Interference of fluctuating condensates

Detection and characterization of many-body quantum phases of ultracold atoms

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Outline

Measuring correlation functions in intereference experiments

1. Interference of independent condensates as a probe of correlation functions
2. Distribution function of intereference fringes
3. Studying coherent dynamics in interference experiments

Quantum noise interferometry in time of flight experiments

Detection of antiferromagnetic ordering and pairing in optical lattices

References:

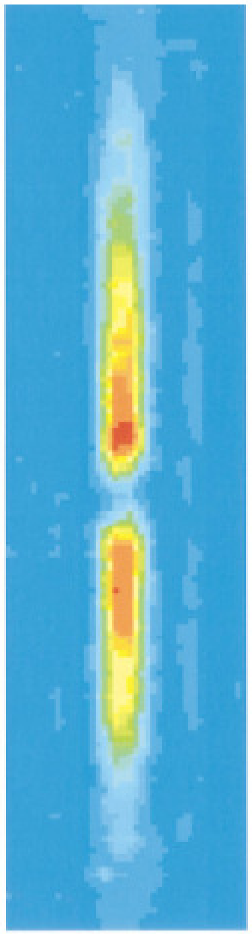
1. Polkovnikov, Altman, Demler, cond-mat/0511675
   Gritsev, Altman, Demler, Polkovnikov, cond-mat/0602475
2. Altman, Demler, Lukin, PRA 70:13603 (2004)

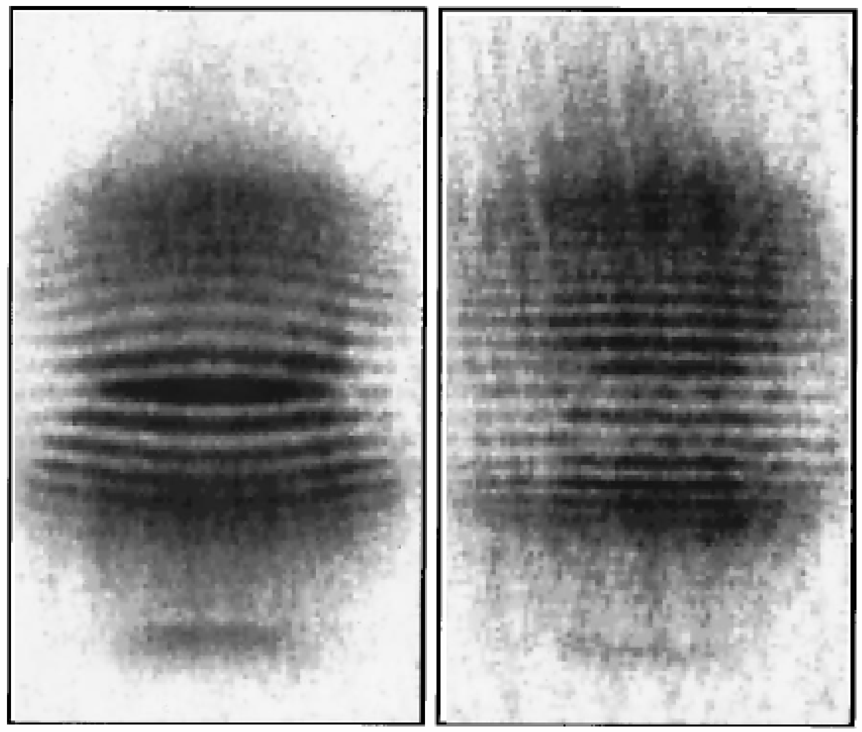
Interference experiments with independent fluctuating condensates

Analysis of correlation functions

Interference of two independent condensates

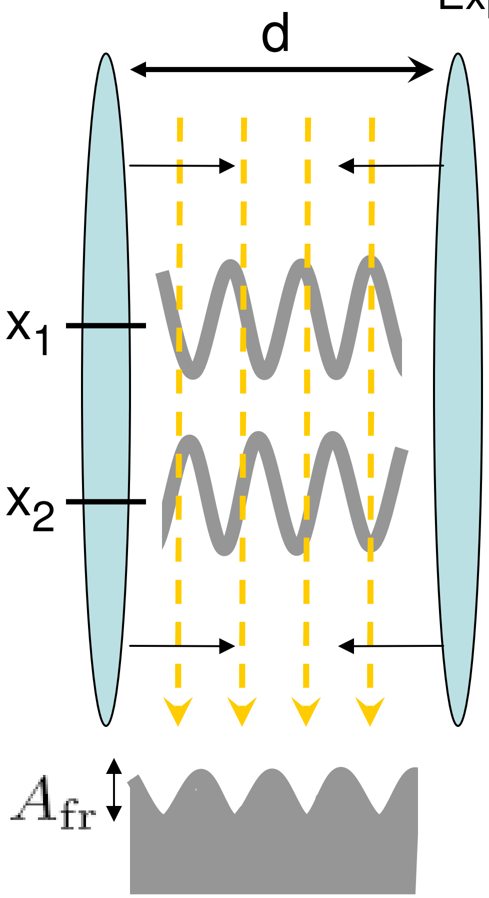
Andrews et al., Science 275:637 (1997)





Interference of one dimensional condensates

Expts: Schmiedmayer et al., Nature Phys. (2005), see also Thywissen et al.



Amplitude of interference fringes, $A_{\rm fr}$

$$|A_{\rm fr}|\, e^{i\Delta\phi} = \int_0^L dx\; a_1^\dagger(x)\, a_2(x)$$

For independent condensates A_{fr} is finite but $\Delta\phi$ is random

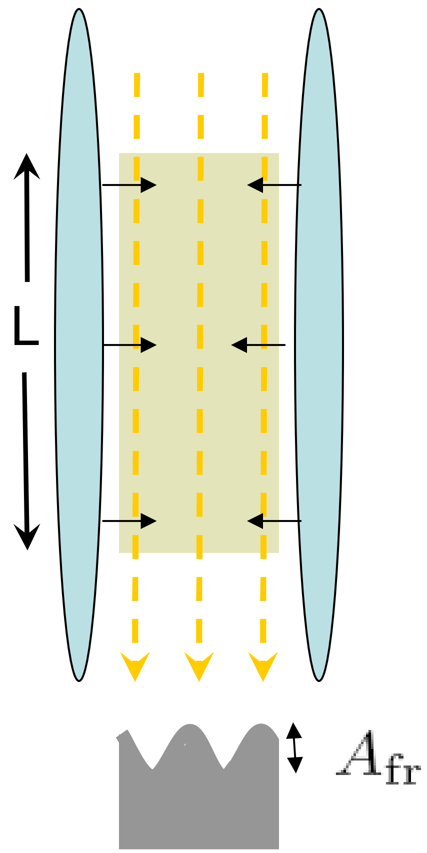
$$\langle |A_{\rm fr}|^2 \rangle = \int_0^L \int_0^L dx\, dy\, \langle a_1^\dagger(x)\, a_2(x)\, a_2^\dagger(y)\, a_1(y) \rangle$$

$$\simeq L \int_0^L dx\, \langle a_1(x)\, a_1^\dagger(0) \rangle \langle a_2(0) a_2^\dagger(x) \rangle$$

For identical condensates $\quad \langle |A_{\rm fr}|^2 \rangle = L \int_0^L dx\, (G(x))^2$

Instantaneous correlation function $\quad G(x) = \langle a(x)\, a^\dagger(0) \rangle$

Interference between 1d interacting bosons



Luttinger liquid at T=0

$$G(x) \sim \rho \left(\frac{\xi_h}{x} \right)^{1/2K}$$

K – Luttinger parameter

$$\langle |A_{\rm fr}|^2 \rangle \sim (\rho \xi_h)^{1/K} (L\rho)^{2-1/K}$$

For non-interacting bosons $K = \infty$ and $A_{\rm fr} \sim L$

For impenetrable bosons $K = 1$ and $A_{\rm fr} \sim \sqrt{L}$

Luttinger liquid at finite temperature

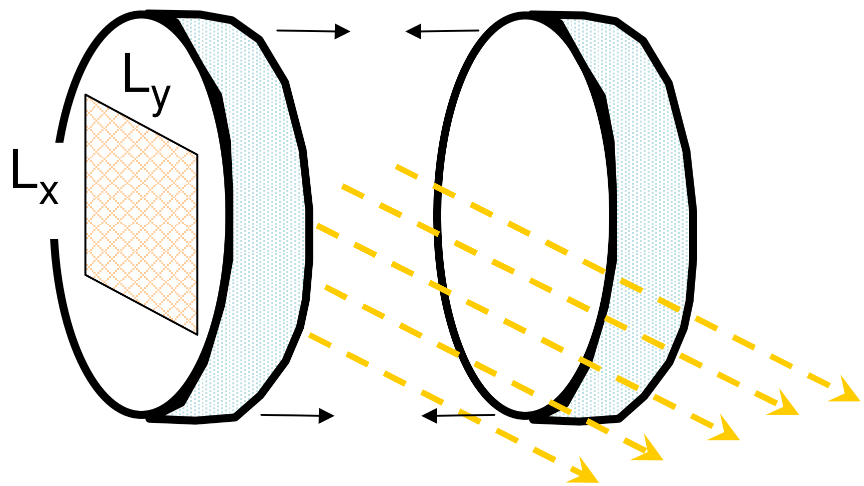
$$\langle |A_{\rm fr}|^2 \rangle \sim L \rho^2 \xi_h \left(\frac{\hbar^2}{m \xi_h^2} \frac{1}{T} \right)^{1-1/K}$$

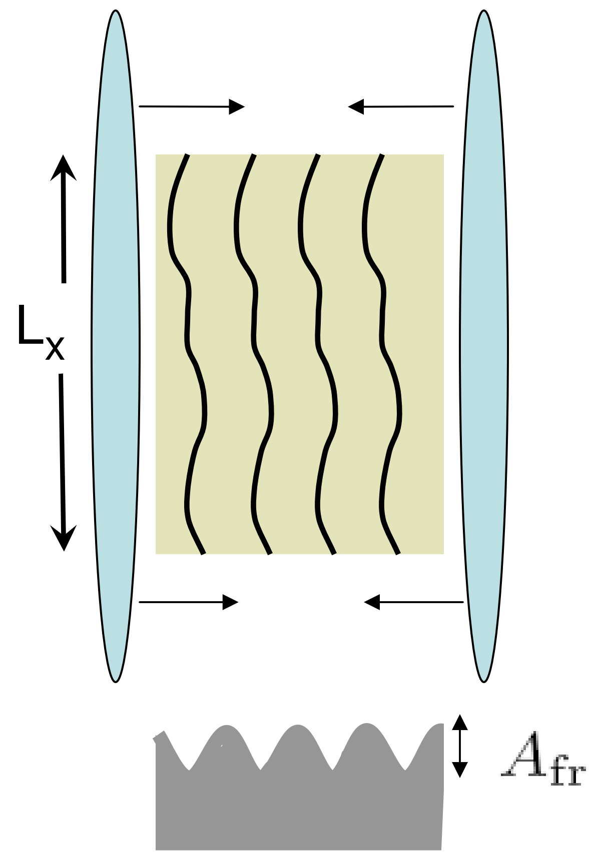
Analysis of $A_{\rm fr}$ can be used for thermometry

Interference of two dimensional condensates

Experiments: Hadzibabic, Dalibard et al. Nature (2006)

Gati, Oberthaler et al., PRL (2006)





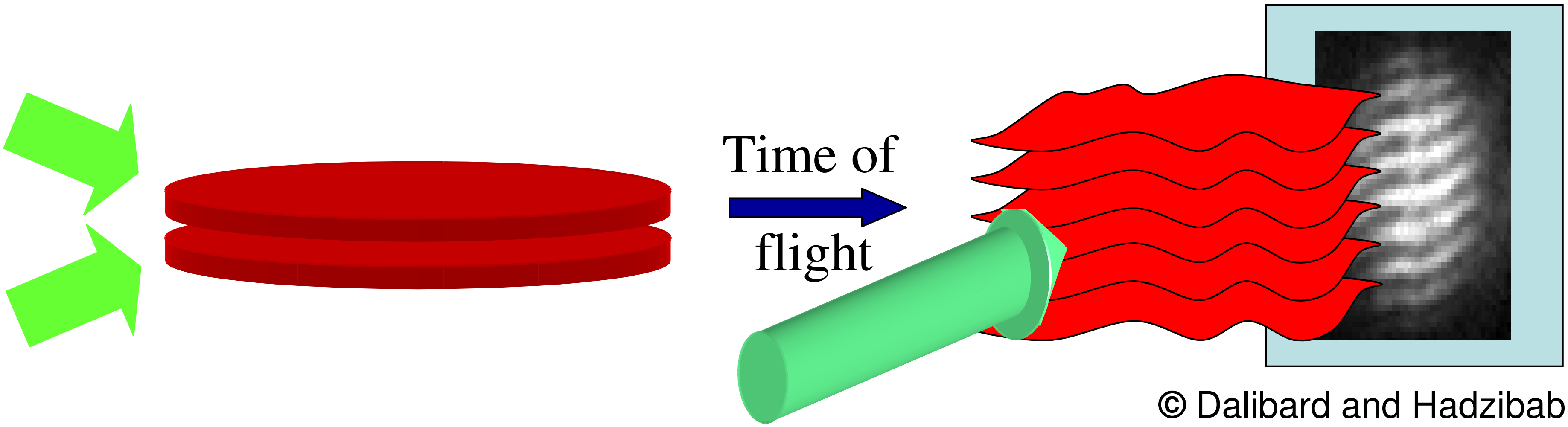
Probe beam parallel to the plane of the condensates

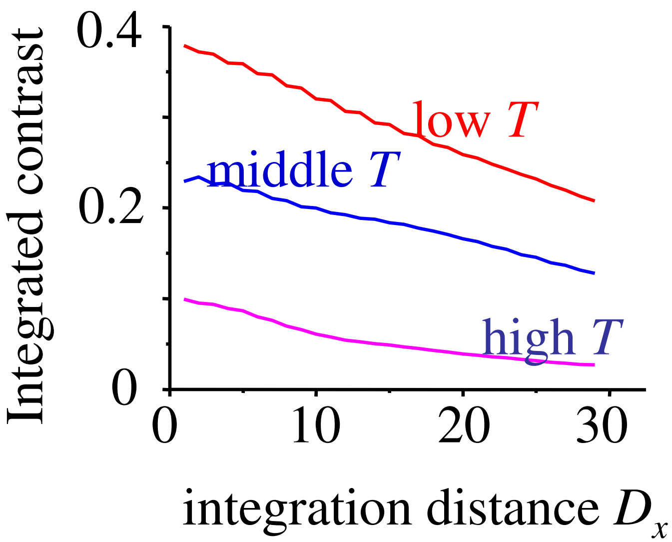
$$\langle |A_{fr}|^2 \rangle = L_x L_y \int_0^{L_x} \int_0^{L_y} d^2\vec{r} \, (G(\vec{r}))^2$$

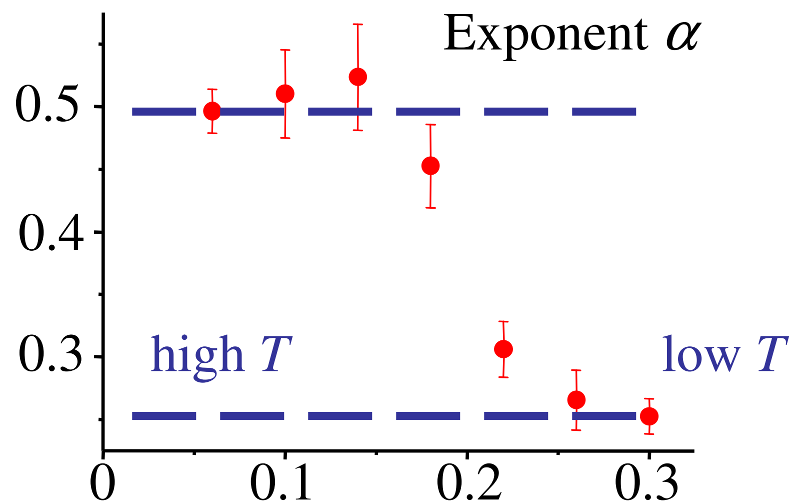
$$G(\vec{r}) = \langle a(\vec{r}) \, a^\dagger(0) \rangle$$

Observation of KT transition in experiments with 2D Bose gas

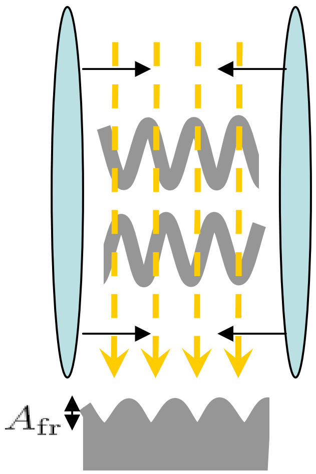
Hadzibabic et al., Nature (2006)



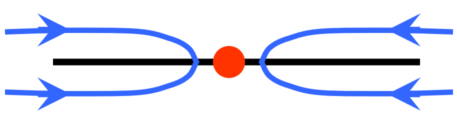




Interference between interacting 1d Bose liquids. Distribution function of the interference amplitude



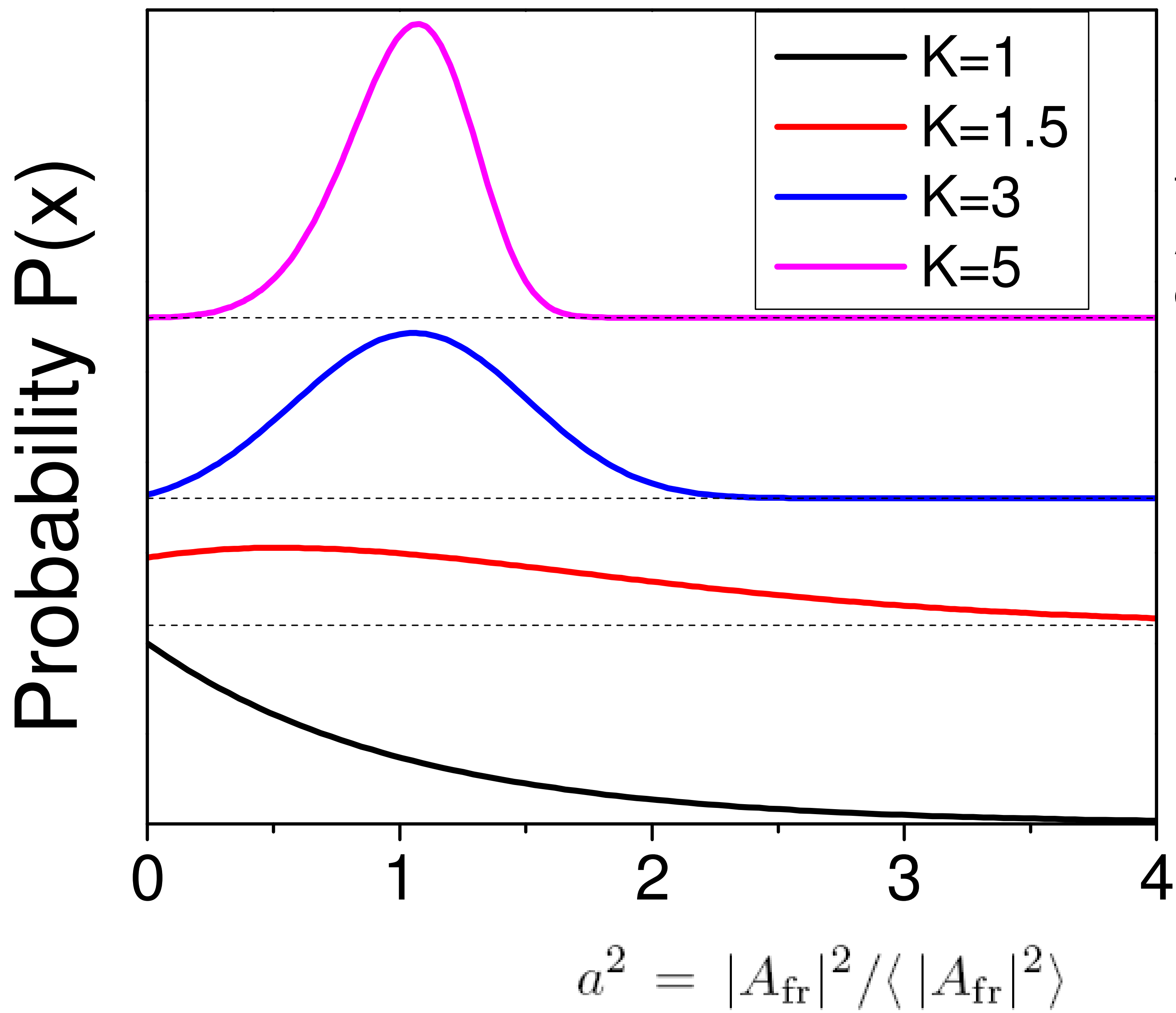
$A_{\rm fr}$ is a quantum operator. The measured value of $|A_{\rm fr}|$ will fluctuate from shot to shot.
How to predict the distribution function of $|A_{\rm fr}|$



Quantum impurity problem: interacting one dimensional electrons scattered on an impurity

Conformal field theories with negative central charges

Evolution of the distribution function



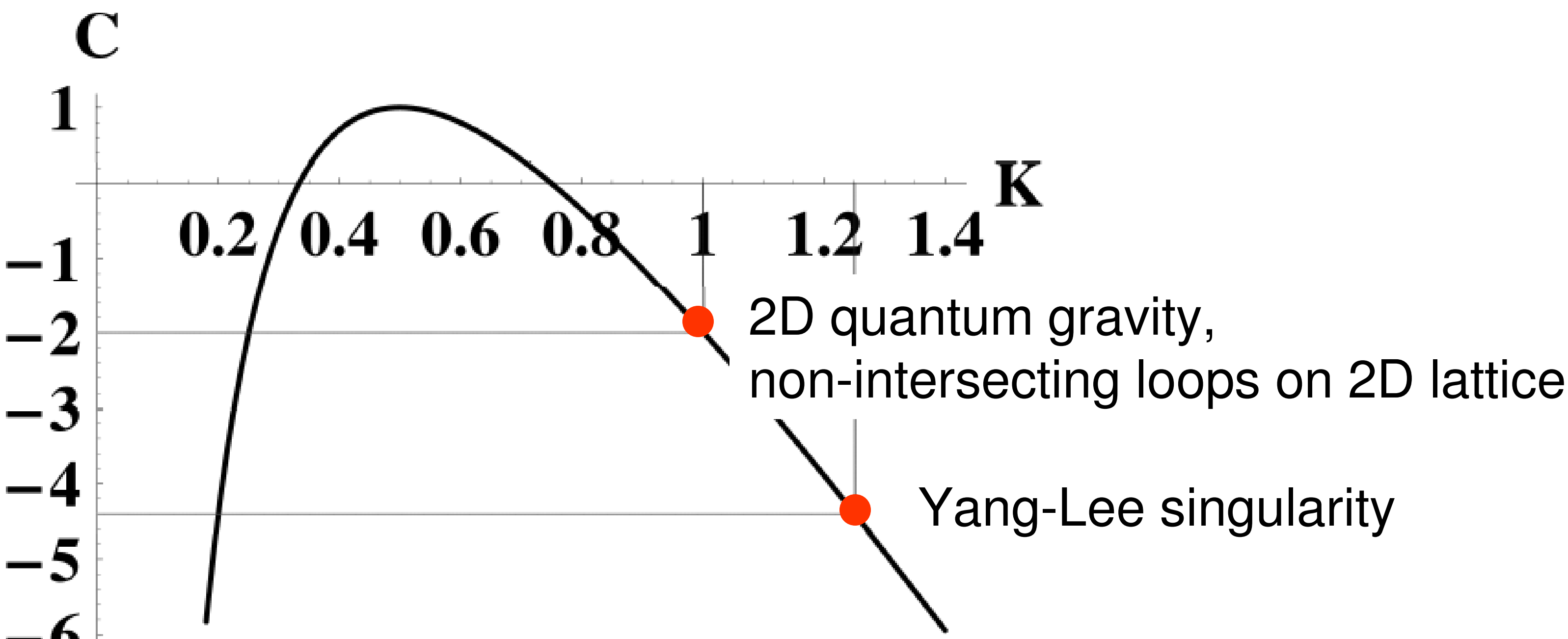
Narrow distribution for $K \to \infty$. Approaches Gumbel distribution. Width

$$\sigma = \frac{\pi}{6K}$$

Wide Poissonian distribution for $K \to 1$

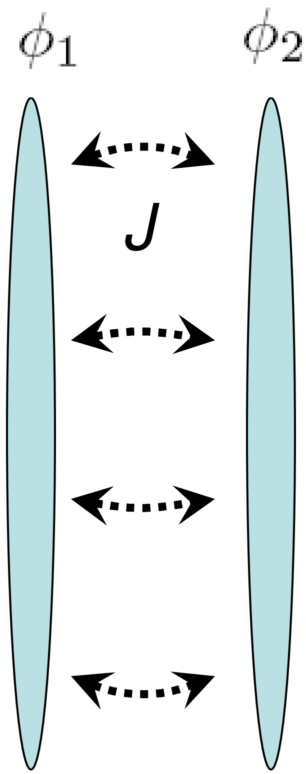
From interference amplitudes to conformal field theories

When K>1, the distribution function of fringe amplitudes is related to Q operators of CFT with c<0. This includes 2D quantum gravity, non-intersecting loop model on 2D lattice, growth of random fractal stochastic interface, high energy limit of multicolor QCD, …



Studying coherent dynamics
of strongly interacting systems
in interference experiments

Coupled 1d Bose liquids



Interactions lead to phase fluctuations within individual condensates

$$\mathcal{H}_0[\phi] = \int dx \left[\frac{1}{2K} n_1^2 + \frac{K}{2} (\partial_x \phi_1)^2\right] + \int dx \left[\frac{1}{2K} n_2^2 + \frac{K}{2} (\partial_x \phi_2)^2\right]$$

The tunneling term favors aligning of the two phases

$$\mathcal{H}_{tun} = -J \int dx \, \cos(\phi_1 - \phi_2)$$

Interference experiments measure only the relative phase

Quantum Sine-Gordon model
for the relative phase $\phi = \phi_1 - \phi_2$

$$\mathcal{H}[\phi] = \int dx \, d\tau \left[\frac{1}{2K}(\Delta n)^2 + \frac{K}{2} (\partial_x \phi)^2\right] - J \int dx \, d\tau \, \cos \phi$$

Quantum Sine-Gordon model

Hamiltonian

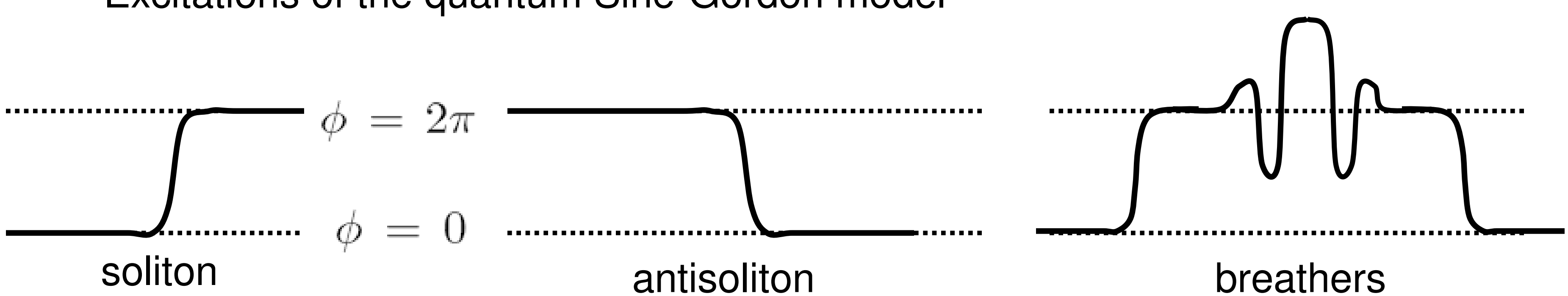
$$\mathcal{H}[\phi] = \int dx \left[\frac{1}{2K} (\Delta n)^2 + \frac{K}{2} (\partial_x \phi)^2 \right] - J \int dx \, \cos\phi$$

Imaginary time action

$$\mathcal{S}[\phi] = \frac{K}{2} \int dx \, d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right] + J \int dx \, d\tau \, \cos\phi$$

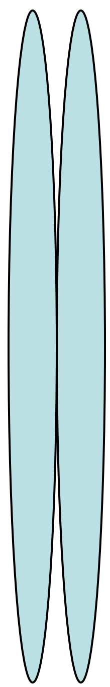
Quantum Sine-Gordon model is exactly integrable

Excitations of the quantum Sine-Gordon model



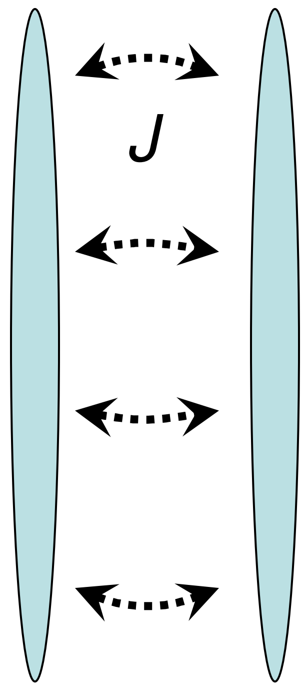
Dynamics of the quantum Sine-Gordon model

Motivated by experiments of J. Schmiedmayer et al.

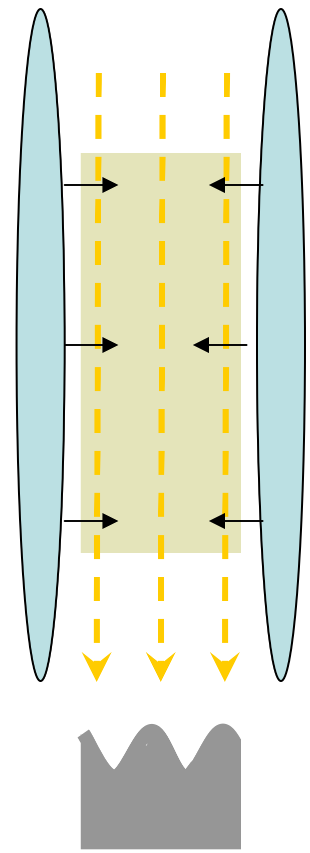


Prepare a system at t=0

$$\phi(x) = 0$$

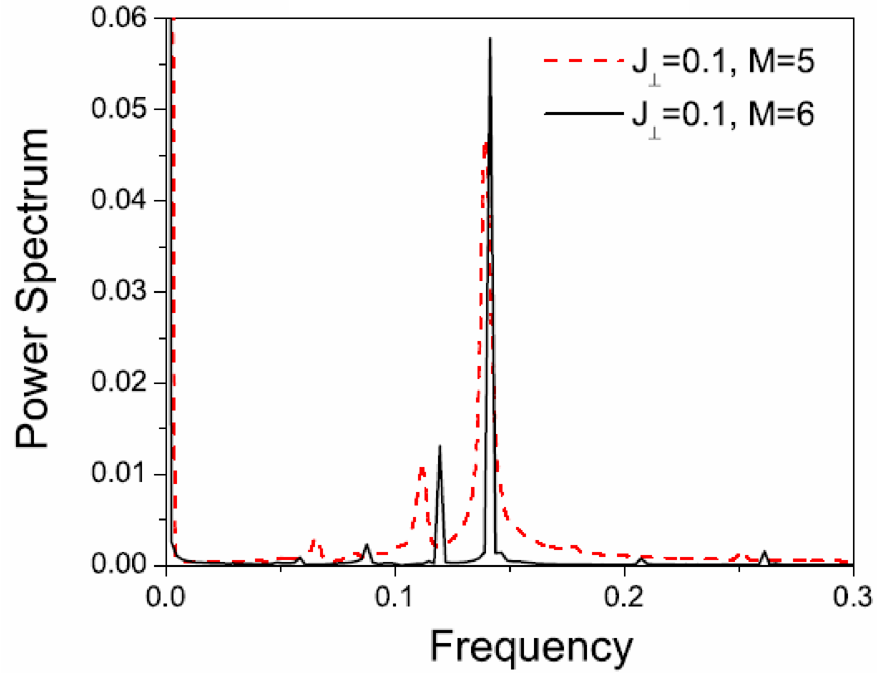


Take to the regime of finite tunneling and let evolve for some time



Measure amplitude of interference pattern

Coherent dynamics of quantum Sine-Gordon model

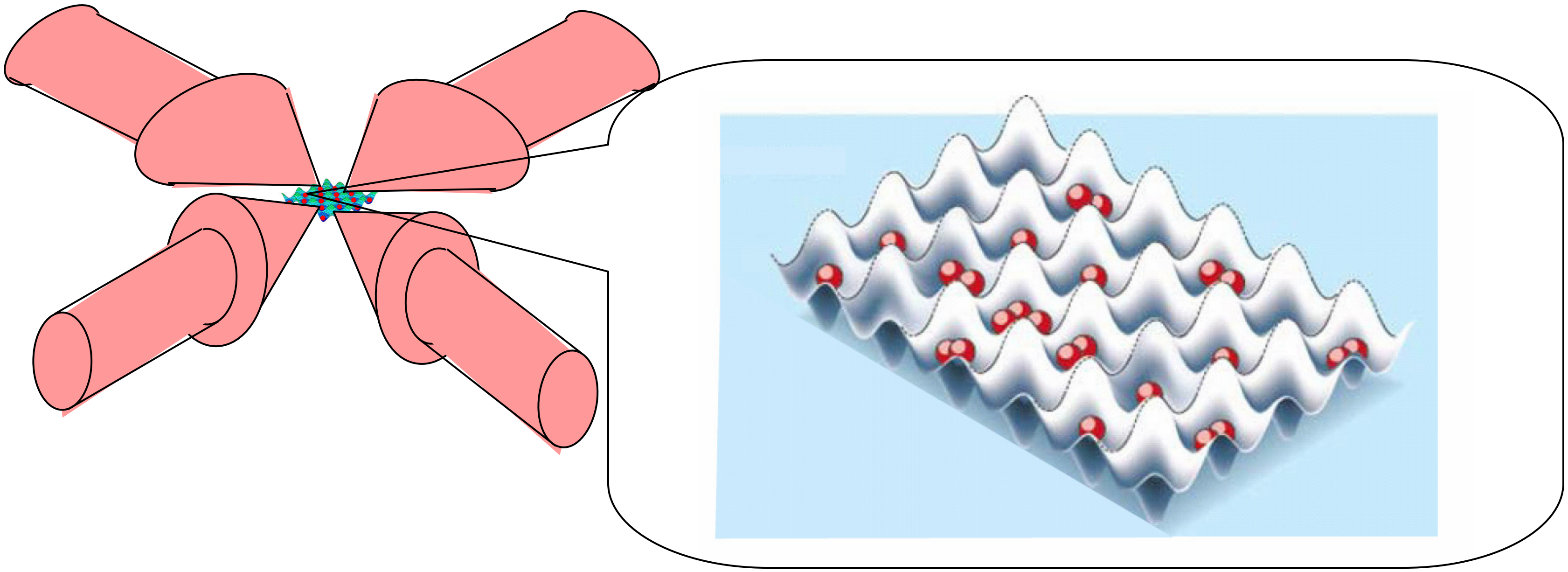


Amplitude of interference fringes shows oscillations at frequencies that correspond to energies of various excitations. Low damping of breather modes

Quantum noise interferometry in time of flight experiments

Detection of many-body phases of ultracold atoms in optical lattices

Atoms in optical lattices

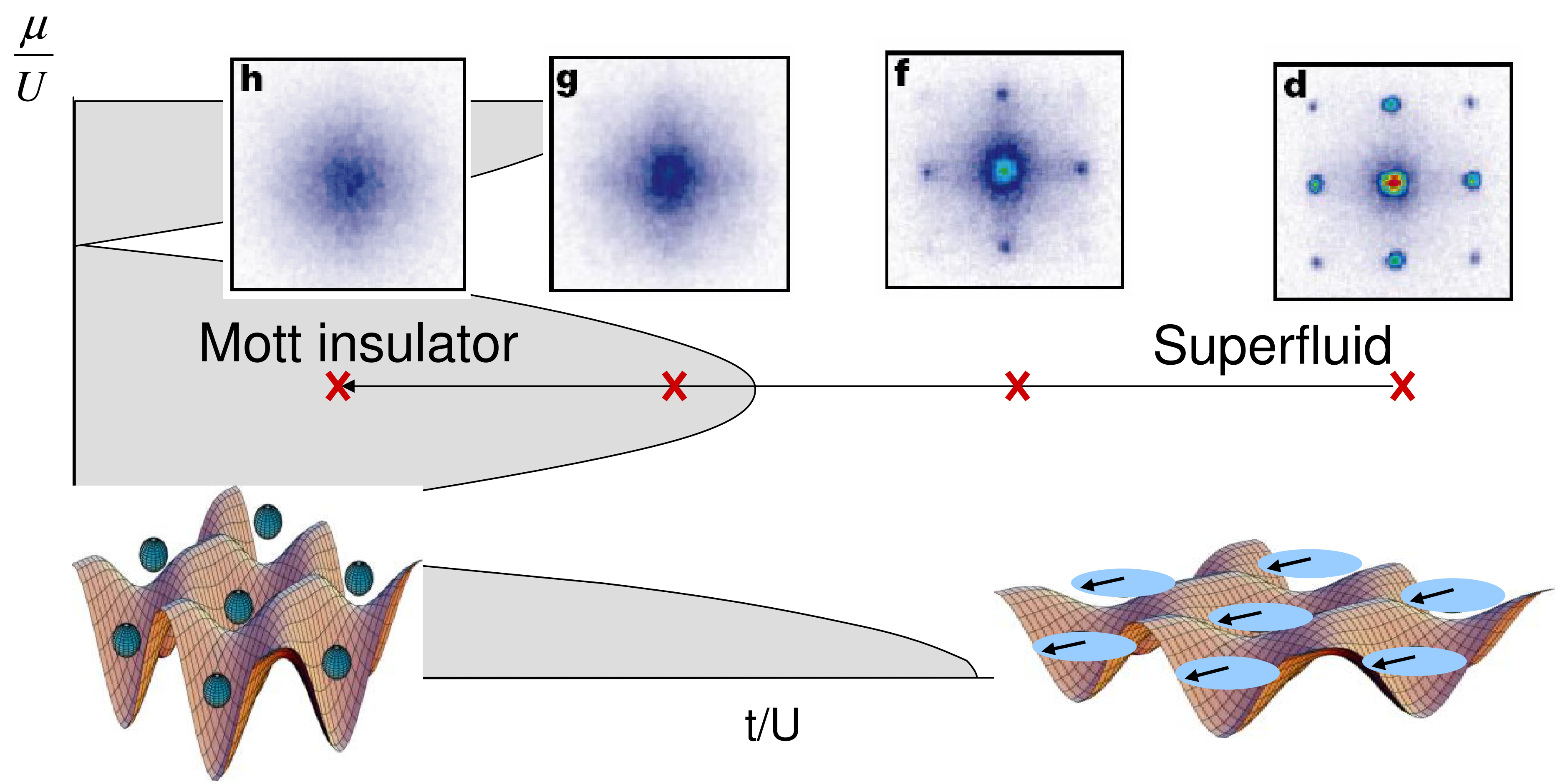


Theory: Jaksch et al. PRL (1998)

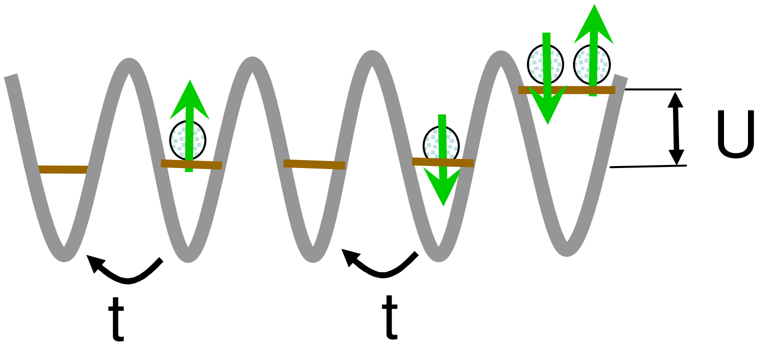
Experiment: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Phillips et al., J. Physics B (2002)
Esslinger et al., PRL (2004);
and many more …

Atoms in an optical lattice. Superfluid to Insulator transition

Greiner et al., Nature 415:39 (2002)



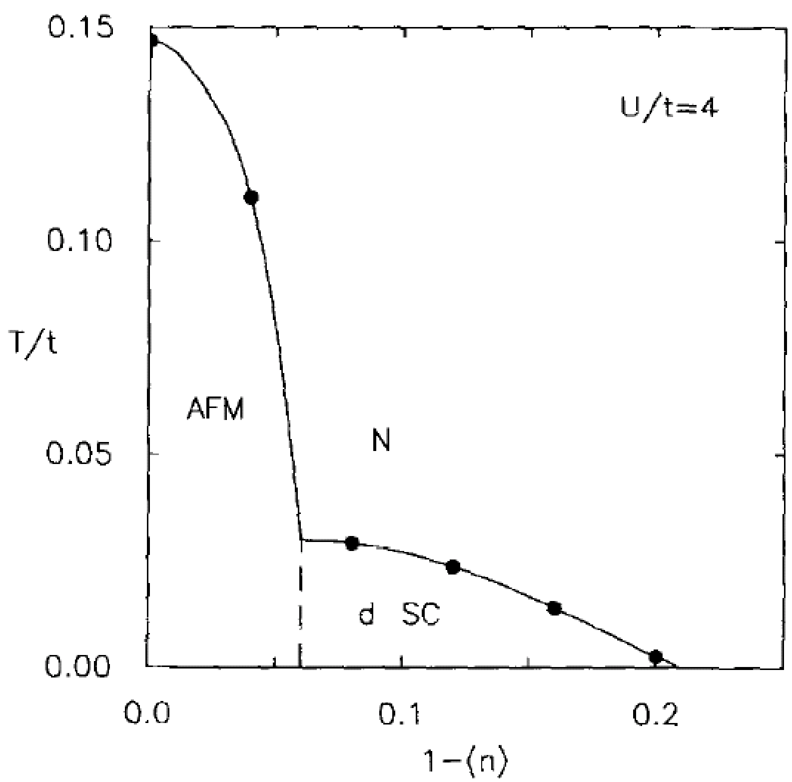
Fermionic Hubbard model and high Tc superconductivity



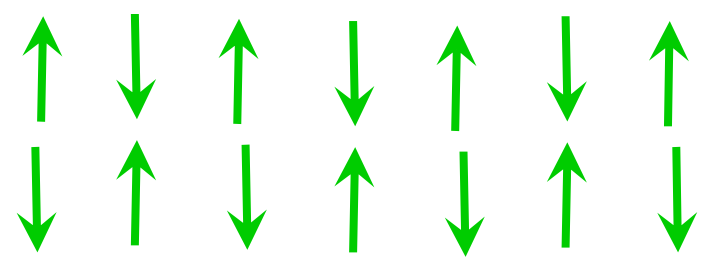
Expts: Kohl et al., PRL (2005)

Fermions with repulsive interactions in an optical lattice can be described by the Hubbard model

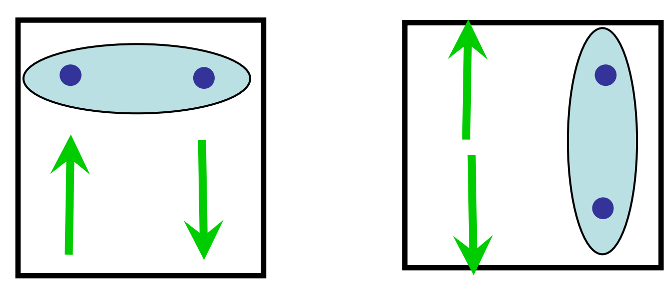
Possible phase diagram. Scalapino, Phys. Rep. 250:329 (1995)



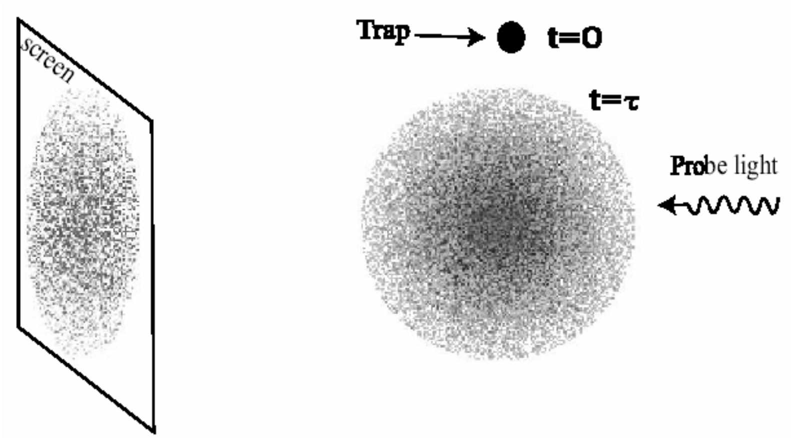
Antiferromagnetic insulator



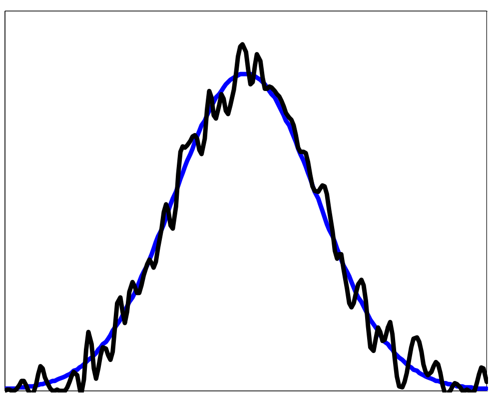
D-wave superconductor

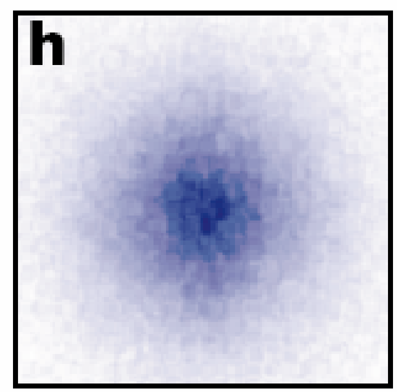


Time of flight experiments



Quantum noise interferometry of atoms in an optical lattice



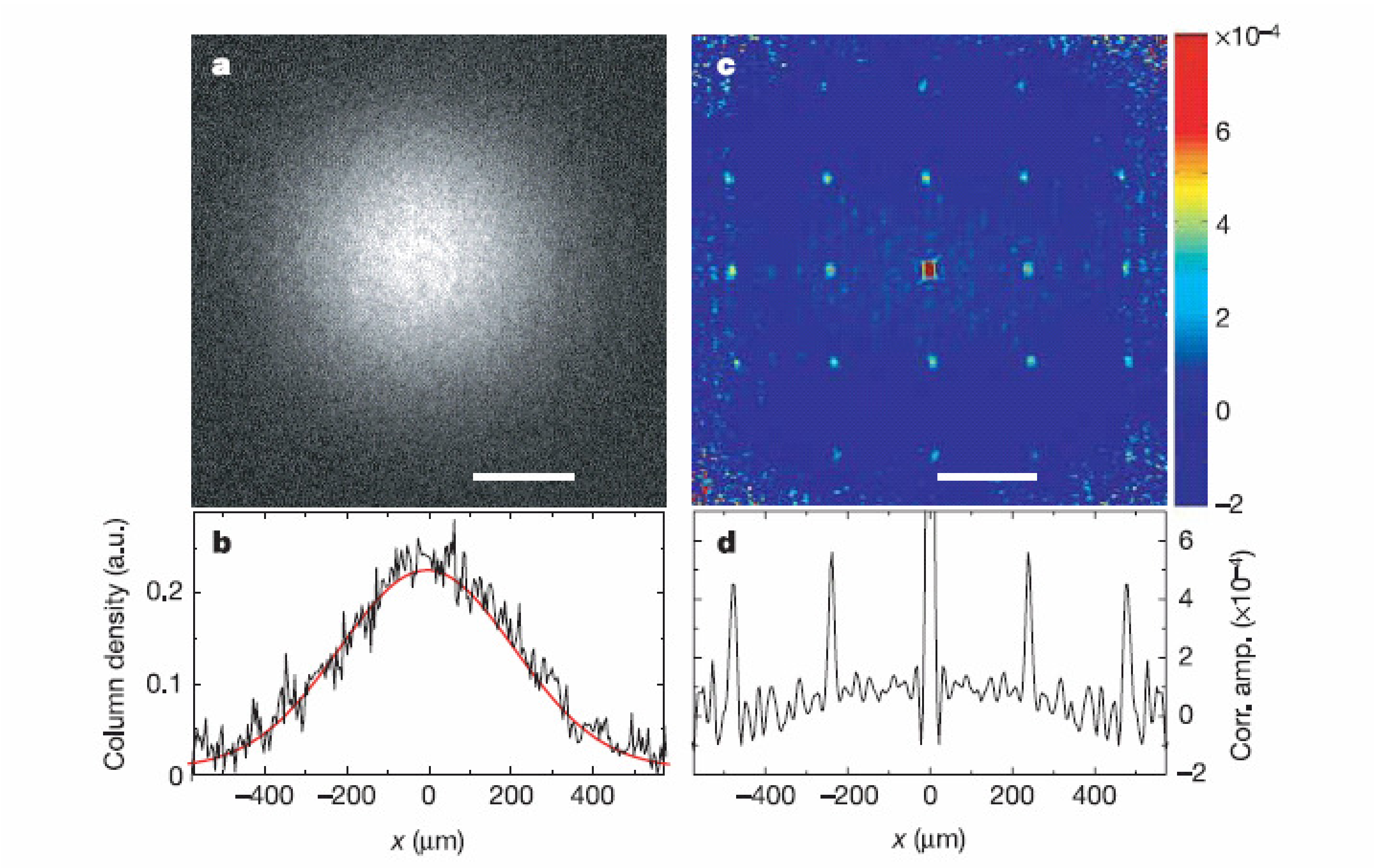


Second order coherence $G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$

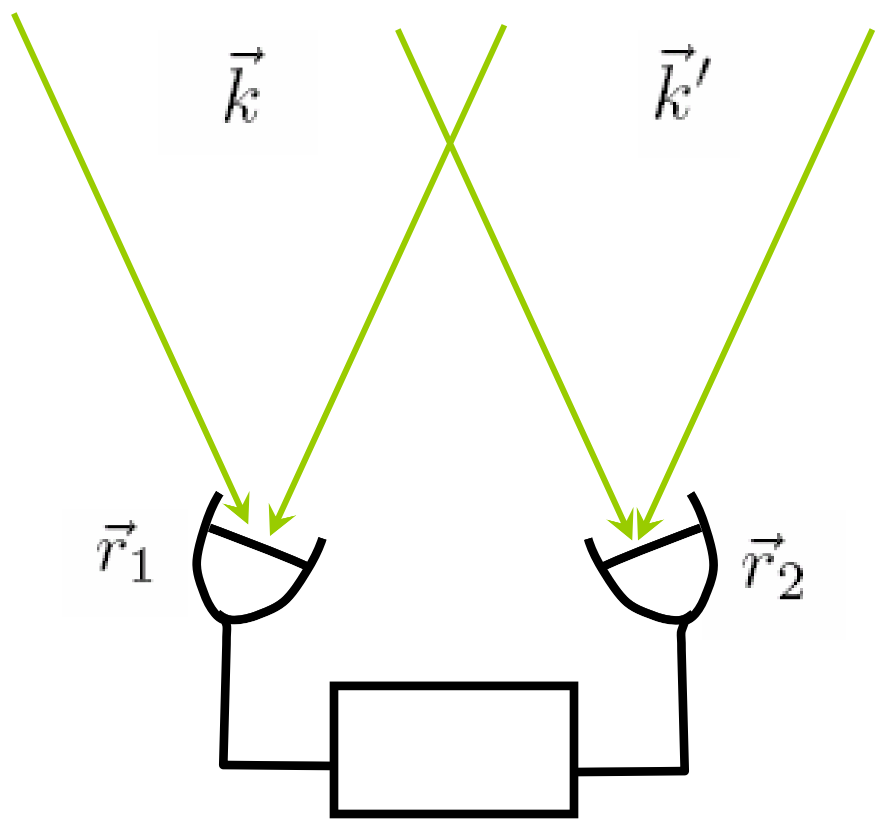
Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

Theory: Altman et al., PRA (2004)

Experiment: Folling et al., Nature (2005)

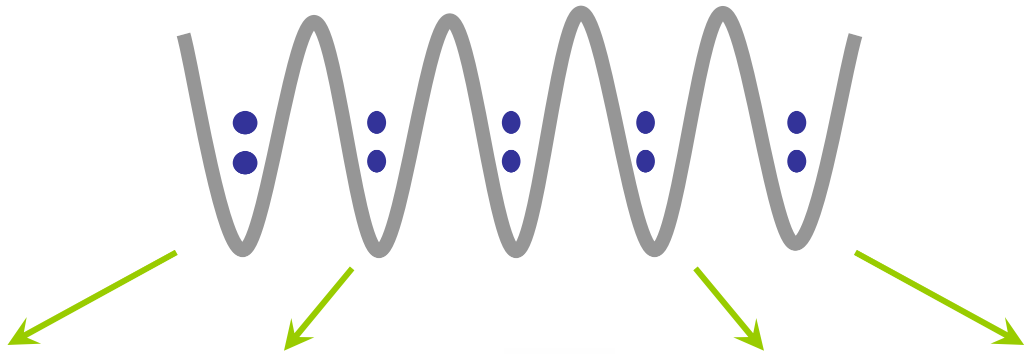


Hanburry-Brown-Twiss stellar interferometer



$$\langle I(\vec{r}_1)\ I(\vec{r}_2)\rangle = A + B\ \cos\Big((\vec{k}-\vec{k}')\ (\vec{r}_1-\vec{r}_2)\Big)$$

Second order coherence in the insulating state of bosons



Bosons at quasimomentum $\vec{k}$ expand as plane waves

with wavevectors $\vec{k},\ \vec{k}+\vec{G}_1,\ \vec{k}+\vec{G}_2$

First order coherence: $\langle\ \rho(\vec{r})\ \rangle$

Oscillations in density disappear after summing over $\vec{k}$

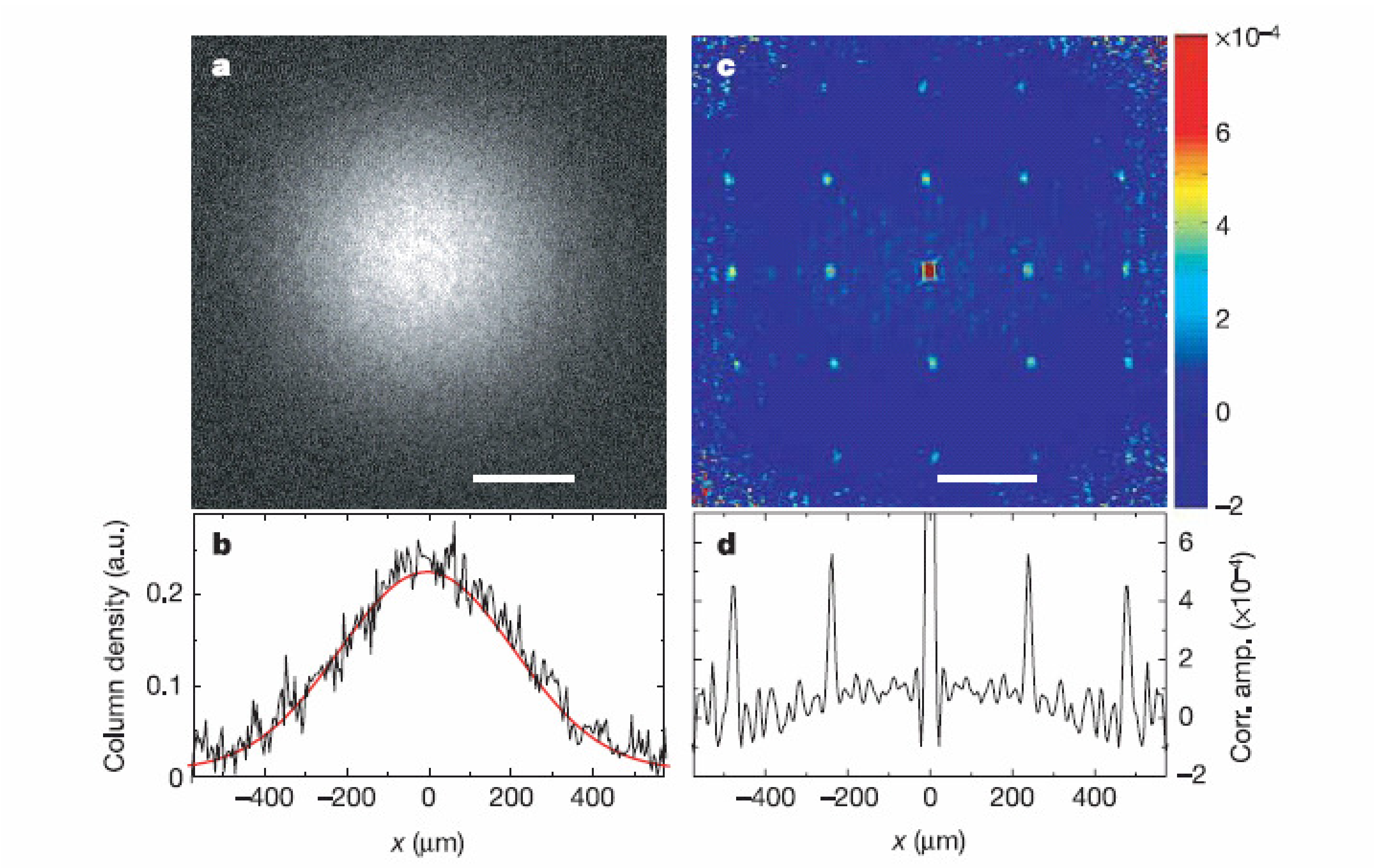
Second order coherence: $\langle\ \rho(\vec{r}_1)\ \rho(\vec{r}_2)\ \rangle$

Correlation function acquires oscillations at reciprocal lattice vectors

$$\langle\ \rho(\vec{r}_1)\ \rho(\vec{r}_2)\ \rangle = A_0 + A_1\ \cos\Big(\ \vec{G}_1(\vec{r}_1-\vec{r}_2)\ \Big) + A_2\ \cos\Big(\ \vec{G}_2(\vec{r}_1-\vec{r}_2)\ \Big) + \dots$$

Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

Experiment: Folling et al., Nature (2005)
see also Hadzibabic et al., PRL (2004)



The Mott insulator transition in two dimensions

I. B. Spielman,[1,*] W. D. Phillips,[1,2] and J. V. Porto[1,2]

cond-mat/0606216

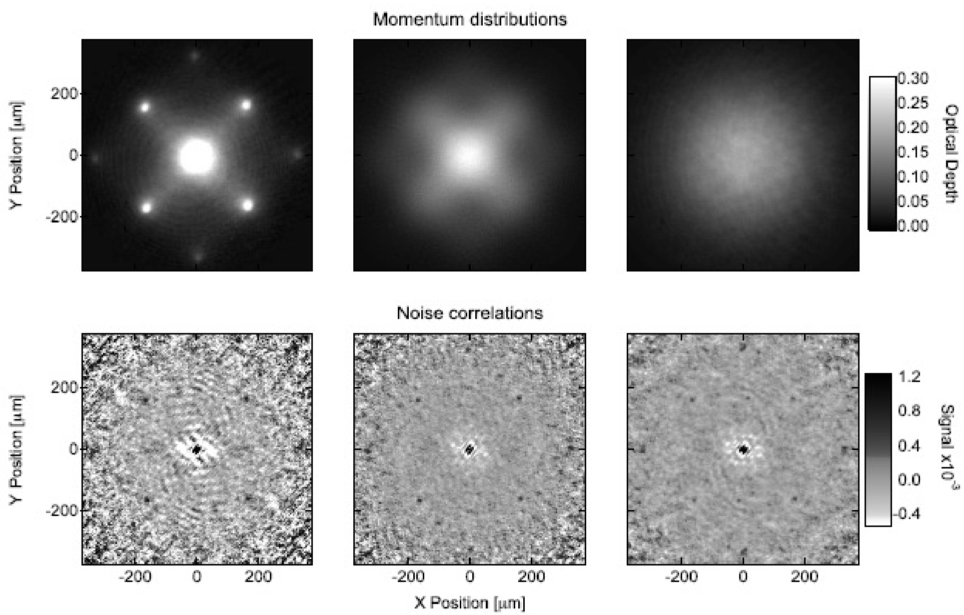
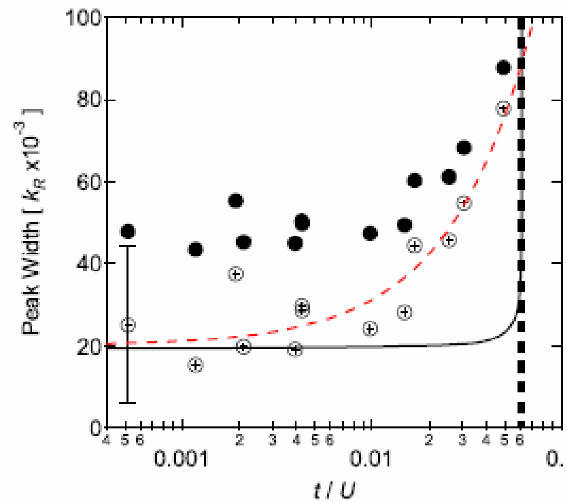


FIG. 1: Top: expected in situ density profile from a LDA caclulation. The extended regions of uniform density denote the portion of the system in the MI phase. Middle: imaged atom density versus final position after TOF. Bottom: noise correlations as a function of final position. Each image represents an average of about 60 raw images and each data set is presented at 3 different values of t/U.

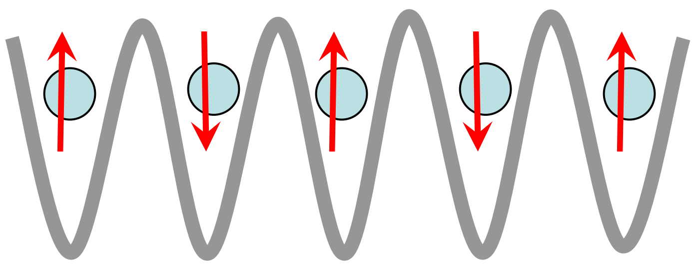


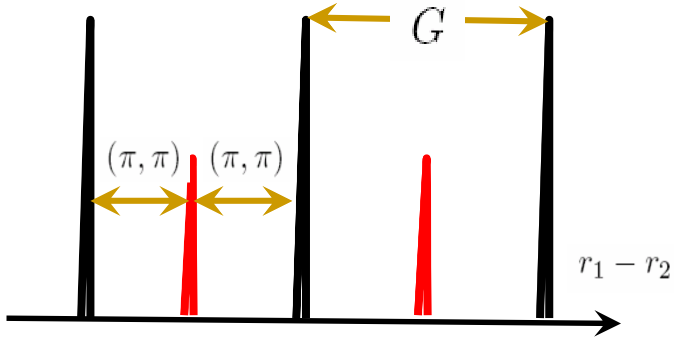
Width of the noise peaks

Potential applications of quantum noise intereferometry

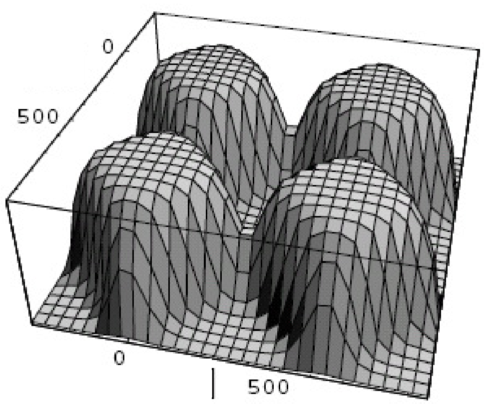
Altman et al., PRA (2004)

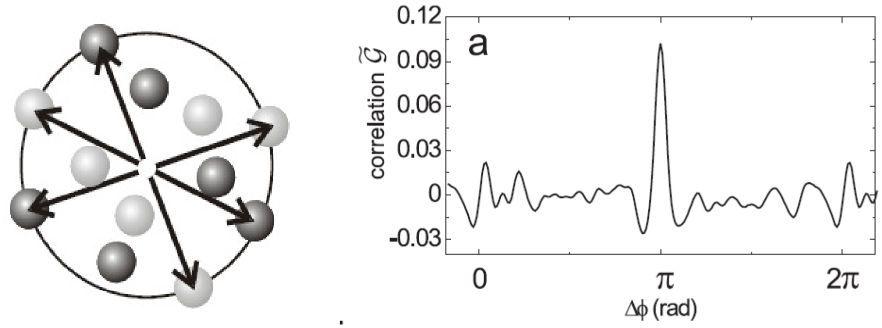
Detection of magnetically ordered Mott states





Detection of paired states of fermions





Fermions without lattice. Pairing correlations seen in experiments of Greiner et al. PRL (2005)

Conclusions

Interference of extended condensates can be used to probe equilibrium correlations as well as out of equilibrium dynamics of low dimensional systems

Noise interferometry in time of flight experiments is a powerful tool for studying quantum many-body states in optical lattices