Ultracold atoms in optical lattices

Eugene Demler

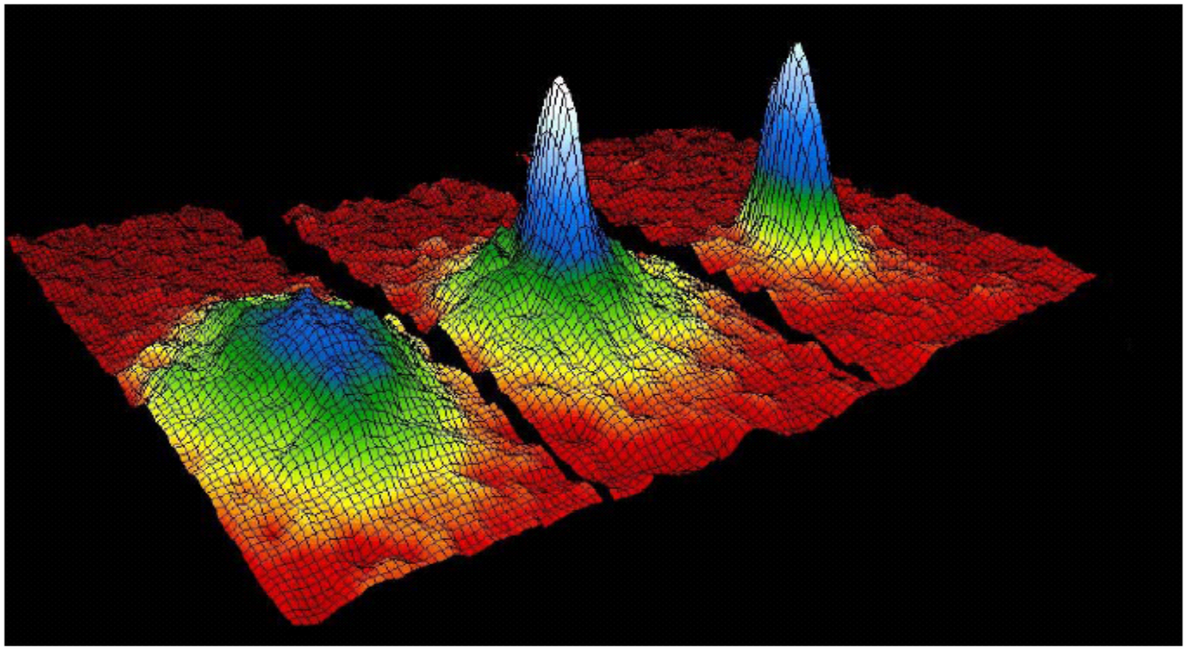
Harvard University

Lectures at the 2013 RQC summer school

Harvard-MIT CUA

$$ NSF, AFOSR MURI, DARPA OLE, MURI ATOMTRONICS

Bose-Einstein condensation of weakly interacting atoms



First BEC of alkali atoms, 1995

Density	10^{13} cm^{-1}
Typical distance between atoms	300 nm
Typical scattering length	10 nm

$$T_{\mathrm{BEC}} \sim 1\mu\mathrm{K}$$

Scattering length is much smaller than characteristic interparticle distances.
Interactions are weak

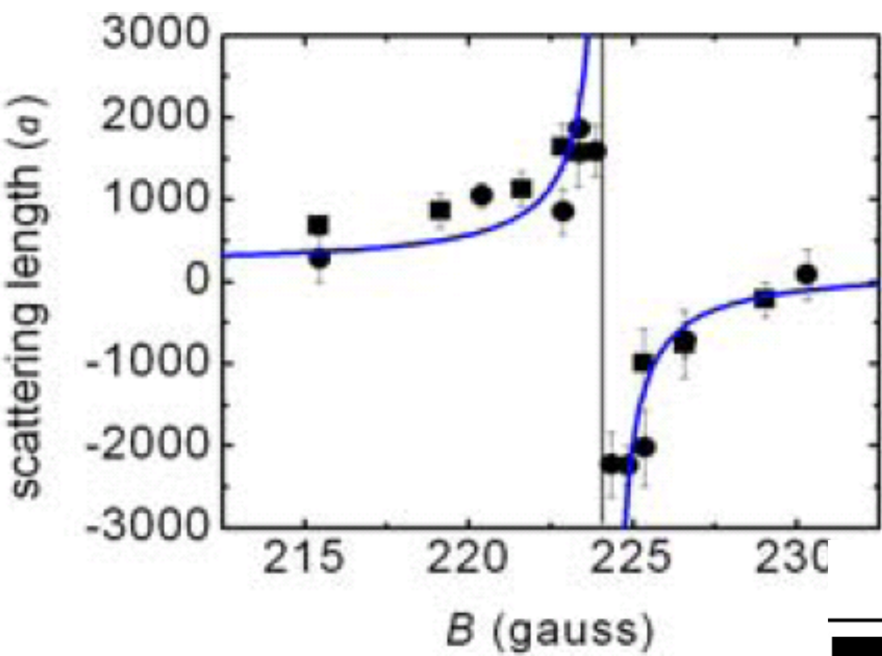
New Era in Cold Atoms Research

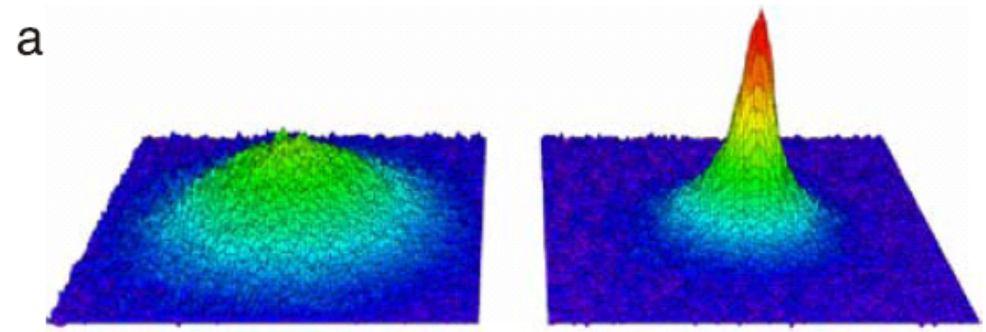
Focus on Systems with Strong Interactions

- Feshbach resonances
- Atoms in optical lattices
- Low dimensional systems
- Systems with long range dipolar interactions
- Rotating systems

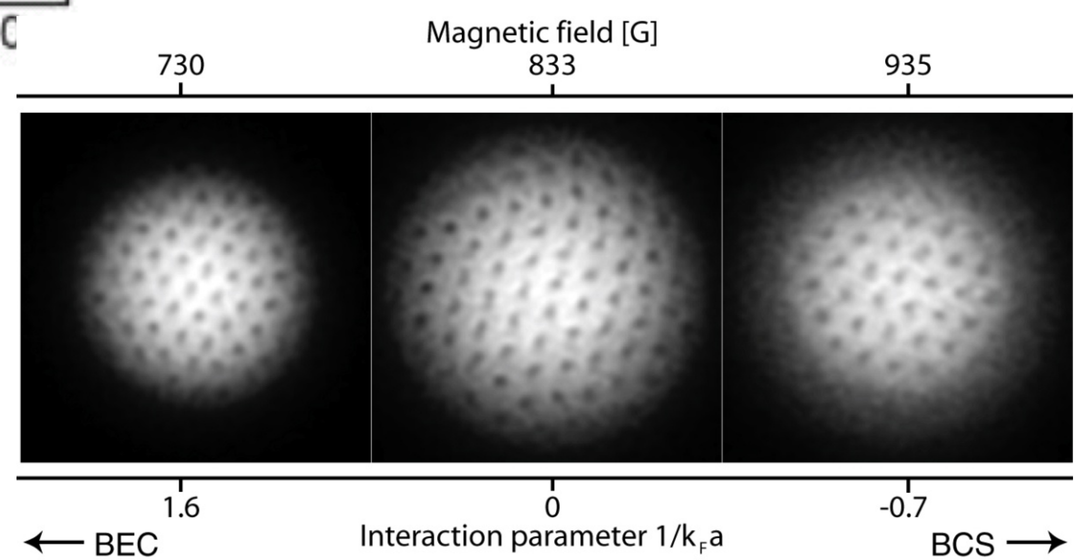
Feshbach resonance and fermionic condensates

Greiner et al., Nature (2003); Ketterle et al., (2003)

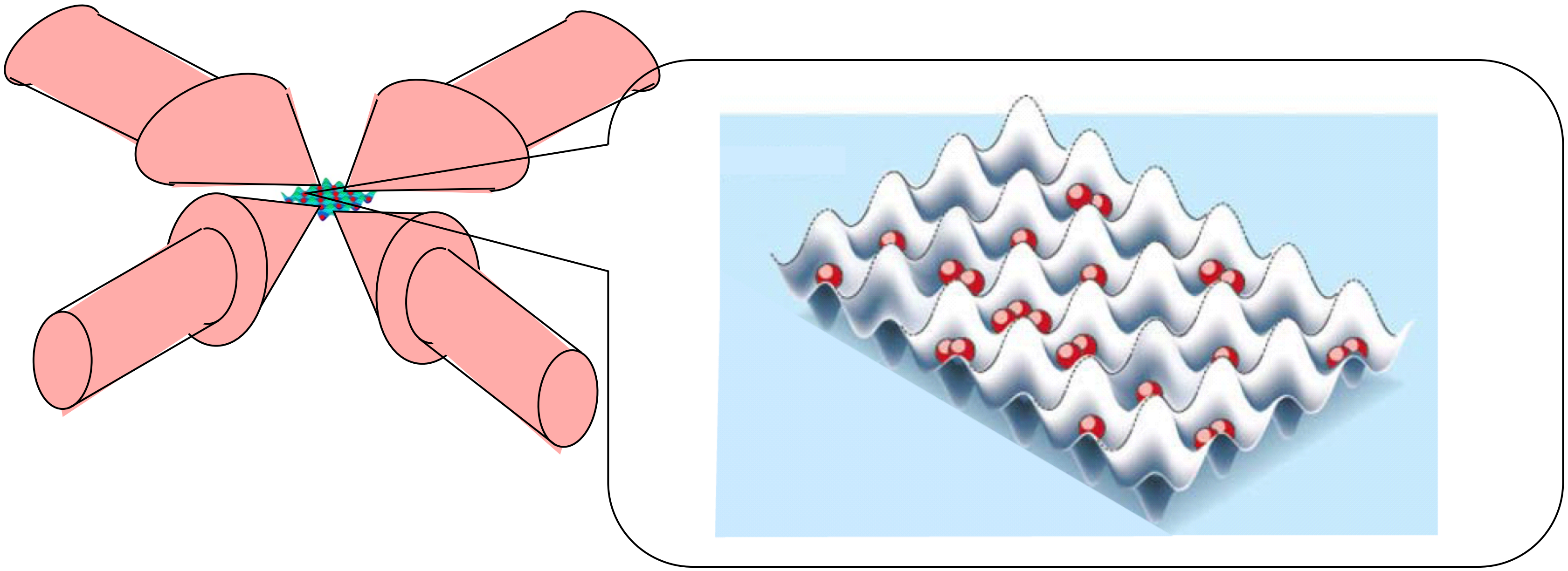




Ketterle et al.,
Nature 435, 1047-1051 (2005)



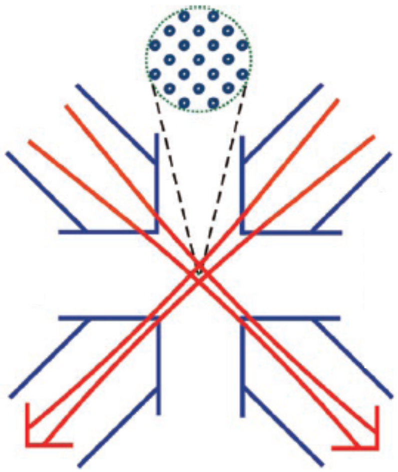
Atoms in optical lattices



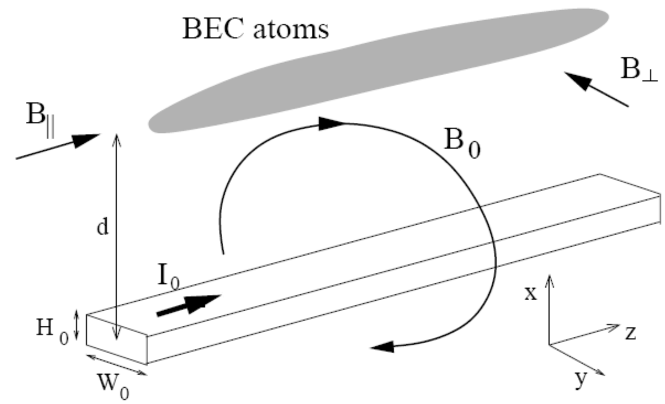
Theory: Jaksch et al. PRL (1998)

Experiment: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Phillips et al., J. Physics B (2002)
Esslinger et al., PRL (2004);
and many more …

One dimensional systems



1D confinement in optical potential
Weiss et al., Science (05);
Bloch et al.,
Esslinger et al.,



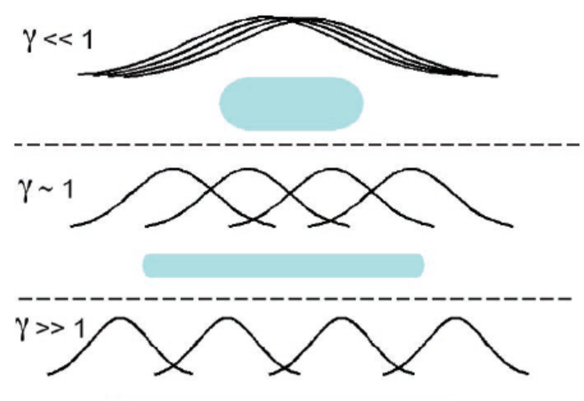
One dimensional systems in microtraps.
Thywissen et al., Eur. J. Phys. D. (99);
Hansel et al., Nature (01);
Folman et al., Adv. At. Mol. Opt. Phys. (02)

$$\mathrm{E_{kin}} \sim \frac{\hbar^2}{m\,d^2} \sim \frac{\hbar^2\,n^2}{m}$$

$$\mathrm{E_{int}} \sim g\,n$$

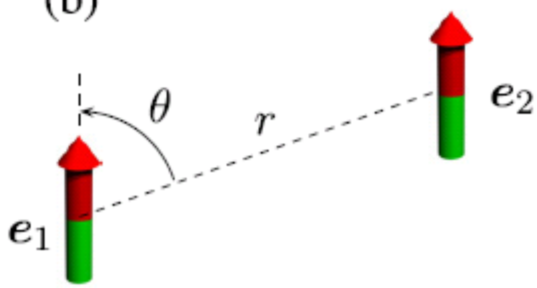
$$\gamma = \frac{\mathrm{E_{int}}}{\mathrm{E_{kin}}} \sim \frac{g\,m}{\hbar^2\,n}$$

Strongly interacting regime can be reached for low densities

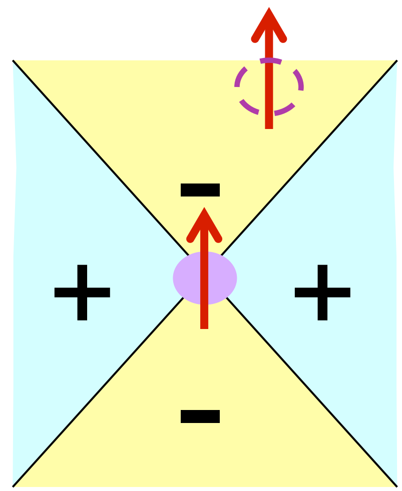


Ultracold polar molecules

(b)



$$U_{dd} = \frac{C_{dd}}{4\pi} \frac{1-3\cos^2\theta}{r^3}$$



Experiments on polar molecules:
Innsbruck, Yale, Harvard,
JILA, UConn,…

Lengthscale to characterize the strength
of dipolar interactions (not the scattering length)

$$a_{dd} = C_{dd} m / 12\pi\hbar^2$$

For typical dipolar moments of the order of 1 Debye

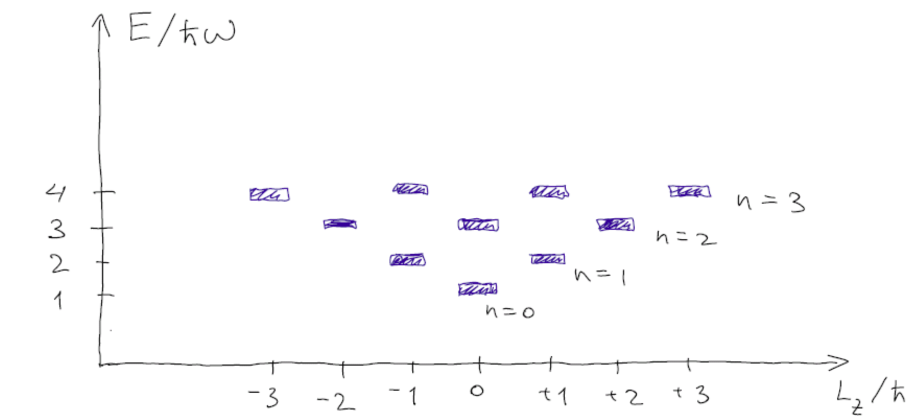
a_{dd} is about 500 nm

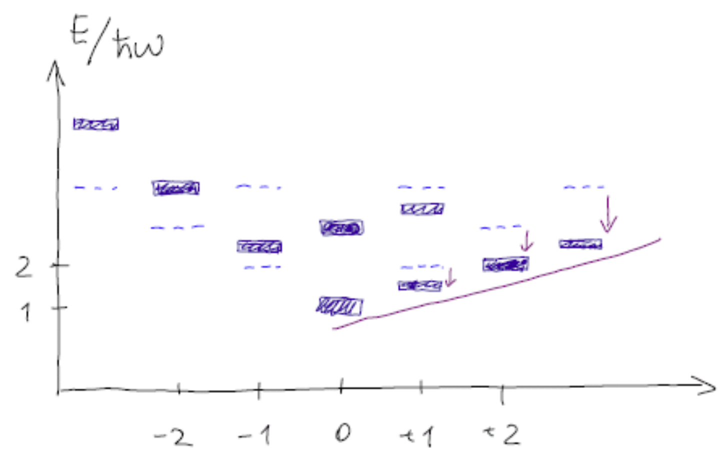
This is much larger than a typical scattering and
of the order of interparticle distances

Rotating atoms: towards FQHE states

Atoms in a two dimensional confining parabolic potential

In the presence of rotation



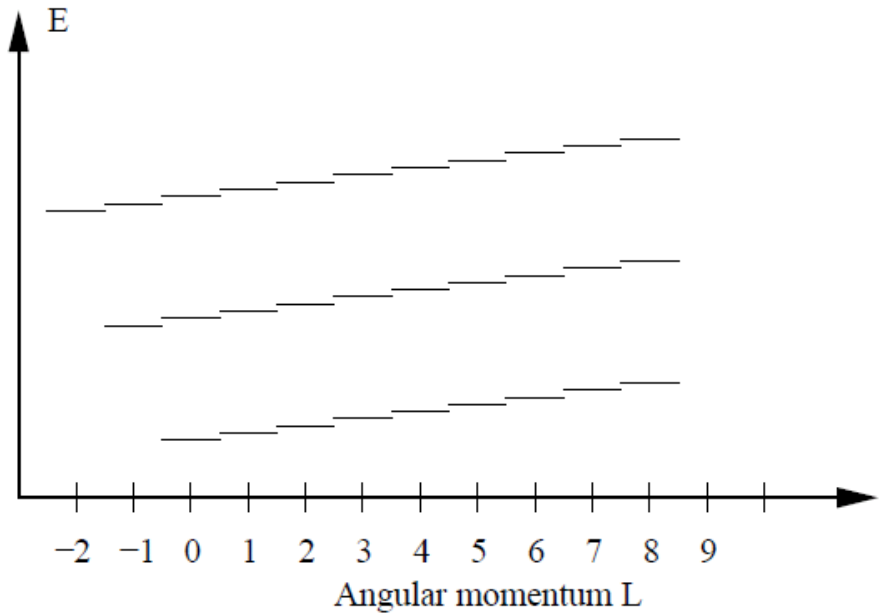


$$E = (n+1)\hbar\omega$$

$$L_z = -n, -n+2, \dots, n-2, n$$

$$\boxed{E = (n+1)\hbar\omega - \Omega L_z}$$

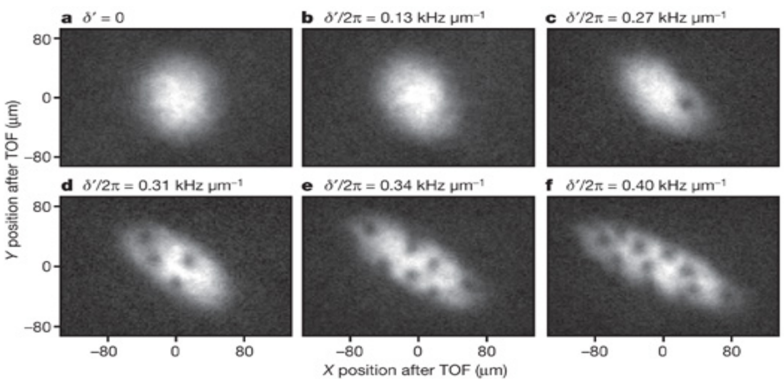
Rotating atoms: towards FQHE states



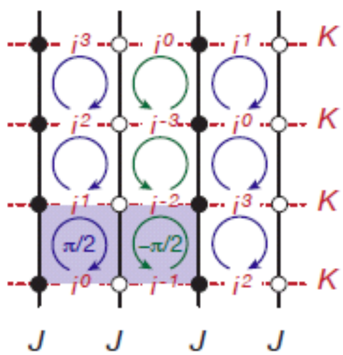
Slope is given by the difference of confinement
and rotation frequencies

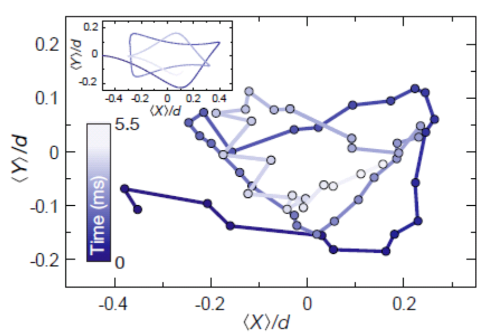
Nearly macroscopic degeneracy for noninteracting particles.
Interactions are crucial to lift the degeneracy

Artificial magnetic fields
Synthetic gauge fields



Vortices induced by synthetic gauge fields
Spielman et al., Nature (2009)





Staggered flux lattice
Bloch et al., PRL (2011)

Strongly correlated systems

Electrons in Solids

$$E_{\text{int}} \sim 1 \div 4 \text{ eV} \sim 10^4 \text{ K}$$

$$E_{\text{kin}} \sim 1 \div 10 \text{ eV} \sim 10^5 \text{ K}$$

Atoms in optical lattices

$$E_{\text{int}} \sim E_{\text{kin}} \sim 10 \text{ kHz} \sim 10^{-6} \text{ K}$$

Simple metals $E_{\text{int}} < E_{\text{kin}}$

Perturbation theory in Coulomb interaction applies.
Band structure methods wotk

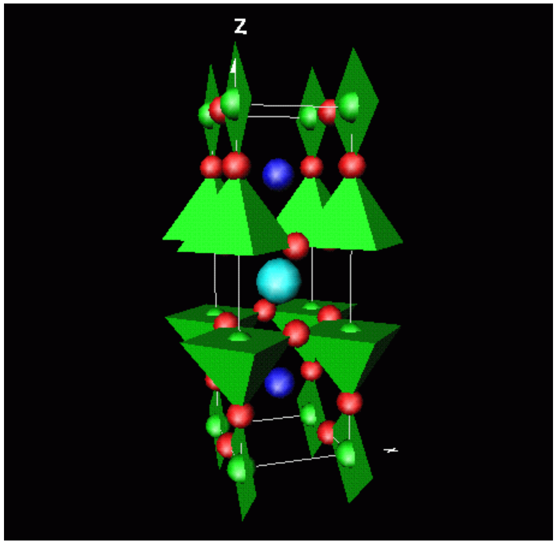
Strongly Correlated Electron Systems $E_{\text{int}} \geq E_{\text{kin}}$

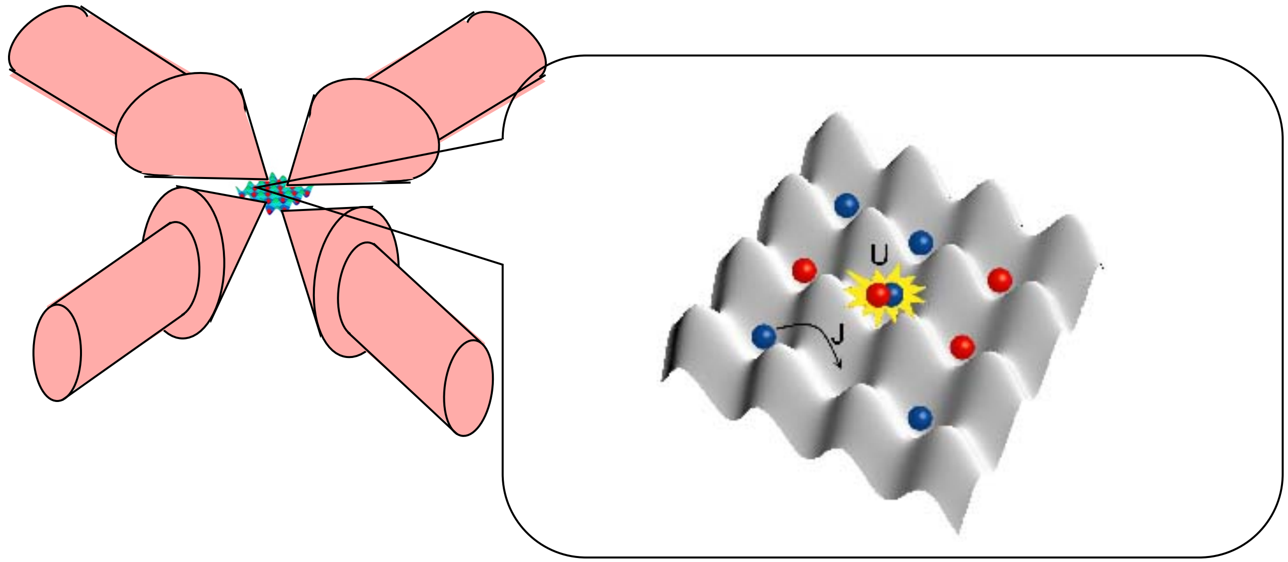
Band structure methods fail.

Novel phenomena in strongly correlated electron systems:

Quantum magnetism, phase separation, unconventional superconductivity, high temperature superconductivity, fractionalization of electrons …

Quantum simulations with ultracold atoms





$\mathrm{YBa_2Cu_3O_7}$

Atoms in optical lattice

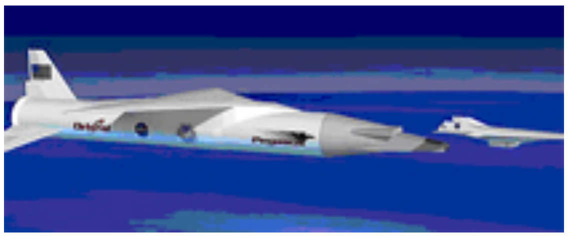
Antiferromagnetic and
superconducting Tc
of the order of 100 K

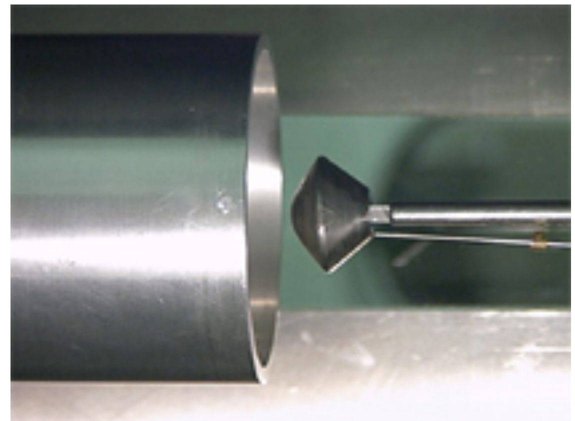
Antiferromagnetism and
pairing at sub-micro Kelvin
temperatures

Same microscopic model

$$\mathcal{H} = -t \sum_{\langle ij \rangle\, \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_i$$

By studying strongly interacting systems of cold atoms we expect to get insights into the mysterious properties of novel quantum materials: Quantum Simulators





BUT

Strongly interacting systems of ultracold atoms and photons:
are NOT direct analogues of condensed matter systems
These are independent physical systems with their own "personalities", physical properties, and theoretical challenges

Strongly correlated systems of ultracold atoms should also be useful for applications in quantum information, high precision spectroscopy, metrology

New detection methods needed

Equilibration and thermalization are important issues

New Frontier in quantum many-body physics: nonequilibrium dynamics

Long intrinsic time scales

- Interaction energy and bandwidth ~ 1kHz
- System parameters can be changed over this time scale

Decoupling from external environment

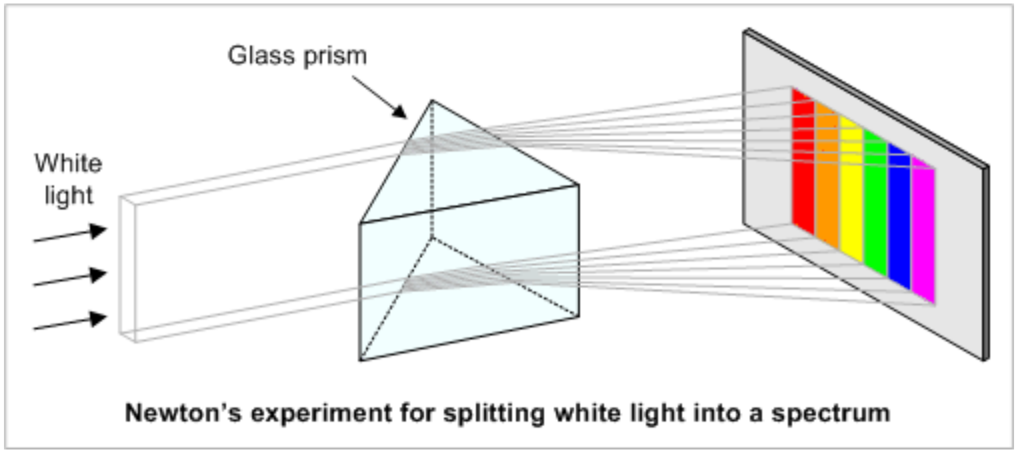
- Long coherence times

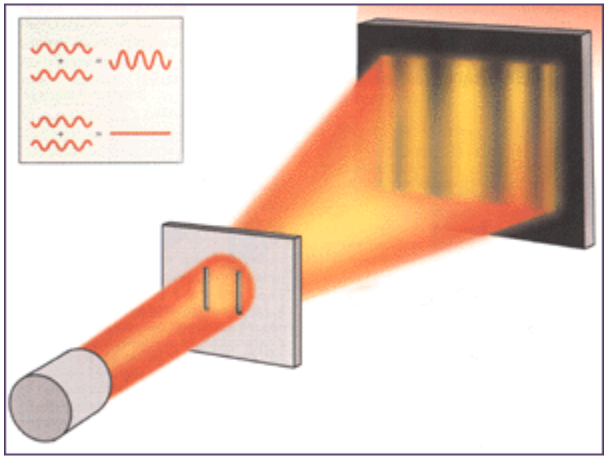
Can achieve highly non equilibrium quantum many-body states

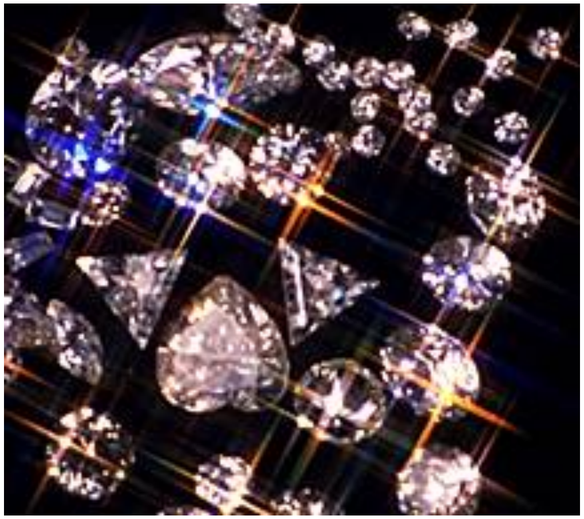
$$H_i \to H_f \qquad\qquad | \Psi(t) \rangle = e^{-iH_f t} | \Psi_i \rangle$$

Strongly correlated many-body systems of photons

Linear geometrical optics

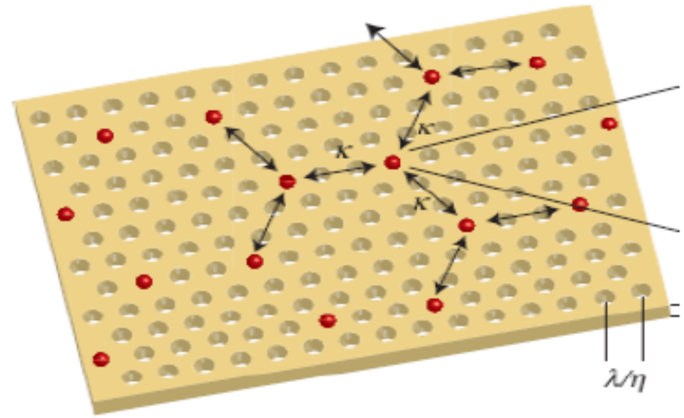




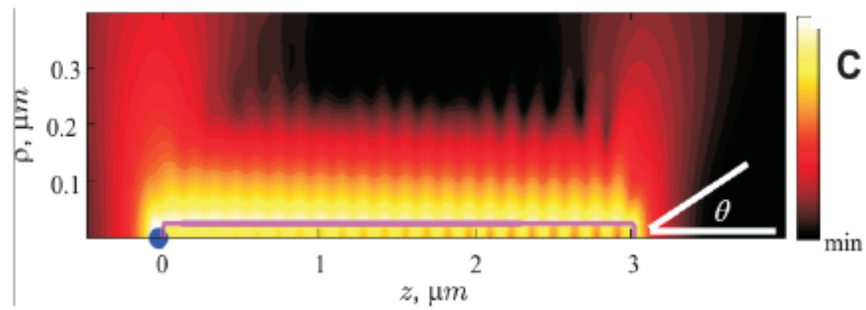


Strongly correlated systems of photons

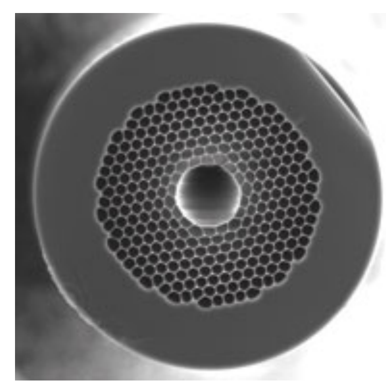
Strongly interacting polaritons in
coupled arrays of cavities
M. Hartmann et al., Nature Physics (2006)



Strong optical nonlinearities in
nanoscale surface plasmons
Akimov et al., Nature (2007)



Crystallization (fermionization) of photons
in one dimensional optical waveguides
D. Chang et al., Nature Physics (2008)



Outline

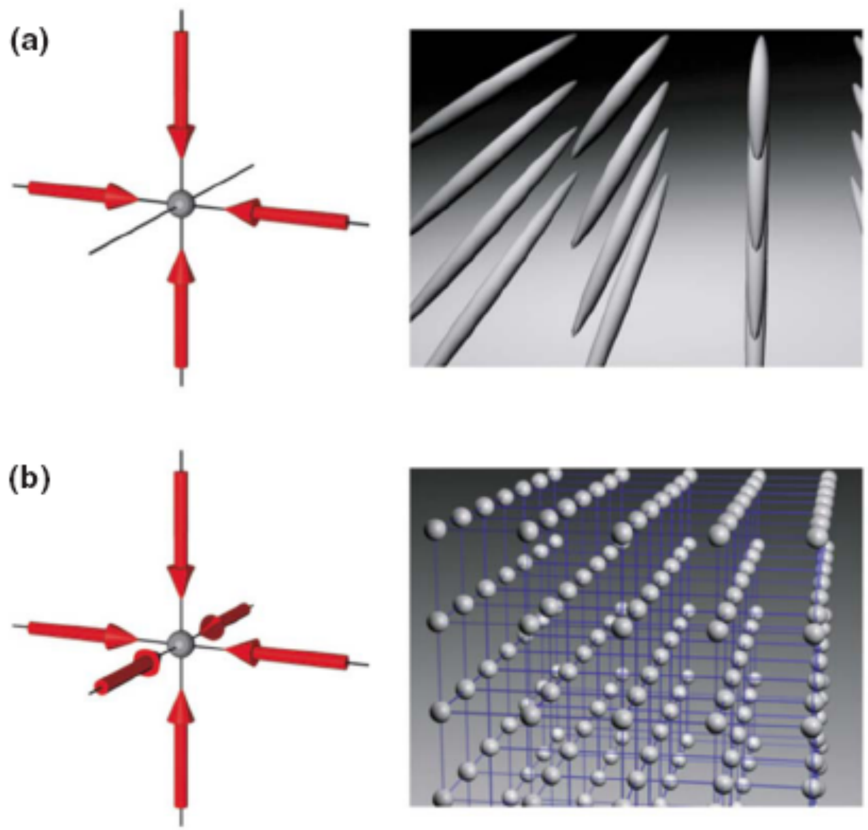
Introduction.

Ultracold atoms in optical lattices

Exploring non-interacting topological states with cold atoms

Exploring topological states with photons

Ultracold atoms in optical lattices. Band structure. Semiclasical dynamics.

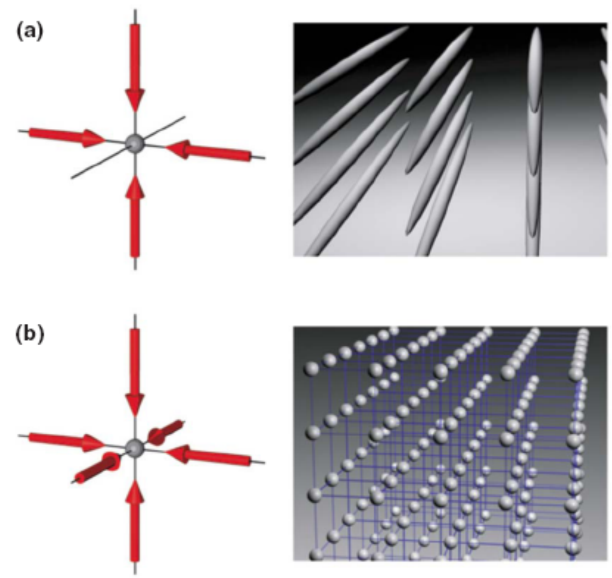


Optical trapping of alkali atoms

Based on AC Stark effect

Dipolar moment induced by the electric field

$$\langle \vec{d}(\omega) \rangle = \alpha(\omega)\, \vec{E}(\omega)$$



Typically optical frequencies.

$\alpha(\omega)$ - polarizability

Potential:

$$H_{ACS} = -\langle \vec{d} \cdot \vec{E} \rangle = -\frac{\alpha}{2} \cdot E^2$$

Far-off-resonant optical trap confines atoms regardless of their hyperfine state

Optical lattice

The simplest possible periodic optical potential is formed by overlapping two counter-propagating beams. This results in a standing wave

$$E(z) = E_0 \sin(kz + \theta) \cos \omega t$$

Averaging over fast optical oscillations (AC Stark effect) gives

$$V(z) = -V_0 \sin^2(kz + \theta)$$

Combining three perpendicular sets of standing waves we get a simple cubic lattice

$$V(r) = -V_0 \cos qx - V_0 \cos qy - V_0 \cos qz$$

This potential allows separation of variables

$$\Psi(x,y,z) = \Psi(x)\,\Psi(y)\,\Psi(z)$$

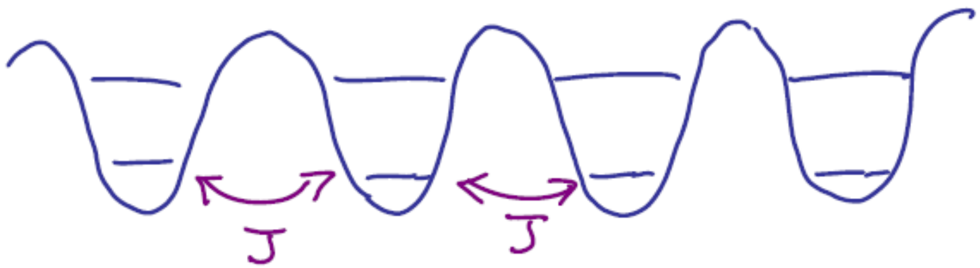
Optical lattice

For each coordinate we have Matthieu equation

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - V_0 \cos qx\right)\psi(x) = E\,\psi(x)$$

Eigenvalues and eigenfunctions are known

In the regime of deep lattice we get the tight-binding model



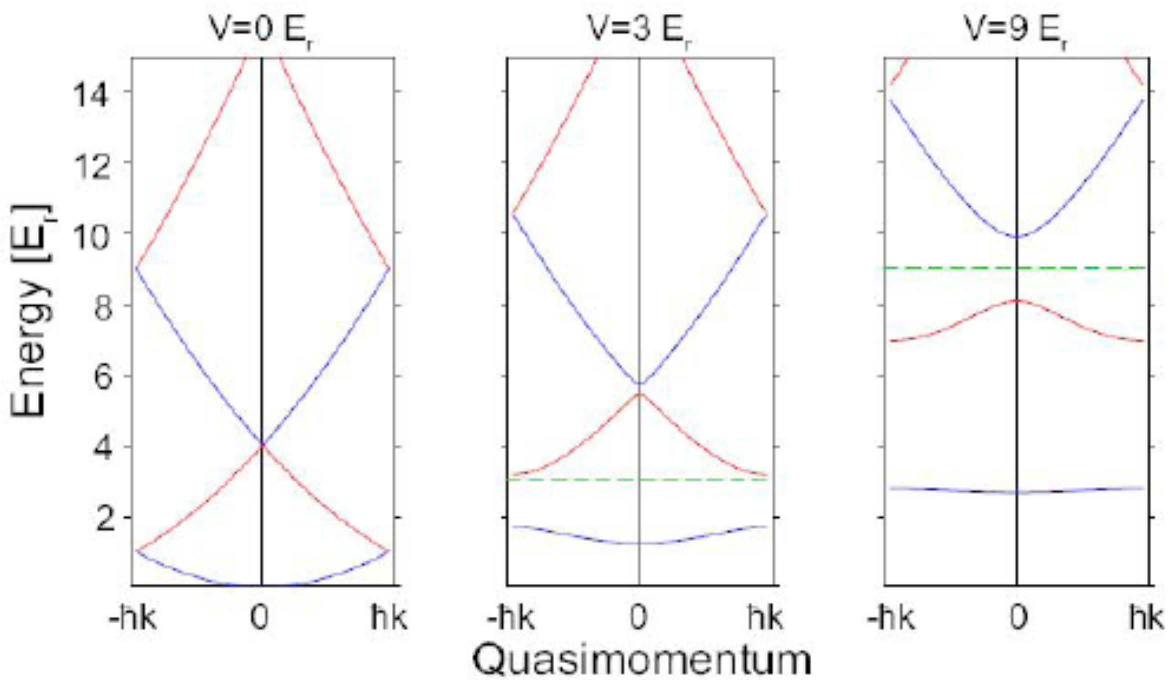
$$E_R = \frac{\hbar^2 K^2}{2m} = \frac{2\hbar^2 q^2}{m}$$ - recoil energy

$$\omega_0 = \frac{1}{\hbar}\left(8 V_0 E_R\right)^{1/2}$$ - bandgap

Lowest band

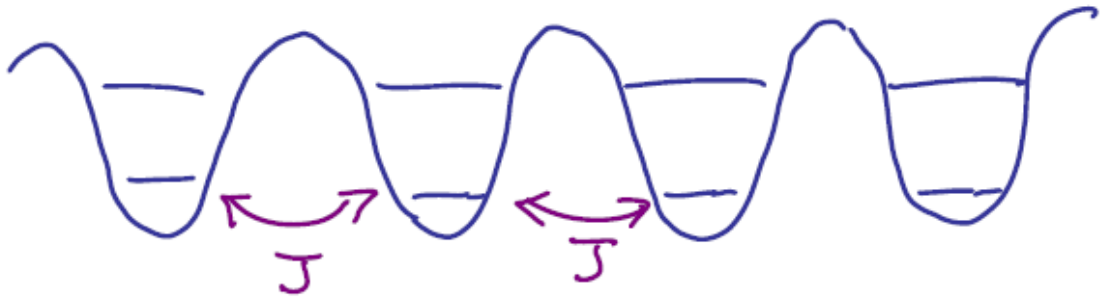
$$J = \frac{4}{\sqrt{\pi}} E_R \left(\frac{V_0}{E_R}\right)^{3/4} \exp\left[-2\left(\frac{V_0}{E_R}\right)^{1/2}\right]$$

Band structure evolution in optical lattices



Tight binding limit of optical lattice

Effective Hamiltonian for non-interacting atoms in the lowest Bloch band



$$\mathcal{H} = -J \sum_{\langle ij \rangle} b_i^{+} b_j = \sum_{\vec{p}} \varepsilon_{\vec{p}}\, b_{\vec{p}}^{+} b_{\vec{p}}$$

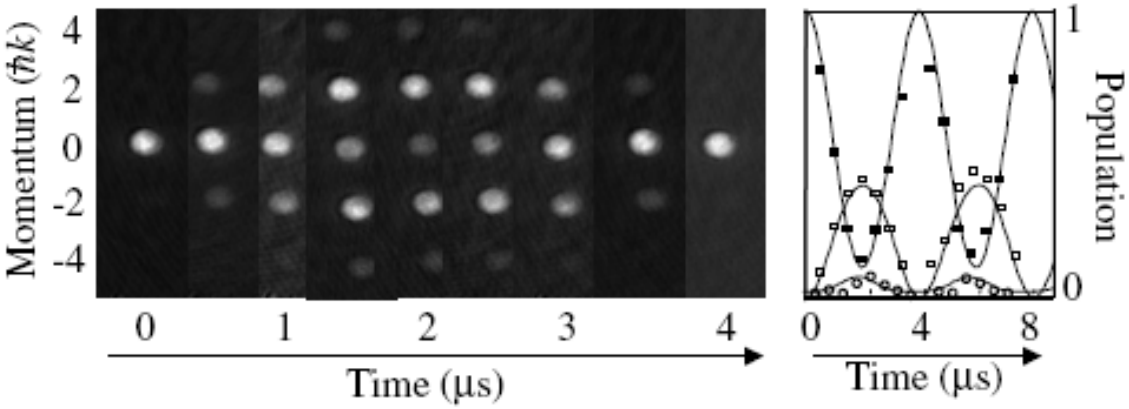
nearest neighbors

$$\varepsilon(\vec{p}) = -2J \left(\cos p_x d + \cos p_y d + \cos p_z d \right)$$

A Bose–Einstein condensate in an optical lattice

J Hecker Denschlag[1], J E Simsarian[2], H Häffner[1], C McKenzie,
A Browaeys, D Cho[3], K Helmerson, S L Rolston and W D Phillips

J. Phys. B: At. Mol. Opt. Phys. **35** (2002) 3095–3110



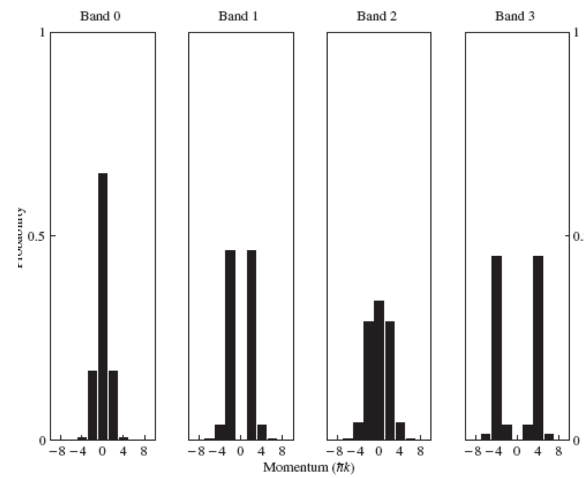


Figure 3. Coherent oscillations of the momentum decomposition of a BEC after suddenly switching on the lattice ($q = 0$, $V_0 = 14E_R$). On the left are time-of-flight images showing the plane-wave decomposition of the lattice state evolving as a function of time held in the lattice. The momentum components have been allowed to spatially separate. Only a single 4 μs cycle is shown. The right-hand-side plot shows the populations of the $0\hbar k$, $+2\hbar k$, and $+4\hbar k$ momentum components (respectively: the filled squares, open squares, and circles) over two cycles. The curves are a theoretical calculation of the populations in each component.

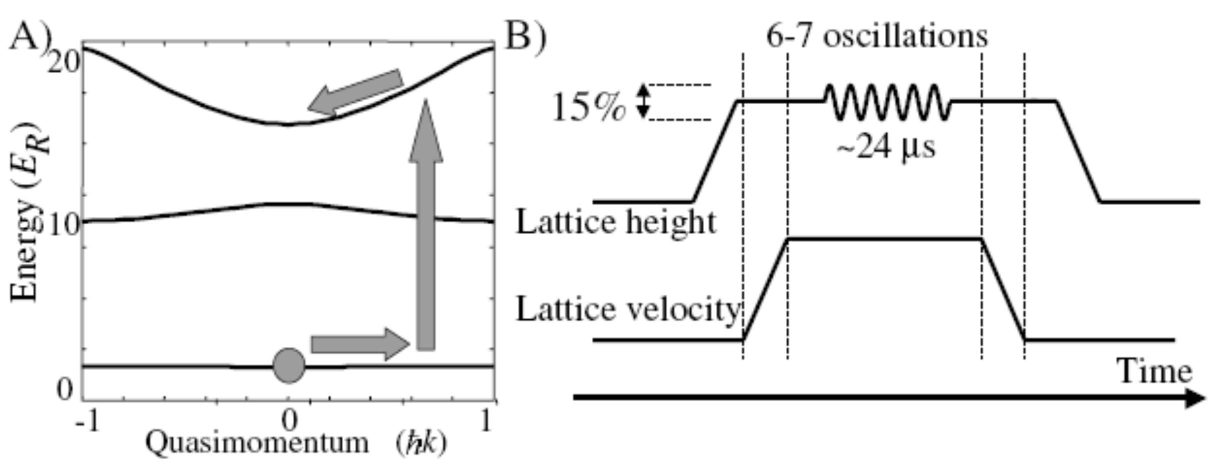


Figure 11. The scheme for transferring population into band 2 at arbitrary q. We start from the ground state at a quasimomentum of 0. We then accelerate the lattice to change the quasimomentum from 0 to the desired value of q. Population is transferred into band 2 by modulating the lattice height over a 20–30 μs period. In order to measure the population in the upper band the acceleration is reversed to bring the quasimomentum back to 0 and the lattice is adiabatically released to map the bands into individual momentum components.

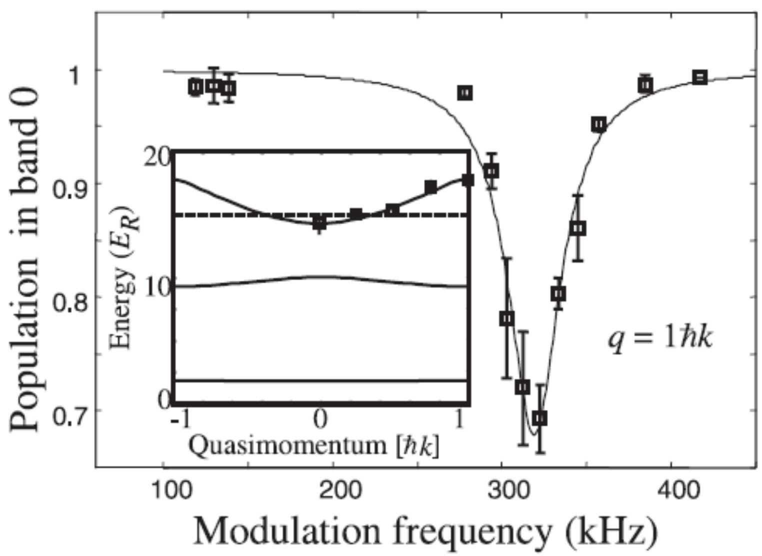


Figure 12. Experimental data showing population transfer from band 0 to band 2 at $q = 1\hbar k$ as a function of excitation frequency. The scheme used is outlined in figure 11. The fit is a Lorentzian. The resonance frequency corresponds to the band gap between band 0 and 2 at $q = 1\hbar k$. Similar measurements at $q = 0\hbar k$, $0.25\hbar k$, $0.5\hbar k$, and $0.75\hbar k$ allow us to map out the curved band structure (see the inset).

Fermionic Atoms in a Three Dimensional Optical Lattice: Observing Fermi Surfaces, Dynamics, and Interactions

Michael Köhl,* Henning Moritz, Thilo Stöferle, Kenneth Günter, and Tilman Esslinger
Institute of Quantum Electronics, ETH Zürich, Hönggerberg, CH-8093 Zürich, Switzerland

PRL **94,** 080403 (2005)

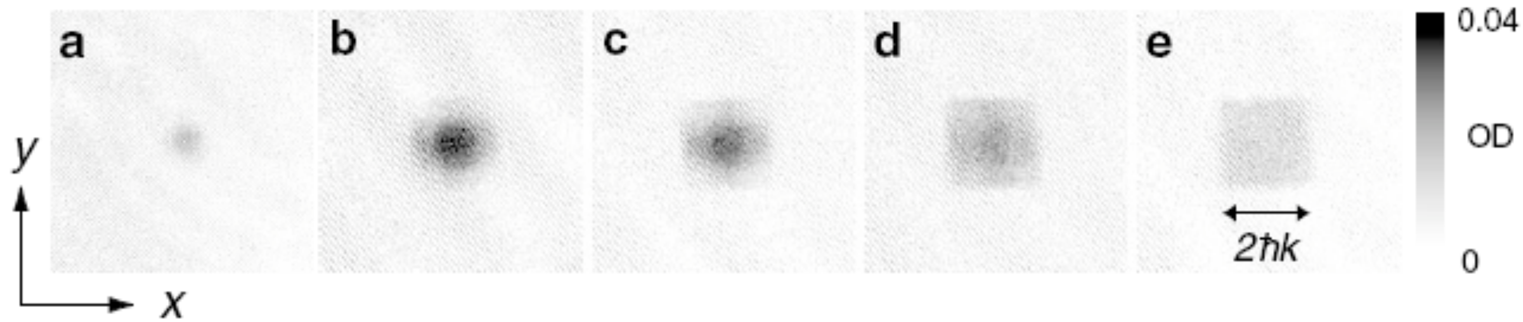
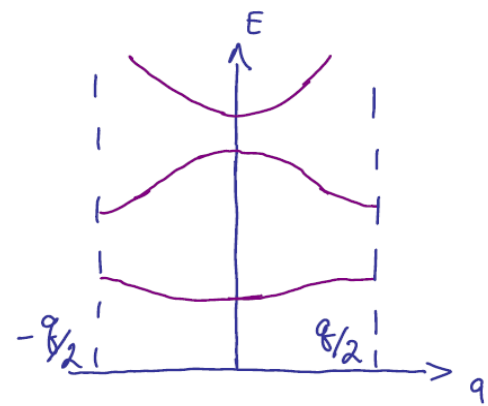


FIG. 1. Observing the Fermi surface. Time of flight images obtained after adiabatically ramping down the optical lattice.

Semiclassical dynamics in optical lattices

1. Band index is constant
2. $$\frac{d\vec{r}}{dt} = \frac{\partial \varepsilon}{\partial \vec{k}} = \vec{V}_g$$
3. $$\frac{d\vec{k}}{dt} = \vec{F}$$



Bloch oscillations

Consider a uniform and constant force

$$\vec{k}(t) = \vec{k}(0) + \vec{F}\cdot t$$

$$\frac{dx}{dt} = 2Jd \sin\left[(q_0 + Ft)d\right]$$

$$x = x_0 - \frac{2J}{F}\cos\left[(q_0 + Ft)d\right]$$

Bloch Oscillations of Atoms in an Optical Potential

Maxime Ben Dahan, Ekkehard Peik, Jakob Reichel, Yvan Castin, and Christophe Salomon
Laboratoire Kastler Brossel, Departement de Physique, Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris Cedex 05, France

PRL 76:4508 (1996)

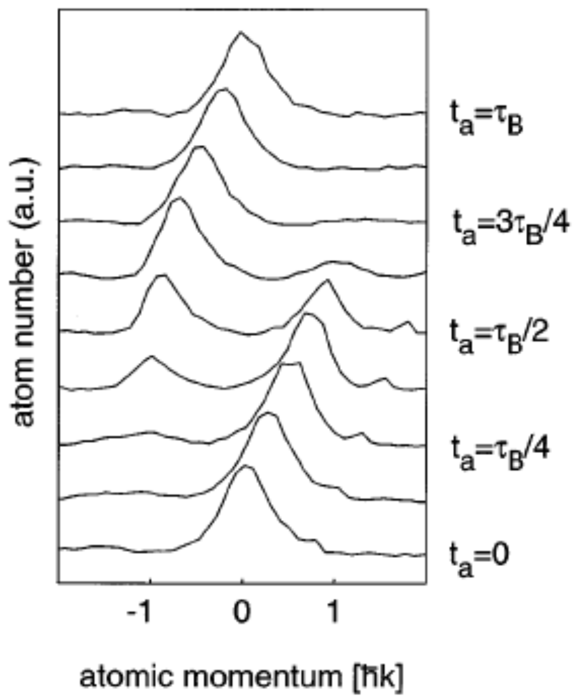


FIG. 2. Bloch oscillations of atoms: momentum distributions in the accelerated frame for equidistant values of the acceleration time t_a between $t_a = 0$ and $t_a = \tau_B = 8.2$ ms. The light potential depth is $U_0 = 2.3E_R$ and the acceleration is $a = -0.85$ m/s^2. The small peak in the right wing of the first five spectra is an artifact.

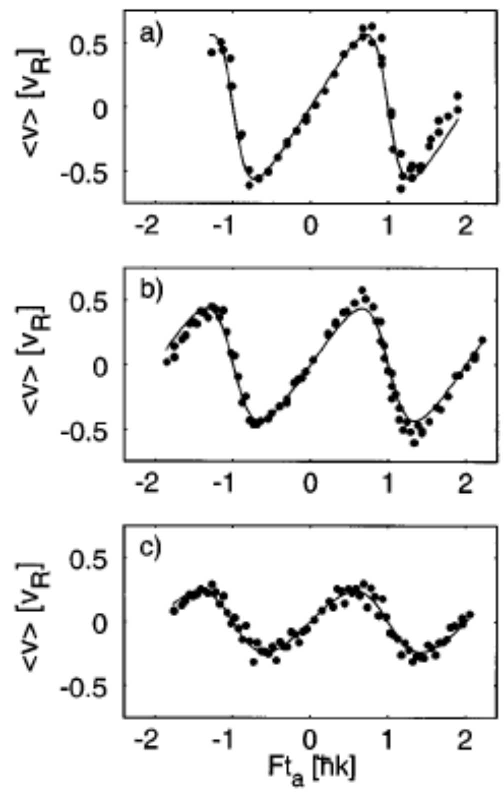


FIG. 3. Mean atomic velocity $\langle v \rangle$ as a function of the acceleration time t_a for three values of the potential depth: (a) $U_0 = 1.4E_R$, (b) $U_0 = 2.3E_R$, (c) $U_0 = 4.4E_R$. The negative values of Ft_a were measured by changing the sign of F. Solid lines: theoretical prediction.

Control of Interaction-Induced Dephasing of Bloch Oscillations

M. Gustavsson, E. Haller, M. J. Mark, J. G. Danzl, G. Rojas-Kopeinig, and H.-C. Nägerl

PRL **100**, 080404 (2008)

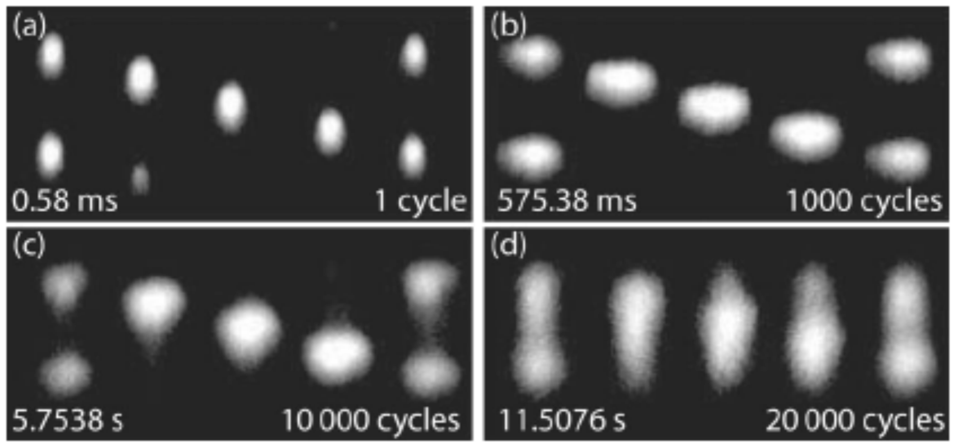


FIG. 1. Long-lived Bloch oscillations for a noninteracting BEC with Cs atoms in the vertical lattice under the influence of gravity. Each picture shows one Bloch cycle in successive time-of-flight absorption images corresponding to the momentum distribution at the time of release from the lattice. Displayed are the first (a), the 1000th (b), the 10 000th (c), and the 20 000th (d) Bloch cycle for minimal interaction near the zero crossing for the scattering length.