2(r_1) \sum_G \delta(r_1 - r_2 - \frac{G\hbar t}{m}) \]

Superfluid State

\[ G_S(r_1, r_2) = G_N(r_1, r_2) + \Psi(r_1) \sum_G \delta(r_1 + r_2 + \frac{G\hbar t}{m}) \]

\[ \Psi(r) = |u(Q(r))v(Q(r))|^2 \text{ measures the Cooper pair wavefunction} \]

\[ Q(r) = \frac{mr}{\hbar t} \text{ One can identify unconventional pairing} \]
Exploring topological states with cold atoms and photons

Theory: Takuya Kitagawa, Fabian Grusdt, Dima Abanin, Immanuel Bloch, Eugene Demler

Experiments: I. Bloch’s group (MPQ/LMU)

Harvard-MIT CUA

NSF, AFOSR MURI, DARPA OLE, MURI ATOMTRONICS
Universality in physics
Spontaneous symmetry breaking and order
Higgs excitations at LHC and in optical lattices

Cold atoms experiments
$10^{-11} - 10^{-10}$ K

Higgs mode in ultracold atoms, 2012

Higgs mode of the standard model, 2012
Order beyond symmetry breaking

In 1980 the first ordered phase beyond symmetry breaking was discovered

**Integer Quantum Hall Effect:** 2D electron gas in strong magnetic field shows plateaus in Hall conductance

Current along x, measure voltage along y.

On a plateau

\[ \sigma_{xy} = \frac{n e^2}{h} \]

with an accuracy of \(10^{-10}\)

Topological order is the “quantum protectorate” of this precise quantization
Order parameters

Magnetization - order parameter in ferromagnets

How to measure topological order parameter?

Berry/Zak phase in 1d

$$P = \frac{e}{\pi} \int A(k) \, dk$$

$$A(k) = \sum_n \langle u_n(k) | \partial_k | u_n(k) \rangle$$

Measure the Berry/Zak phase itself, not its consequence

$$P = \frac{\text{dipole moment}}{\text{length}}$$

Vanderbilt, King-Smith PRB 1993
Su-Schrieffer-Heeger Model

\[ H(k) = \mathbf{d}(k) \cdot \vec{\sigma} \]

\[ d_x(k) = (t + \delta t) + (t - \delta t) \cos ka \]
\[ d_y(k) = (t - \delta t) \sin ka \]
\[ d_z(k) = 0 \]

When \( d_z(k) = 0 \), states with \( \delta t > 0 \) and \( \delta t < 0 \) are topologically distinct.
Domain wall states in SSH Model

An interface between topologically different states has protected midgap states

\[ \delta t > 0 \quad \pm \frac{e}{2} \quad \delta t < 0 \]

Absorption spectra on neutral and doped trans-(CH)_x
Probing band topology with Ramsey/Bloch interference
Tools of atomic physics: Bloch oscillations

C. Salomon et al., PRL (1996)

More than 20,000 Bloch oscillations: Innsbruck, Florence
Tools of atomic physics: Ramsey interference

\( \pi/2 \) pulse

\[ | \downarrow \rangle \rightarrow \frac{1}{\sqrt{2}} | \downarrow \rangle + \frac{1}{\sqrt{2}} | \uparrow \rangle \]

Evolution

\[ | \Psi(t) \rangle = \frac{1}{\sqrt{2}} e^{-i\mathcal{H}_t} | \downarrow \rangle + \frac{1}{\sqrt{2}} e^{-i\mathcal{H}_t} | \uparrow \rangle \]

\( \pi/2 \) pulse + measurement of \( S_z \) gives relative phase accumulated by the two spin components

Used for atomic clocks, gravitometers, accelerometers, magnetic field measurements
Zak phase probe of band topology in 1d

One dimensional superlattices
Su-Schrieffer-Heeger model


Experiments Marcos Atala, Monika Aidelsburger, Julio Barreiro, I. Bloch (LMU/MPQ)

arXiv:1212.0572
SSH model of polyacetylene

Su, Schrieffer, Heeger, 1979

\[ H = \sum_i (t + \delta t) c_{Ai}^\dagger c_{Bi} + (t - \delta t) c_{Ai}^\dagger c_{Bi+1} + h.c. \]

Analogous to bichromatic optical lattice potential

I. Bloch et al., LMU/MPQ
Dimerized model

\[ H(k) = d(k) \cdot \vec{\sigma} \]

\[ d_x(k) = (t + \delta t) + (t - \delta t) \cos ka \]
\[ d_y(k) = (t - \delta t) \sin ka \]
\[ d_z(k) = 0 \]

\( \delta t > 0 \) : Berry phase 0
\( \delta t < 0 \) : Berry phase \( \pi \)
Characterizing SSH model using Zak phase

Two hyperfine spin states experience the same optical potential

\[ \varphi_{\text{tot}} = \varphi_{\text{Zak}} + \varphi_{\text{dyn}} + \varphi_{\text{Zeeman}} \]

\[ \frac{1}{i} \int_{-\pi}^{\pi} dk \langle \psi_k | \partial_k | \psi_k \rangle = \pi \]

Problem: experimentally difficult to control Zeeman phase shift
Spin echo protocol for measuring Zak phase

Dynamic phases due to dispersion and magnetic field fluctuations cancel. Interference measures the difference of Zak phases of the two bands in two dimerizations. Expect phase $\pi$. 
Bloch oscillations measurements in LMU/MPQ

With $\pi$-pulse but no swapping of dimerization
Bloch oscillations measurements in LMU/MPQ
With $\pi$-pulse and with swapping of dimerization
Zak phase measurements in LMU/MPQ

Phase of final MW pulse $\varphi_{MW}$

$$\delta \varphi = 0.97(2) \pi$$
Zak phase measurements can be used to probe topological properties of Bloch bands in 2D

D. Abanin, T. Kitagawa, I. Bloch, E. Demler
arXiv:1212.0572
Berry phase in hexagonal lattice

\[ H(k) = -J \begin{pmatrix} 0 & A(k) \\ A^*(k) & 0 \end{pmatrix} \]

\[ A(k) = e^{i k \cdot a_1} + e^{i k \cdot a_2} + e^{-i k \cdot (a_1 + a_2)} \]

- Eigenvectors lie in the XY plane
- Around each Dirac point eigenvector makes \(2\pi\) rotation
- Integral of the Berry phase is \(\pi\)

\[ \frac{1}{i} \int_C d\vec{k} \left\langle \psi_k \mid \partial_{\vec{k}} \mid \psi_k \right\rangle = \pi \]
Berry’s phase of Dirac fermions

Shifted positions of Integer quantum Hall plateaus

\[ \nu = \pm 4 \left(N + \frac{J}{2}\right) \]
Measurement of the Berry and Zak phases in 2D with Ramsey/Bloch method

Experimental realization
Tarruell et al., Nature (2012)
How to measure the $\pi$-Berry phase of Dirac fermions

“Parallel” measurements with a cloud of fermions

In Ramsey interference

$$S'_z = \cos \phi$$

When $\phi$ jumps by $\pi$

$S'_z$ changes sign
Measurement of the Berry and Zak phases in 2D
Spin echo protocol
Measurement of the Berry and Zak phases in 2D
Modified spin echo protocol
Summary

Experiments with ultracold atoms provide a new perspective on the physics of strongly correlated many-body systems. Quantum noise is a powerful tool for analyzing many body states of ultracold atoms.

Thanks to: