Modulation spectroscopy
BEC polarons
Quasicrystals

Everything is work in progress

Harvard-MIT

$$ \text{NSF, MURI ATOMTRONICS, MURI QuISM, DARPA OLE} $$
Lattice modulation experiments as a probe of collective mode dispersion

Based on recent discussion with Immanuel Bloch
Recent results by Michael Knap (Harvard)
Earlier results by David Pekker (Harvard, Caltech)
Modulation experiments in optical lattice
Observation of the Anderson-Higgs mode

Lattice modulation probe of collective mode dispersion

\[ H_0 = -t \sum_{\langle i,j \rangle} b_i^+ b_j^{} + \frac{U}{2} \sum_i n_i(n_i-1) \]

\[ H_{\text{drive}} = \cos \omega t A_x \sum_i (b_i^+ b_{i+x} + \text{c.c.}) \]

\[ + \cos \omega t A_y \sum_i (b_i^+ b_{i+y} + \text{c.c.}) \]

Modulation performed at zero wavevector.
How can this probe finite momentum modes?
Faraday waves

M. Faraday, Philosophical Transactions of the Royal Society (London), vol. 121, pages 299–318 (1831)
Faraday waves in BEC

Bogoliubov theory

\[ \mathcal{H}_0 = \sum_k E_k \gamma^+_k \gamma_k \]

\[ \mathcal{H}_{\text{drive}} = \cos \omega t \sum_k \left( A_k \gamma^+_k \gamma^-_k + B_k \gamma^+_k \gamma_k \right) \]

\[ A_k = u_k v_k \left( A_x \cos k_x + u_k v_k A_y \cos k_y \right) \]

Time evolution can be solved exactly

\[ |\Psi(t)\rangle = C(t) \prod_k e^{\alpha_k(t) \gamma^+_k \gamma^-_k} |0\rangle \]

Resonance

\[ \omega = 2 E_k \]
Faraday waves in BEC

\[ \langle \rho(x) \rho(y) \rangle = \sum u_{k_1} u_{k_2} u_{k_3} u_{k_4} \langle \delta_{k_1}^+ \delta_{k_2}^+ \delta_{k_3}^+ \delta_{k_4}^+ \rangle e^{i(k_1-k_2)x} e^{i(k_3-k_4)y} + \ldots \]

From wavefunction

\[ |\psi(t)\rangle = C(t) \prod_k e^{\alpha_k(t) \gamma_k^+ \gamma_k^+} |0\rangle \]

we expect the appearance of \( \langle \gamma_k^+ \gamma_{-k}^+ \rangle \) and \( \langle \delta_k \delta_{-k} \rangle \)

Density acquires ripples

\[ \langle \rho(x) \rho(y) \rangle = \sum_k \langle \gamma_k^+ \gamma_{-k}^+ \rangle \langle \delta_k \delta_{-k} \rangle \cos(2k \cdot r) + \ldots \]

We expect ripples peaked at the resonance wavevector \( \omega = 2k E_k \)
Faraday waves in BEC.
One dimensional case

U=0.1
n=1

(b) $A = 0.3J, t = 20 \times 2\pi/\omega$

(c) $A = J, t = 20 \times 2\pi/\omega$
Faraday waves in BEC. Two dimensional case

Different driving geometries

\[ \log \langle \rho_k \rho_{-k} \rangle \]
Faraday waves in BEC.
Two dimensional case

Different driving amplitudes
Faraday waves in BEC.
Two dimensional case

Different pulse durations

\[ \log \langle \rho_k \rho_{-k} \rangle \]
Modulation spectroscopy

Detecting phonons and persistent currents in toroidal Bose-Einstein condensates by means of pattern formation

M. Modugno,1,2 C. Tozzo,3 and F. Dalfovo1,4

We theoretically investigate the dynamic properties of a Bose-Einstein condensate in a toroidal trap. A periodic modulation of the transverse confinement is shown to produce a density pattern due to parametric amplification of phonon pairs. By imaging the density distribution after free

FIG. 2. Top. Amplitude of the $\perp k$ components of the Fourier transform of the order parameter along the torus, where the wavevector $k$ satisfies the resonance condition $\omega(k) = \Omega/2$. 
Modulation spectroscopy

Modulation of interaction strength

PHYSICAL REVIEW A 80, 023625 (2009)

Parametric generation of density and spin modes with formation of stationary condensed states in ultracold two-component quasi-one-dimensional Fermi gases

Yu. Kagan and L. A. Manakova*

Faraday patterns in coupled one-dimensional dipolar condensates

Kazimierz Lakomy, 1 Rejish Nath, 2 and Luis Santos 1

Interference of parametrically driven one-dimensional ultracold gases

Susanne Pielawa
Modulation spectroscopy

Interesting applications

Particle-hole excitations in the Mott state, excitations in the critical region

Probe of spin waves in a ferromagnetic state
Quantum simulations with ultracold atoms

\[ H = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i \]

YBa$_2$Cu$_3$O$_7$

Antiferromagnetic and superconducting Tc of the order of 100 K

Atoms in optical lattice

Antiferromagnetism and pairing at sub-micro Kelvin temperatures

Same microscopic model
Positive U Hubbard model

Signature of d-wave pairing of fermions: dispersion of quasiparticles

Superconducting gap

\[ \Delta_k = \Delta_0 \left( \cos k_x - \cos k_y \right) \]

Normal state dispersion of quasiparticles

\[ \epsilon_k \]

Quasiparticle energies

\[ E_k = \sqrt{\epsilon_k^2 + \Delta_k^2} \]

Low energy quasiparticles correspond to four Dirac nodes

Observed in:

- Photoemission
- Raman spectroscopy
- T-dependence of thermodynamic and transport properties, \( c_V, \kappa, \lambda_L \)
- STM
- and many other probes
Modulation spectroscopy

Probe of quasiparticles in d-wave paired state

D. Pekker et al., arXiv:0906.093
Probing polarons in bosonic systems

Dima Abanin (Harvard Perimeter),
Adiya Shashi (Rice/Harvard)
Eugene Demler
+ recent discussion with Immanuel Bloch
Polarons in ultracold atoms: fermionic environment

3D Zwierlein et al., PRL (2009)

2D Kohl et al., Nature (2012)
Polarons in BEC

\[ H_{\text{imp}} = \frac{1}{2M} \cdot p^2 \]

\[ H_{\text{int}} = g \sum_k e^{ikx} \rho_k^+ \]

BEC regime

\[ \rho_q^+ = \sum_k b_{k+q}^+ b_k = \sqrt{N_0} \left( b_q^+ + b_{-q}^+ \right) \]

\[ = \sqrt{N_0} \left( \nu_q - \nu_{-q} \right) \left( \gamma_q + \gamma_{-q}^+ \right) \]
Polarons in BEC

\[ H_{\text{imp}} = \frac{1}{2M} \cdot \mathbf{P}^2 \]

\[ H_{\text{phon}} = \sum \omega_k \gamma_k^+ \gamma_k \]

\[ H_{\text{int}} = \sum_k V_k e^{-ik\mathbf{x}} (\gamma_k + \gamma_k^+) \]
Dynamics of BEC polarons. Spectral function

RF spectroscopy of spin states

Interacting

Noninteracting

Transitions of localized impurity atoms

Change of the impurity wavefunction excites Bogoliubov phonons in BEC
Dynamics of BEC polarons.
Lang-Firsov transformation approach

\[ H_{\text{imp}} = \frac{1}{2M} \cdot p^2 \]
\[ H_{\text{phon}} = \sum \omega_k \gamma_k^+ \gamma_k \]
\[ H_{\text{int}} = \sum_k V_k e^{-ikx} (\gamma_k + \gamma_{-k}^+) \]

Lang-Firsov transformation

\[ S = \bar{R} \sum_k \gamma_k^+ \gamma_k \]
\[ \tilde{H} = e^{iS} H e^{-iS} \]
Dynamics of BEC polarons.
Langs-Firsoov transformation approach

\[ \hat{H}_p = \frac{1}{2M} \left( \hat{p}^2 - \sum_k \hat{\gamma}_k^+ \hat{\gamma}_k \right)^2 \]

\[ + \sum_k V_k \left( \hat{\gamma}_k^+ \hat{\gamma}_k^- \right) + \sum_k \omega_k \hat{\gamma}_k^+ \hat{\gamma}_k \]

Conserved total momentum of the system p is now explicitly separated

Interaction between impurity and phonons was traded for interactions between phonons
Dynamics of BEC polarons.
Spectral function

\[ \tilde{H}_p = \frac{1}{2M} \left( \vec{p} - \sum_k \vec{k} \gamma^+_k \gamma_k \right)^2 \]

\[ + \sum_k V_k (\gamma_k + \gamma^+_k) + \sum_k \omega_k \gamma^+_k \gamma_k \]

\[ G(t) = \langle 0_{\text{phon}} | e^{-i \tilde{H}_p t} | 0_{\text{phon}} \rangle \]

\[ A(\omega) = \int_{-\infty}^{+\infty} e^{i \omega t} G(t) \, dt \]
Dynamics of BEC polarons. RF absorption for infinite impurity mass

$d=3$

- **Coherent Polaron peak**

\[ Z = e^{-\frac{n_0 g_1^2}{\sqrt{3} \pi^2 a^2 \xi}} \]

- **Incoherent part, power-laws**
  - low freq \( \sim \omega \)
  - high freq \( \sim \omega^{3/2} \)
Dynamics of BEC polarons. RF absorption of localized impurity

\[
\frac{1}{\sqrt{m\Omega\xi}} = 0.1.
\]
\[
\frac{gIBn}{2\sqrt{2}\pi c^2\xi} = 1
\]

<table>
<thead>
<tr>
<th>$d_{\text{st Rb-se Rb}}$</th>
<th>$\approx 5.3$ nm</th>
</tr>
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<tbody>
<tr>
<td>$a_s$</td>
<td>$\approx 130$ nm [3]</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>$\approx 20$ kHz.</td>
</tr>
<tr>
<td>$n_2$</td>
<td>$\approx \frac{500 \text{ atoms}}{(5 \mu \text{m})^2 \times 100} = 0.17 \times 10^{-4}$ nm$^{-2}$ [4]</td>
</tr>
<tr>
<td>$\xi$</td>
<td>$= \sqrt{\frac{2\pi}{8\pi n_2 a}} \approx 1000$ nm.</td>
</tr>
<tr>
<td>$\sqrt{\frac{\hbar}{m\Omega}}$</td>
<td>$\approx 200$ nm.</td>
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</tbody>
</table>
Dynamics of BEC polarons.
RF absorption of localized impurity
Dynamics of BEC polarons. RF absorption for finite impurity mass

Signatures of self-trapping transition?
Quantum quasicrystals of spin orbit coupled bosons

Sarang Gopalakrishnan (Harvard)
Ivar Martin (Los Alamos/Argonne)
Eugene Demler (Harvard)
What are quasicrystals

Associated with fivefold (or sevenfold) symmetry: pentagons in 2D, dodecahahedra/icosahedra in 3D

Translational order without periodicity
(position of tile determines the entire tiling, but the tiling is aperiodic)

Sharply defined Bragg peaks that are inconsistent with space group (spatial periodicity)

Conjectured in models with multiple incommensurate length scales (e.g. Mermin, Troian, PRL 1985)

Origin is still a subject of debate (equilibrium or metastable states)
Phase diagram of Bose-Fermi mixture with spin orbit coupling for bosons

One species of fermions

Two species of fermions
Spin-orbit coupled BECs

- Hamiltonian (spin-$\frac{1}{2}$ bosons):
  \[ H = \frac{p^2}{2m} + 1 + \alpha(k_x \sigma_y - k_y \sigma_x) + H_{\text{int}}. \]

- Properties:
  - Circular dispersion minimum
  - Experimentally realizable

- Why is this interesting?
  - Degenerate single-particle minimum
  - Interactions determine nature of condensate
  - Condensation at finitely many $k$ implies crystallization
Mean-field phase diagram

- Spin-dependence of interactions gives rise to momentum-dependence
- Interactions weakest when momenta either the same or opposite
- Two possibilities:
  - Same-momentum repulsion weaker: plane-wave state
  - Opposite-momentum repulsion weaker: stripes
- But are these the real interactions?
- No: low-dimensional system, therefore T-matrix corrections are crucial
Idea: Bose-Fermi mixtures

Use Fermi momentum to engineer momentum dependent interactions

Make $2k_F$ to provide 5-fold covering of the spin-orbit circle
Quasicrystals in systems with competing lengthscales

Mermin, Troian, PRL 1985

\[ f = -t \psi^2 - \psi^3 + \psi^4 + \sum_i [\tau_i \phi_i^2 - \phi_i \psi^2] \]

Order parameter $\psi$ has preferred wave vector $q$

Order parameters $\phi_i$ have preferred wave vectors $k_i$

Many possible phases depending on $q$, $k_i$

Competition of Bravais lattices and quasicrystals
Fermion mediated interaction (RKKY)

\[ V(|\mathbf{k} - \mathbf{k}'|)^2 \chi(|\mathbf{k} - \mathbf{k}'|) \]

Superficially, \( V \) is a constant in \( \mathbf{k} \); so RKKY interaction structure depends on susceptibility

Bad news: susceptibility is constant for all \( k < 2 k_F \) in two dimensions

We need to include renormalization of \( V(k-k') \)
Renormalization of RKKY interaction
T-matrix renormalization of Bose-Fermi interaction

\[ V_r \sim \frac{V_0}{1 + V_0 \times \text{stuff}} \rightarrow \frac{1}{\text{stuff}} \]

“Stuff” is angle dependent
T-matrix renormalization of Bose-Fermi interaction

Intermediate-state momenta
kinematically forced to live
near intersection of the Fermi and Rashba circles
Further constrained by Pauli
Renormalized Bose-Fermi interaction

\[ V_\alpha(q) = \frac{V_0}{1 + \frac{mV_0 A(\phi)}{2\pi} \log \left( \frac{A}{m v_F^\alpha} \right)} \sim \frac{1}{A(\phi) \log \left( \frac{A}{m v_F^\alpha} \right)} \]

\[ A(\phi) = \left[ \frac{q}{2k_F} \sqrt{1 - \frac{q^2}{4k_0^2}} \right]^{-1} \]

Renormalized interaction is peaked at 2k_f
Mean-field theory with renormalized interactions

Minimize interaction Hamiltonian for states condensed at one or more pairs of momenta

General result (schematic) for energy density per particle

\[ E = U(2 - 1/m) - \sum_{i,j} V_{\text{RKKY}} (\theta_i - \theta_j)/m^2 \]

\( m = \# \) of different pairs of momenta

Bose-Bose interaction penalizes condensation at multiple momenta; RKKY favors it

Energy of striped phase always \( U \); other states come down in energy as RKKY is ramped up

Stable system so long as \( E > 0 \)
Mean-field theory with renormalized interactions
One species of fermions

Several competing Bravais lattices
No quasicrystals
Mean-field theory with renormalized interactions
Two species of fermions

The strongly interacting species has RKKY peaked at $\pi/5$.
The weakly interacting species has RKKY uniform until $3\pi/5$.

Quasicrystal is the only configuration that can benefit from both
Toward 3d quasicrystals

Mermin-Troian: an “RKKY”-type interaction peaked at 1.0515 $k_0$ gives rise to an icosahedral quasicrystal

How do we do this? Two difficulties

- How to get a spherical dispersion minimum? (Various possible schemes exist, including generalizations of the NIST idea with six lasers)
- The RKKY interaction is peaked at zero momentum transfer

We shall ignore the first problem and focus on the second
Interaction renormalization in 3d

Two spheres intersect on a circle. Intermediate-leg phase space maximized when fermion scatters through a large angle, minimized for small momentum transfer.

Cf. 2D in which the max. phase space was for small momentum transfer.

Thus, for repulsive Bose-Fermi interactions the renormalized RKKY is even more strongly peaked at $k = 0$.

However, for attractive Bose-Fermi interactions there is still hope...

Increasing attractive $V$
Renormalized RKKY interaction in 3d

\[ V(q) \sim - \left( \frac{-V_0}{1 - mV_0 \frac{k_0}{8\pi k_F} \sqrt{4k_F^2 - q^2 \log \frac{k_F}{mV_F}}} \right)^2 \chi(q) \]

\[ \chi(q) \approx 1/E_F \left[ 1 - q^2/(2k_F)^2 \right] \]
Quantum quasicrystals beyond mean-field

New aspects of crystal order intertwined with superfluidity. Example FFLO

Significance of wavefunction sign
Hybrid defects, e.g. half-vortex half-dislocation or half-vortex half-disclination

Even richer phenomena for quantum quasicrystals
Summary

Modulation spectroscopy

BEC polarons

Quasicrystals