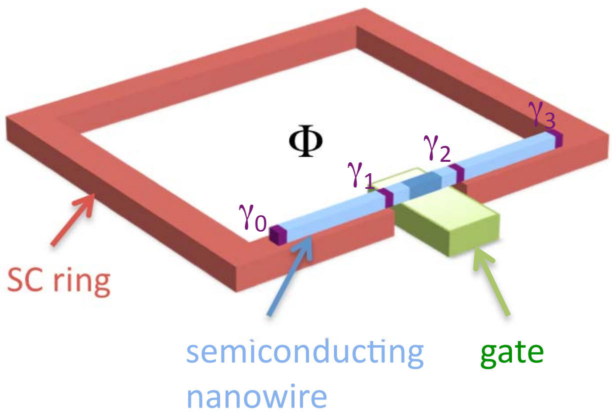
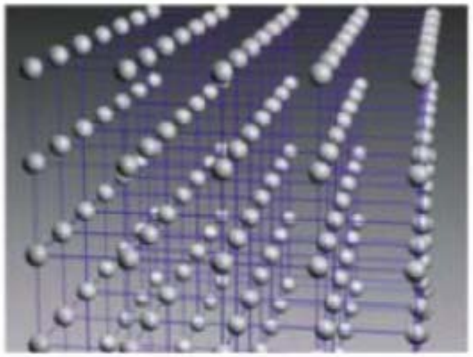
Probing topological states with qubits





Solid State: D. Pekker, C.-Y. Hou, V. Manucharyan, E. Demler, arXiv:1301.3161

Cold Atoms: M. Atala, M. Aidelsburger, J. Barreiro, I. Bloch, D. Abanin, T. Kitagawa, E. Demler, arXIv:1212.0572

Harvard-MIT CUA

$$ NSF, MURI QUISM, AFOSR MURI, DARPA OLE, MURI ATOMTRONICS

Outline

Probing Majorana fermions wth fluxonium qubits in topologcal superconductors

D. Pekker, C.-Y. Hou, V. Manucharyan, E. Demler, arXiv:1301.3161

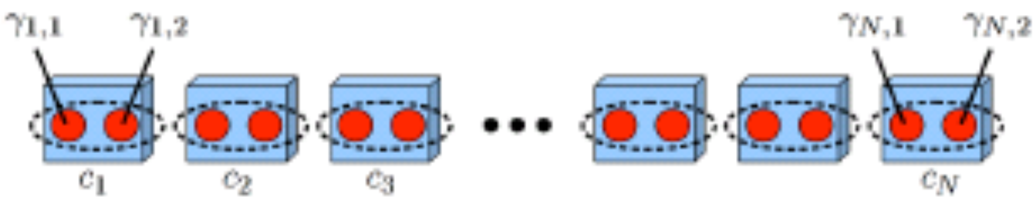
Probing band topology with cold atoms qubits

Theory: D. Abanin, T. Kitagawa , I. Bloch, E. Demler,
Experiments: M. Atala, M. Aidelsburger, J. Barreiro, I. Bloch,
arXIv:1212.0572

Probing Majorana fermions with fluxonium qubits in topologcal superconductors

Kitaev Model and Majorana fermions

$$\mathcal{H}_{\text{chain}} = -\mu \sum_{i=1}^{N} n_i - \sum_{i=1}^{N-1} \left(t c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + h.c. \right)$$



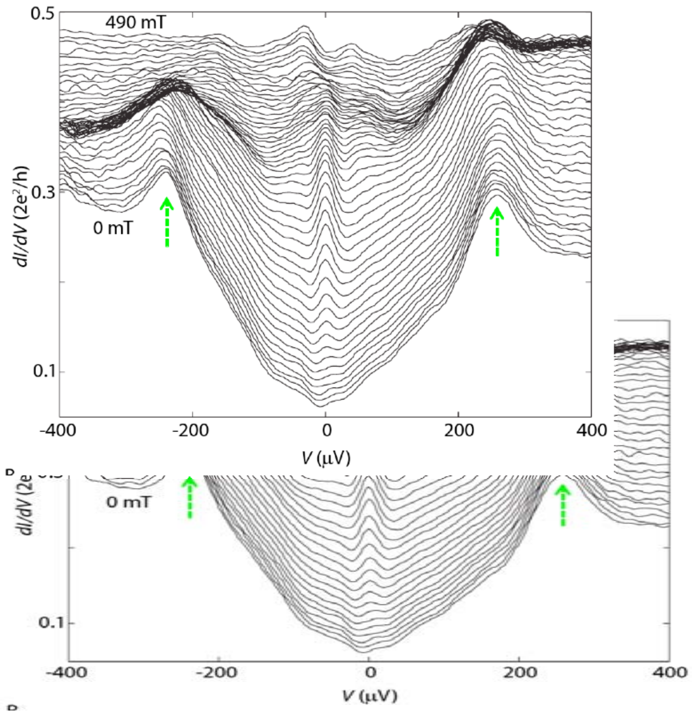
$$\gamma_{i,1} = c_i^\dagger + c_i \quad \gamma_{i,2} = i\left(c_i^\dagger - c_i\right)$$

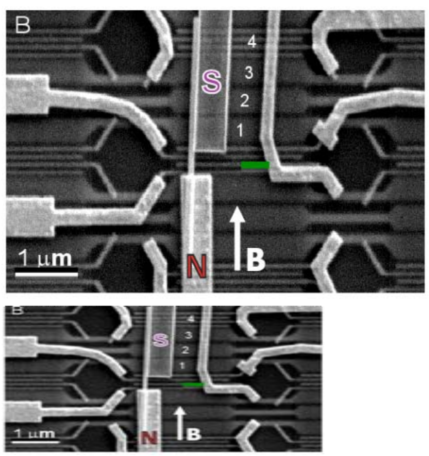
Free Majorana states at the ends of the wire

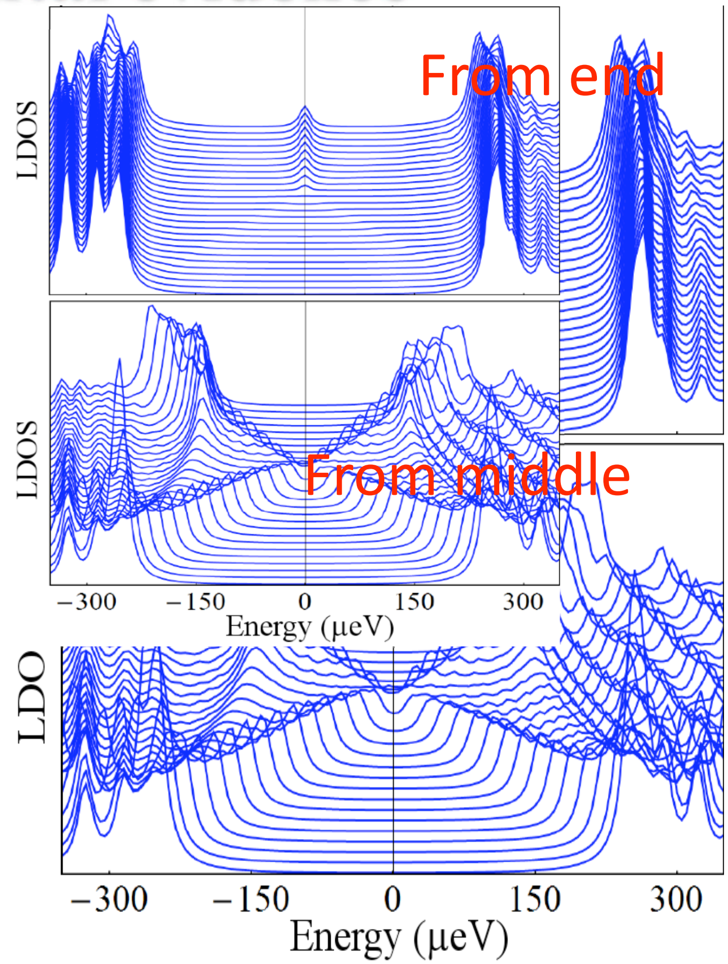
One Dirac fermion localized on the opposite ends of the wire $\tilde{c}_M = (\gamma_{N,2} + i\gamma_{1,1})/2$

Degeneracy of states with even and odd number of fermions

Experimental evidence







Stanescu, Tewari, Sau, Das Sarma arxiv (2012)

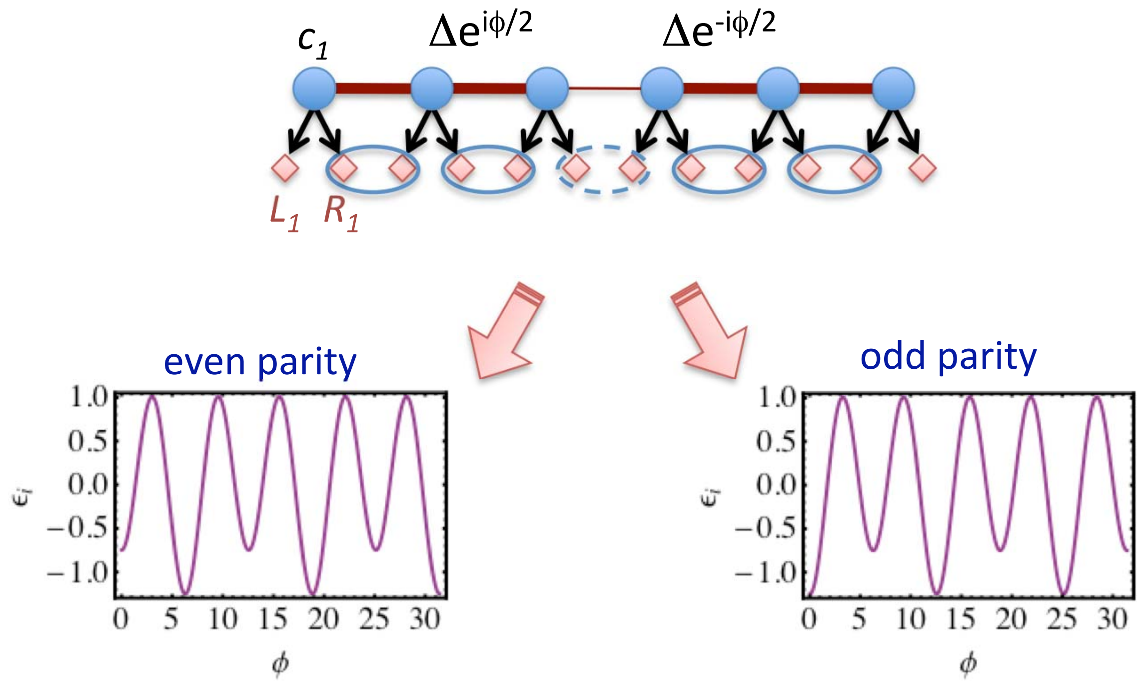
Mourik, Zuo, Frolov, Plissard, Bakkers, Kouwenhoven Science(2012);
See also Heiblum, Marcus group and H. Q. Xu group

Topological wires

Kitaev Model:

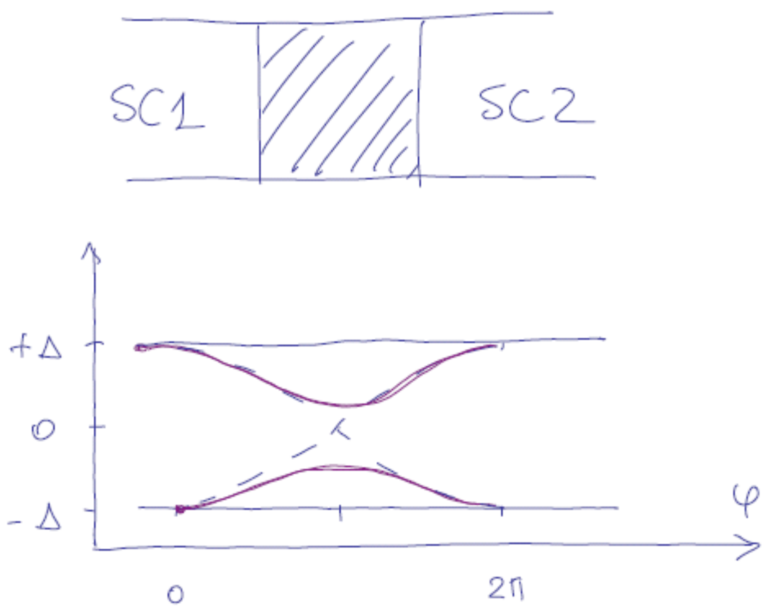
$$H = \sum_i \left(t_i c_i^\dagger c_{i+1} + \Delta_i c_i c_{i+1} + h.c. \right)$$

Weak link fermion



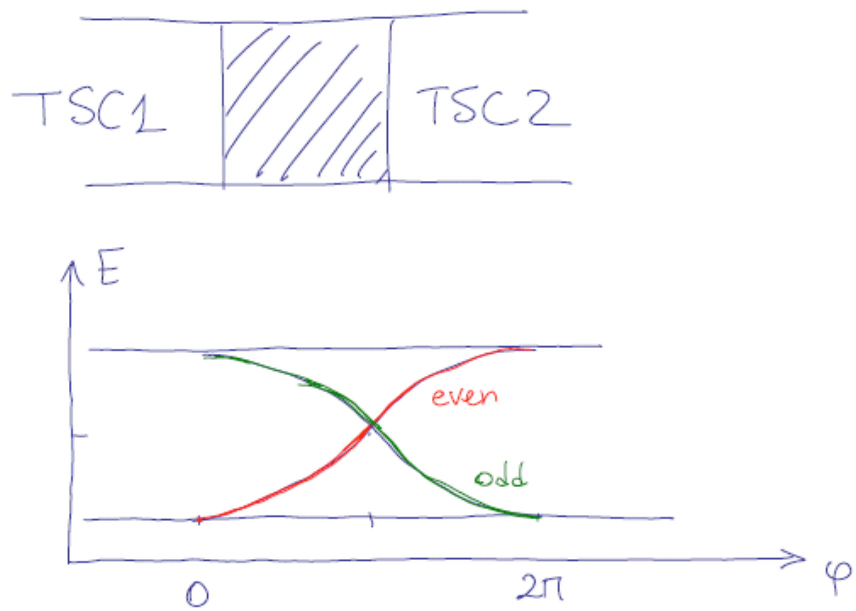
Andreev-bound states and the Fractional ac Josephson effect

Conventional superconductor



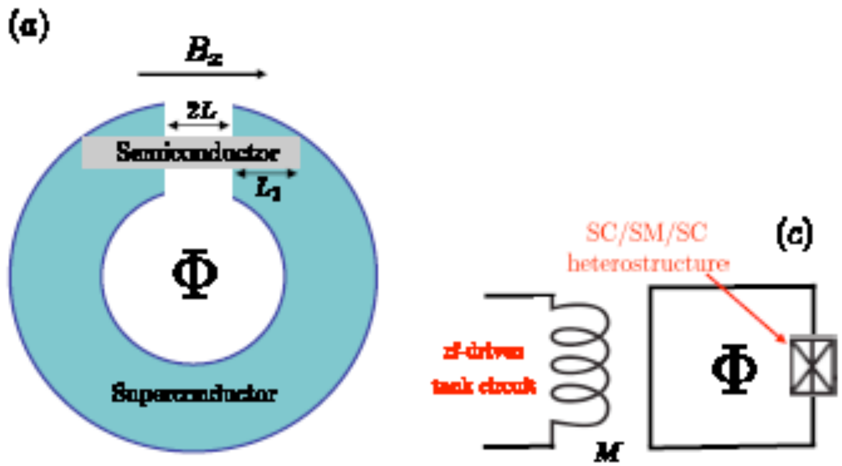
$$E(\varphi) = -E_J \cos\varphi$$

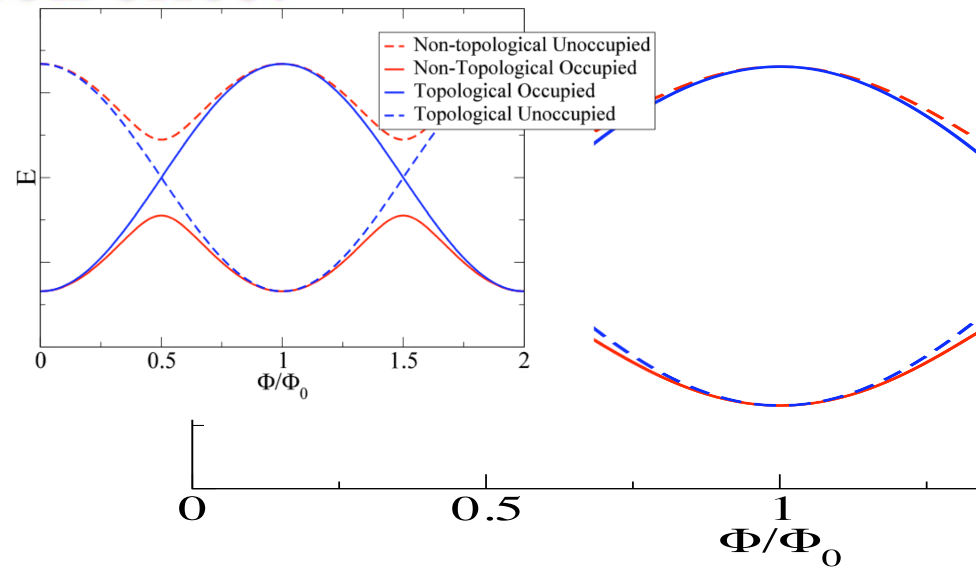
Topologcal superconductor



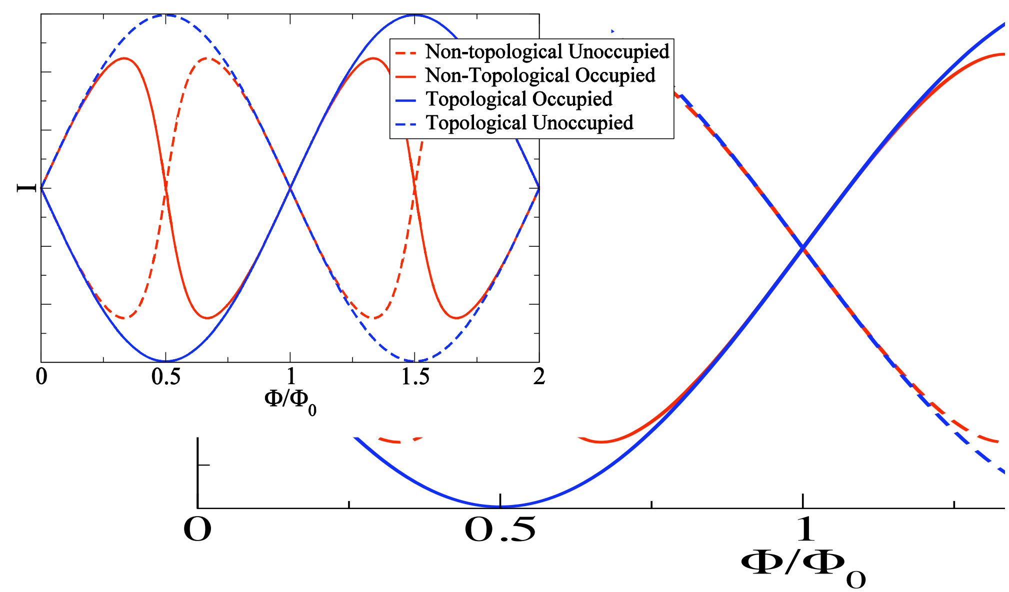
$$E(\varphi) = -E \cos\left(\frac{\varphi}{2}\right)$$

Andreev-bound states and the Fractional ac Josephson effect





Fermion-parity protected crossing of ABSs leads to fractional Josephson effect

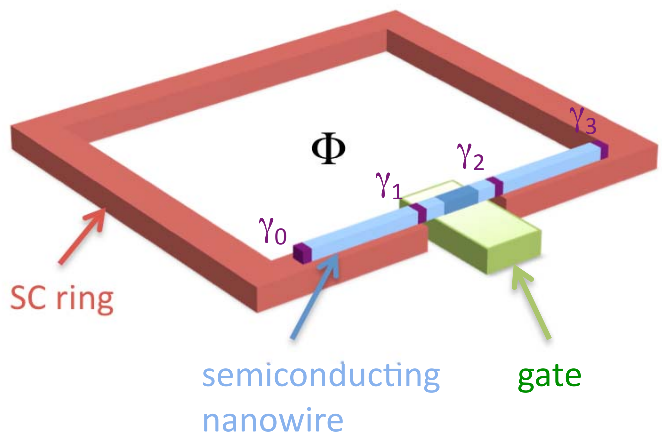


- Experiments see evidence for fractional ac Josephson (Shapiro steps) effect only in high B-field

(*Rokhinson et al Nat Phys (2012)*)

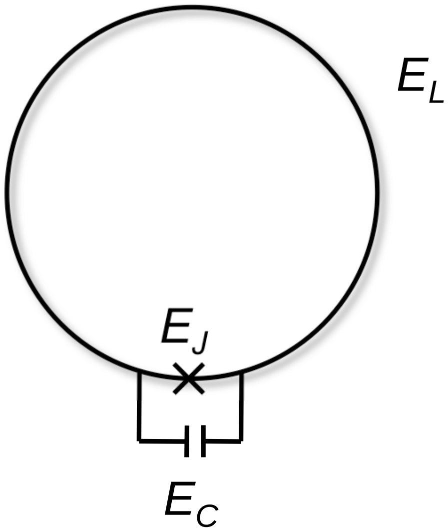
Proposal for coherent coupling of Majorana and fluxonium qubit

D. Pekker, C.-Y. Hou, V. Manucharyan, E. Demler arXiv:1301.3161

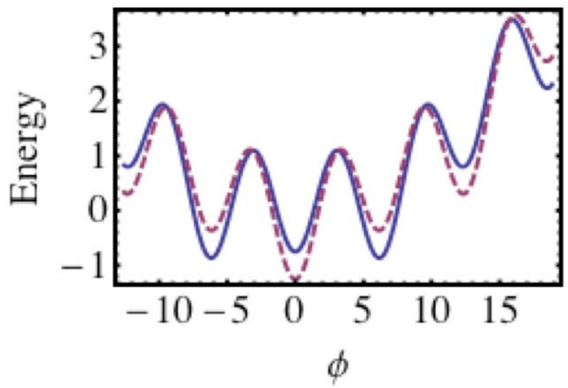


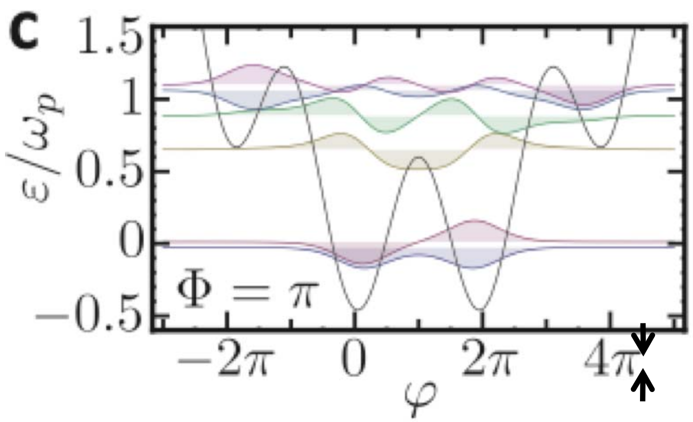
Fluxonium qubit

Superconducting ring interrupted by a capacitance shunted Josephson junction



(1) Large inductance – multiple minima
(small EL compared to EJ)

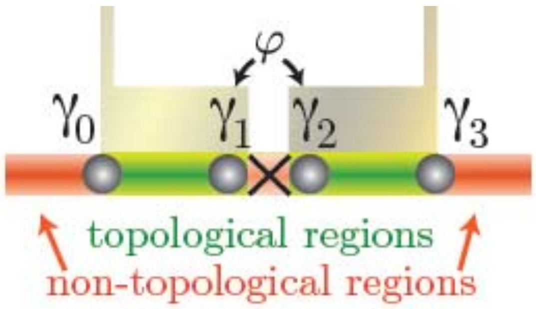




(2) Light Mass (Capacitance) – phase slips
splitting of degeneracy at π

$$H_F(\varphi, \Phi) = -4E_C\,\partial_\varphi^2 + \frac{1}{2}E_L(\varphi - \Phi)^2 - E_J\cos\varphi$$

Topological qubit

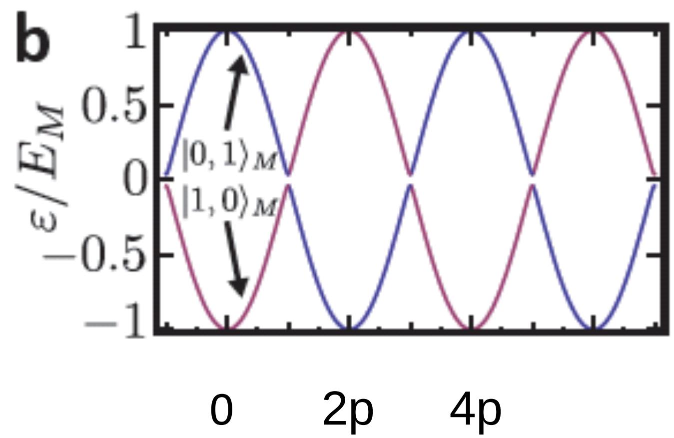


$$H_{M,c} = E_M(2c_w^\dagger c_w - 1)\cos(\varphi/2) - 2\left[(g_{01} + g_{23})c_w^\dagger c_e + (g_{01} - g_{23})c_w^\dagger c_e^\dagger + \text{h.c.}\right]$$

$$c_w = \gamma_1 + i\gamma_2$$

$$c_e = \gamma_3 + i\gamma_0$$

$$|n_w, n_e\rangle_M$$



$$H_M = g_{01} i\gamma_0\gamma_1 + E_M i\gamma_1\gamma_2 \cos(\varphi/2 + \Theta_M) + g_{23} i\gamma_2\gamma_3$$

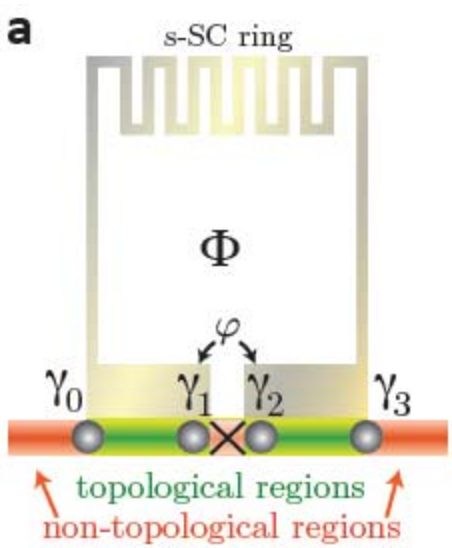
Topological qubit

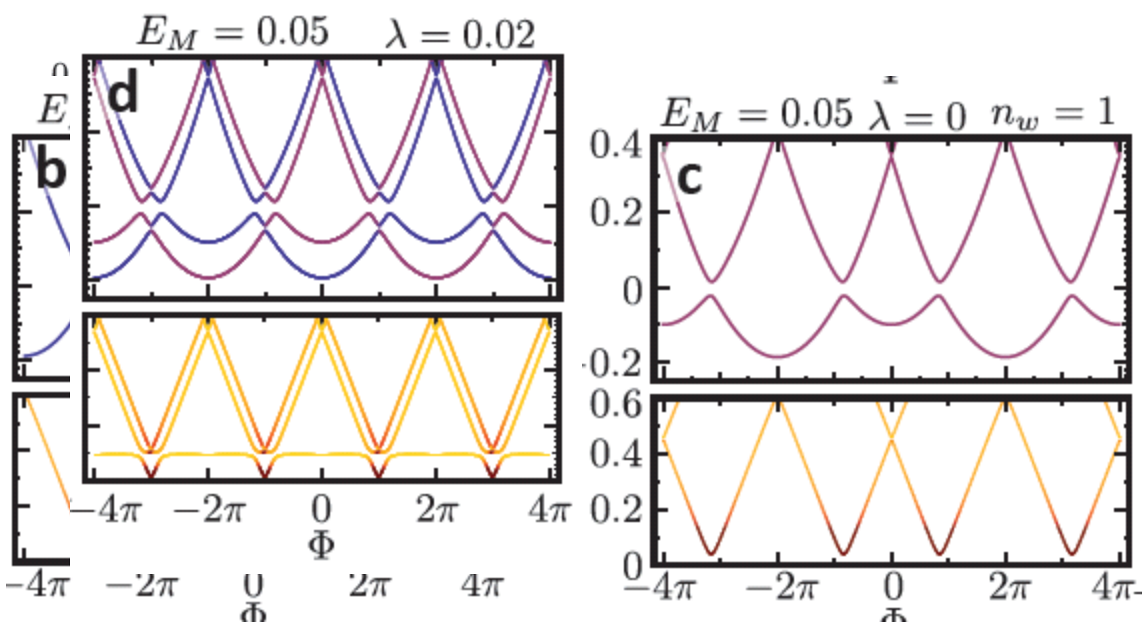
$$|0, 1\rangle_M, |1, 0\rangle_M$$

Coupling Majorana qubit & Fluxonium qubit

$$H_F(\varphi, \Phi) = -4E_C\,\partial_\varphi^2 + \frac{1}{2}E_L(\varphi - \Phi)^2 - E_J \cos\varphi,$$

$$H_{M-F} = H_F(\varphi, \Phi)\mathbb{1} - E_M \cos(\varphi/2)\sigma_z + \lambda\sigma_x.$$





$$|n_w, n_e\rangle_M$$

$$|\uparrow\rangle = |1,0\rangle_M$$

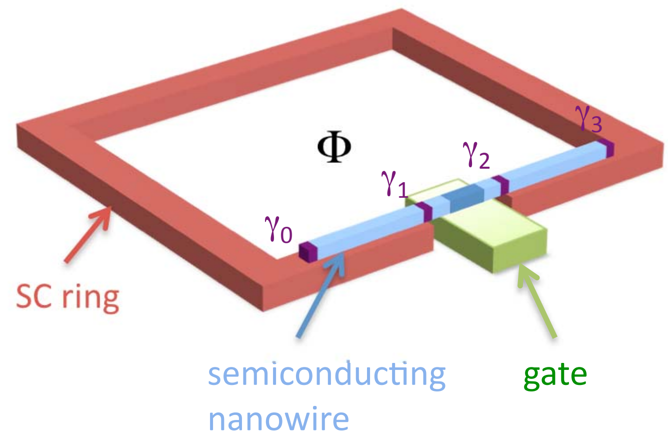
$$|\downarrow\rangle = |0,1\rangle_M$$

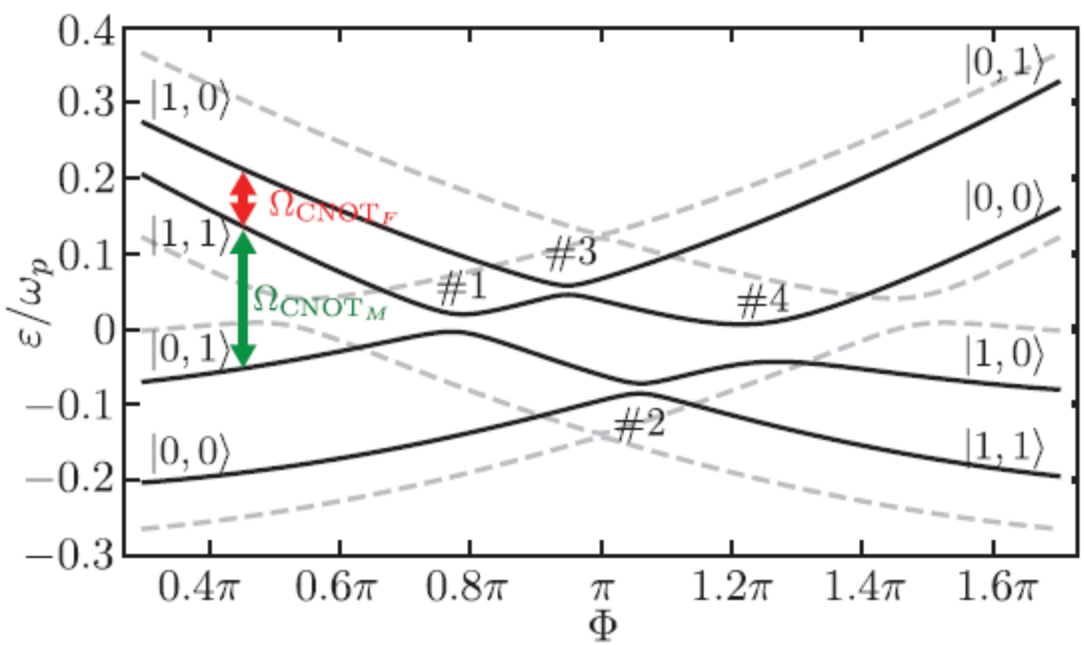
Can detect very small 4π Josephson currents quickly (before poisoning)

Coupling Majorana qubit & Fluxonium qubit

Combined basis $|n_\varphi, n_w\rangle$

$$H^{\text{eff}}_{M-F} = \frac{E_L}{2}(2\pi n_\varphi - \Phi)^2 + 4E_M(-1)^{n_\varphi}\sigma_z - \frac{1}{2}E_S(T^+_{n_\varphi} + T^-_{n_\varphi}) + \lambda\sigma_x$$





SWAP = $\text{CNOT}_F\ \text{CNOT}_M\ \text{CNOT}_F$

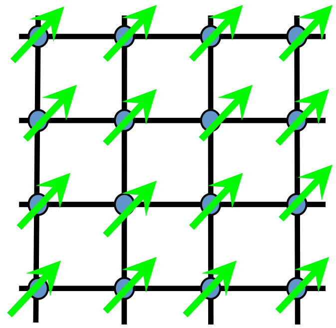
Implement Quantum Memory

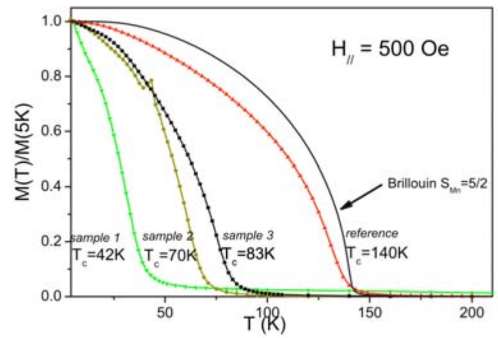
(1) SWAP data into Majorana qubit
(2) Decouple via gate under junction
(3) Store information
(4) Readout in reverse order

Probing band topology with cold atoms qubits

Order parameters

Magnetization - order parameter in ferromagnets



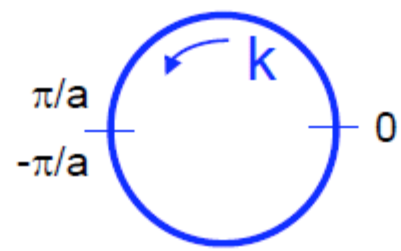


How to measure topological order parameter?

Berry/Zak phase in 1d

$$\boxed{P = \frac{e}{\pi} \oint A(k)\, dk}$$

$$A(k) = \sum_n \langle\, u_n(k)\, |\, \partial_k \,|\, u_n(k)\,\rangle$$

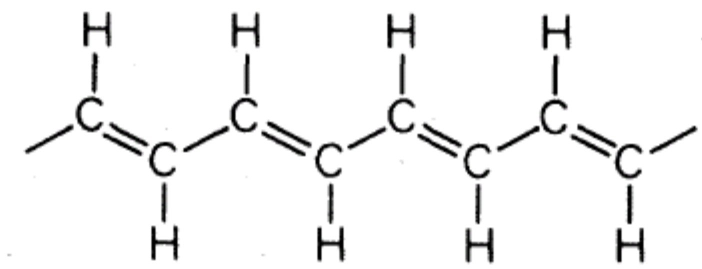


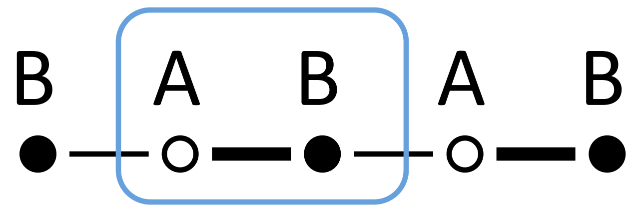
Measure the Berry/Zak phase itself, not its consequence

$$P = \frac{\text{dipole moment}}{\text{length}}$$

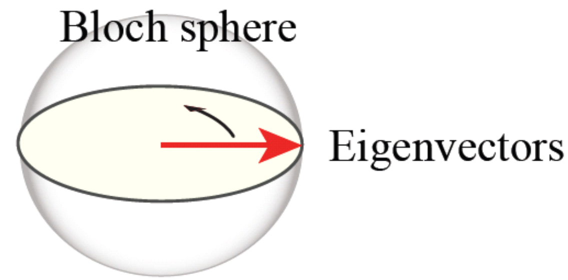
Vanderbilt, King-Smith
PRB 1993

Su-Schrieffer-Heeger Model





$$H=\sum_i (t+\delta t)c^\dagger_{Ai}c_{Bi}+(t-\delta t)c^\dagger_{Ai+1}c_{Bi}+h.c.$$

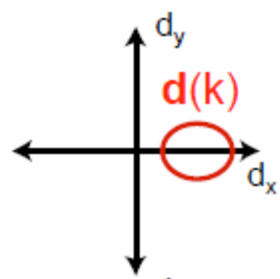


$$H(k)=\mathbf{d}(k)\cdot\vec{\sigma}$$

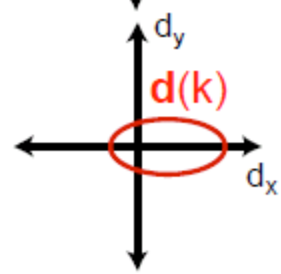
$$d_x(k)=(t+\delta t)+(t-\delta t)\cos ka$$

$$d_y(k)=(t-\delta t)\sin ka$$

$$d_z(k)=0$$



$\delta t>0$: Berry phase 0

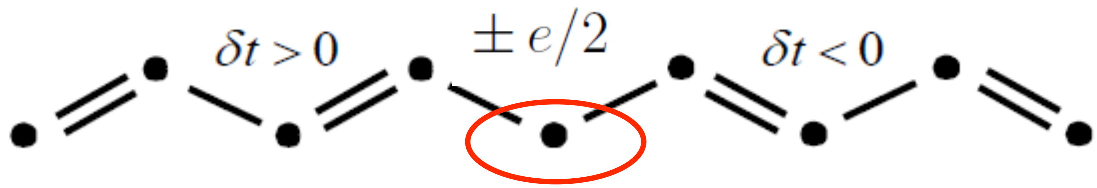


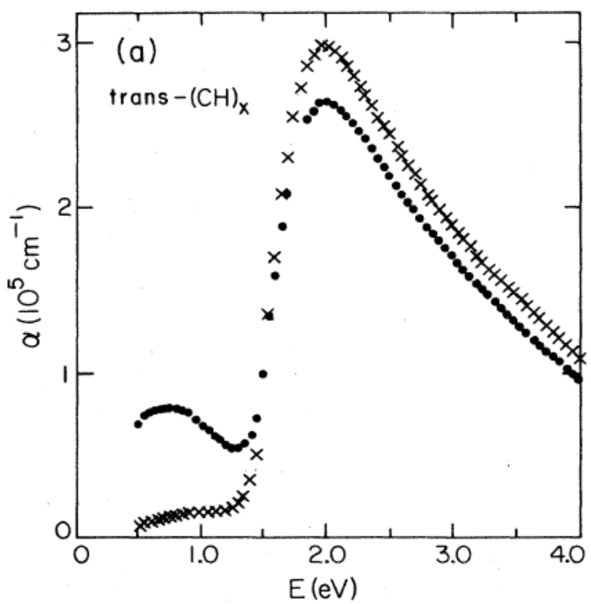
$\delta t<0$: Berry phase π

When $d_z(k)=0$, states with dt>0 and dt<0 are topologically distinct.
We can not deform two paths into each other without closing the gap.

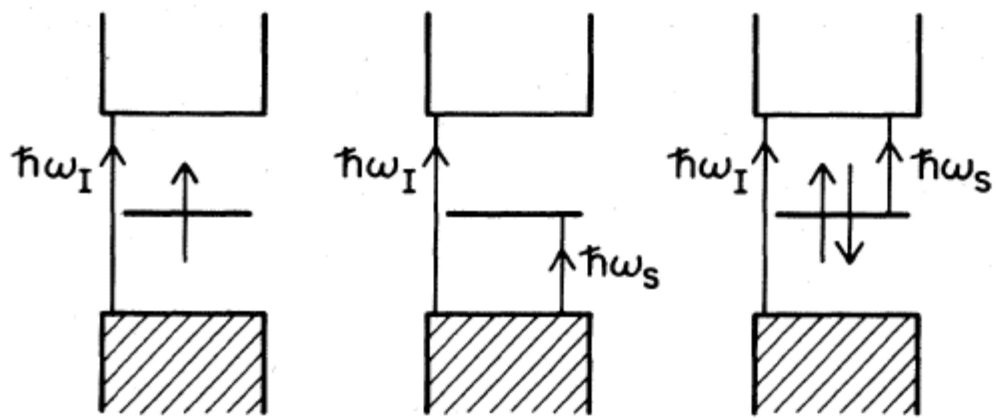
Domain wall states in SSH Model

An interface between topologically different states has protected midgap states



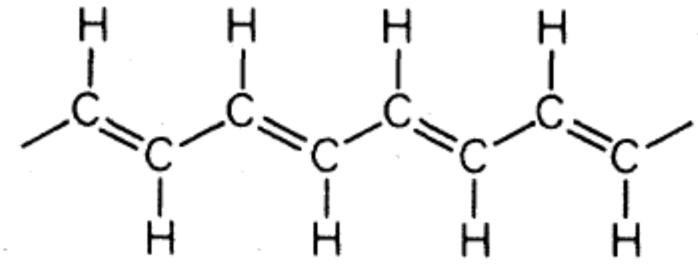


Absorption spectra on neutral and doped trans-$(CH)_x$

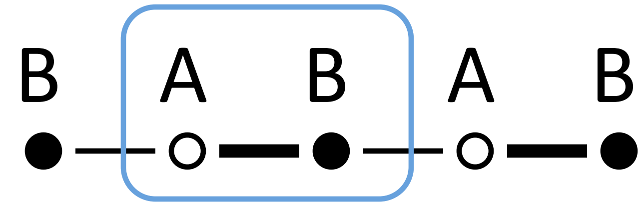


Probing band topology with Ramsey/Bloch interference

SSH model in bichromatic lattices

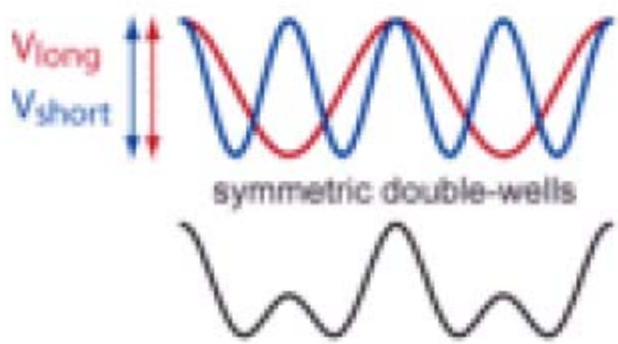


Su, Schrieffer, Heeger, 1979



$$H = \sum_i (t+\delta t) c_{Ai}^{\dagger} c_{Bi} + (t-\delta t) c_{Ai+1}^{\dagger} c_{Bi} + h.c.$$

Analogous to bichromatic optical lattice potential

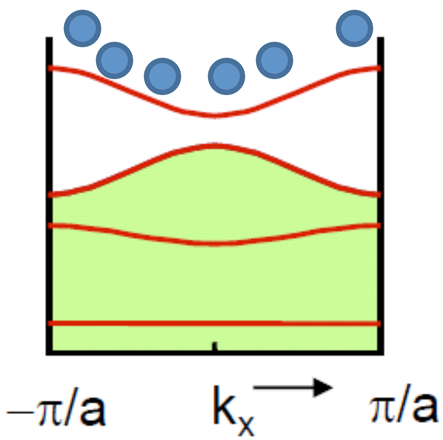


I. Bloch et al.,
LMU/MPQ

Tools of atomic physics: Bloch oscillations

C. Salomon et al., PRL (1996)

$$\boxed{\frac{dk}{dt} = F}$$



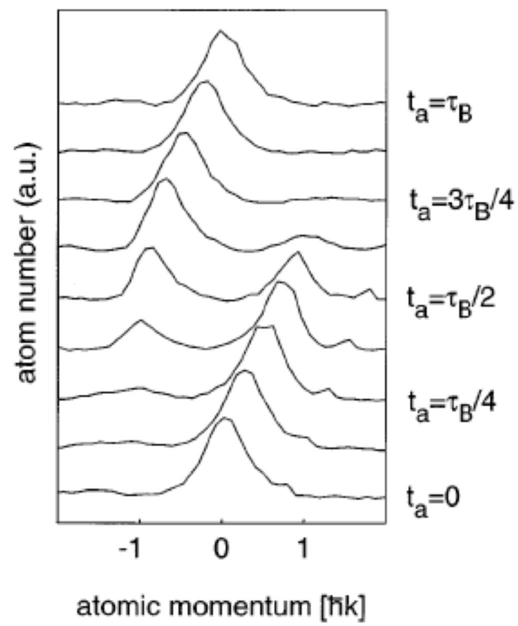


FIG. 2. Bloch oscillations of atoms: momentum distributions in the accelerated frame for equidistant values of the acceleration time t_a between $t_a = 0$ and $t_a = \tau_B = 8.2$ ms. The light potential depth is $U_0 = 2.3E_R$ and the acceleration is $a = -0.85$ m/s^2. The small peak in the right wing of the first five spectra is an artifact.

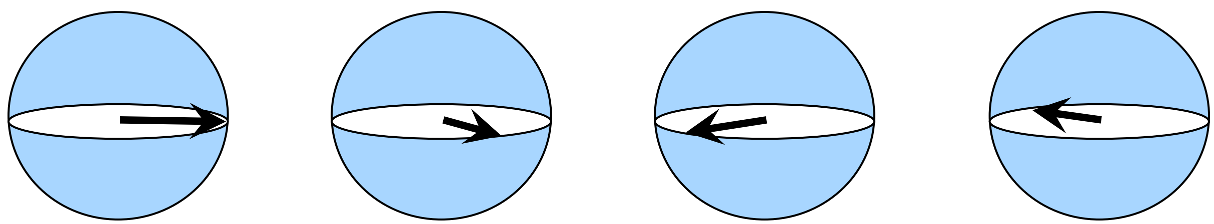
Tools of atomic physics: Ramsey interference

p/2 pulse

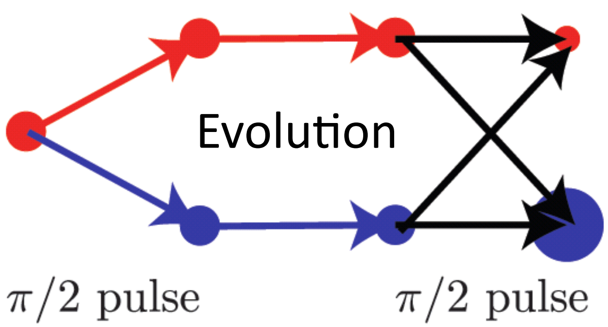
$$|\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}|\downarrow\rangle + \frac{1}{\sqrt{2}}|\uparrow\rangle$$

Evolution

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\mathcal{H}_{\downarrow}t}|\downarrow\rangle + \frac{1}{\sqrt{2}} e^{-i\mathcal{H}_{\downarrow}t}|\uparrow\rangle$$



p/2 pulse + measurement ot S_z gives relative phase accumulated by the two spin components



Used for atomic clocks, gravitometers, accelerometers, magnetic field measurements

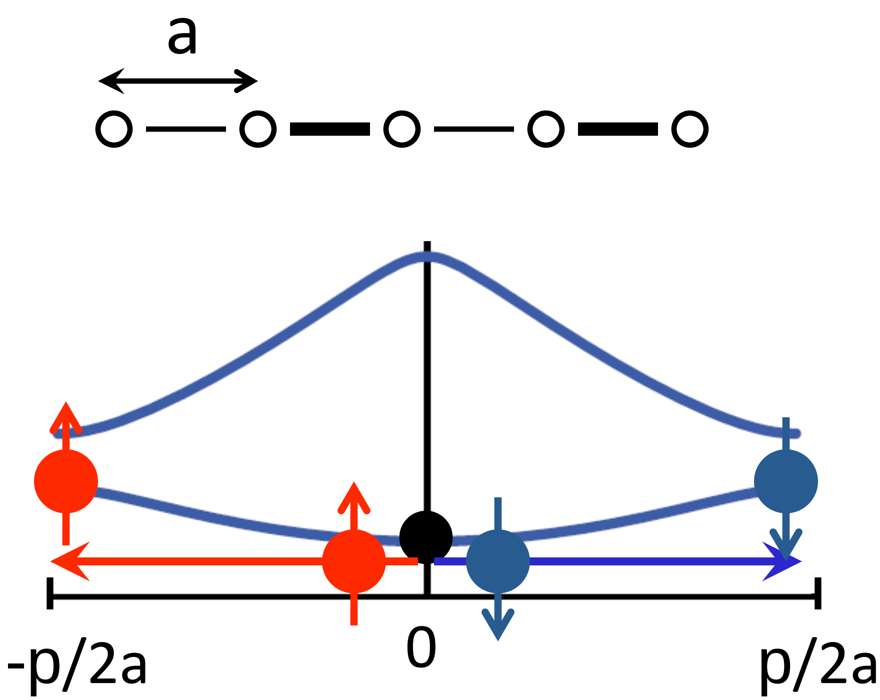
Measurements of Zak/Berry phase in one dimensional Bloch band

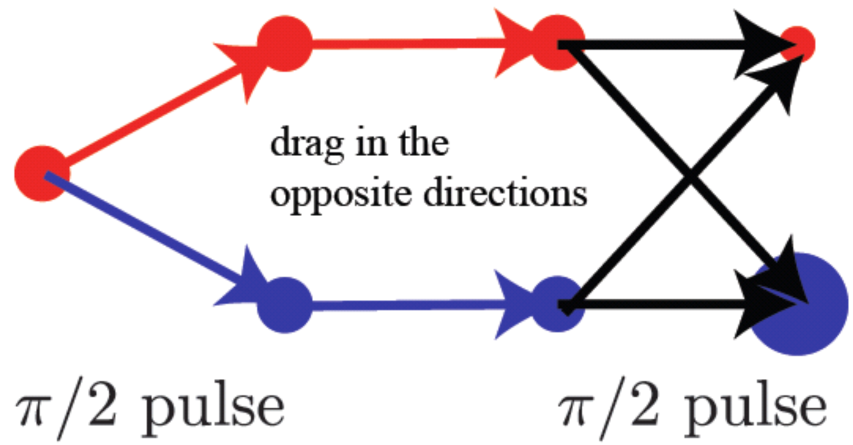
One dimensional superlattices
Su-Schrieffer-Heeger model

M. Atala et al., arXIv:1212.0572

Characterizing SSH model using Zak phase

Two hyperfine spin states experience the same optical potential





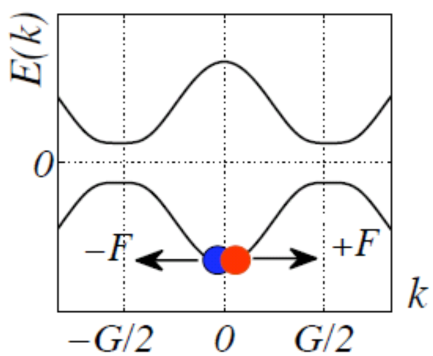
Zak phase is equal to p

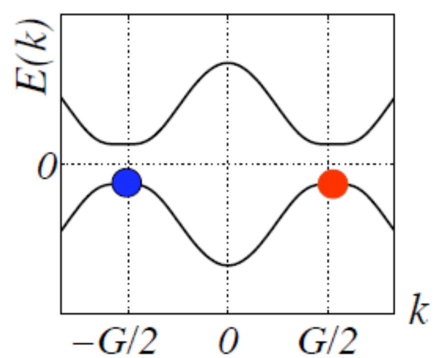
$$\varphi_{\mathrm{tot}} = \varphi_{\mathrm{Zak}} + \varphi_{\mathrm{dyn}} + \varphi_{\mathrm{Zeeman}}$$

$$\frac{1}{i}\int_{-\pi}^{\pi} dk \, \langle \psi_k \, | \, \partial_k \, | \, \psi_k \rangle \; = \; \pi$$

Problem: experimentally difficult to control Zeeman phase shift

Spin echo protocol for measuring Zak phase



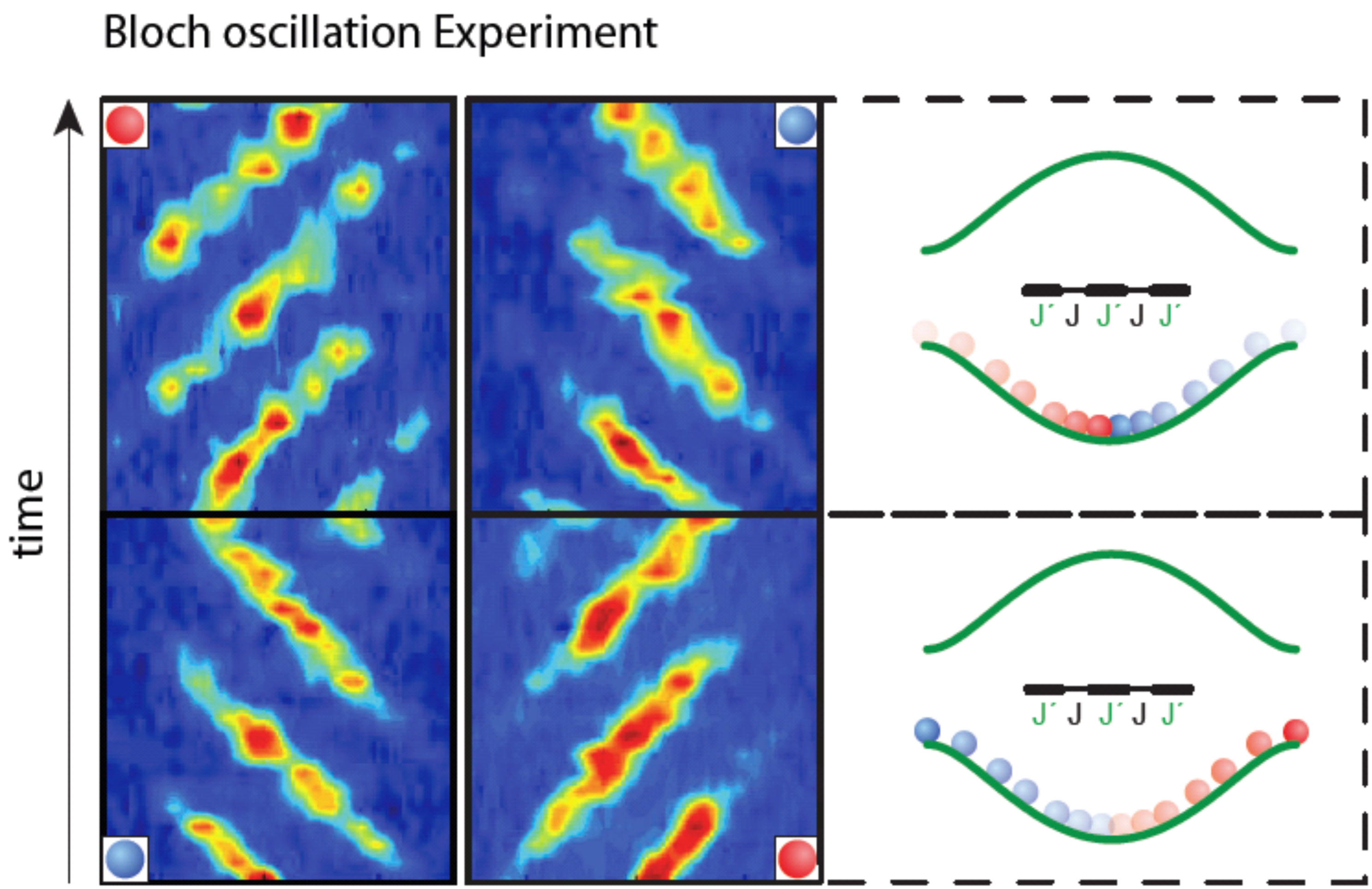


Dynamic phases due to dispersion and magnetic field fluctuations cancel. Interference measures the difference of Zak phases of the two bands in two dimerizations.

Expect phase p

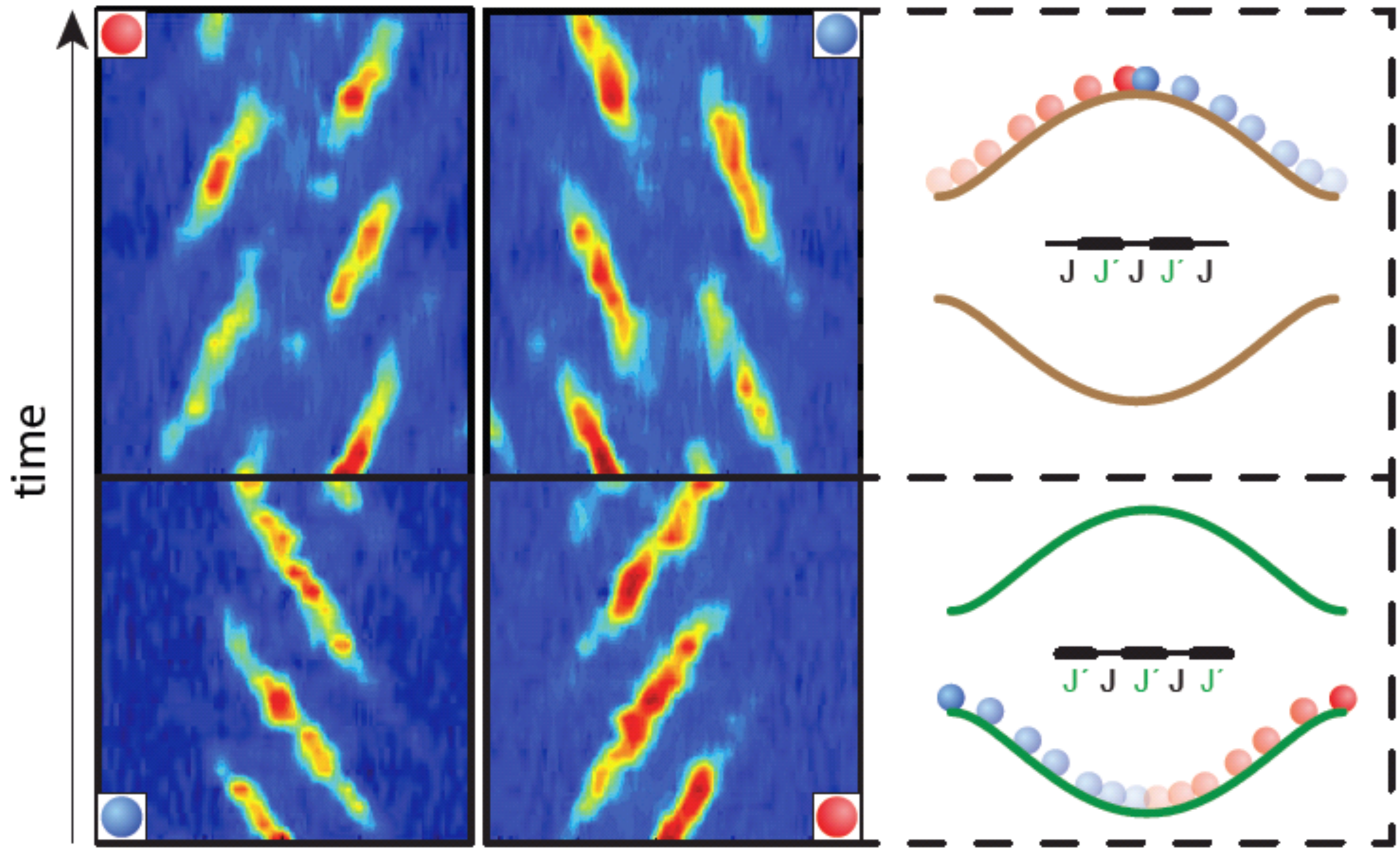
Bloch oscillations measurements

With p-pulse but no swapping of dimerization

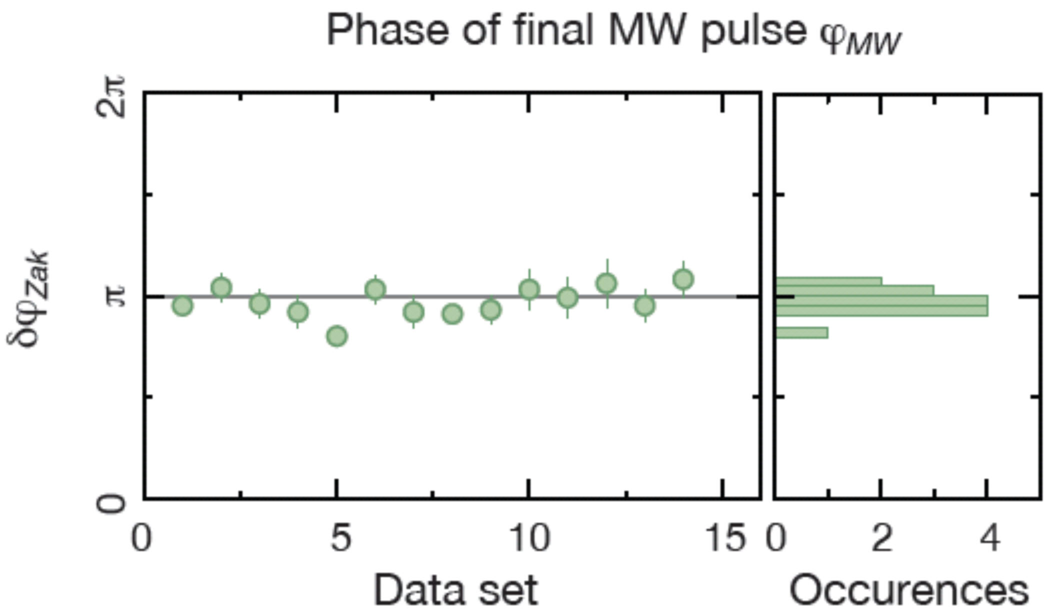


Bloch oscillations measurements

With p-pulse and with swapping of dimerization

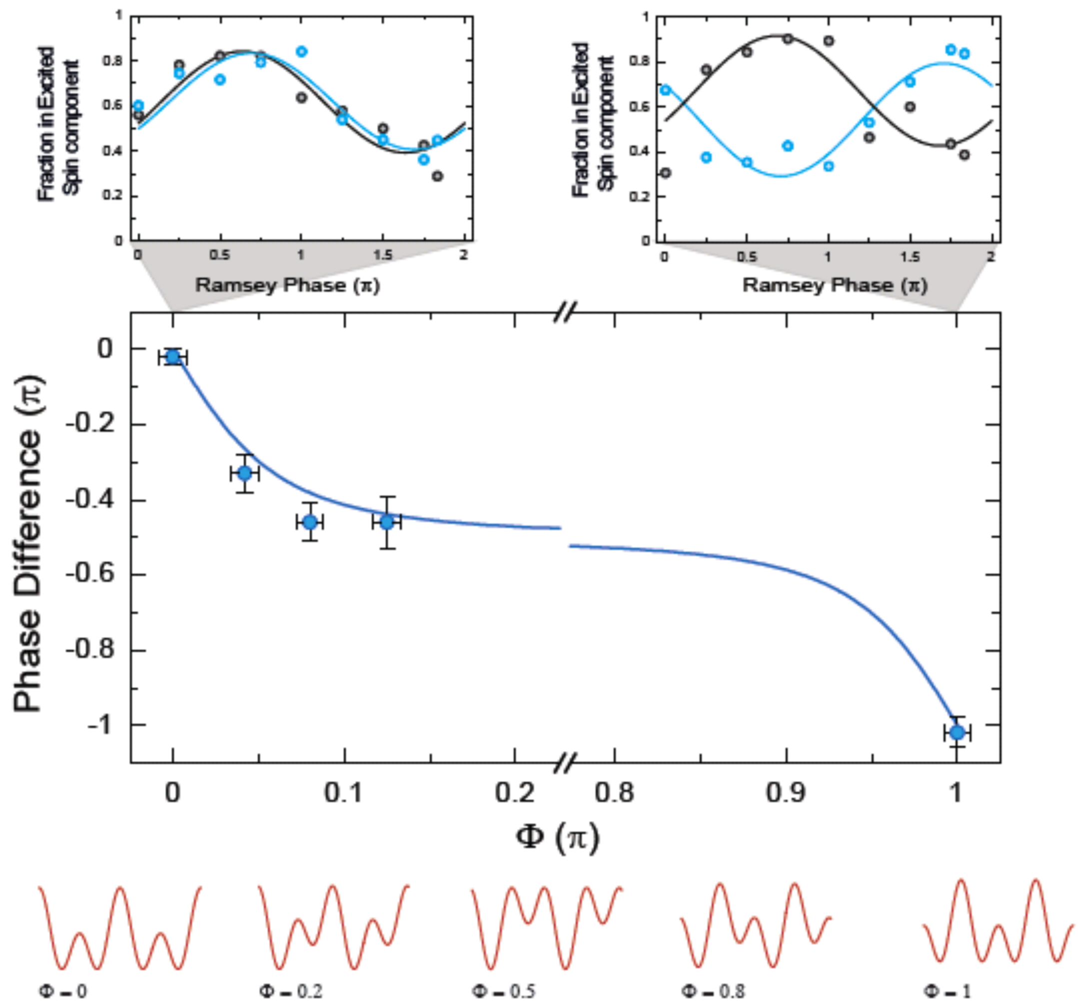


Zak/Berry phase measurements



$$\delta\varphi = 0.97(2)\pi$$

Zak/Berry phase measurements extended



Summary

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