Exploring dynamics of 1d systems using interference experiments of ultracold atoms
Analogues of CMB anisotropy

Theory:
Ehud Altman, Anatoli Polkovnikov, Vladimir Gritsev,
Adilet Imambekov, Takuya Kitagawa, Kartiek Agarwal,
Emanuele Dalla Torre
Experiment: Joerg Schmiedmayer et al.
Adilet Imambekov, 1981 - 2012
Graduate student at Harvard, 2002-2007

Photo courtesy of Vladimir Gritsev
Adilet at Harvard

Photo courtesy of Dima Abanin
Photo courtesy of Dima Abanin
Spin-exchange interactions of spin-one bosons in optical lattices: Singlet, nematic, and dimerized phases

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(Received 8 June 2003; published 4 December 2003)

\[ \mathcal{H} = -t \sum \langle ij \rangle a_{im}^\dagger a_{jm} + U_0 \sum_i n_i^2 + U_2 \sum_i \vec{S}_i^2 - \mu \sum_i n_i \]

Effective S=1 spin model

\[ \mathcal{H} = -J_1 \sum \langle ij \rangle \vec{S}_i \vec{S}_j - J_2 \sum \langle ij \rangle \left( \vec{S}_i \vec{S}_j \right)^2 \]
Exactly solvable case of a one-dimensional Bose–Fermi mixture

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Applications of exact solution for strongly interacting one-dimensional Bose–Fermi mixture: Low-temperature correlation functions, density profiles, and collective modes

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Interference of independent condensates
Experiments with 2D Bose gas


Experiments with 1D Bose gas

Hofferberth et al., Nat. Physics 2008
Interference experiments with condensates

\[ A \psi(x) = \psi_1(x) + \psi_2(x) \]

\[ \rho_{\text{int}}(x) = \psi_1^*(x) \psi_2(x) + \text{c.c.} \]

\[ \rho_{\text{int}}(x) = e^{i\frac{mdx}{\hbar t}} e^{i(\phi_2 - \phi_1)} + \text{c.c.} \]

Interference of fluctuating condensates

Polkovnikov et al. (2006)

Amplitude of interference fringes

\[ C = \int_{-\frac{l}{2}}^{\frac{l}{2}} dr \ e^{i(\phi_1(r) - \phi_2(r))} \]

\[ C = |C| \ e^{i\theta} \]
DistribuGon funcGon of phase and contrast

Polkovnikov et al. (2006), Gritsev et al. (2006), Imambekov et al. (2007)

$C$ is a quantum operator.

The measured value of $C$ will fluctuate from shot to shot.

Higher moments reflect higher order correlation functions.

Experiments analyze distribution function of $C$.
FDF of phase and contrast

- Matter-wave interferometry
FDF of phase and contrast

• Matter-wave interferometry

• Plot as circular statistics
FDF of phase and contrast

- Matter-wave interferometry: repeat

\[ \text{contrast}_i \]

\[ \text{phase} \]

accumulate statistics
FDF of phase and contrast

• Matter-wave interferometry: repeat

\[ \text{accumulate statistics} \]

\[ \text{phase}_i \quad \text{contrast}_i \]

This is the full distribution function of phase & contrast
Mapping of Coulomb gases and sine-Gordon models to statistics of random surfaces

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\[ S = \frac{K}{2} \int_{\Omega} d^2 x \ (\nabla \phi)^2 + g \int_{\omega} d^2 x \cos \phi \]
Probing quantum and thermal noise in an interacting many-body system

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AND J. SCHMIEDMAYER$^{1,2,*}$

Nature Physics (2008)
Measurements of dynamics of split condensate

- **Initial gas**: $\phi_{\text{initial}}(z)$
- **Relative phase**: $\Delta \phi(z) = \phi_1(z) - \phi_2(z)$
  - Phase correlation length $\lambda_\phi$ → $\infty$
- **Matterwave interference after time-of-flight**: Integrated contrast $C(L)$
- **Many repeats**: Full contrast distribution functions $P(C^2)$

Initial state

$t_e = 0 \text{ ms}$
Theoretical analysis of dephasing Luttinger liquid model
Luttinger liquid model of phase dynamics

Condensate 1  Condensate 2

\[ \hat{\phi}_s(r) = \hat{\phi}_1(r) - \hat{\phi}_2(r) \]
\[ 2\hat{n}_s(r) = \hat{n}_1(r) - \hat{n}_2(r) \]
\[ [\hat{n}_s(r), \hat{\phi}_s(r')] = -i\delta(r - r') \]

For identical average densities, phase difference modes decouple from the phase sum mode

\[ H_s = \frac{c_s}{2} \int \left[ \frac{K_s}{\pi} (\nabla \hat{\phi}_s(r))^2 + \frac{\pi}{K_s} \hat{n}_s^2(r) \right] \, dr \]
Luttinger liquid model of phase dynamics

\[ H_s = \frac{c_s}{2} \int \left[ \frac{K_s}{\pi} (\nabla \phi_s(r))^2 + \frac{\pi}{K_s} \hat{n}_s^2(r) \right] \, dr \]

\[ = \frac{c_s}{2} \sum_k \left[ \frac{K_s k^2}{\pi} \hat{\phi}_s^\dagger(k) \hat{\phi}_s(k) + \frac{\pi}{K_s} \hat{n}_s^\dagger(k) \hat{n}_s(k) \right] \]

For each k-mode we have simple harmonic oscillators

Fast splitting prepares states with small fluctuations of relative phase

Time evolution

\[ \hat{n}_s(k) \]

\[ \phi_s(k) \]

\[ n_s(k) \]

\[ \phi_s(k) \]
Energy distribution

Initially the system is in a squeezed state with large number fluctuations

\[ \langle \phi_s(r_1) \phi_s(r_2) \rangle = \phi_0^2 \delta(r_1 - r_2) \]

Energy stored in each mode initially

\[ E_k = g n_s^2(k, t = 0) = \frac{g}{\phi_0^2} \]

Equipartition of energy

For 2d quasi-condensates pointed out by Mathey, Polkovnikov (2010)

The system should look thermal like after different modes dephase.
Effective temperature is not related to the physical temperature

Analogous to thermalization of Calabrese and Cardy in CFT, PRL (2006)
Prethermalization

Expt: M. Gring et al., Science 2012

\[ k_B T_{\text{eff}} = \frac{g \rho}{2} \]
Exploring Lorenz transformations of temperature in split atomic condensates

K. Agarwl, E. Dalla Torre, EAD, in preparation
How do basic thermodynamic quantities transform under a Lorentz boost?

• For example, temperature $T$ of a moving body, which in its rest frame, appears thermal at temperature $T_0$:
  - Einstein (‘08), Planck (‘08) : $T = T_0 / \gamma$
  - Ott (‘63), Arzieles (‘65), Gamba (‘65), Kibble (‘66) : $T = \gamma T_0$
  - Landsberg (‘66) : $T = T_0$
  - Pathria (‘66) : $T = T_0 / \gamma$ : Planck and Einstein were right!
  - Callen, Horwitz (‘71) : $T = T_0$ is most sensible.
  - Landsberg (‘86) : does not exist
  - Sewell (2008) : does not exist
Ultracool approach to the old problem
The dynamical problem can be analyzed in a Lorentz Boosted frame, where the problem is equivalent to the sudden splitting case.

For such a case, we know that the system thermalizes at a temperature $T_0$.

Making measurements in the lab frame is equiv. to making measurements on a *moving body*, at rest frame temperature $T_0$. 

**T_0 = g \rho \quad t' \rightarrow \infty**
Interpretation: Measurement of Doppler-shifted ripples

equal time correlations are affected by left and right movers equally.
unequal time correlations sample the waves differently. Special cases: \( v_r = \pm c \)
**Interpretation : Measurement of Doppler-shifted ripples**

\[ u_s = \frac{c^2}{v_s} < c \]

\[ T_0 = g \rho \quad t' \to \infty \]

Momента are Doppler shifted in the Lab frame. Note: Right and left movers shifted differently.

\[ T = ? \quad t \to \infty \]

Due to linear dispersion, the energy is also doppler shifted giving rise to doppler shifting of temperature

\[ \eta : \text{Relativistic doppler shift} \]

\[ \eta = \sqrt{\frac{1 + c/v_s}{1 - c/v_s}} = \sqrt{\frac{1 + u_s/c}{1 - u_s/c}} \]
Effective Temperature(s)

\[ T(v_s, \pm \infty) = \frac{T_0}{(\eta + 1/\eta)/2} = \gamma_{v_s} T_0 \quad \text{(equal time)} \]

Equal time correlations measure an average of the temperatures of left and right movers, as shown above. This transformation is in agreement with Ott-Kibble.

Note: \( \gamma_{v_s} = \frac{1}{\sqrt{1 - c^2/v_s^2}} = \frac{1}{\sqrt{1 - u_s^2/c^2}} \)

\( u_s = c^2/v_s \): Lorentz Boost velocity
Experimental signatures of different Effective Temperatures

(a) $\tilde{v}/c = 10$, (b) $\tilde{v}/c = 2$ and (b) $\tilde{v}/c = 1.2$

K=10
Analogy to the Cosmic Microwave Background Radiation

Depending on how you orient your telescopes, you measure, different effective temperatures.
Analogy to the Cosmic Microwave Background Radiation

\[ \eta T_0 \]

\[ \gamma T_0 = (\eta + \frac{1}{\eta}) T_0 / 2 \]

\[ T_0 / \eta \]

The U2 Anisotropy Experiment
http://aether.lbl.gov/www/projects/u2/

\[ \eta \] here, as before, is the relativistic doppler shift. In the context of the CMBR, it gives an estimate of the speed at which our galaxy is currently traveling away from the CMBR rest frame.
Summary: interesting effects due to interplay of interactions, disorder, and temperature in quantum many-body systems

Quantum noise is a powerful tool for studying 1d many-body systems in and out of equilibrium

• Exploring Lorenz transformations of temperature in split atomic condensates

K. Agarwal, E. Dalla Torre, EAD, in preparation
We cheered you on last year
At the top of heart Break Hill.
Not knowing that you so young
Will run away so far too soon,
breaking our hearts

Olga Demler