Interplay of interactions, disorder, and temperature in quantum many-body systems

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Outline

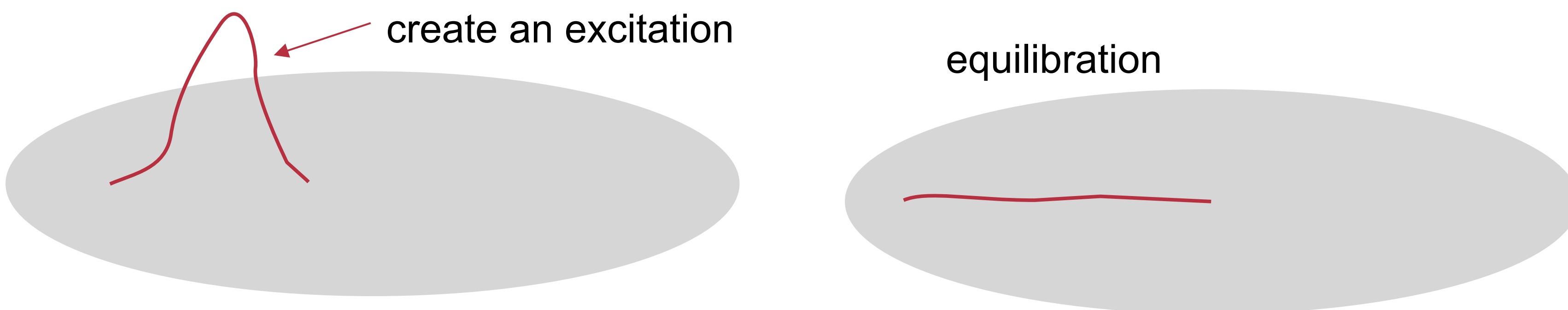
Introduction. Many-Body Localization

Real space RG to MBL states in 1d. Hilbert glass phase

Probing MBL states with Double Electron Electron Resonance type probes

Many-Body Localization

Ergodicity: equivalence of temporal and ensemble averaging
Equilibration is exchange of particles, energy, ...
Thermalization means system acts as it's own bath

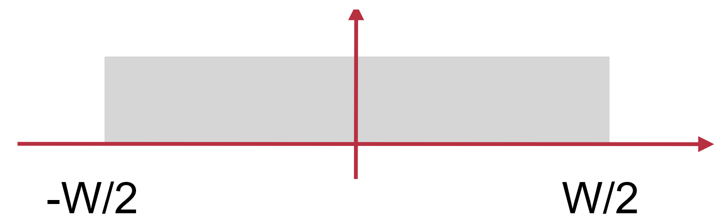


Many-Body Localized states: phases of interacting many-body systems, which do not exhibit ergodicity

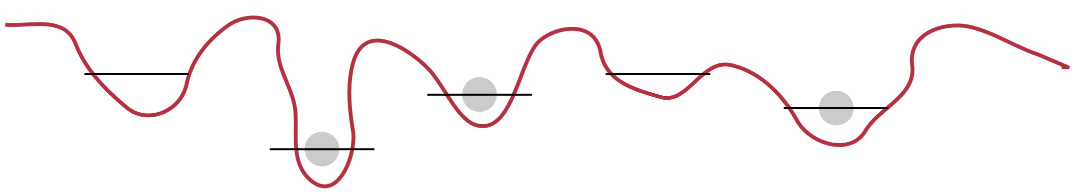
Single-particle localization

Non-interacting particles in quenched disorder

$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \sum_i V_i n_i \qquad V_i \in [-W/2, W/2]$$



hopping cannot overcome disorder P. W. Anderson, Phys. Rev. (1958)



← **zero conductivity**

(critical strength of disorder depends on dimension)

wave-functions are exponentially localized

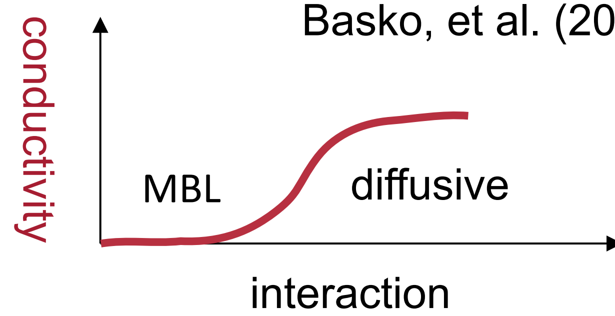
$\psi(r) \sim e^{-r/\xi}$ ← localization length

Many-body localization (MBL)

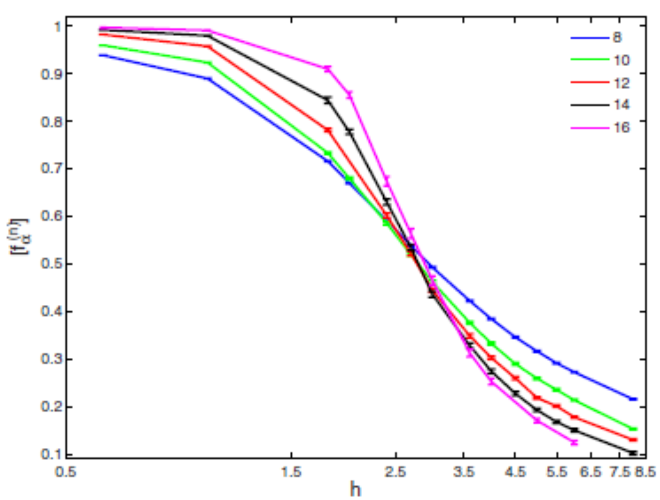
add interactions V: system can still be localized

Basko, et al. (2006)

system does not act as its own bath (discrete local spectrum)
→ fails to thermalize



Many-body localization in spin systems in 1d



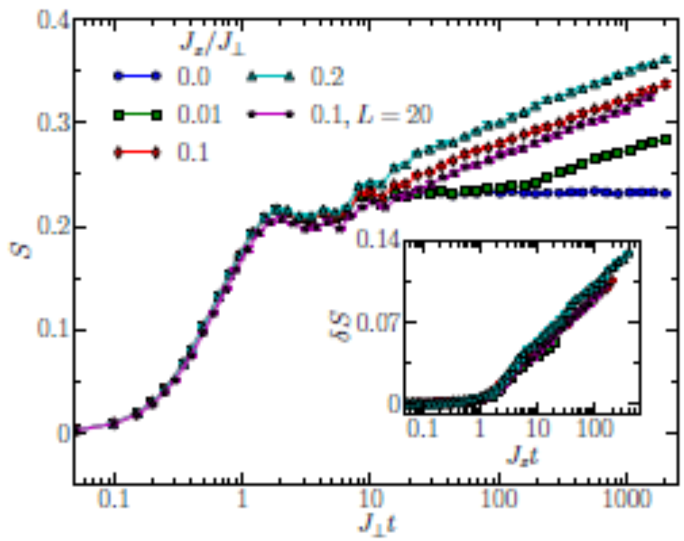
A. Pal, D. Huse (2006)

$$H = \sum_{i=1}^{L} [h_i \hat{S}_i^z + J \hat{\vec{S}}_i \cdot \hat{\vec{S}}_{i+1}]$$

The fraction of the initial spin polarization that is dynamic

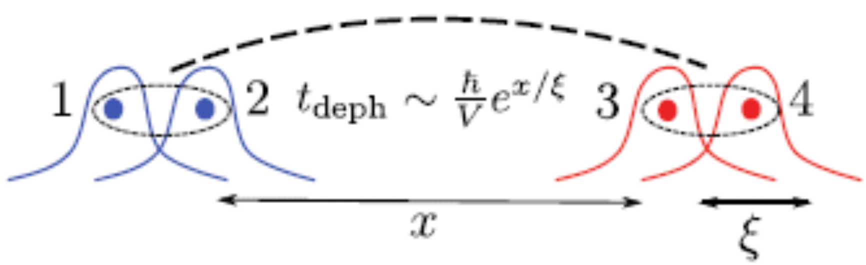
Entanglment growth in quenches with random spin XXZ model

Bardarson, Pollman, Moore, PRL 2012



Exponentially small interaction induced corrections to energies

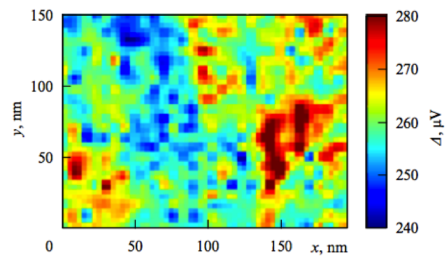
Serbin, Papic, Abanin, PRL 2013



Systems with interactions and disorder

Granular superconductors and Josephson junction networks

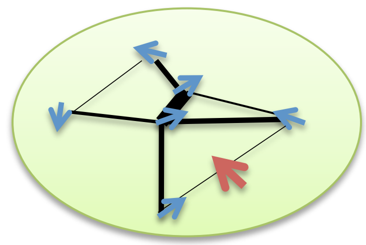
- Crane et al. (2007): AC response
- Bouadim et al. (2011): numerics
- Baturina, Sacepe et. al. (2008): STM
- Trivedi et. al. (2012 review)



Gap map in TiN film

Central spin problem in q-dots
NV centers in diamond

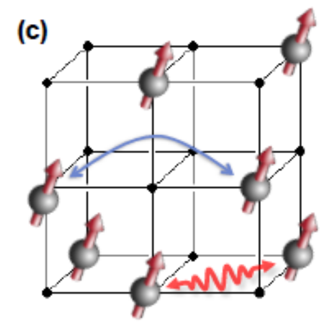
- Marcus et al. (2004)
- Lukin et al. (2006)
- Jelezko et a. (2007)
- Awschalom et al. (2007)



Nuclear spin interactions mediated by electron spin

Polar molecules in optical lattices

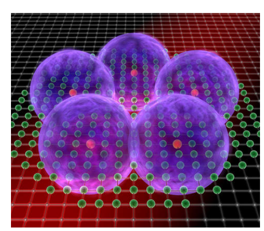
- Ye et al. (2013)



Angular momentum as spin degree of freedom

Rydberg atoms

- Ryabtsev et al. (2010)
- Bloch et al. (2012)



Strong interactions due to large electric dipole moment

Many Body localized phases:

How to understand them

How to probe them in experiments

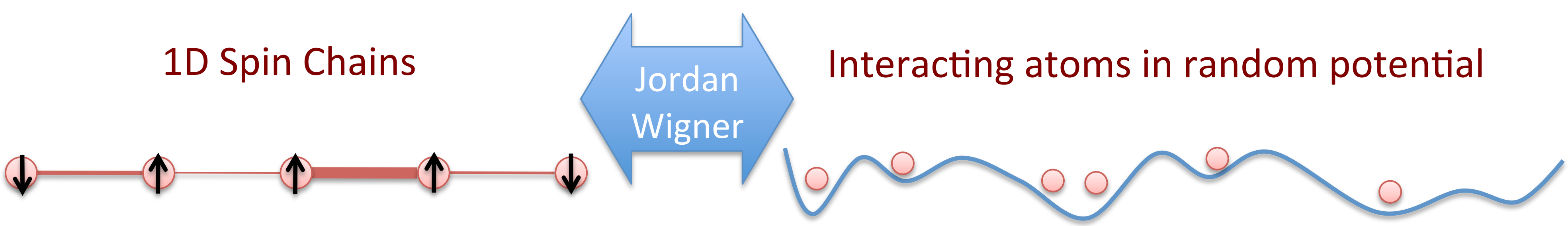
The Hilbert-glass transition: new universality of temperature-tuned many-body dynamical quantum criticality

D. Pekker, G. Refael, E. Altman, EAD, V. Oganesyan, arXiv:1307.3253

Hierarchical structure of excited many-body states in disordered systems

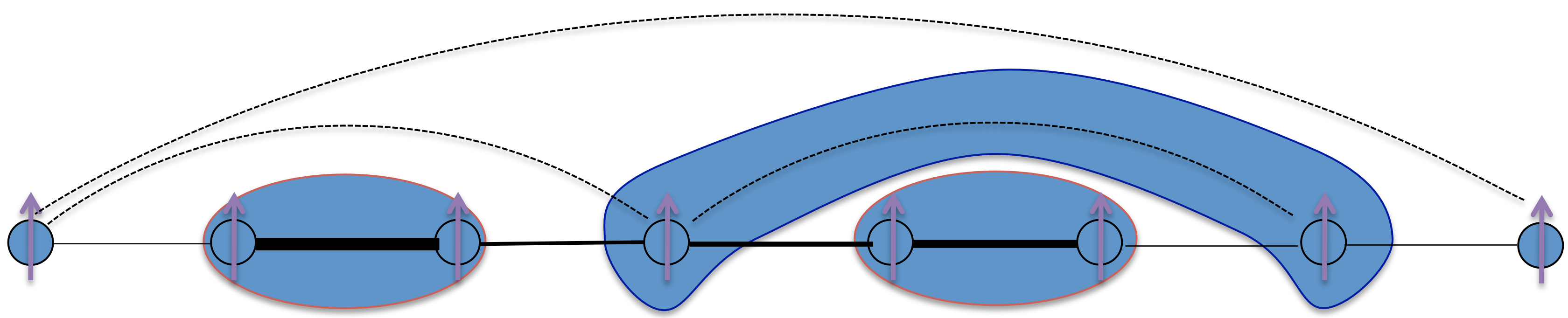
- Premise:
  - Ground states have hierarchical structure described by power law distributions of couplings and gaps (D. Fisher)

- Conjecture:
  - Excited states can share similar hierarchical structure (e.g. MBL states are essentially integrable)



Implications of the conjecture

- Strongly coupled spins precess fast around each other
- Mediate coupling between outer spins

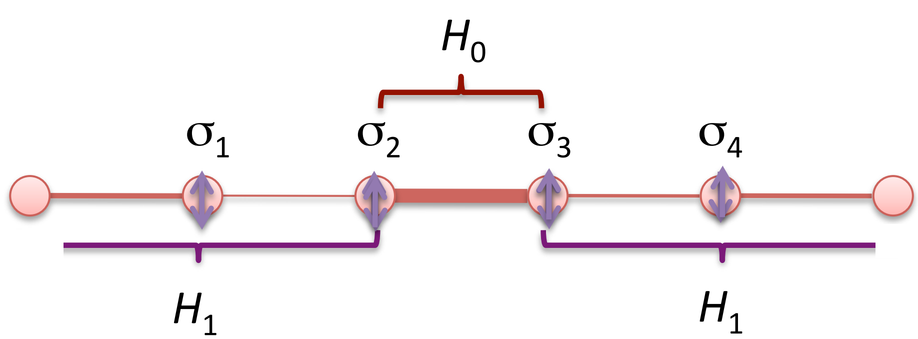


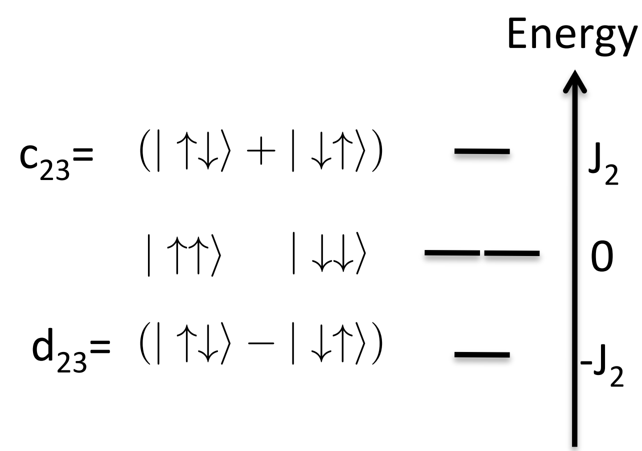
XY model: Beyond the ground state

- Real space RG following the idea of Vosk & Altman (2012)
- Considered the 1D XY chain (free fermions with random hopping)

$$H = \sum_i J_i \left(\sigma_i^- \sigma_{i+1}^+ + \sigma_i^+ \sigma_{i+1}^- \right)$$

- RG decimation step: perturbation theory



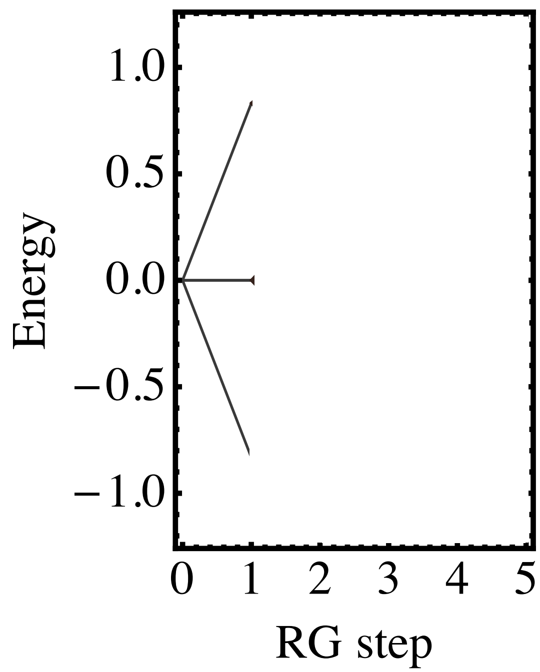


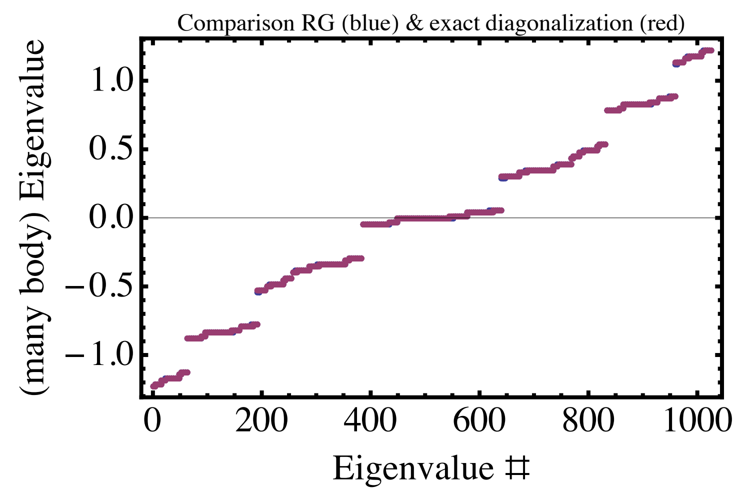
Effective coupling across cluster:

$$H^{(2)} = -\frac{J_1 J_3}{J_2} \left(\sigma_1^+ \sigma_4^- + \sigma_1^- \sigma_4^+ \right) \sigma_2^z \sigma_3^z + \frac{J_1^2 + J_3^2}{2 J_2} \left(P[d_{23}] - P[c_{23}] \right)$$

Results of the RG procedure

- Construct spectrum via choice of branch





What is our RG good for ?

(3) RG Procedure can be made generic

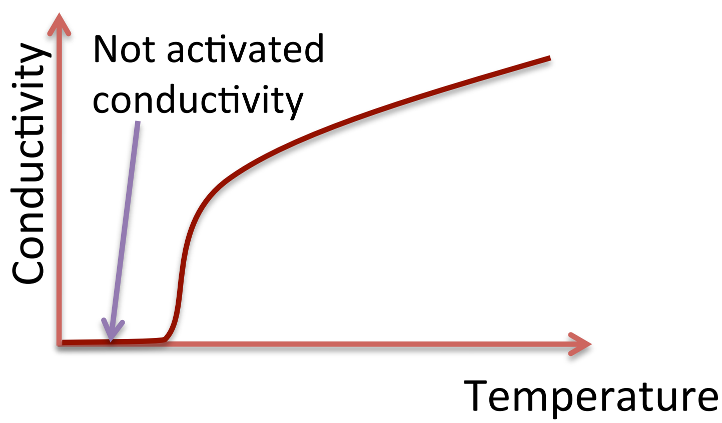
- construct ground and excited many-body wave functions

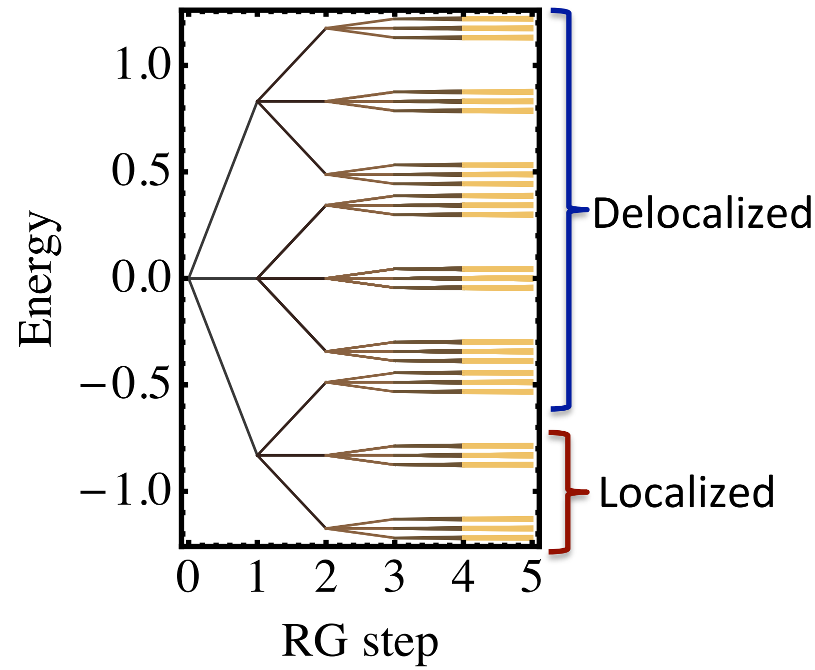
(1) Full spectrum → dynamics

- objects like conductivity $\sigma(\omega, T)$
- use MC to probe the tree

(2) Many-body localization ?

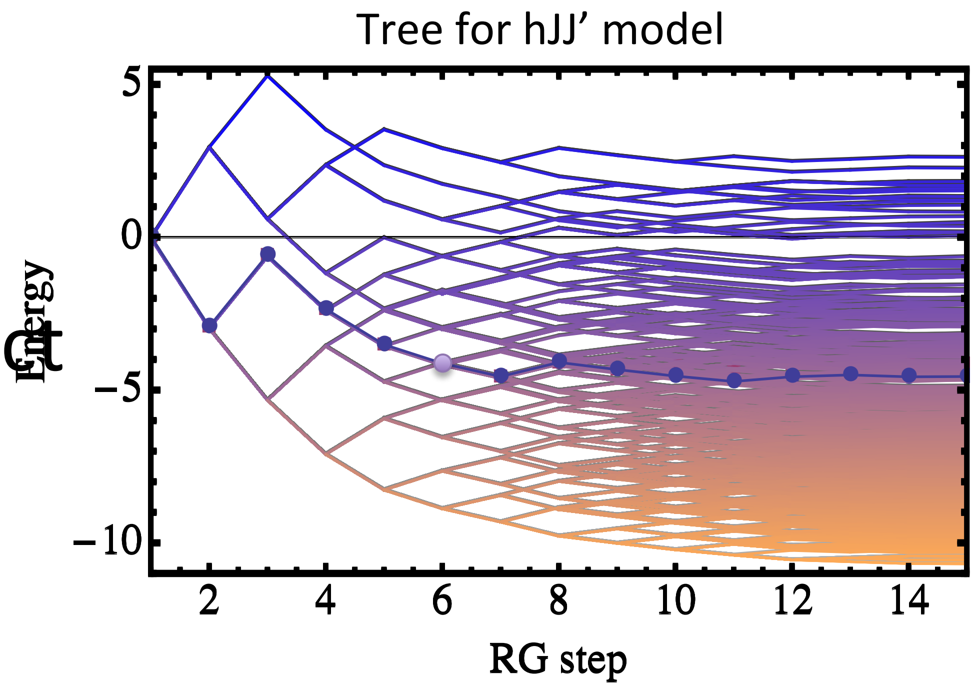
- Anderson
- Basko, Aleiner, Altshuler





Sampling the tree using Monte Carlo

- Start with a branch
- Propose a new branch
- Metropolis accept/reject



- Example of sampling:
  - finite freq. conductivity
  - run RG to ω scale

Adding interactions: hJJ' model

- Twist on the random transverse field Ising model

$$H = \sum_i h_i \sigma_i^z + J_i \sigma_i^x \sigma_{i+1}^x + J_i' \sigma_i^z \sigma_{i+1}^z$$

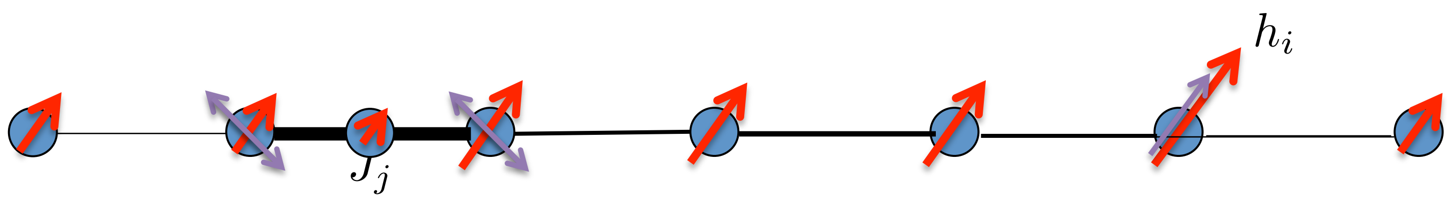
- Without *J*': solved by D. Fisher (equivalent to free fermions)
  - transition between *h*-dominated phase and a *J*-dominated phase
- With *J*': model becomes interacting
  - above transition becomes temperature tuned

Random transverse field Ising model

- D. Fisher 1992

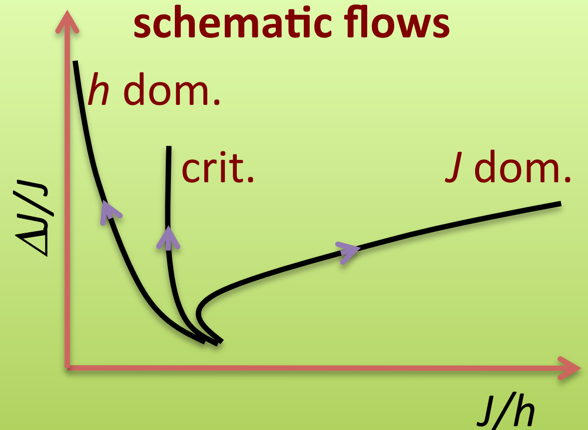
free fermions:
excited states identical except signs

$$H = \sum_i h_i \sigma_i^z + J_i \sigma_i^x \sigma_{i+1}^x$$

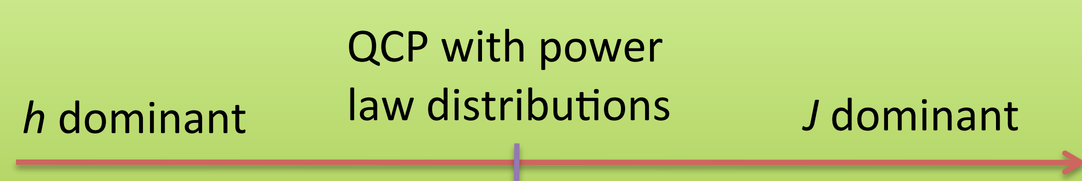


$$h_{\text{eff}} = \frac{h_j h_{j+1}}{J_j} \qquad J_{\text{eff}} = \frac{J_{i-1} J_i}{h_i}$$

schematic flows

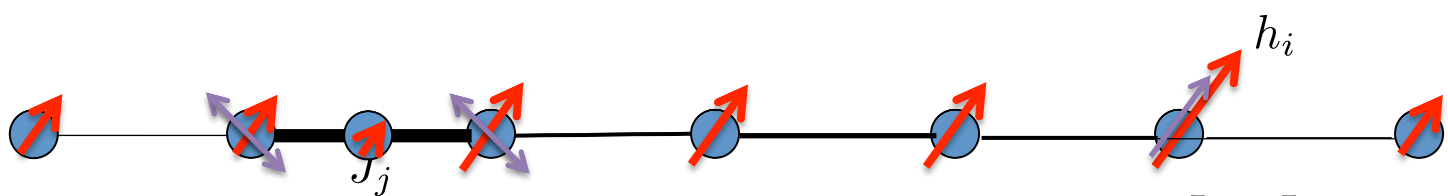


schematic phase diagram



Effect of J'

$$H = \sum_i h_i \sigma_i^z + J_i \sigma_i^x \sigma_{i+1}^x + J'_i \sigma_i^z \sigma_{i+1}^z$$



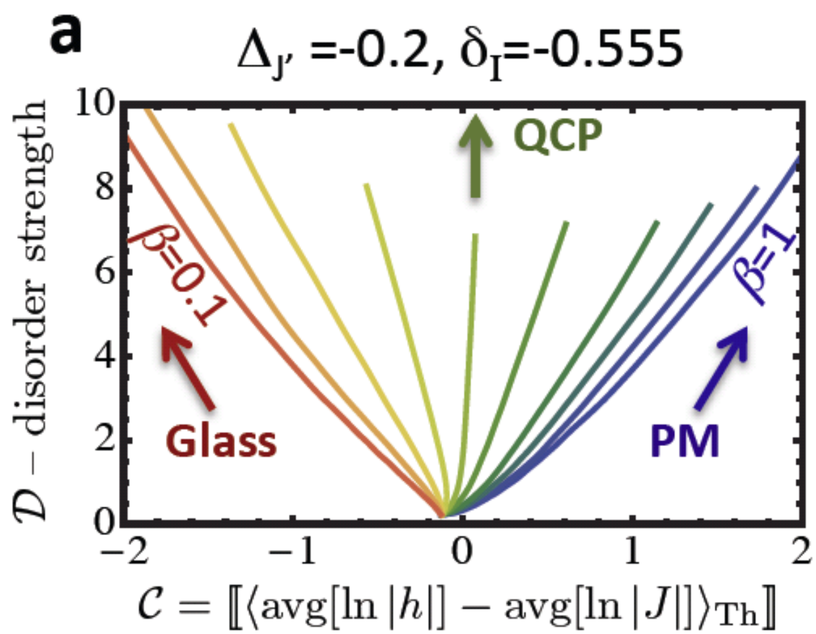
$$h_{\text{eff}} = J'_j \pm \frac{h_j h_{j+1}}{J_j}$$

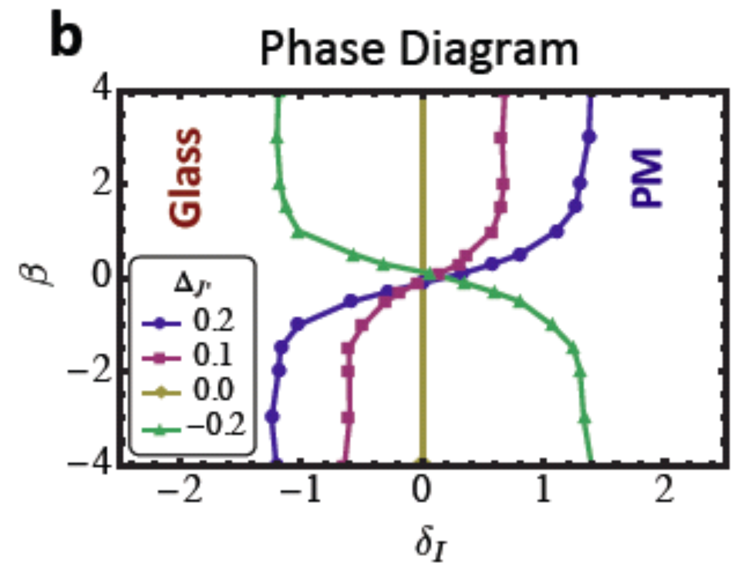
$$J_{\text{eff}} = \pm \frac{J_{i-1} J_i}{h_i}$$

$$J'_{\text{eff}} = 0$$

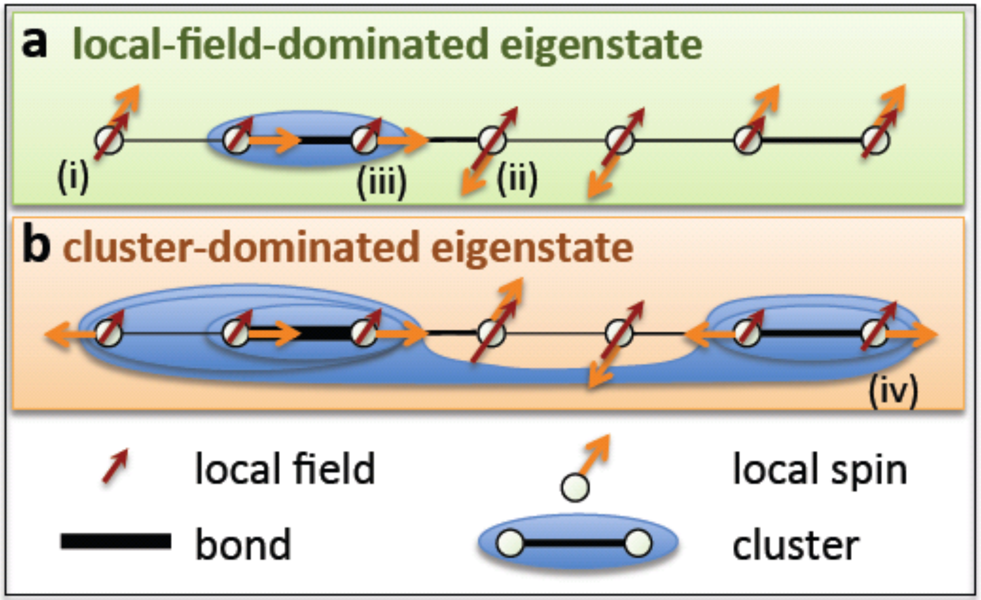
RG flows of JJ' model

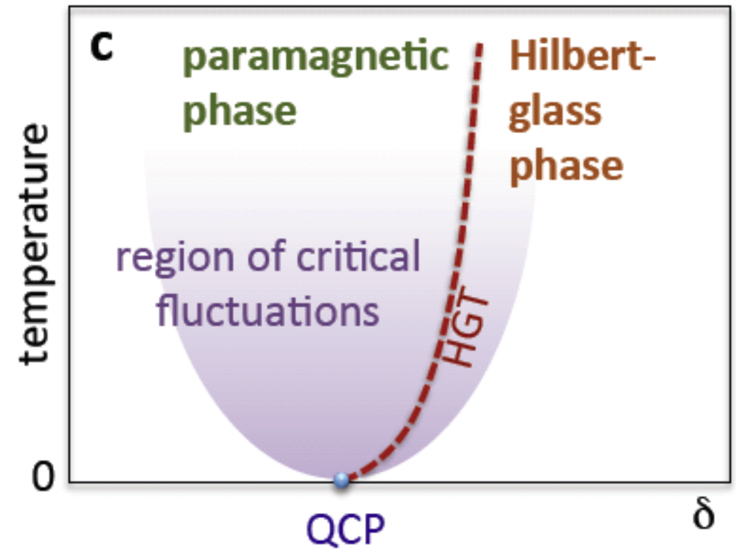
$$\delta = \mathcal{C}/\mathcal{D}$$



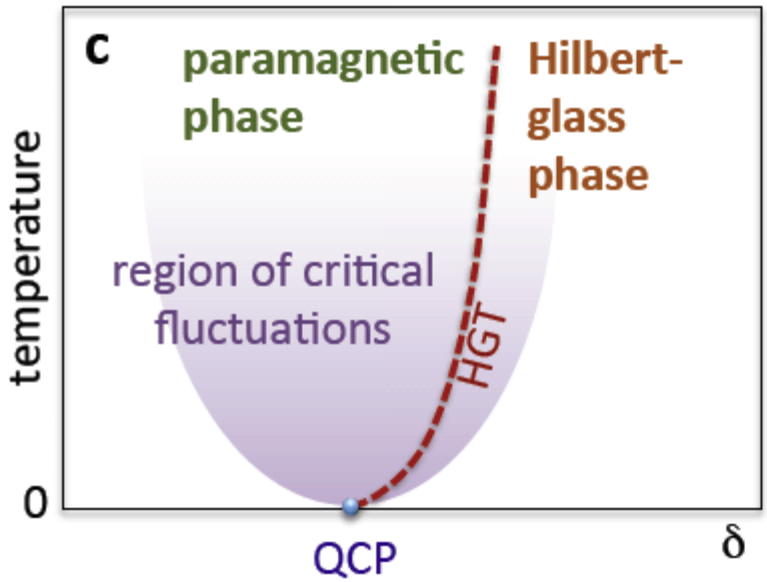


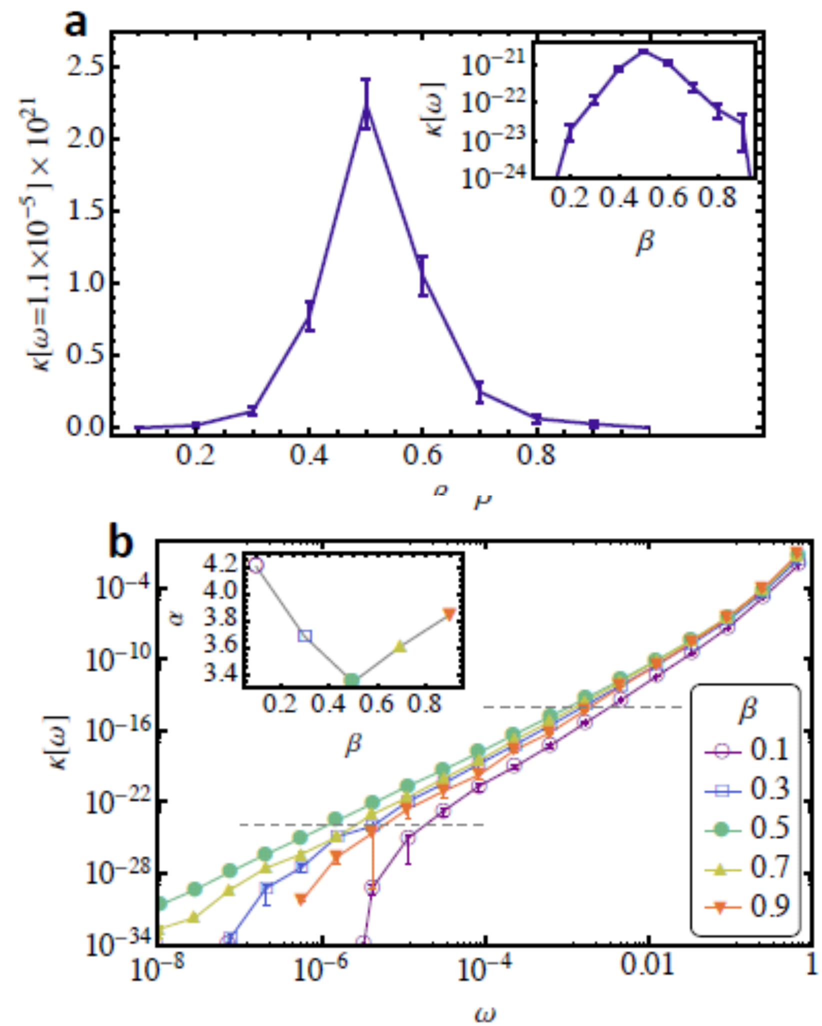
Phase diagram of JJ' model





Manifestations of Hilbert Glass transition





$$\kappa(\omega) \sim \omega^{\alpha}$$

$$\alpha \sim 3 + \text{const} \times |\beta - \beta_c|$$

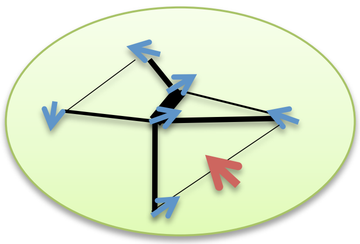
Looking for smoking-gun experimental signatures of Many Body Localization

M. Serbyn, M. Knap, S. Gopalakrishnan, D. Abanin, M. Lukin, ED

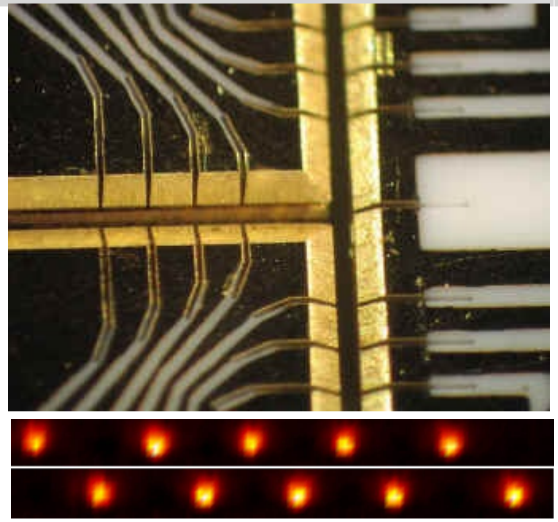
work in progress
see also M. Knap et al., PRL (2013)

Probing spin dynamics in synthetic matter

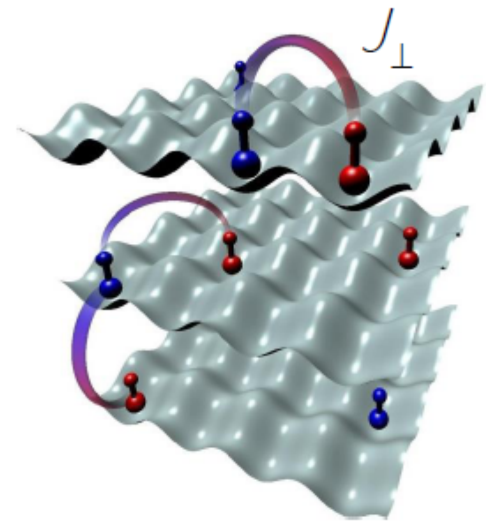
NVs in diamond	Trapped ions	Polar molecules



Suttgart, Harvard, UCSB, …



JQI group



JILA group

- **Central spin system**

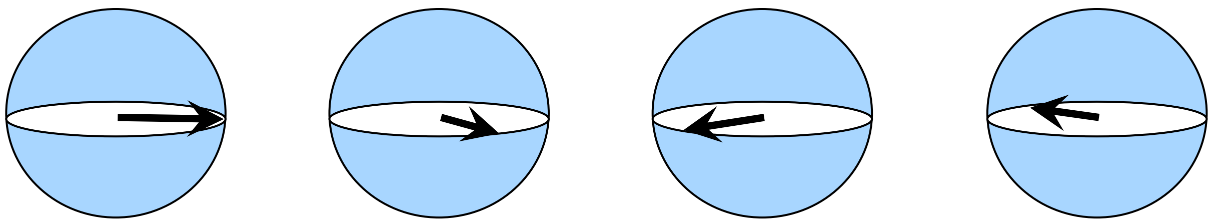
- **LR transverse field Ising model**
- interactions mediated by phonons
- e.g. ^{171}Yb

- **LR XX model**
- dipolar interactions
- e.g. KRb

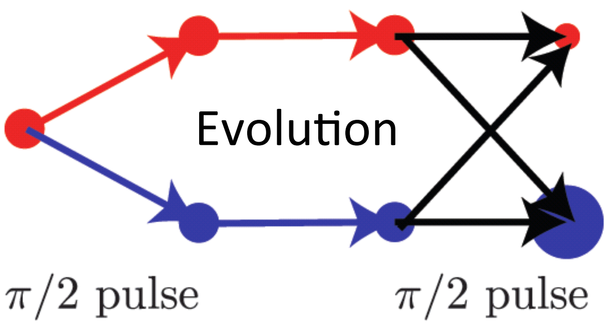
Tools of atomic physics: Ramsey interference

$\frac{\pi}{2}$ pulse $\quad |\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}|\downarrow\rangle + \frac{1}{\sqrt{2}}|\uparrow\rangle$

Evolution $\quad |\Psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\mathcal{H}_{\downarrow}t}|\downarrow\rangle + \frac{1}{\sqrt{2}} e^{-i\mathcal{H}_{\downarrow}t}|\uparrow\rangle$



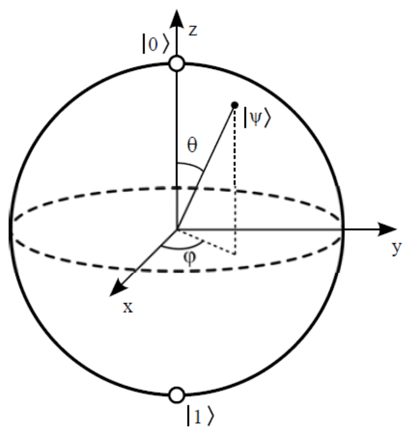
$\frac{\pi}{2}$ pulse + measurement ot S_z gives relative phase accumulated by the two spin components



Used for atomic clocks, gravitometers, accelerometers, magnetic field measurements

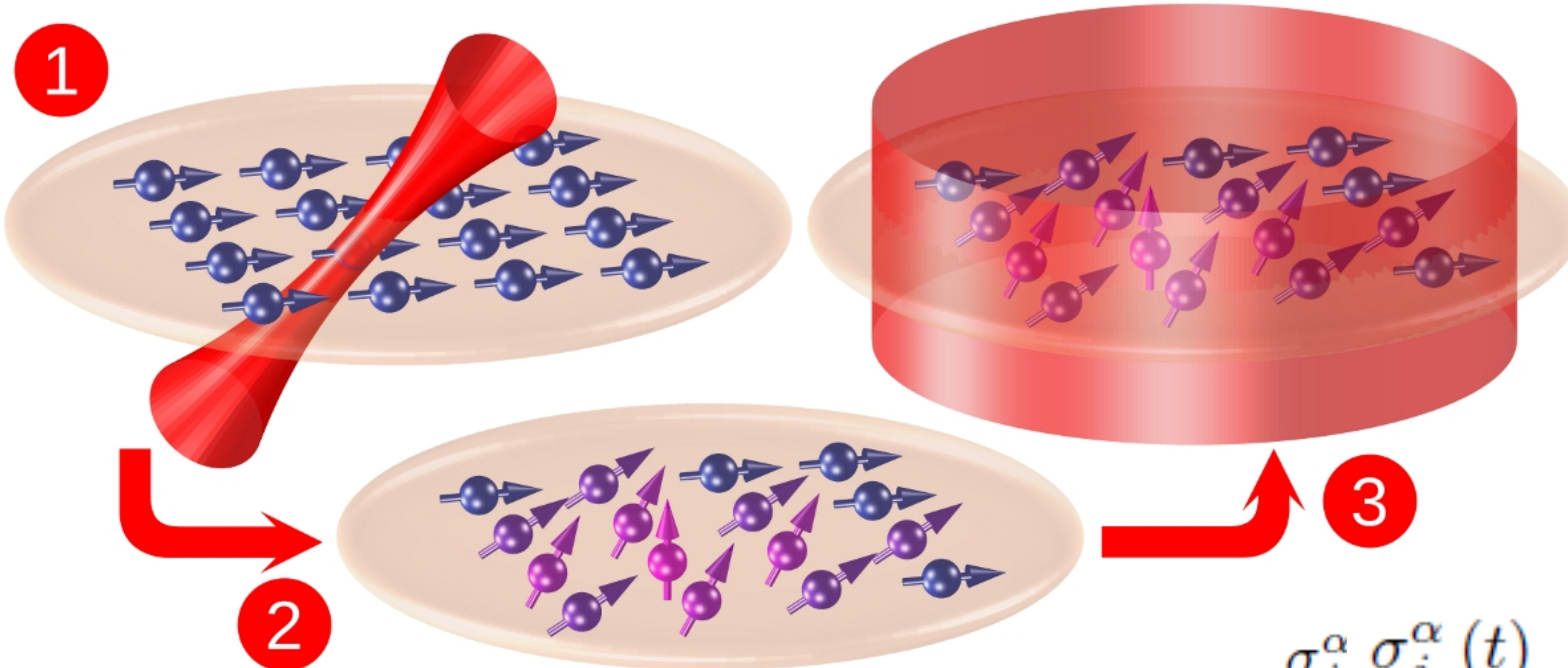
Spin rotations

$$R_j(\theta, \phi) = \hat{1} \cos\frac{\theta}{2} + i(\sigma_j^x \cos\phi - \sigma_j^y \sin\phi) \sin\frac{\theta}{2}$$



p/2 pulse: $R_j(\frac{\pi}{2}, \phi) = \frac{1}{\sqrt{2}}(1 + e^{i\phi}\sigma_j^+ + e^{-i\phi}\sigma_j^-)$

Many-body spin Ramsey protocol



$\sigma_j^\alpha \, \sigma_j^\alpha(t)$

Many-body spin Ramsey protocol

$$M_{ij}(\phi_1,\phi_2,t) = \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| R_i^\dagger(\phi_1) e^{i\hat{H}t} R^\dagger(\phi_2) \sigma_j^z R(\phi_2) e^{-i\hat{H}t} R_i(\phi_1) |n\rangle$$

$$\begin{aligned} M_{ij}(\phi_1,\phi_2,t) = \frac{1}{2}\big(&\cos\phi_1 \sin\phi_2 G_{ij}^{xx,-} + \cos\phi_1 \cos\phi_2 G_{ij}^{xy,-} \\ &- \sin\phi_1 \sin\phi_2 G_{ij}^{yx,-} - \sin\phi_1 \cos\phi_2 G_{ij}^{yy,-}\big) \\ &+ \text{terms with odd number of } \sigma^{x,y} \text{ operators} \,, \end{aligned}$$

- for many relevant cases terms with odd number of spin-x/spin-y operators vanish
- additional degree of freedom:

$\rightarrow$ phases of the laser field

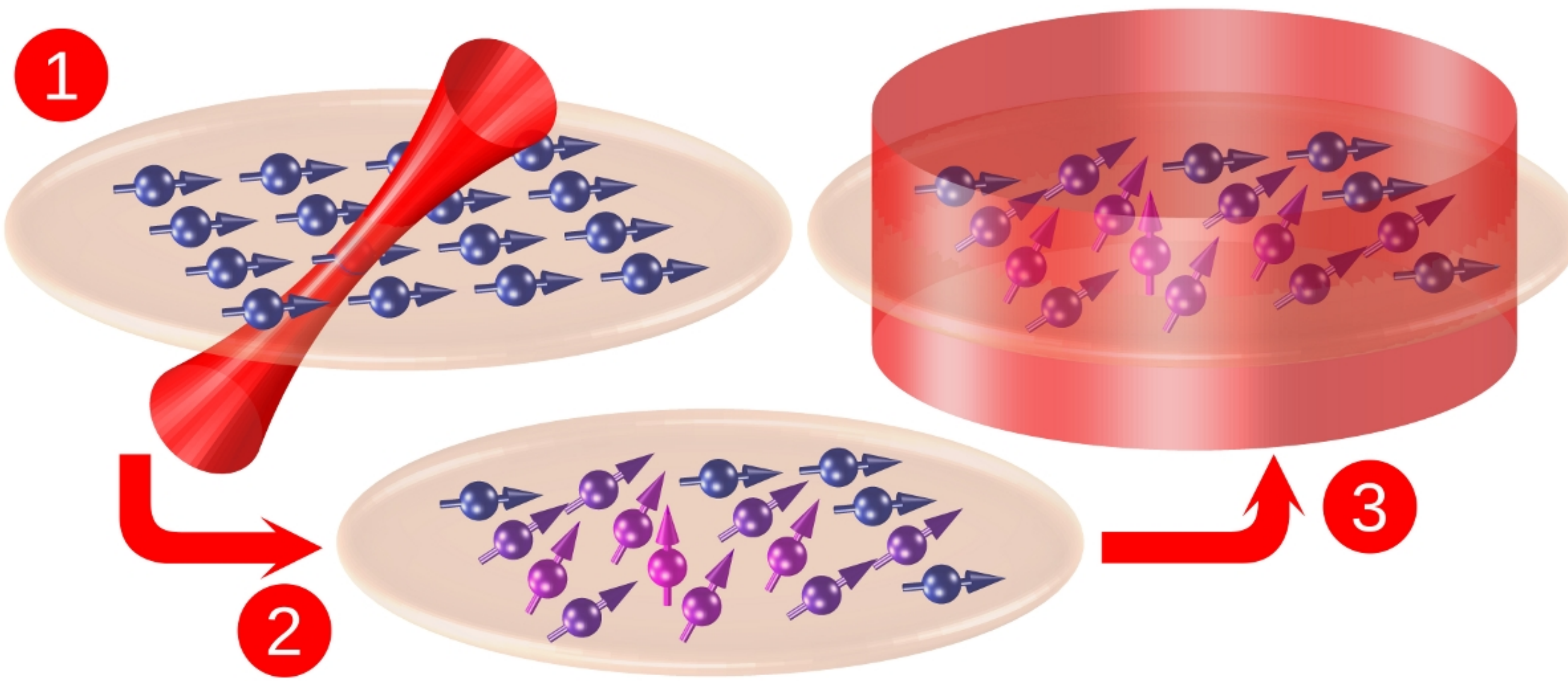
Heisenberg model

$$\hat{H}_{\mathrm{Heis}} = \sum_{i<j} J_{ij}^{\perp}(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) + J_{ij}^z \sigma_i^z \sigma_j^z$$

- global symmetry $\sigma^x \to -\sigma^x \quad \sigma^y \to -\sigma^y \quad \sigma^z \to \sigma^z$
- U(1) symmetry around z axis

$$\begin{aligned} M_{ij}(\phi_1,\phi_2,t) = \frac{1}{4}\{ & \sin(\phi_1+\phi_2)(G_{ij}^{xx,-} - G_{ij}^{yy,-}) \\ & - \sin(\phi_1-\phi_2)(G_{ij}^{xx,-} + G_{ij}^{yy,-}) \\ & + \cos(\phi_1+\phi_2)(G_{ij}^{xy,-} + G_{ij}^{yx,-}) \\ & + \cos(\phi_1-\phi_2)(G_{ij}^{xy,-} - G_{ij}^{yx,-})\} \,. \end{aligned}$$

Many-body spin Ramsey protocol



Measures the usual retarded spin correlation function

$$\theta(t)\cdot\frac{1}{Z}\sum_n e^{-\beta E_n}\langle n|\, S_i^x(0)\, S_i^x(t) - S_i^x(t)\, S_i^x(0)\,|n\rangle$$

Spin correlation function as quantum quench

$$\langle u| S_i^x(t) S_i^x(0) |u\rangle = \langle u| e^{iHt} S_i^x \cdot e^{-iHt} S_i^x |u\rangle$$

$$= \langle u| e^{iHt} e^{-i\tilde{H}_i t} |u\rangle$$

$\tilde{H}_i$ differs from H by

$$S_i^y \to -S_i^y$$

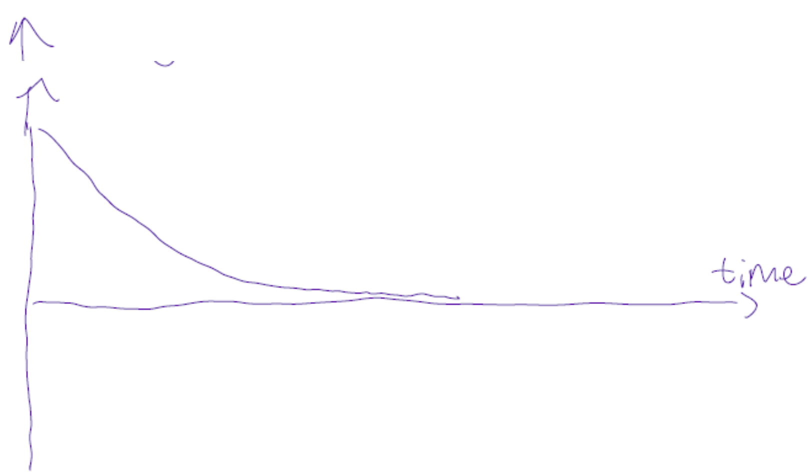
$$S_i^z \to -S_i^z$$

Spin correlation function as quantum quench

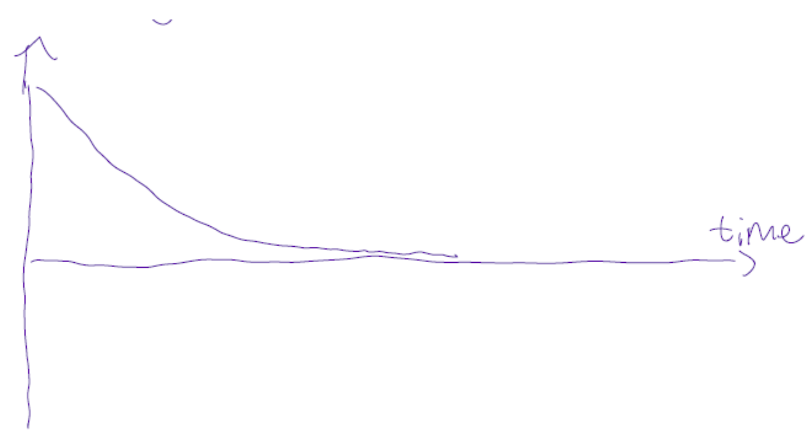
$$\langle n | e^{iHt} e^{-i\tilde{H}_i t} | n \rangle$$

In a localized phase, local quench affects only a few excitations. For each eigenstate expect non-decaying oscillations

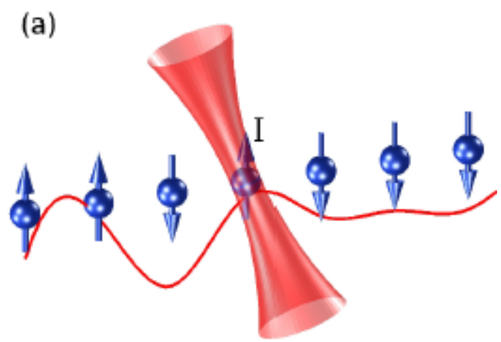
After averaging over thermal ensemble (and/or disorder realizization) find decay



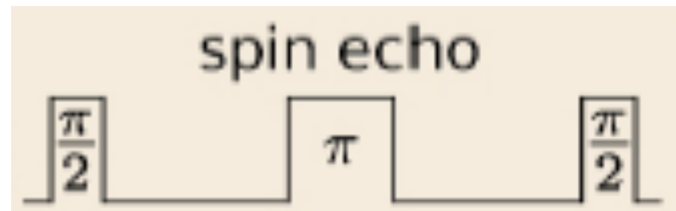
In a delocalized phase (diffusive regime), local quench affects all excitations. Expect decay akin orthogonality catastrophe



Ramsey + spin echo



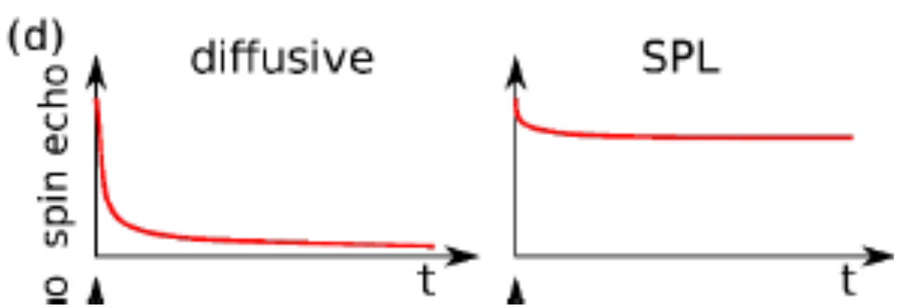
M. Knap, S. Gopalakrishnan, M. Serbyn, et al.



"Cartoon" model of the localized phase $H_{SPL} = \sum h_i^z S_i^z$

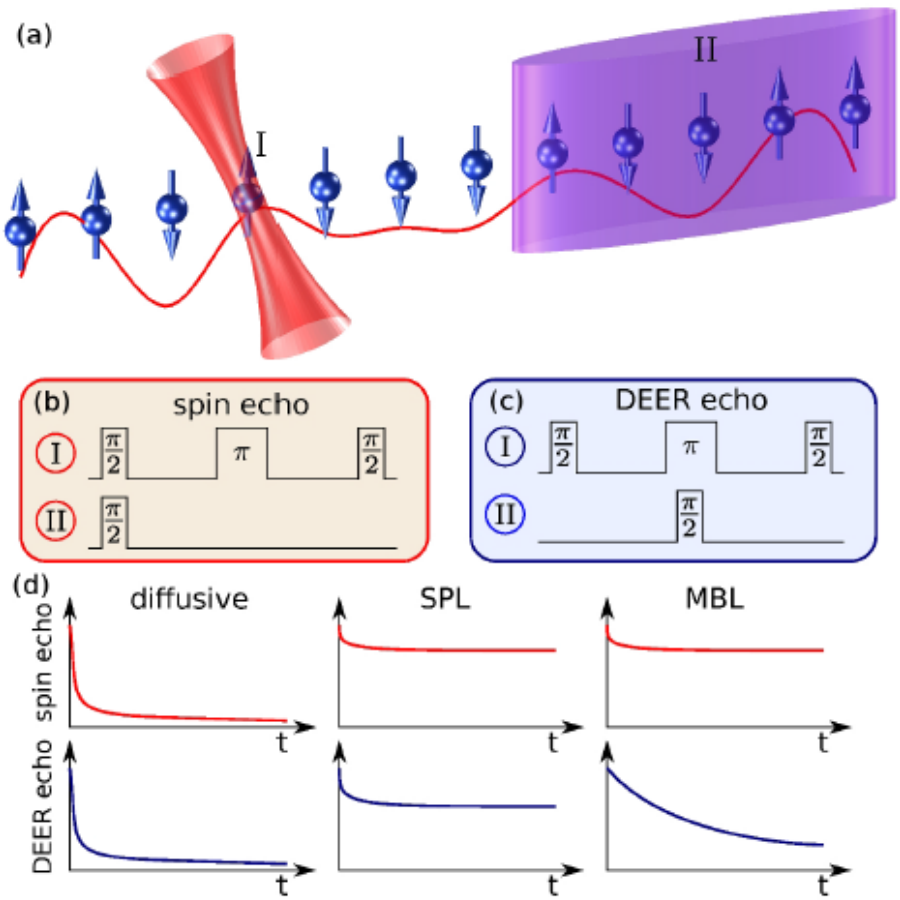
Spin echo $S_i^z \to -S_i^z$

$$|\psi_{z_i}(t)\rangle = e^{+iH_{z_i}\frac{t}{2}} e^{-iH_{z_i}\frac{t}{2}} |\psi_{z_i}\rangle$$



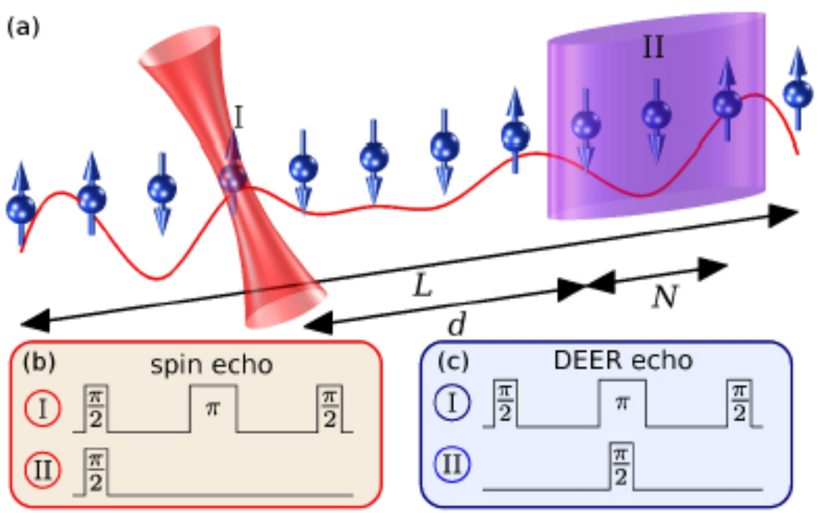
Double Electron-Electron Resonance Ramsey sequence

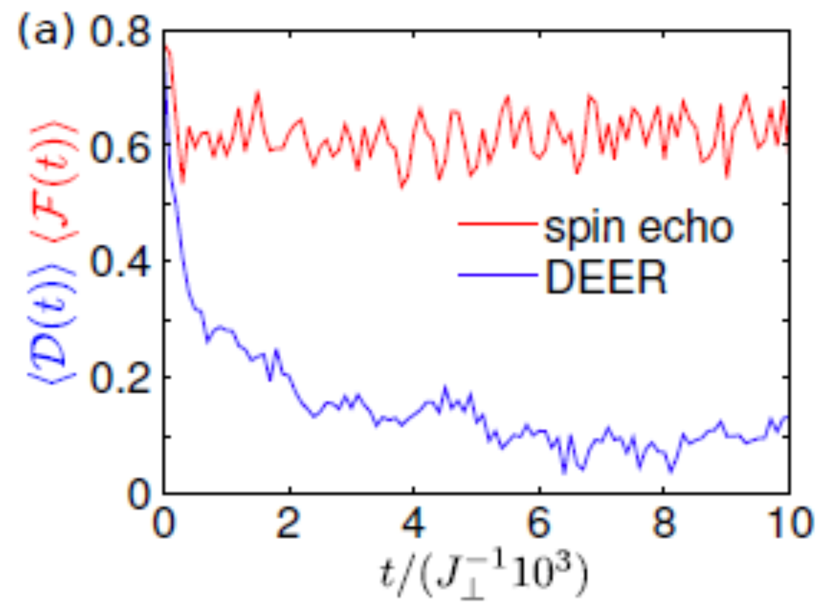
M. Knap, S. Gopalakrishnan, M. Serbyn, et al.



Double Electron-Electron Resonance Ramsey sequence

$$\hat{H} = \frac{J_\perp}{2}\sum_{\langle ij\rangle}(\hat{S}_i^+\hat{S}_j^- + \hat{S}_j^+\hat{S}_i^-) + J_z\sum_{\langle ij\rangle}\hat{S}_i^z\hat{S}_j^z + \sum_i h_i\hat{S}_i^z$$

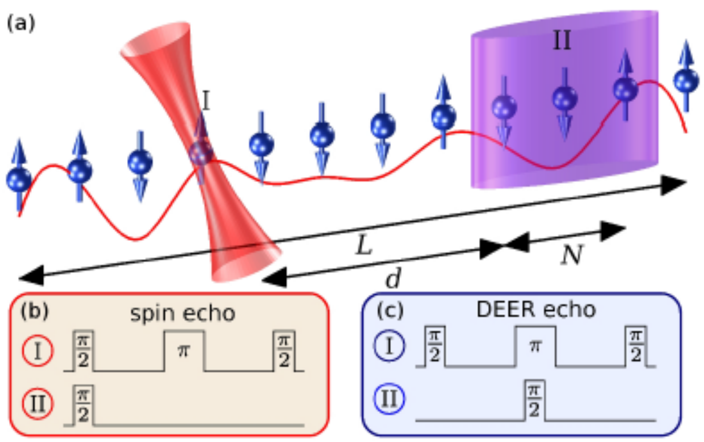


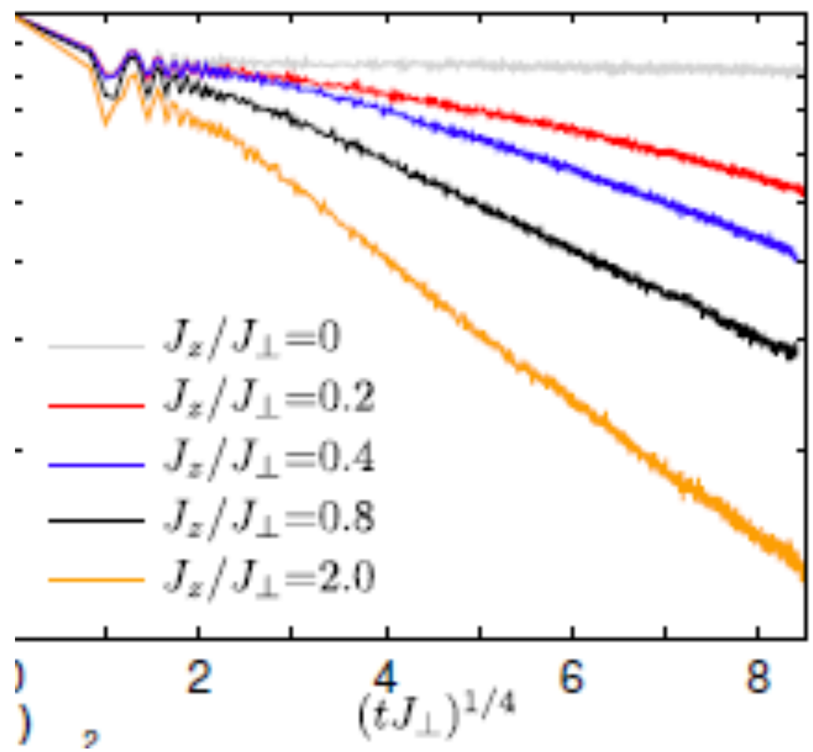


single realization

thermal averaging over 50 eigenstates

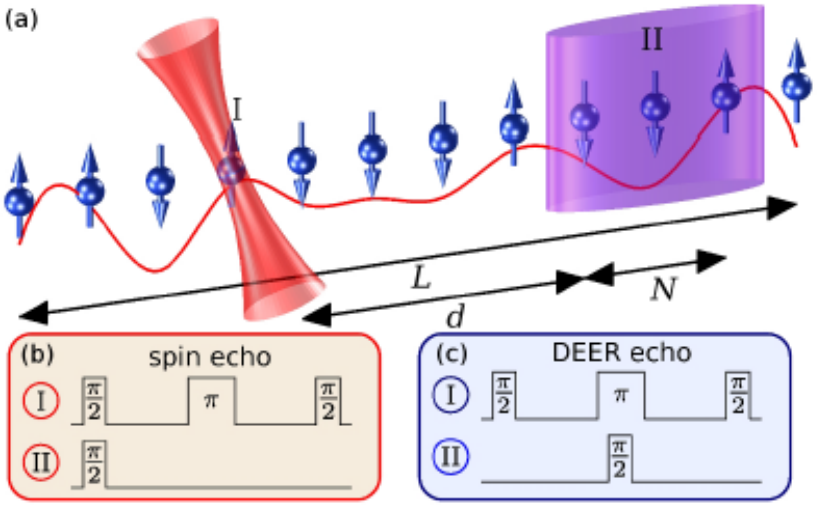
Double Electron-Electron Resonance
Ramsey sequence

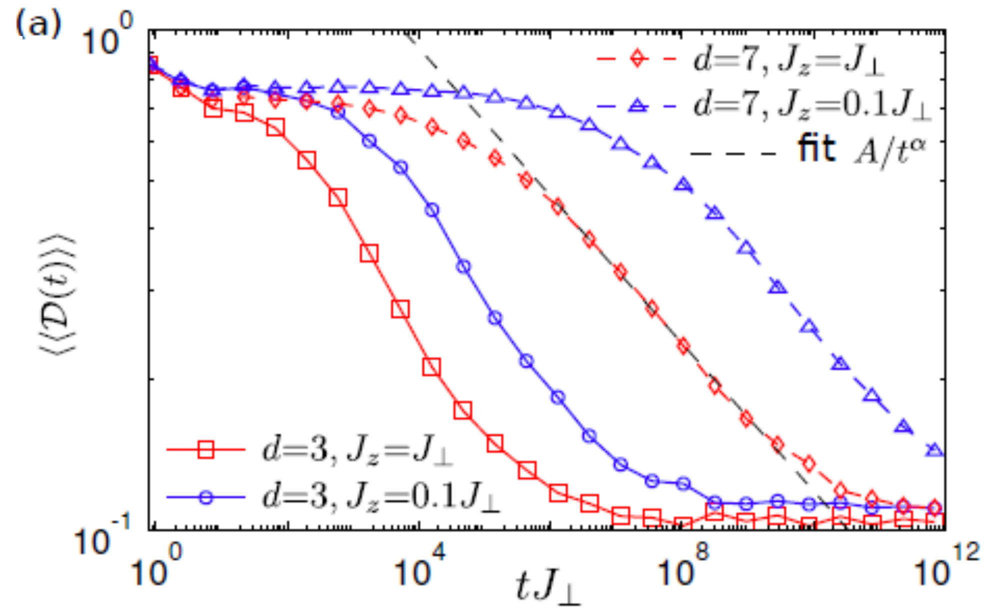




Double Electron-Electron Resonance Ramsey sequence

Power law decay of DEER signal with time





thermal and ensemble averaging

MBL as integrable model

$$\hat{H} = \sum_i \tilde{h}_i \tau_i^z + \sum_{ij} J_{ij} \tau_i^z \tau_j^z$$

"Cartoon" model of MBL phase

Interaction strength decays as $J_{\mathrm{I}j} \propto \exp(-|j - \mathrm{I}|/\xi)$

$$\mathcal{D}(t) \equiv \langle \psi(t) | \hat{\tau}_{\mathrm{I}}^z | \psi(t) \rangle = \frac{1}{2^N} \mathrm{Re} \prod_{j \in \mathrm{II}} \{1 + e^{2iJ_{\mathrm{I}j}\tau_j t}\}$$

For a given time *t* we can separate fast modes $J_{\mathrm{I}j} t \gg 1$

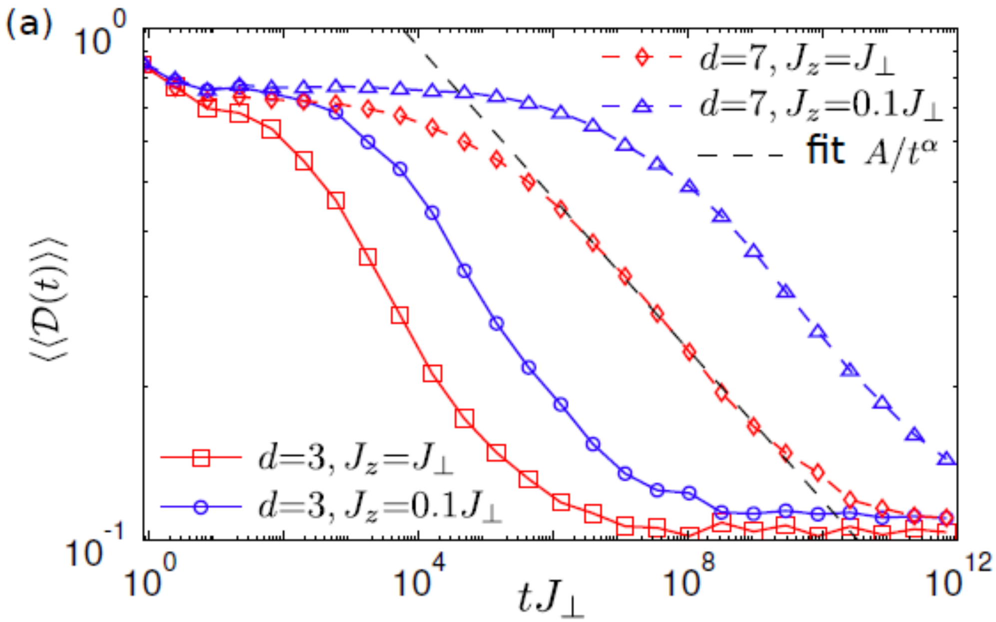
and slow modes $J_{\mathrm{I}j} t \ll 1$

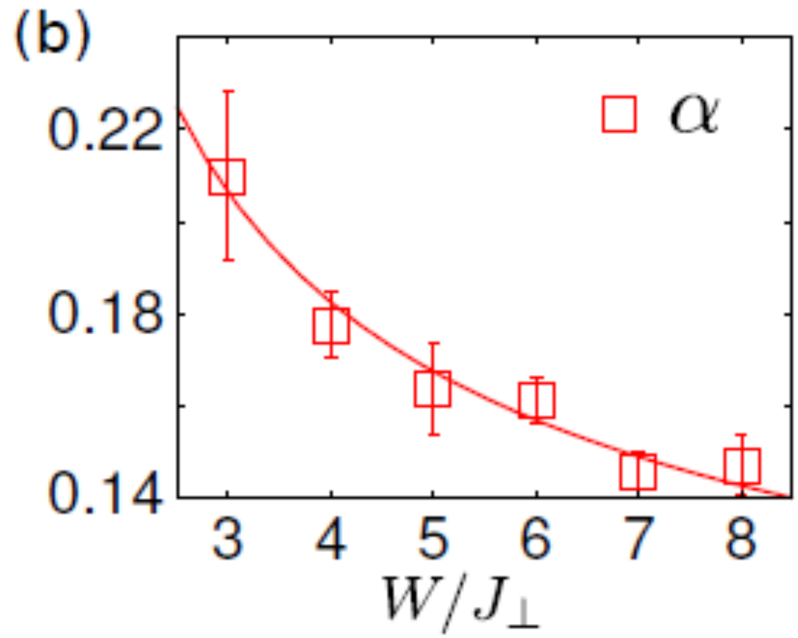
$$\mathcal{D}(t) = \bar{\mathcal{D}}(t) + \mathcal{D}_{\mathrm{osc}}(t), \qquad \bar{\mathcal{D}}(t) = 1/2^{N_{\mathrm{fast}}(t)}$$

$$N_{\mathrm{fast}}(t) \sim \xi \log t$$

$$\bar{\mathcal{D}}(t) \sim \frac{1}{(1 + t/t_0)^{\alpha}} \qquad \alpha = \xi \ln 2$$

Double Electron-Electron Resonance Ramsey sequence





Summary

Many-Body Localized phases are essentially integrable systems. In 1d they can be conveniently analyzed using real space RG.

Interesting phase diagram for disordered transverse field Ising model with interactions. New Hilbert glass phase. Possibility of temperature tuning of the transition.

MBL states can be probed with Double Electron Electron Resonance type sequence