Measuring correlation functions in interacting systems of cold atoms

Ehud Altman
Ryan Barnett
Mikhail Lukin
Dmitri Petrov
Anatoli Polkovnikov
Eugene Demler

Physics Department,
Harvard University

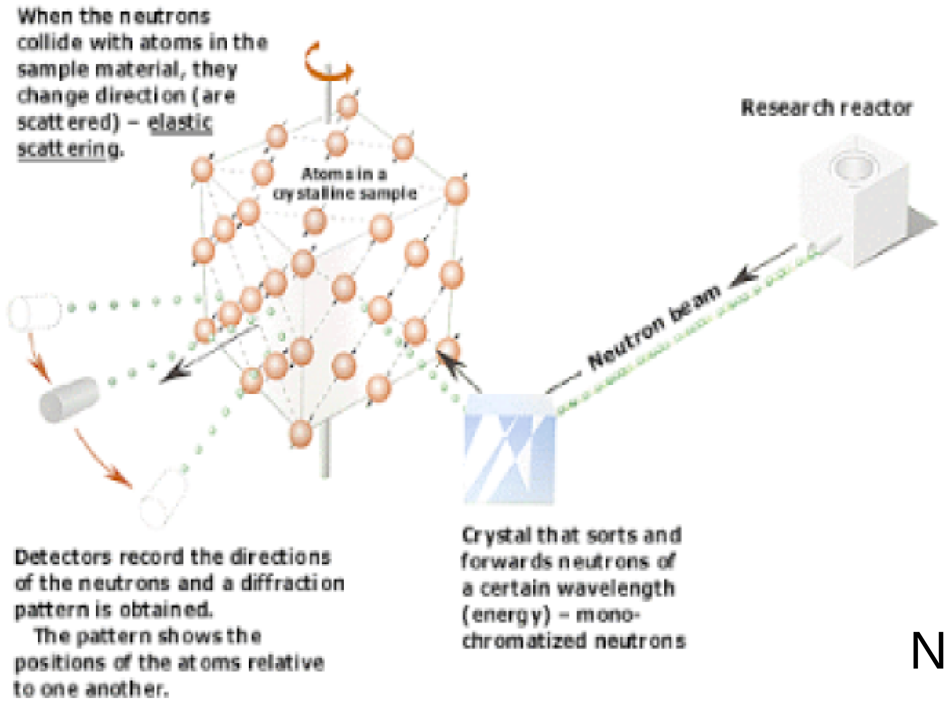
Thanks to: J. Schmiedmayer, M. Oberthaler, V. Vuletic, M. Greiner

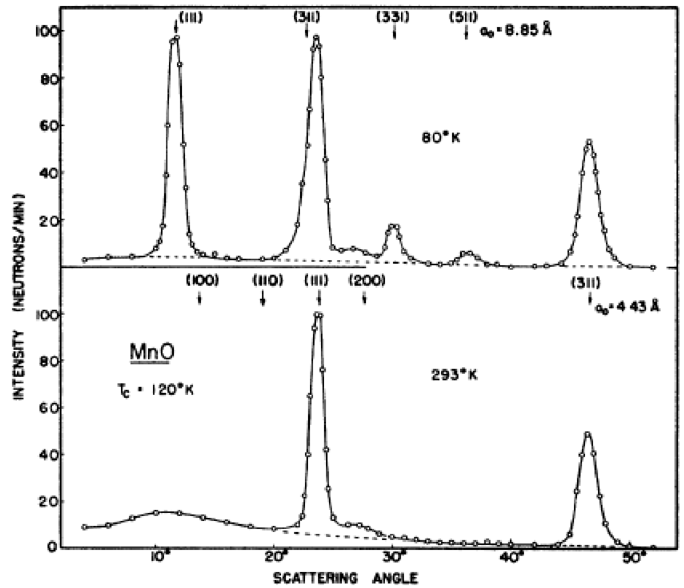
Correlation functions in condensed matter physics

Most experiments in condensed matter physics measure correlation functions

Example: neutron scattering measures spin and density correlation functions

$$S_{\mathrm{s}}(q) = \int d\mathbf{r}\, e^{i\,\mathbf{q}\,\mathbf{r}} \langle\, S^{+}(\mathbf{r})\, S^{-}(0)\,\rangle \qquad S_{\rho}(q) = \int d\mathbf{r}\, e^{i\,\mathbf{q}\,\mathbf{r}} \langle\, \rho(\mathbf{r})\, \rho(0)\,\rangle$$





Neutron diffraction patterns for MnO

Shull et al., Phys. Rev. 83:333 (1951)

Outline

1. Measuring correlation functions in intereference experiments
   Introduction: interference of independent condensates
   1D systems: Luttinger liquid behavior
   2D systems: quasi long range order and the KT transition

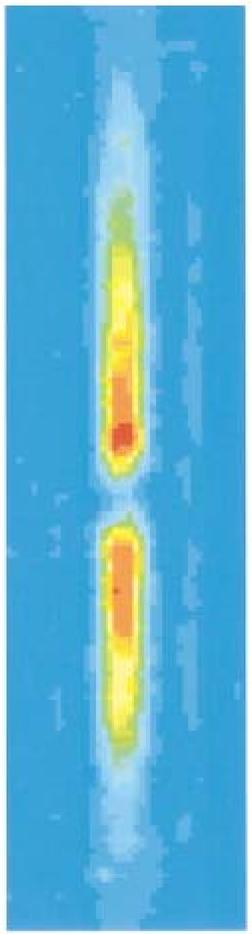
2. Quantum noise interferometry of atoms in an optical lattice

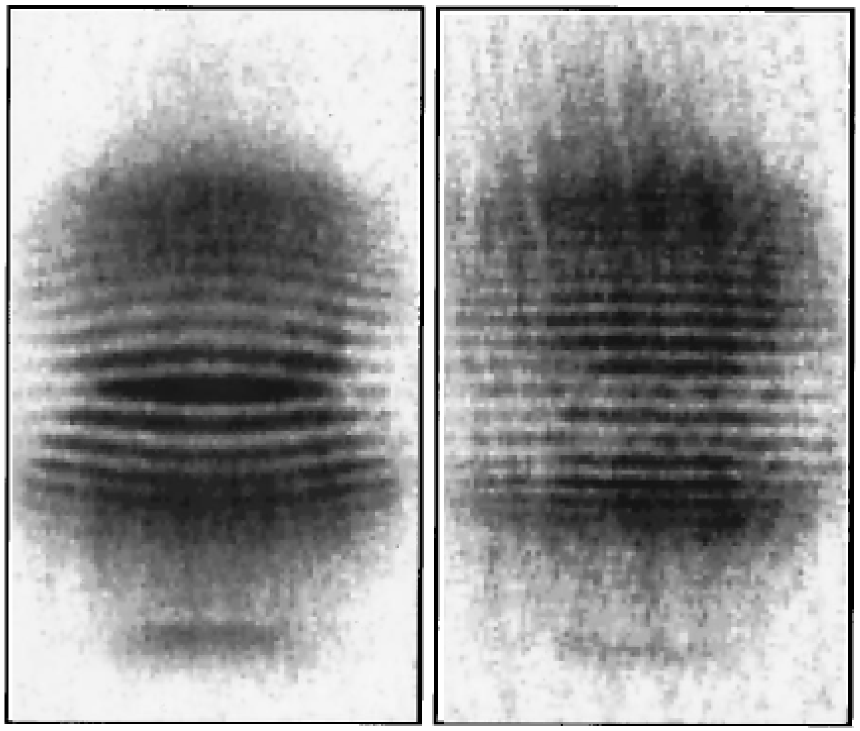
3. Applications of quantum noise interferometry
   Spin order in Mott states of atomic mixtures
   Polar molecules in optical lattices. Charge and spin order

Measuring correlation functions in intereference experiments

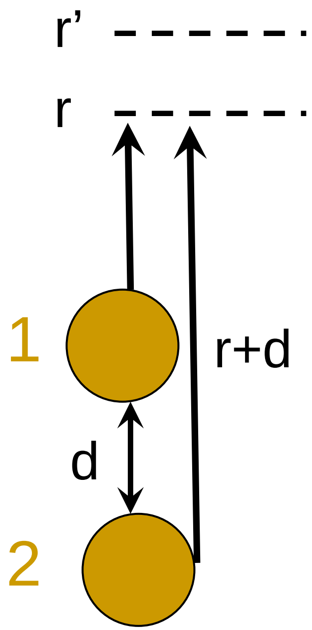
Interference of two independent condensates

Andrews et al., Science 275:637 (1997)





Interference of two independent condensates



$$\rho_{\text{int}}(r) = a_1^\dagger(r)\, a_2(r) + \text{c.c.}$$

$$a_1(r) = e^{i\phi_1 + i k_1 r} \qquad k_1 = \frac{m v_1}{\hbar} = \frac{m r}{\hbar t}$$

$$a_2(r) = e^{i\phi_2 + i k_2 r} \qquad k_2 = \frac{m v_2}{\hbar} = \frac{m(r+d)}{\hbar t}$$

$$\rho_{\text{int}}(r) = e^{i(k_2 - k_1) r}\, e^{i(\phi_2 - \phi_1)} + \text{c.c.}$$

$$\rho_{\text{int}}(r) = e^{i \frac{m d r}{\hbar t}}\, e^{i(\phi_2 - \phi_1)} + \text{c.c.}$$

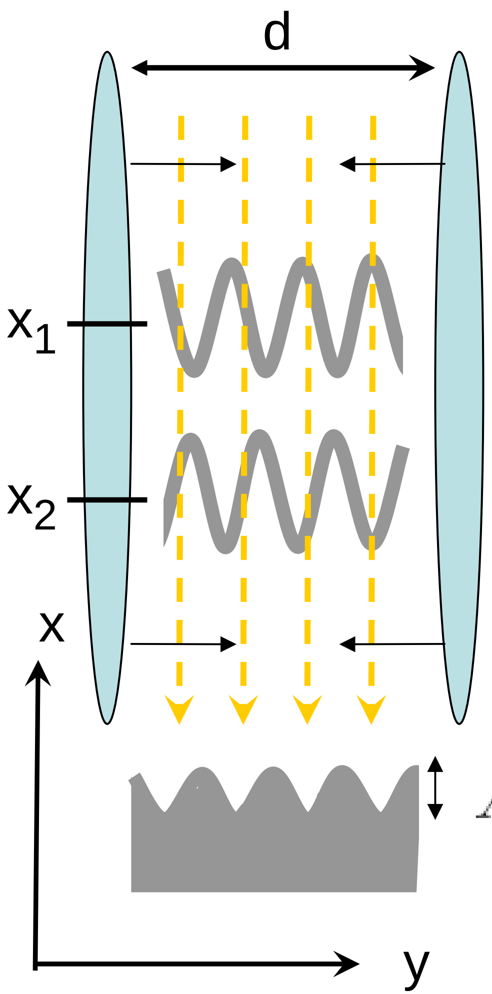
Clouds 1 and 2 do not have a well defined phase difference.
However each individual measurement shows an interference pattern

$$\langle \rho_{\text{int}}(r) \rangle = 0$$

$$\langle \rho_{\text{int}}(r)\, \rho_{\text{int}}(r') \rangle = e^{i \frac{m d}{\hbar t}(r - r')} + \text{c.c.}$$

Interference of one dimensional condensates

Similar experimental setup: Schmiedmayer et al.



$$d\rho_{\rm int}(x,y) = \left(e^{i\frac{mdy}{\hbar t}}\, a_1^\dagger(x)\, a_2(x) + {\rm c.c.}\right) dx$$

$$\sim \left(e^{i\frac{mdy}{\hbar t}}\, e^{i(\phi_2(x)-\phi_1(x))} + {\rm c.c.}\right) dx$$

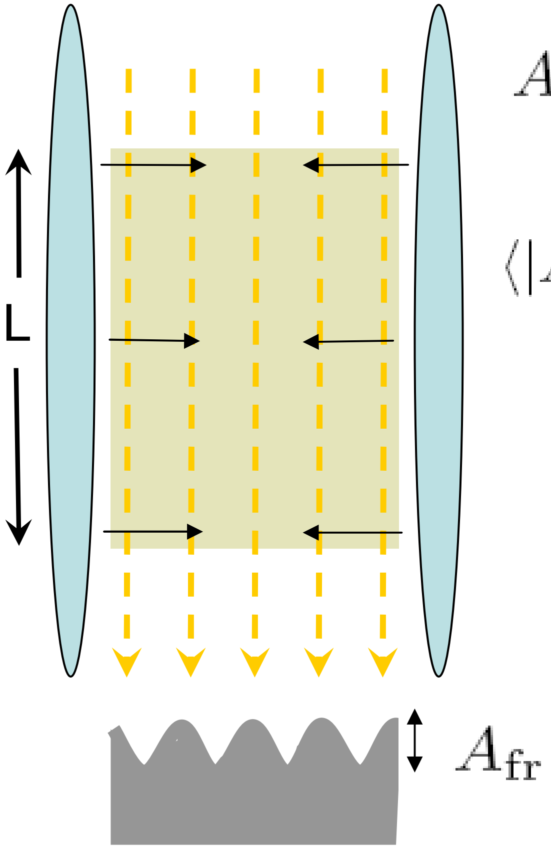
$$\rho_{\rm int}(y) = e^{i\frac{mdy}{\hbar t}} \int_0^L dx\, a_1^\dagger(x)\, a_2(x) + {\rm c.c.}$$

$A_{\rm fr}$

$$\rho_{\rm int}(y) = A_{\rm fr}\, e^{i\Delta\phi + i\frac{mdy}{\hbar t}} + {\rm c.c.}$$

Amplitude of the interference fringes, $A_{\rm fr}$, contains information about phase fluctuations within individual condensates

Interference of one dimensional condensates



$$A_{\rm fr}\, e^{i\Delta\phi} = \int_0^L dx\, a_1^\dagger(x)\, a_2(x)$$

$$\langle |A_{\rm fr}|^2 \rangle = \int_0^L \int_0^L dx\, dy\, \langle a_1^\dagger(x)\, a_2(x)\, a_2^\dagger(y)\, a_1(y) \rangle$$

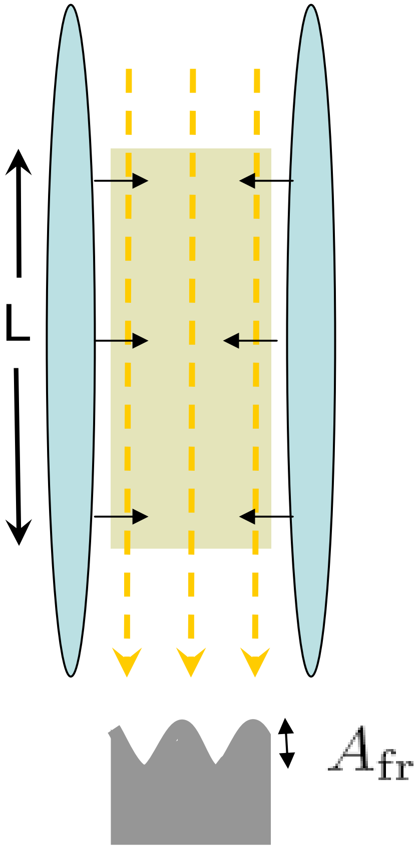
$$\simeq L \int_0^L dx\, \langle a_1(x)\, a_1^\dagger(0) \rangle \langle a_2(0) a_2^\dagger(x) \rangle$$

For identical condensates

$$\langle |A_{\rm fr}|^2 \rangle = L \int_0^L dx\, (G(x))^2$$

Instantaneous correlation function $G(x) = \langle a(x)\, a^\dagger(0) \rangle$

Interference of one dimensional condensates



Luttinger liquid at T=0

$$G(x) \sim \rho \left(\frac{\xi_h}{x} \right)^{2-1/K}$$

K – Luttinger parameter

$$\langle |A_{\rm fr}|^2 \rangle \sim (\rho \xi_h)^{1/K} (L\rho)^{2-1/K}$$

For non-interacting bosons $K = \infty$ and $A_{\rm fr} \sim L$

For impenetrable bosons $K = 1$ and $A_{\rm fr} \sim \sqrt{L}$

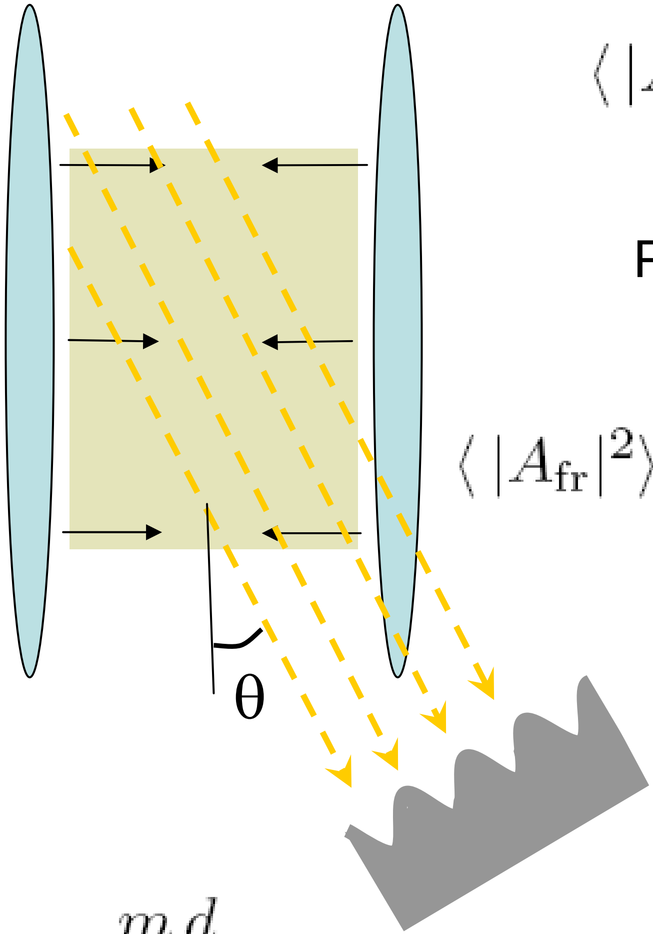
Luttinger liquid at finite temperature

$$\langle |A_{\rm fr}|^2 \rangle \sim L \rho^2 \xi_h \left(\frac{\hbar^2}{m \xi_h^2} \frac{1}{T} \right)^{1-1/K}$$

Luttinger parameter *K* may be extracted from the *L* or *T* dependence of $A_{\rm fr}$

Interference of one dimensional condensates

Luttinger liquid at T=0. Rotated probe beam experiment



$$\langle |A_{\rm fr}|^2 \rangle = L \int_0^L dx \;\cos(\, qx\,)\,(\,G\,(x)\,)^2$$

For large imaging angle, $q\,L >> 1$

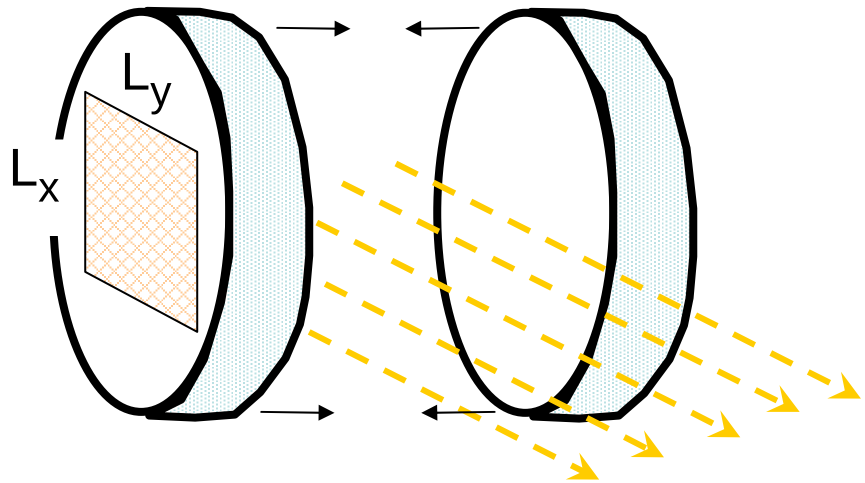
$$\langle |A_{\rm fr}|^2 \rangle \sim L\,\rho^2\,\xi_h\,\sin(\frac{\pi}{K})\,\Gamma(1-\frac{2}{K})\,(\,\xi_h\,q\,)^{1/K-1}$$

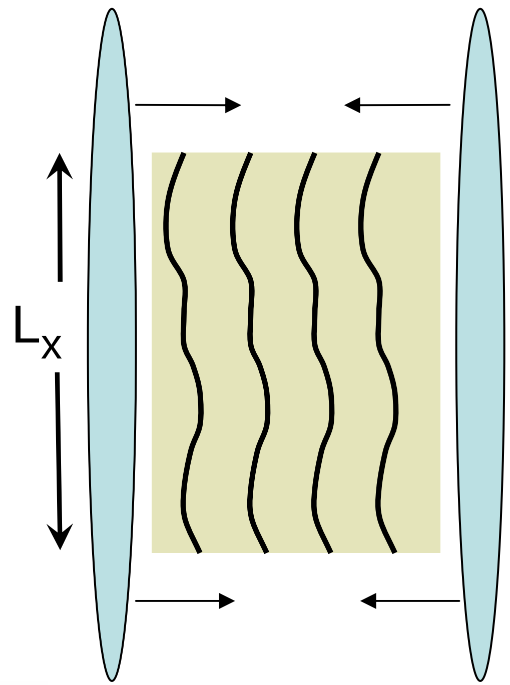
Luttinger parameter *K* may be extracted from the angular dependence of $A_{\rm fr}\,(\,\theta\,)$

$$q = \frac{m\,d}{\hbar\,t}\tan\theta$$

Interference of two dimensional condensates

Similar experimental setup: Stock et al., cond-mat/0506559



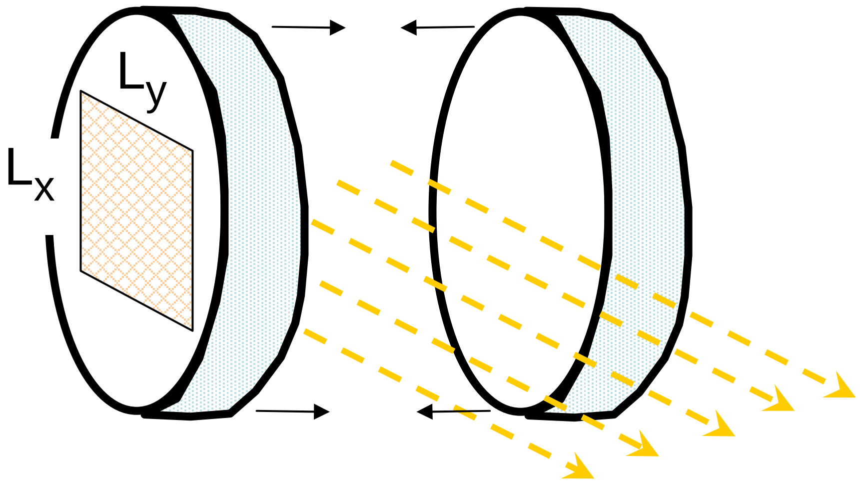


Probe beam parallel to the plane of the condensates

$$\langle\, |A_{\rm fr}|^2 \rangle \,=\, L_x\, L_y \int_0^{L_x} \int_0^{L_y} d^2\vec{r}\; (\, G(\vec{r}\,)\,)^2$$

$$G(\vec{r}\,) \,=\, \langle\, a(\vec{r}\,)\, a^\dagger(0)\,\rangle$$

Interference of two dimensional condensates. Quasi long range order and the KT transition



Above KT transition

$$G(r) \sim e^{-r/\xi}$$

$$\langle |A_{\rm fr}|^2 \rangle \sim L_x L_y$$

$$\log \xi(T) \sim 1/\sqrt{T - T_{\rm KT}}$$

Below KT transition

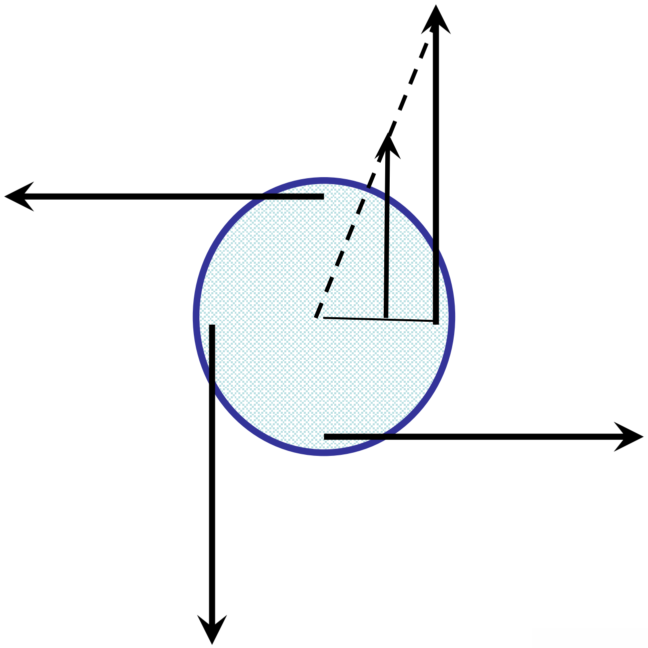
$$G(r) \sim \rho \left(\frac{\xi_h}{r} \right)^{\alpha}$$

$$\alpha(T) = \frac{mT}{2\pi \rho_s(T) \hbar^2}$$

$$\langle |A_{\rm fr}|^2 \rangle \sim (L_x L_y)^{2-\alpha}$$

One can also use rotated probe beam experiments to extract α from the angular dependence of $A_{\rm fr}$

Rapidly rotating two dimensional condensates



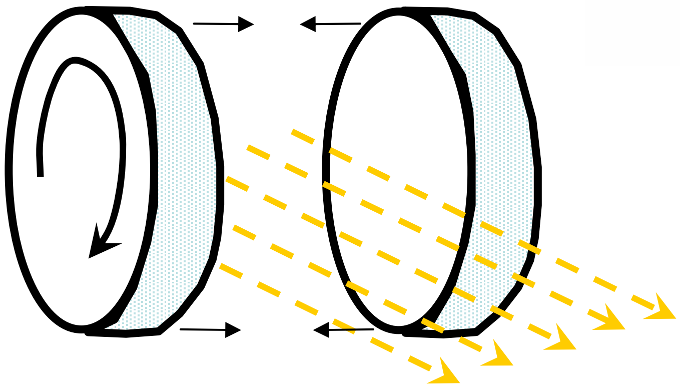
$$\langle \rho(r) \rangle$$

$$\langle \rho(r) \rho(r') \rangle$$

Time of flight experiments with rotating condensates correspond to density measurements

$$\langle |A_{\mathrm{fr}}|^2 \rangle = L_x L_y \int_0^{L_x} \int_0^{L_y} d^2\vec{r} \cos(\vec{q}\vec{r}) \, (G(\vec{r}))^2$$

$$G(\vec{r}) = \langle a(\vec{r}) \, a^\dagger(0) \rangle$$

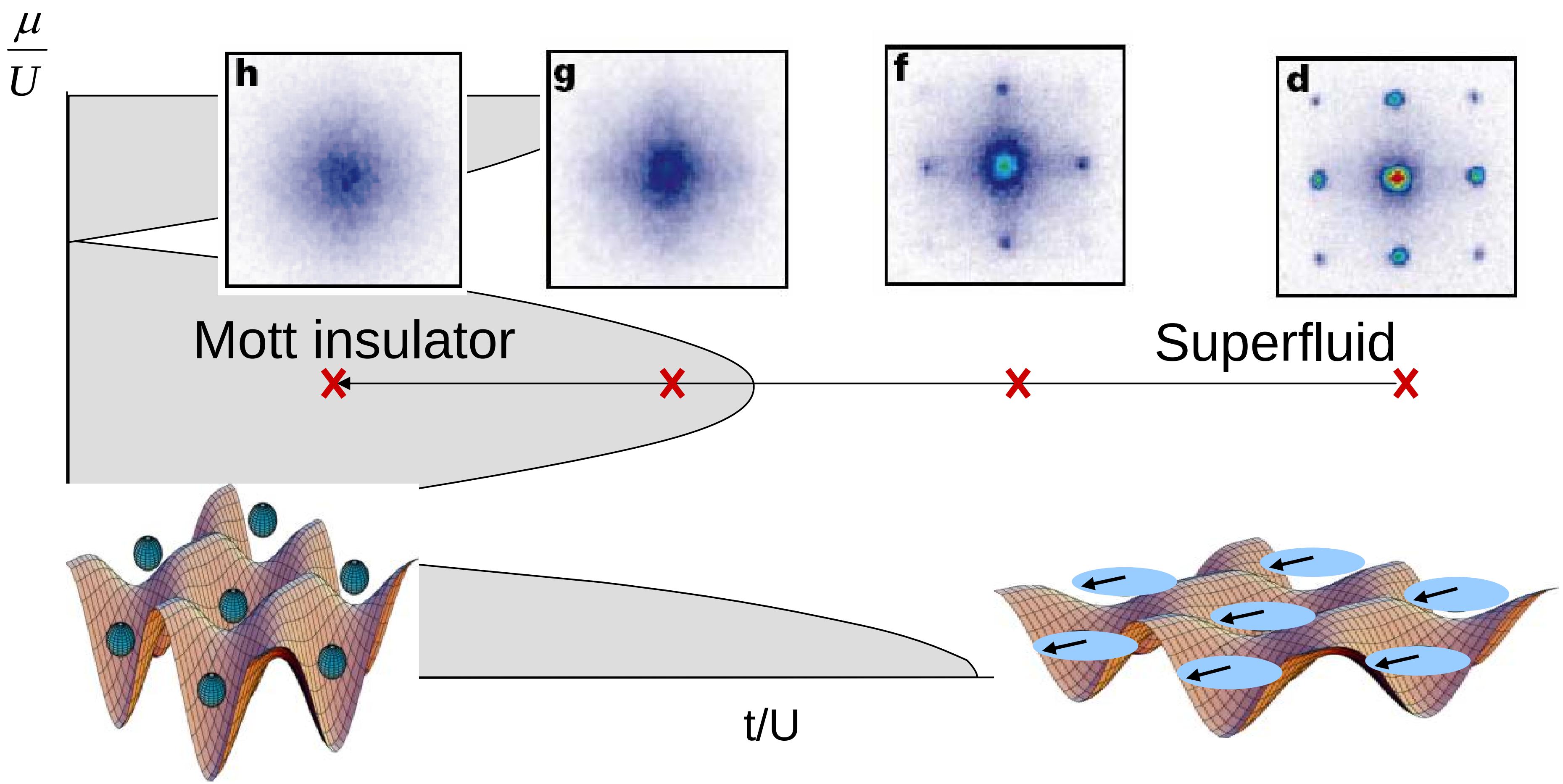


Interference experiments measure single particle correlation functions in the rotating frame

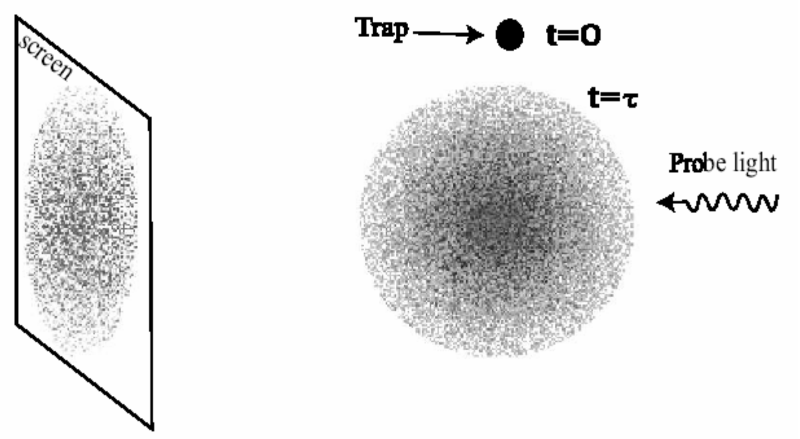
Quantum noise interferometry of atoms in an optical lattice

Atoms in an optical lattice.
Superfluid to Insulator transition

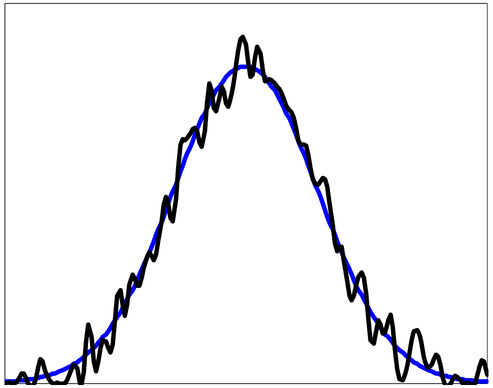
Greiner et al., Nature 415:39 (2002)

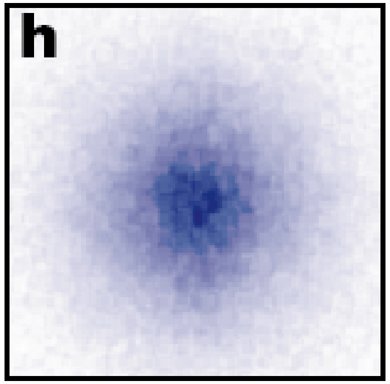


Time of flight experiments



Quantum noise interferometry of atoms in an optical lattice



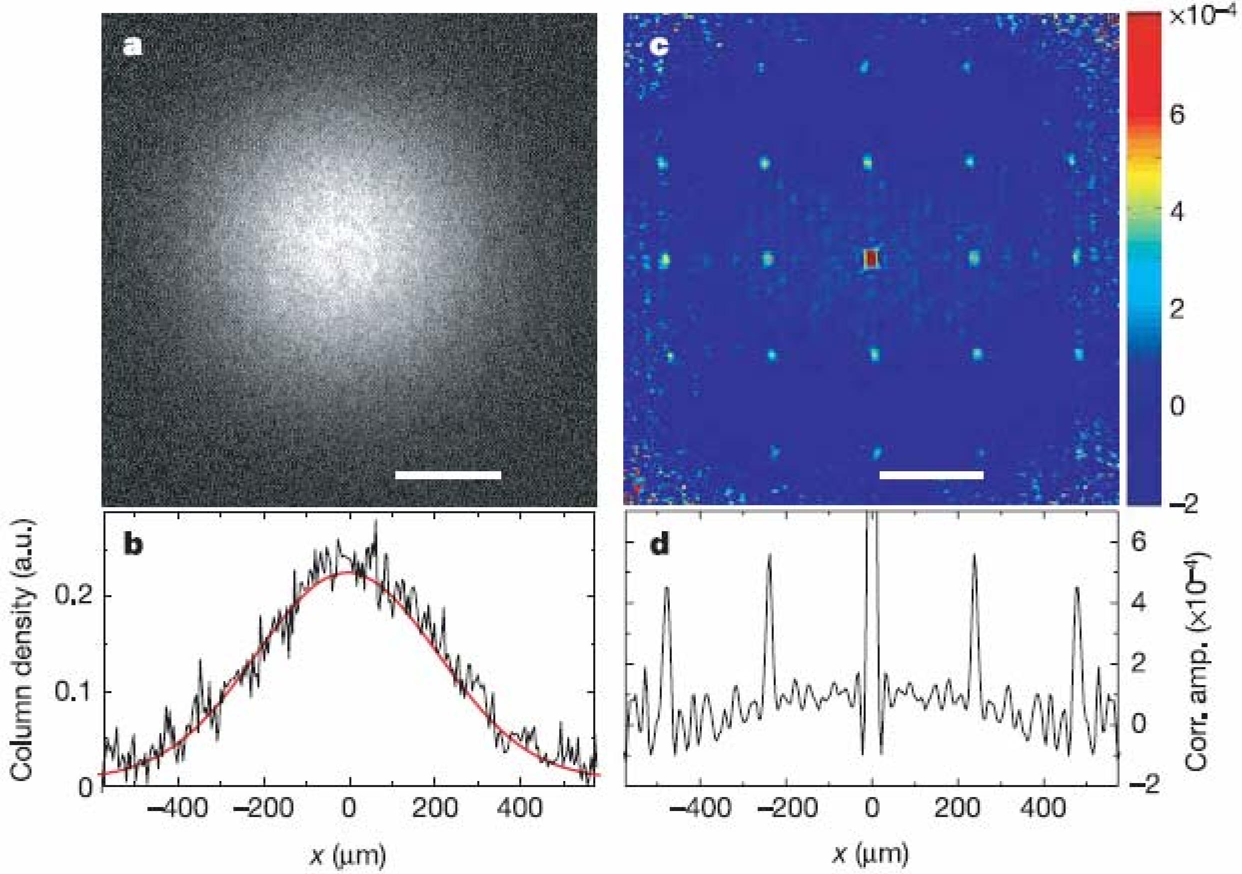


Second order coherence $G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$

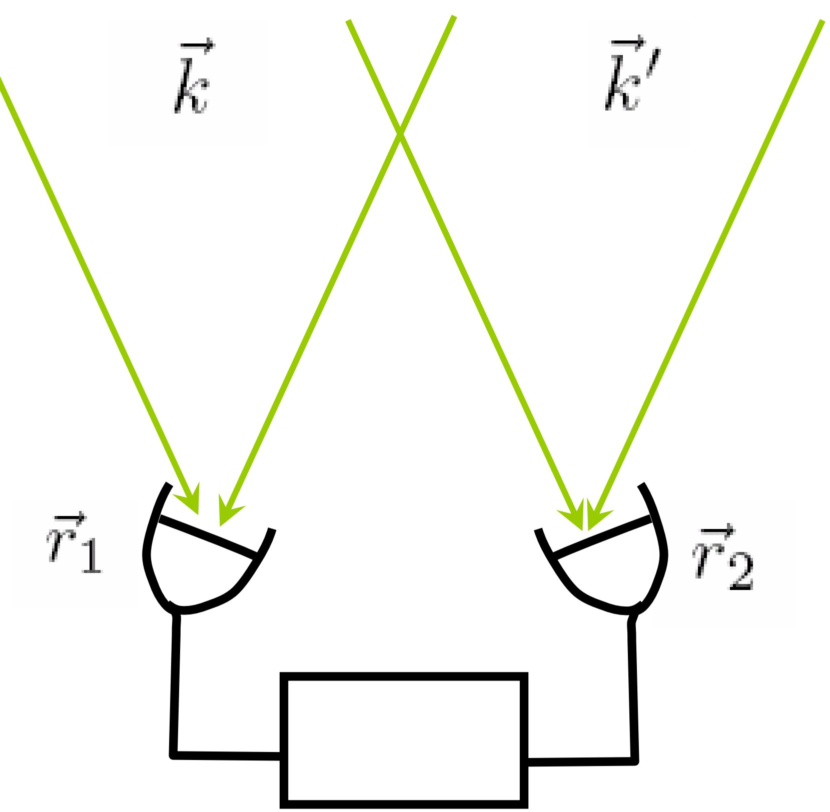
Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

Theory: Altman et al., PRA 70:13603 (2004)

Experiment: Folling et al., Nature 434:481 (2005)

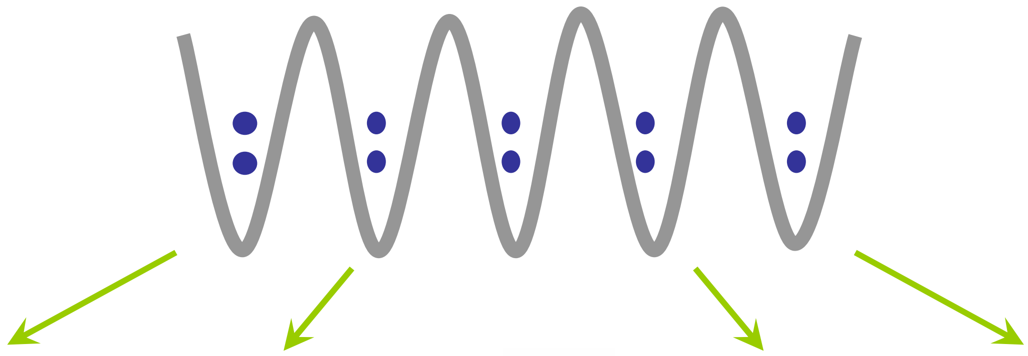


Hanburry-Brown-Twiss stellar interferometer



$$\langle\, I(\vec{r}_1)\; I(\vec{r}_2)\rangle = A + B\;\cos\Big((\vec{k}-\vec{k}')\;(\vec{r}_1-\vec{r}_2)\Big)$$

Second order coherence in the insulating state of bosons



Bosons at quasimomentum $\vec{k}$ expand as plane waves

with wavevectors $\vec{k},\ \vec{k}+\vec{G}_1,\ \vec{k}+\vec{G}_2$

First order coherence: $\langle\ \rho(\vec{r})\ \rangle$

Oscillations in density disappear after summing over $\vec{k}$

Second order coherence: $\langle\ \rho(\vec{r}_1)\ \rho(\vec{r}_2)\ \rangle$

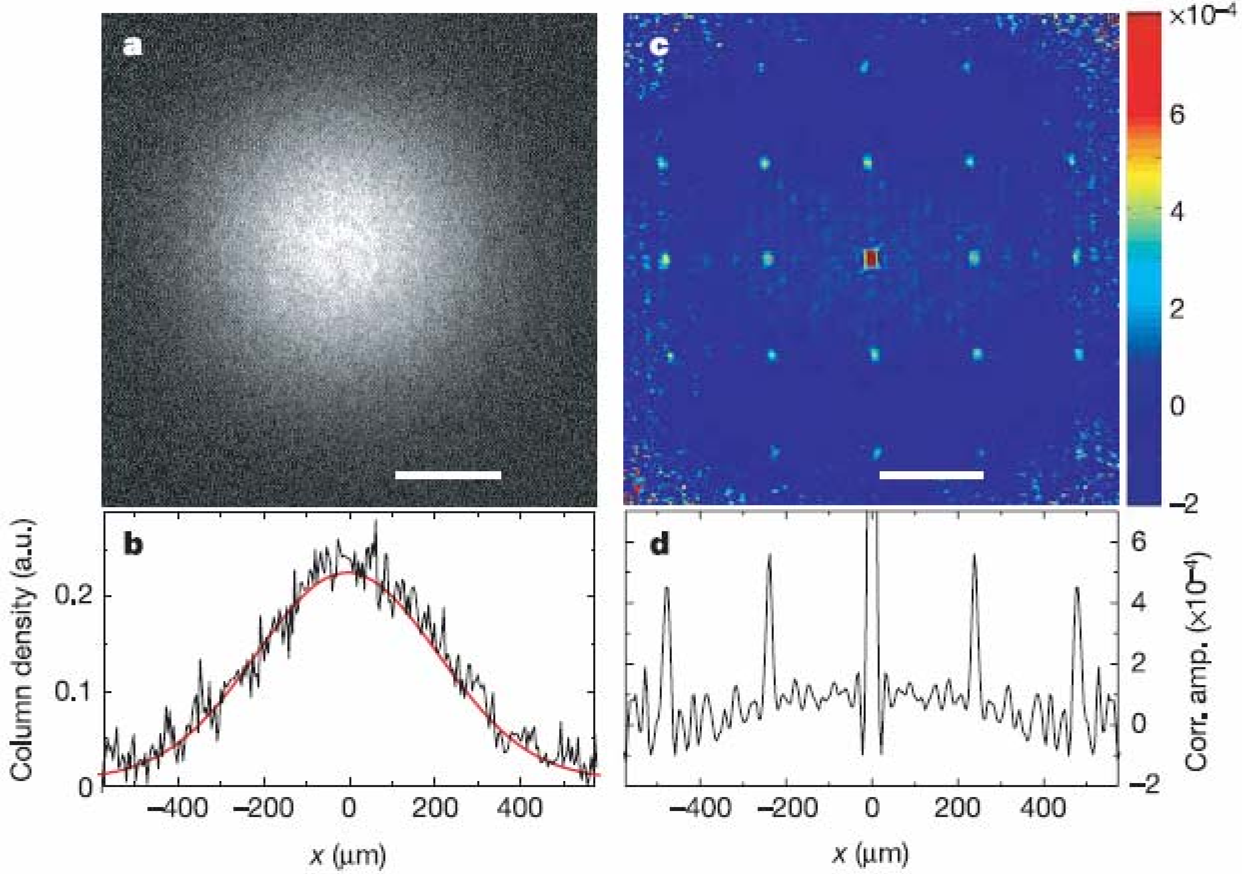
Correlation function acquires oscillations at reciprocal lattice vectors

$$\langle\ \rho(\vec{r}_1)\ \rho(\vec{r}_2)\ \rangle = A_0 + A_1\ \cos\left(\ \vec{G}_1(\vec{r}_1-\vec{r}_2)\ \right) + A_2\ \cos\left(\ \vec{G}_2(\vec{r}_1-\vec{r}_2)\ \right) + \ldots$$

Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

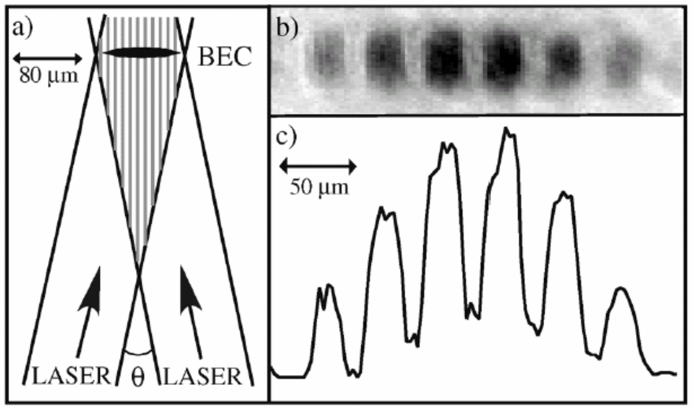
Theory: Altman et al., PRA 70:13603 (2004)

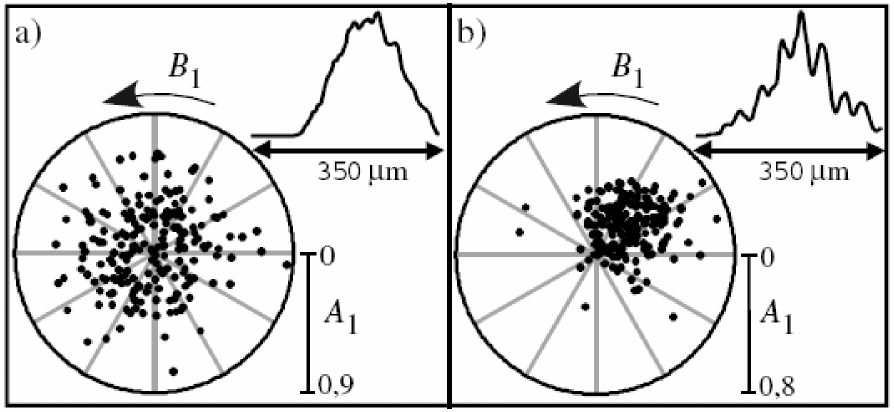
Experiment: Folling et al., Nature 434:481 (2005)



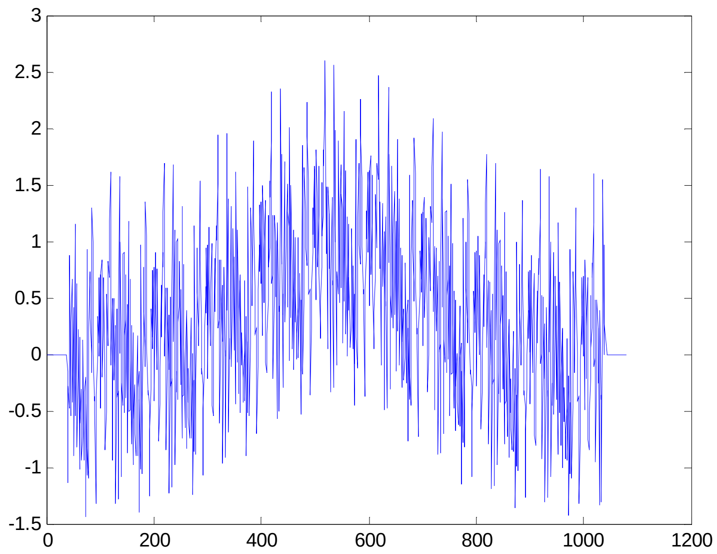
Interference of an array of independent condensates

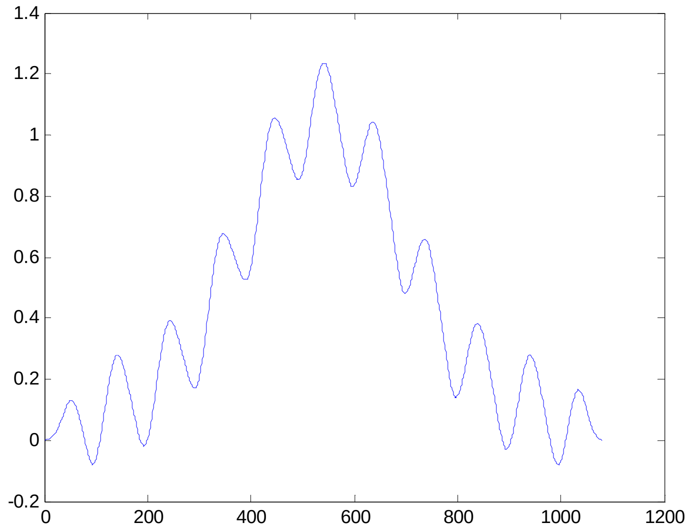
Hadzibabic et al., PRL 93:180403 (2004)





Smooth structure is a result of finite experimental resolution (filtering)



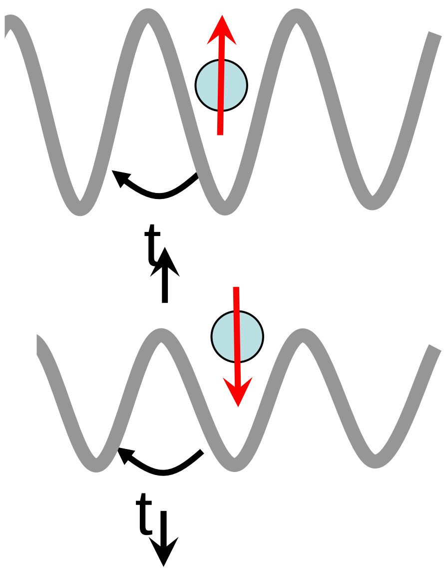


Applications of quantum noise interferometry

Spin order in Mott states of atomic mixtures

Two component Bose mixture in optical lattice

Example: $^{87}\mathrm{Rb}$.   Mandel et al., Nature 425:937 (2003)



$$|\uparrow\rangle = |F=1, m_F=-1\rangle$$

$$|\downarrow\rangle = |F=2, m_F=-2\rangle$$

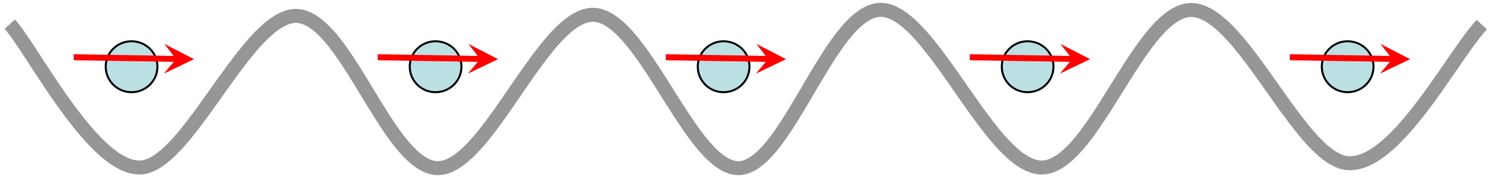
Two component Bose Hubbard model

$$\mathcal{H} = -t_\uparrow \sum_{\langle ij \rangle} b^\dagger_{i\uparrow} b_{j\uparrow} - t_\downarrow \sum_{\langle ij \rangle} b^\dagger_{i\downarrow} b_{j\downarrow} + U_{\uparrow\uparrow} \sum_i n_{i\uparrow}(n_\uparrow - 1) + U_{\downarrow\downarrow} \sum_i n_{i\downarrow}(n_\downarrow - 1) + U_{\uparrow\downarrow} \sum_i n_{i\uparrow} n_\downarrow$$

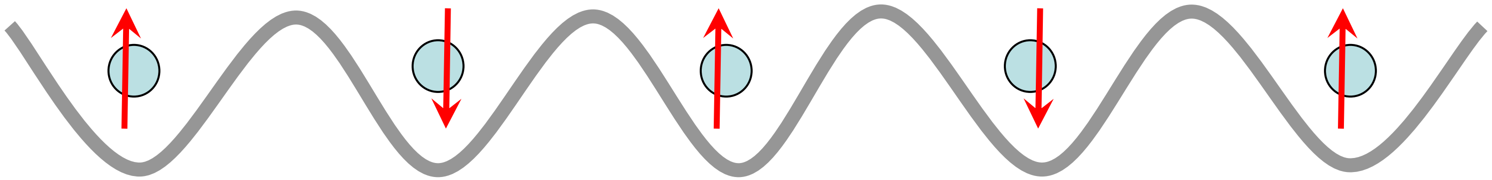
Two component Bose mixture in optical lattice. Magnetic order in an insulating phase

Insulating phases with N=1 atom per site. Average densities $n_\uparrow = n_\downarrow = \frac{1}{2}$

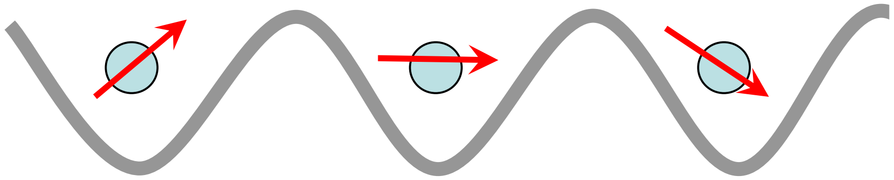
Easy plane ferromagnet $| \Psi \rangle = \prod_i \left(b^\dagger_{i\uparrow} + e^{i\phi} b^\dagger_{i\downarrow} \right) | 0 \rangle$



Easy axis antiferromagnet $| \Psi \rangle = \prod_{i \in A} b^\dagger_{i\uparrow} \prod_{i \in B} b^\dagger_{i\downarrow}$



Quantum magnetism of bosons in optical lattices



Kuklov and Svistunov, PRL (2003)

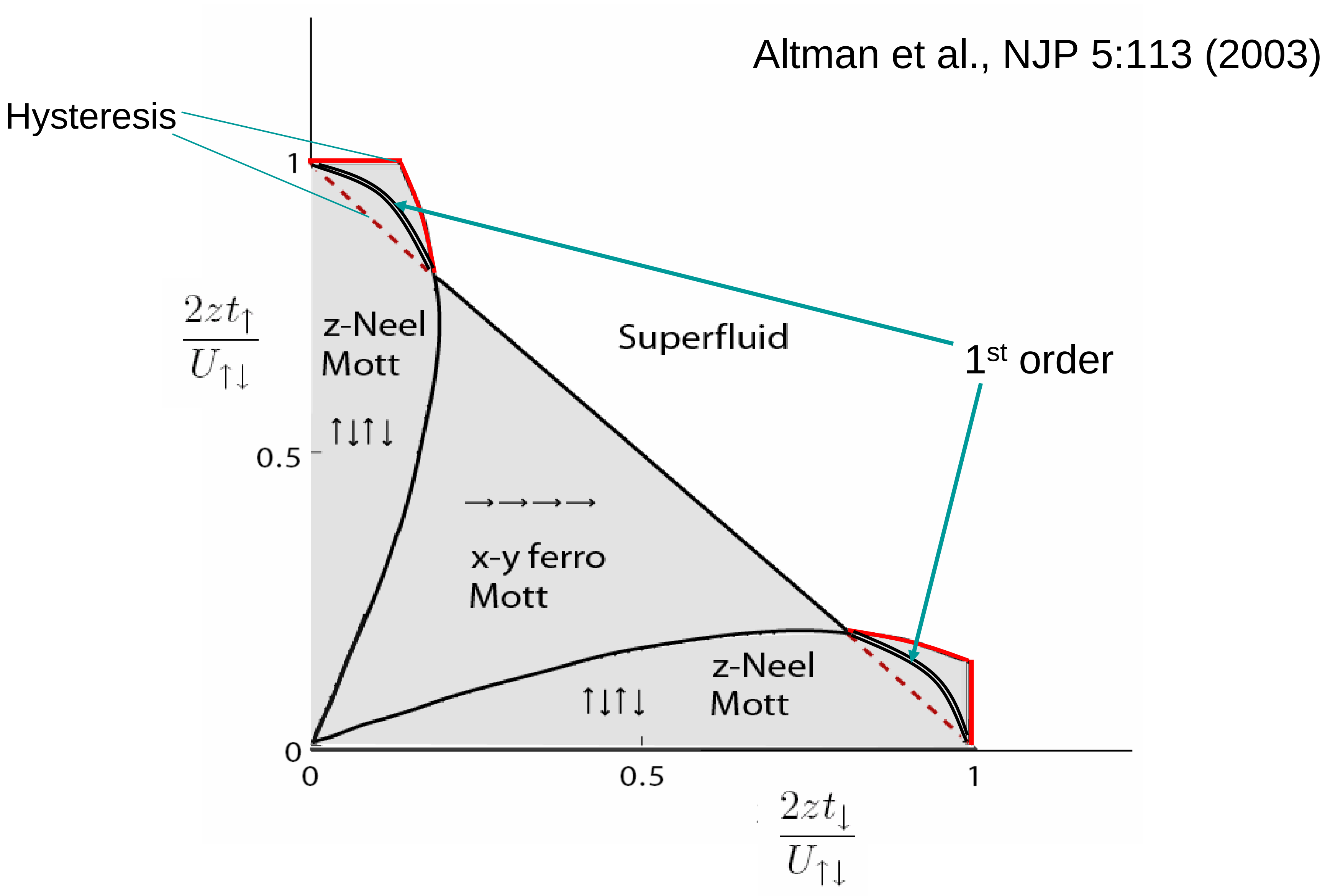
Duan et al., PRL (2003)

$$\mathcal{H} = J_z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + J_\perp \sum_{\langle ij \rangle} \left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right)$$

$$J_z = \frac{t_\uparrow^2 + t_\downarrow^2}{2U_{\uparrow\downarrow}} - \frac{t_\uparrow^2}{U_{\uparrow\uparrow}} - \frac{t_\downarrow^2}{U_{\downarrow\downarrow}} \qquad J_\perp = -\frac{t_\uparrow t_\downarrow}{U_{\uparrow\downarrow}}$$

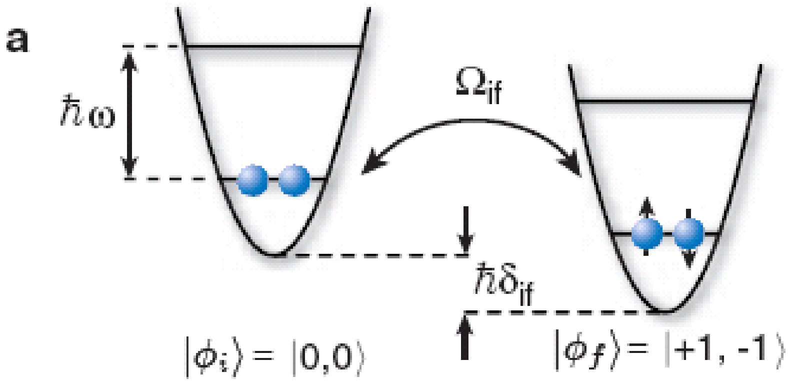
- Ferromagnetic $\quad U_{\uparrow\downarrow} >> U_{\uparrow\uparrow},\, U_{\downarrow\downarrow}$
- Antiferromagnetic $\quad U_{\uparrow\downarrow} << U_{\uparrow\uparrow},\, U_{\downarrow\downarrow}$

Two component Bose mixture in optical lattice.
Mean field theory + Quantum fluctuations

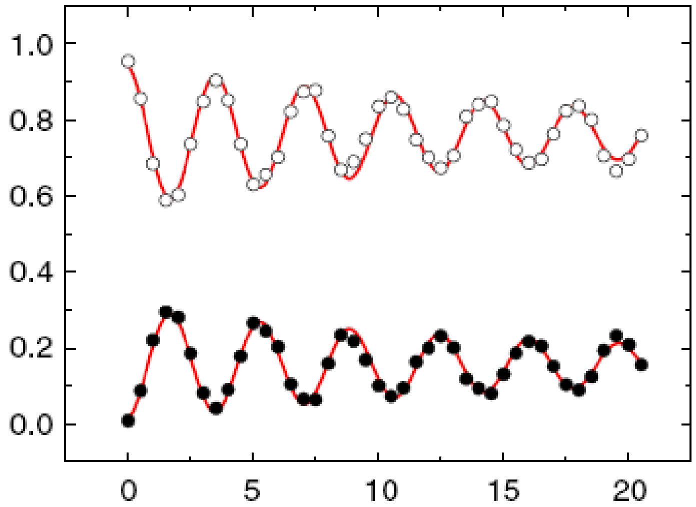


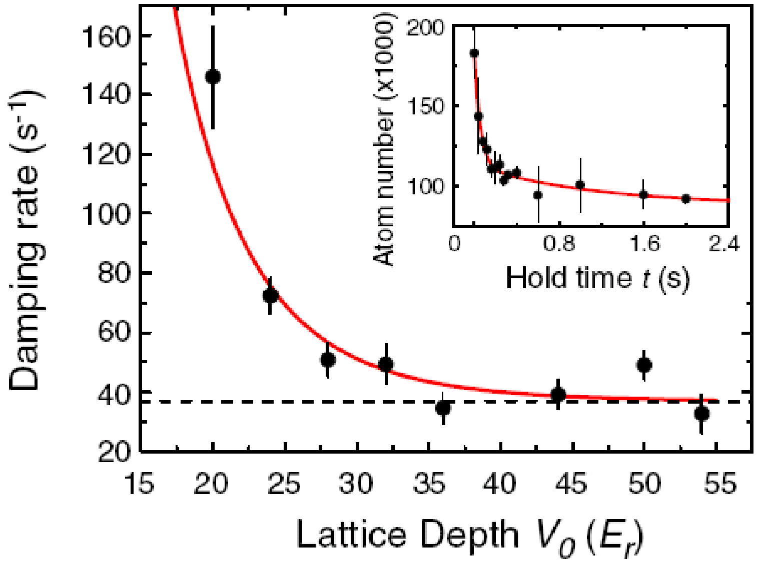
Coherent spin dynamics in optical lattices

Widera et al., cond-mat/0505492

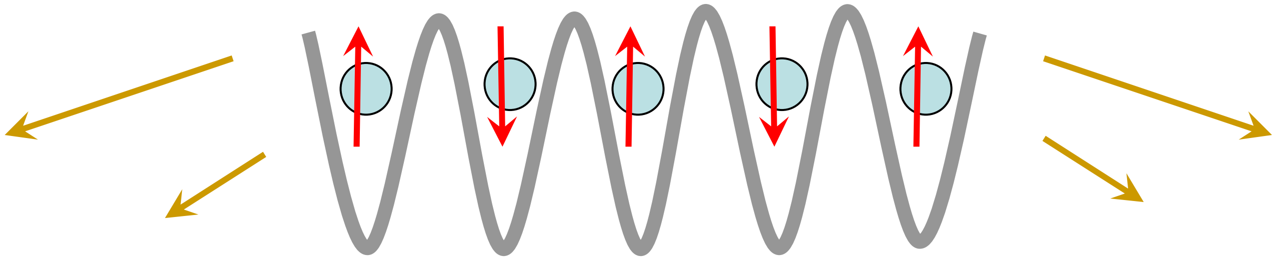


$^{87}\mathrm{Rb}$ **atoms in the F=2 state**





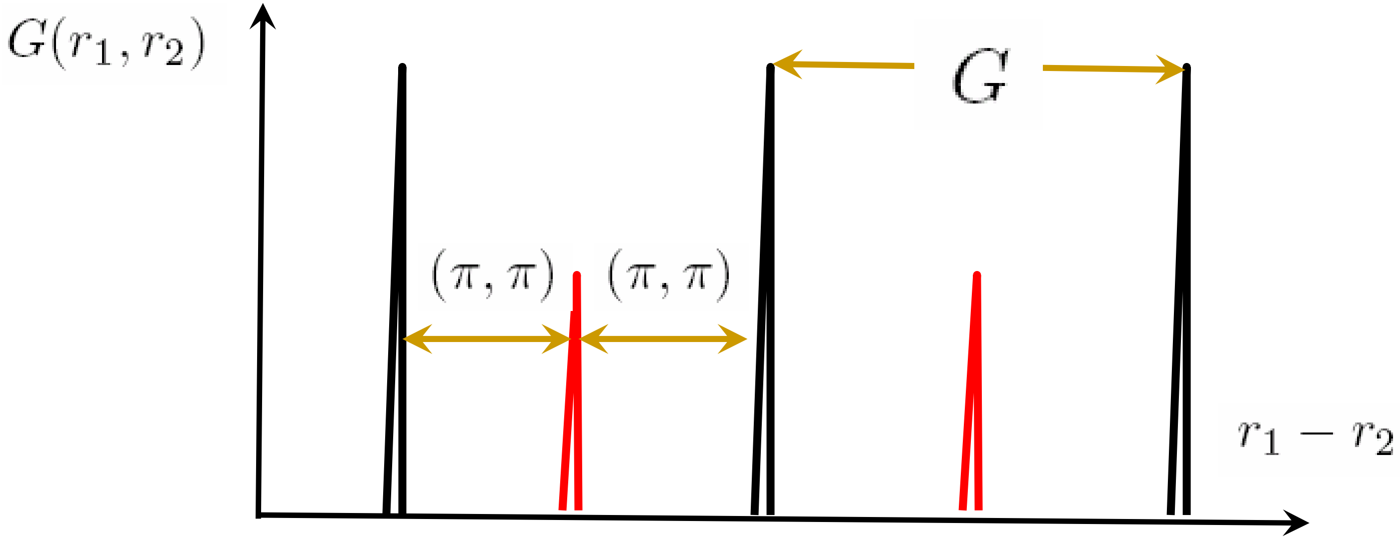
Probing spin order of bosons



$$r = \frac{\hbar\ k\ t}{m}$$

Correlation Function Measurements

$$G(r_1, r_2) = \langle\ n(r_1)\ n(r_2)\ \rangle_{\mathrm{TOF}} - \langle\ n(r_1)\ \rangle_{\mathrm{TOF}}\ \langle\ n(r_2)\ \rangle_{\mathrm{TOF}}$$
$$\sim \langle\ n(k_1)\ n(k_2)\ \rangle_{\mathrm{LAT}} - \langle\ n(k_1)\ \rangle_{\mathrm{LAT}}\ \langle\ n(k_2)\ \rangle_{\mathrm{LAT}}$$



Extra Bragg peaks appear in the second order correlation function in the AF phase

Applications of quantum noise interferometry

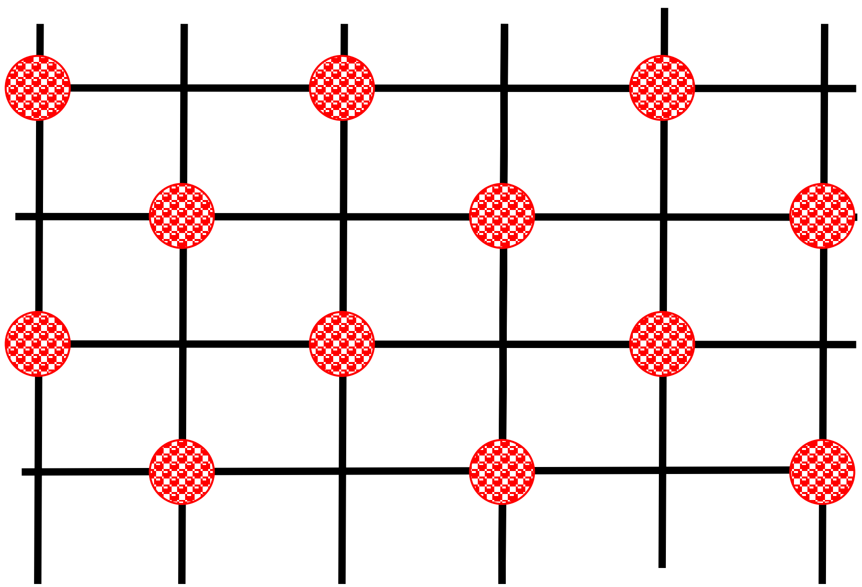
Polar molecules in optical lattices.
Charge and spin order

Extended Hubbard model

$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \sum_{ij} n_i U_{ij} n_j - \mu \sum_i n_i$$

U_0 - on site repulsion

U_1 - nearest neighbor repulsion

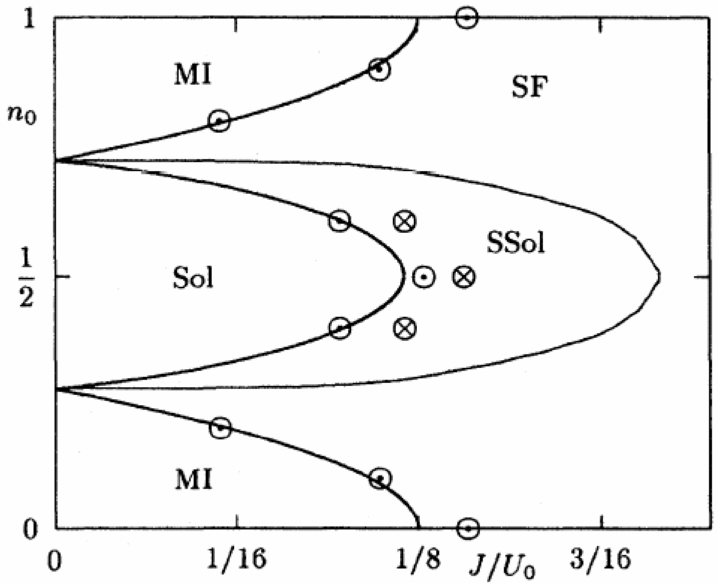


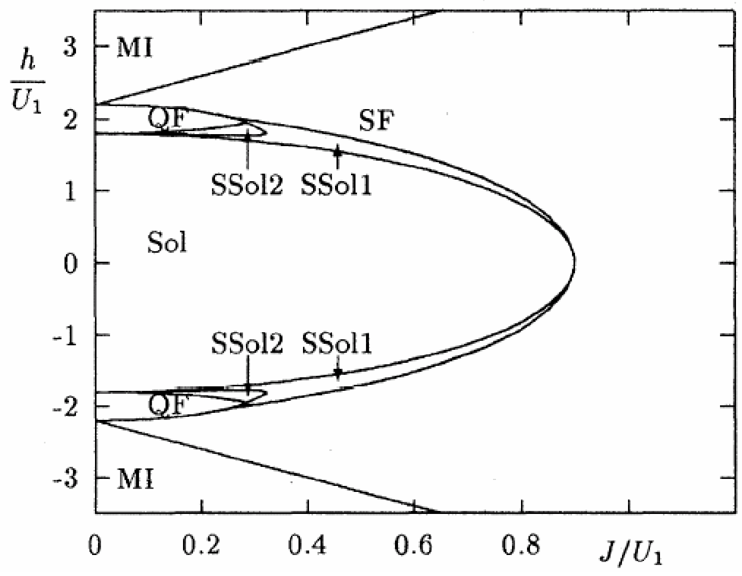
Checkerboard phase:

Crystal phase of bosons.
Breaks translational symmetry

Extended Hubbard model. Mean field phase diagram

van Otterlo et al., PRB 52:16176 (1995)



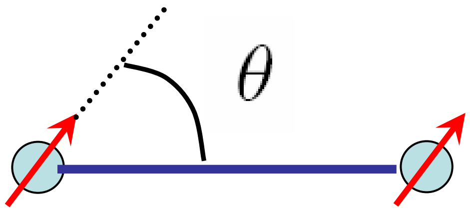


$$\frac{U_1}{U_0} = \frac{1}{5}$$

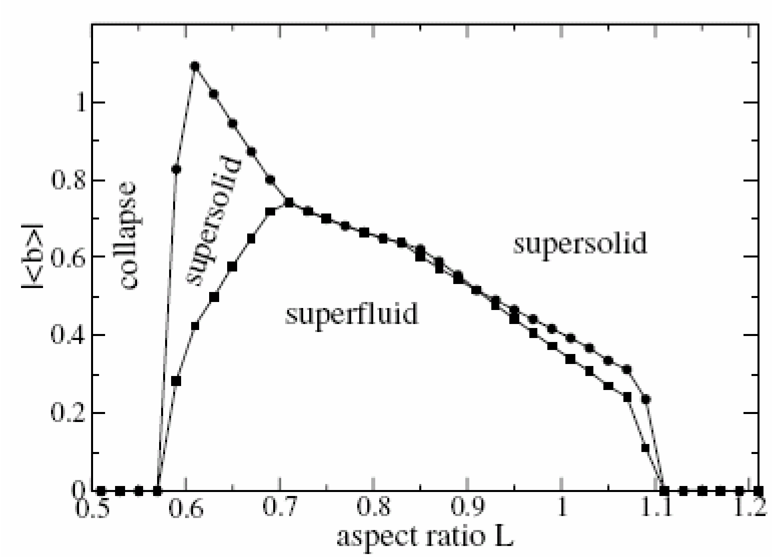
Hard core bosons. $\frac{U_2}{U_1} = \frac{1}{10}$

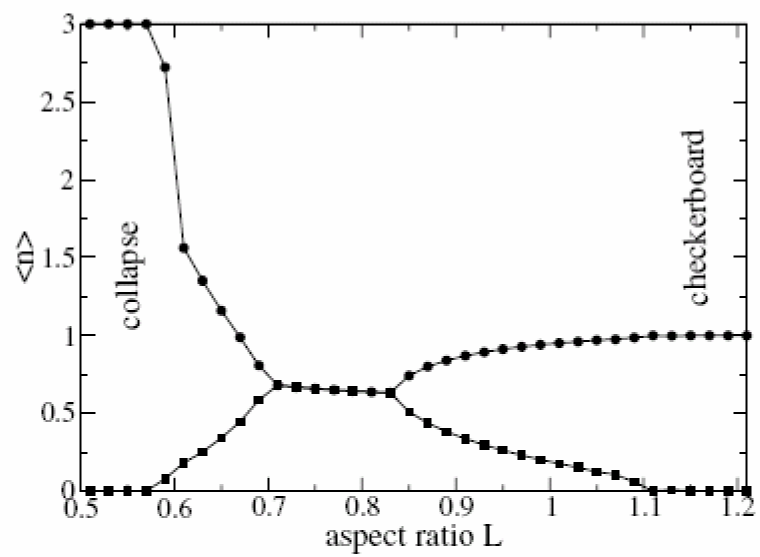
Supersolid – superfluid phase with broken translational symmetry

Dipolar bosons in optical lattices



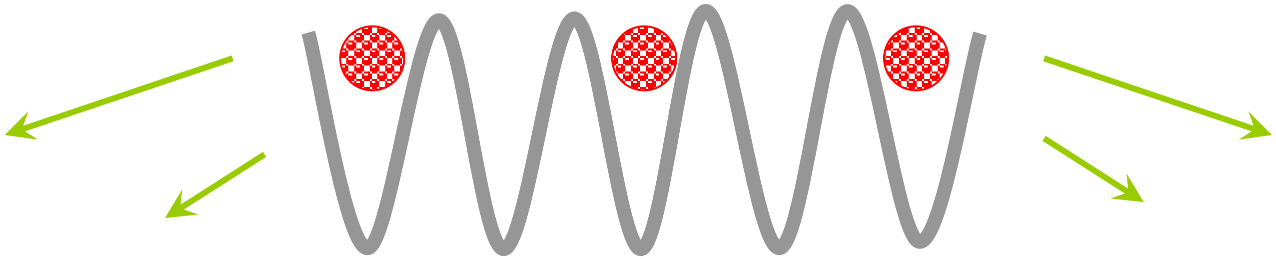
$$V_{\text{int}} = d^2\,\frac{1 - 3\cos^2\theta}{|\mathbf{r} - \mathbf{r}'|^3} + \frac{4\pi\hbar^2 a}{m}\,\delta(\mathbf{r} - \mathbf{r}')$$





Goral et al., PRL88:170406 (2002)

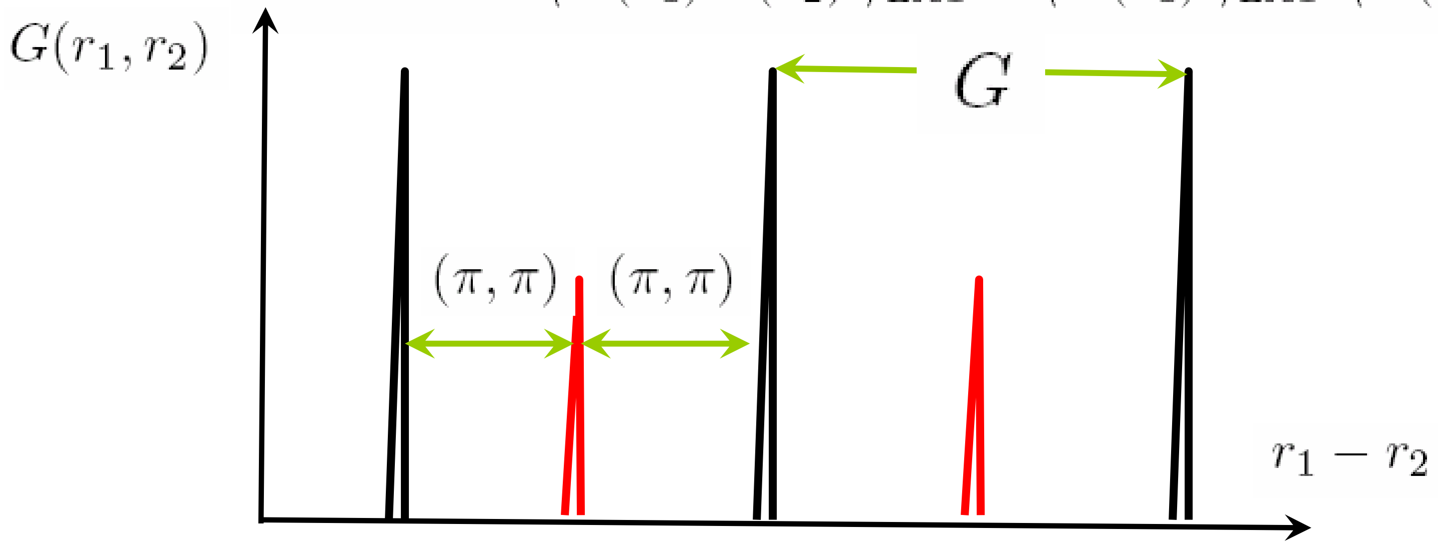
Probing a checkerboard phase



$$r = \frac{\hbar\, k\, t}{m}$$

Correlation Function Measurements

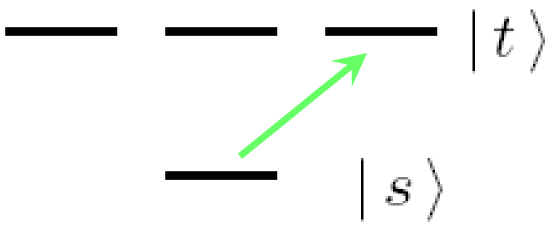
$$\begin{aligned} G(r_1, r_2) &= \langle\, n(r_1)\; n(r_2)\, \rangle_{\text{TOF}} - \langle\, n(r_1)\, \rangle_{\text{TOF}} \; \langle\, n(r_2)\, \rangle_{\text{TOF}} \\ &\sim \langle\, n(k_1)\; n(k_2)\, \rangle_{\text{LAT}} - \langle\, n(k_1)\, \rangle_{\text{LAT}} \; \langle\, n(k_2)\, \rangle_{\text{LAT}} \end{aligned}$$



Multicomponent polar molecules in an optical lattice. Long range interactions and quantum magnetism

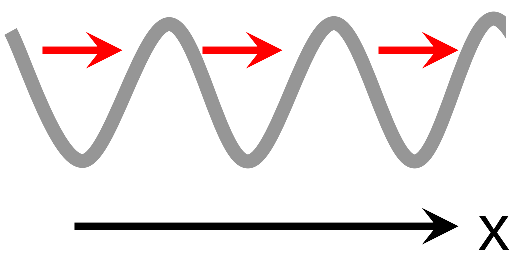
Barnett, Petrov, Lukin, Demler

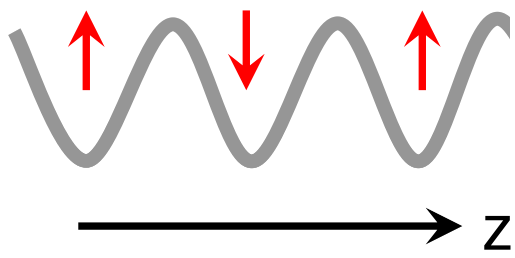
Electric dipolar interactions. Heteronuclear molecules.
Mixture of $l=0$ and $l=0$, $l_z=+1$ states.



$$d^x = d_0 \left(s^\dagger t + t^\dagger s \right)$$

$$d^y = \frac{d_0}{i} \left(s^\dagger t - t^\dagger s \right)$$





Conclusions

Interference of extended condensates can be used to probe correlation functions in one and two dimensional systems

Noise interferometry is a powerful tool for analyzing quantum many-body states in optical lattices