Probing topological states with qubits


Harvard-MIT

$$ NSF, MURI QUISM, AFOSR MURI, DARPA OLE, MURI ATOMTRONICS $$
Outline

Probing Majorana fermions with fluxonium qubits in topological superconductors


Probing band topology with cold atoms qubits

Theory: D. Abanin, T. Kitagawa, I. Bloch, E. Demler,
Experiments: M. Atala, M. Aidelsburger, J. Barreiro, I. Bloch,
arXiv:1212.0572
Probing Majorana fermions with fluxonium qubits in topological superconductors
Kitaev Model and Majorana fermions

\[ H_{\text{chain}} = -\mu \sum_{i=1}^{N} n_i - \sum_{i=1}^{N-1} \left( t c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + h.c. \right) \]

Free Majorana states at the ends of the wire

\[ \gamma_{i,1} = c_i^\dagger + c_i \quad \gamma_{i,2} = i \left( c_i^\dagger - c_i \right) \]

One Dirac fermion localized on the opposite ends of the wire

\[ \tilde{c}_M = (\gamma_{N,2} + i \gamma_{1,1})/2 \]

Degeneracy of states with even and odd number of fermions
Experimental evidence

Stanescu, Tewari, Sau, Das Sarma arxiv (2012)

Mourik, Zuo, Frolov, Plissard, Bakkers, Kouwenhoven Science(2012);
See also Heiblum, Marcus group and H. Q. Xu group
Topological wires

Kitaev Model:

\[ H = \sum_i \left( t_i c_i^\dagger c_{i+1} + \Delta_i c_i c_{i+1} + h.c. \right) \]

Weak link fermion
Andreev-bound states and the Fractional ac Josephson effect

Conventional superconductor

\[ E(\varphi) = - E_\text{J} \cos \varphi \]

Topological superconductor

\[ E(\varphi) = - E \cos \left( \frac{\varphi}{2} \right) \]
Andreev-bound states and the Fractional ac Josephson effect

Fermion-parity protected crossing of ABSs leads to fractional Josephson effect

- Experiments see evidence for fractional ac Josephson (Shapiro steps) effect only in high B-field
  
  (Rokhinson et al Nat Phys (2012))
Proposal for coherent coupling of Majorana and fluxonium qubit

Fluxonium qubit

Superconducting ring interrupted by a capacitance shunted Josephson junction

1) Large inductance – multiple minima (small $E_L$ compared to $E_J$)

2) Light Mass (Capacitance) – phase slips splitting of degeneracy at $\pi$

$$H_F(\varphi, \Phi) = -4E_C \frac{\partial^2}{\partial \varphi^2} + \frac{1}{2} E_L (\varphi - \Phi)^2 - E_J \cos \varphi$$
Topological qubit

\[ c_w = \gamma_1 + i\gamma_2 \]
\[ c_e = \gamma_3 + i\gamma_0 \]
\[ |n_w, n_e\rangle_M \]

Topological qubit

\[ |0, 1\rangle_M, |1, 0\rangle_M \]

\[ H_{M,c} = E_M(2c_w^\dagger c_w - 1)\cos(\varphi/2) \]
\[ - 2 \left[ (g_{01} + g_{23})c_w^\dagger c_e + (g_{01} - g_{23})c_w^\dagger c_e^\dagger + \text{h.c.} \right] \]

\[ H_M = g_{01}i\gamma_0\gamma_1 + E_M i\gamma_1\gamma_2 \cos(\varphi/2 + \Theta_M) + g_{23}i\gamma_2\gamma_3 \]
Coupling Majorana qubit & Fluxonium qubit

\[ H_F(\varphi, \Phi) = -4E_C \partial_\varphi^2 + \frac{1}{2}E_L(\varphi - \Phi)^2 - E_J \cos \varphi, \]

\[ H_{MF} = H_F(\varphi, \Phi) \mathbb{1} - E_M \cos(\varphi/2)\sigma_z + \lambda \sigma_x. \]

\[
\begin{align*}
|\uparrow\rangle &= |1, 0\rangle_M, \\
|\downarrow\rangle &= |0, 1\rangle_M
\end{align*}
\]

Can detect very small $4\pi$ Josephson currents quickly (before poisoning)
Coupling Majorana qubit & Fluxonium qubit

Combined basis $|n_\varphi, n_w\rangle$

$$H_{M-F}^{\text{eff}} = \frac{E_L}{2} (2\pi n_\varphi - \Phi)^2 + 4E_M(-1)^{n_\varphi} \sigma_z$$

$$\hspace{3cm} - \frac{1}{2} E_S (T_{n_\varphi}^+ + T_{n_\varphi}^-) + \lambda \sigma_x$$

SWAP = CNOT_F CNOT_M CNOT_F

Implement Quantum Memory

(1) SWAP data into Majorana qubit
(2) Decouple via gate under junction
(3) Store information
(4) Readout in reverse order
Probing band topology with cold atoms qubits
Order parameters

Magnetization - order parameter in ferromagnets

How to measure topological order parameter?

Berry/Zak phase in 1d

\[ P = \frac{e}{\pi} \int A(k) \, dk \]

\[ A(k) = \sum_n \langle u_n(k) | \partial_k | u_n(k) \rangle \]

Measure the Berry/Zak phase itself, not its consequence

\[ P = \frac{\text{dipole moment}}{\text{length}} \]

Vanderbilt, King-Smith
PRB 1993
**Su-Schrieffer-Heeger Model**

When $\delta t(k) = 0$, states with $t > 0$ and $t < 0$ are topologically distinct. We cannot deform two paths into each other without closing the gap.

$$H = \sum_i (t + \delta t) c_{Ai}^\dagger c_{Bi} + (t - \delta t) c_{Ai+1}^\dagger c_{Bi} + h.c.$$  

$$H(k) = \mathbf{d}(k) \cdot \hat{\mathbf{\sigma}}$$

$$d_x(k) = (t + \delta t) + (t - \delta t) \cos ka$$

$$d_y(k) = (t - \delta t) \sin ka$$

$$d_z(k) = 0$$

When $d_z(k) = 0$, states with $\delta t > 0$ and $\delta t < 0$ are topologically distinct. We cannot deform two paths into each other without closing the gap.
Domain wall states in SSH Model

An interface between topologically different states has protected midgap states

\[ \delta t > 0 \quad \pm e/2 \quad \delta t < 0 \]

Absorption spectra on neutral and doped trans-(CH)_x
Probing band topology with Ramsey/Bloch interference
SSH model in bichromatic lattices

Su, Schrieffer, Heeger, 1979

\[ H = \sum_i (t + \delta t)c_{Ai}^\dagger c_{Bi} + (t - \delta t)c_{Ai+1}^\dagger c_{Bi} + h.c. \]

Analogous to bichromatic optical lattice potential

I. Bloch et al., LMU/MPQ
Tools of atomic physics: Bloch oscillations

\[ \frac{dk}{dt} = F \]

C. Salomon et al., PRL (1996)

FIG. 2. Bloch oscillations of atoms: momentum distributions in the accelerated frame for equidistant values of the acceleration time \( t_a \) between \( t_a = 0 \) and \( t_a = \tau_B = 8.2 \) ms. The light potential depth is \( U_0 = 2.3E_R \) and the acceleration is \( a = -0.85 \) m/s\(^2\). The small peak in the right wing of the first five spectra is an artifact.
Tools of atomic physics: Ramsey interference

**p/2 pulse**

\[ |\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}} |\downarrow\rangle + \frac{1}{\sqrt{2}} |\uparrow\rangle \]

**Evolution**

\[ |\Psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-iH_{\downarrow}t} |\downarrow\rangle + \frac{1}{\sqrt{2}} e^{-iH_{\uparrow}t} |\uparrow\rangle \]

**p/2 pulse + measurement of S_z gives relative phase accumulated by the two spin components**

Used for atomic clocks, gravitometers, accelerometers, magnetic field measurements
Measurements of Zak/Berry phase in one dimensional Bloch band

One dimensional superlattices
Su-Schrieffer-Heeeger model

M. Atala et al., arXiv:1212.0572
Characterizing SSH model using Zak phase

Two hyperfine spin states experience the same optical potential

\[ \varphi_{\text{tot}} = \varphi_{\text{Zak}} + \varphi_{\text{dyn}} + \varphi_{\text{Zeeman}} \]

\[ \frac{1}{i} \int_{-\pi}^{\pi} dk \langle \psi_k | \partial_k | \psi_k \rangle = \pi \]

Problem: experimentally difficult to control Zeeman phase shift
Spin echo protocol for measuring Zak phase

Dynamic phases due to dispersion and magnetic field fluctuations cancel. Interference measures the difference of Zak phases of the two bands in two dimerizations. Expect phase $\phi$
Bloch oscillations measurements
With p-pulse but no swapping of dimerization
Bloch oscillations measurements
With p-pulse and with swapping of dimerization
Zak/Berry phase measurements

\[ \delta \varphi = 0.97(2) \pi \]
Zak/Berry phase measurements extended
Summary

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