Quantum noise studies of ultracold atoms

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Outline

Introduction. Historical review

Hanburry-Brown-Twiss experiments
with atoms in optical lattices

Quantum noise in interference experiments
with independent condensates

Quantum noise

Classical measurement:

collapse of the wavefunction into eigenstates of x

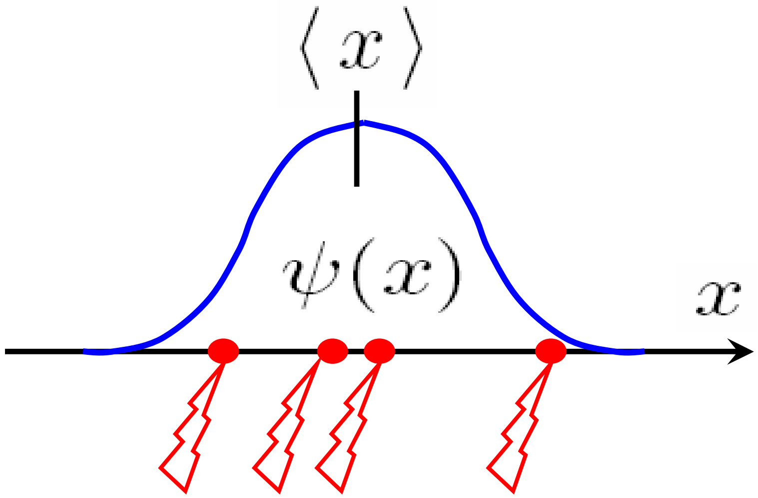
$$\langle x \rangle = \int dx\, x\, |\psi(x)|^2$$

$$\langle x^2 \rangle = \int dx\, x^2\, |\psi(x)|^2$$

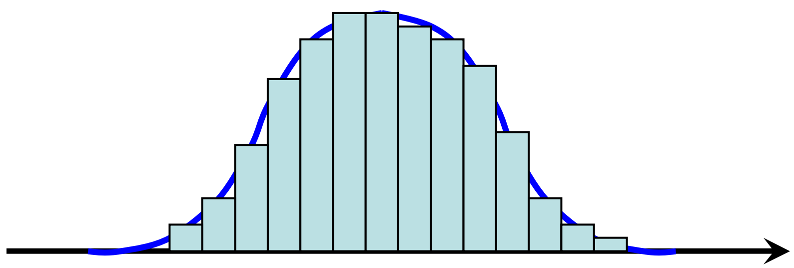
$$\dots$$

$$\langle x^n \rangle = \int dx\, x^n\, |\psi(x)|^2$$

$$\dots$$

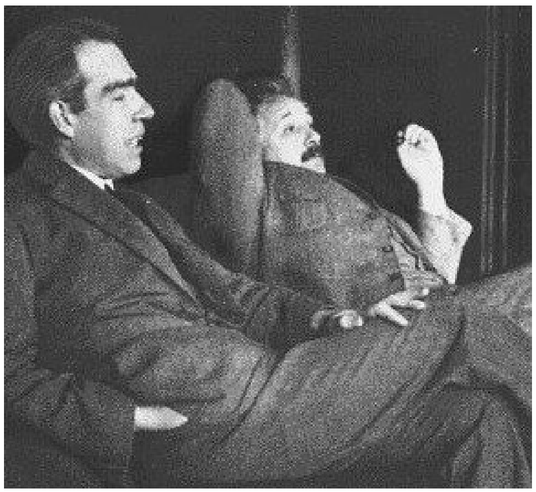


Histogram of measurements of x

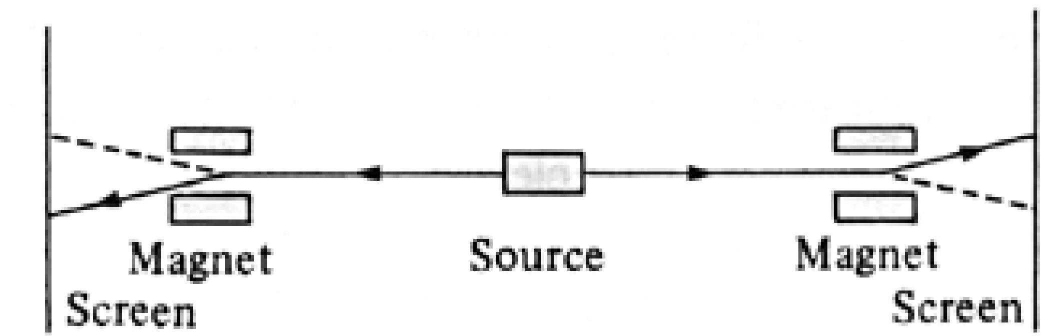


Probabilistic nature of quantum mechanics

Bohr-Einstein debate on spooky action at a distance



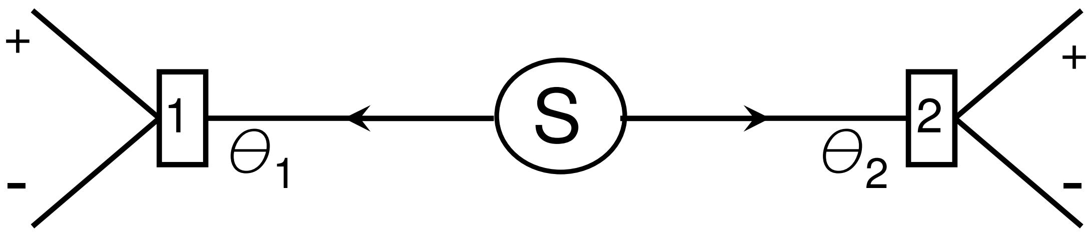
Einstein-Podolsky-Rosen experiment

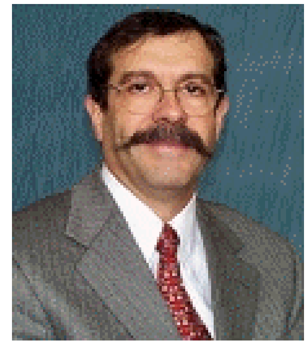


$$|S = 0\rangle = |\uparrow\rangle_L |\downarrow\rangle_R - |\downarrow\rangle_L |\uparrow\rangle_R$$

Measuring spin of a particle in the left detector
instantaneously determines its value in the right detector

Aspect’s experiments: tests of Bell’s inequalities





Correlation function $$E(\theta_1, \theta_2) = \frac{\langle (I_1^+ - I_1^-)(I_2^+ - I_2^-) \rangle}{\langle (I_1^+ + I_1^-)(I_2^+ + I_2^-) \rangle}$$

Classical theories with hidden variable require

$$B = E(\theta_1, \theta_2) - E(\theta_1, \theta_2') + E(\theta_1', \theta_2') - E(\theta_1', \theta_2) \le 2$$

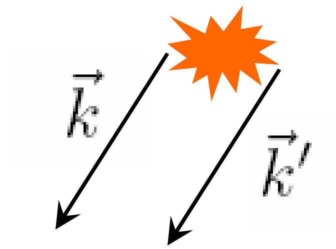
Quantum mechanics predicts B=2.7 for the appropriate choice of Θ‘s and the state

$$|\psi\rangle = |+\rangle_L |+\rangle_R + |-\rangle_L |-\rangle_R$$

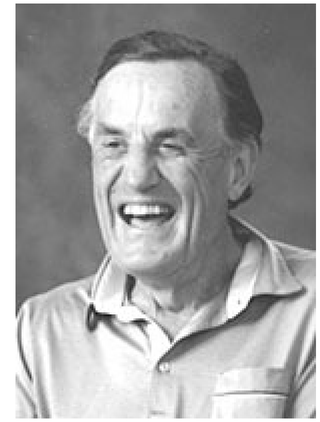
Experimentally measured value B=2.697. Phys. Rev. Let. 49:92 (1982)

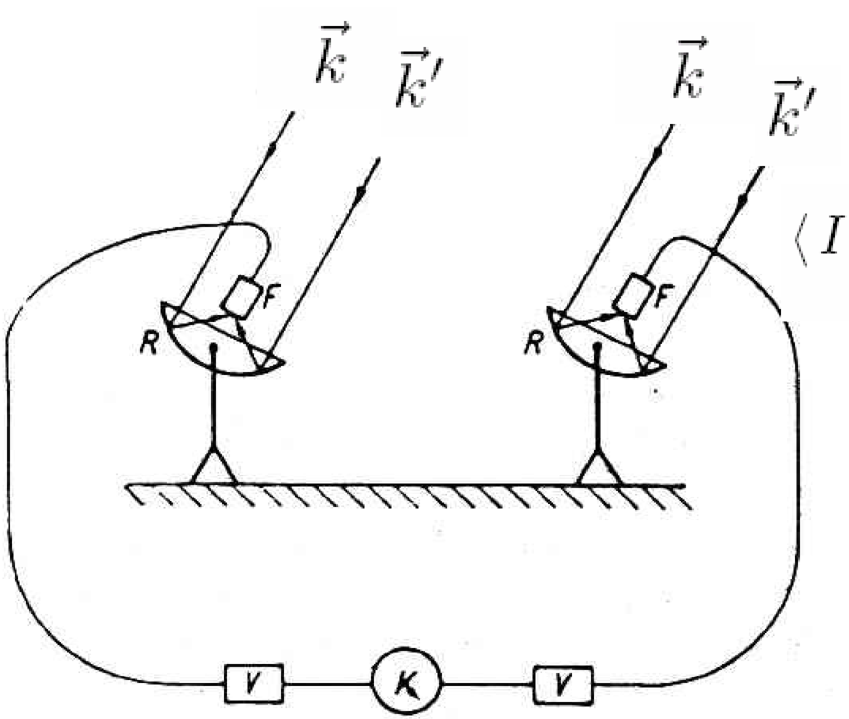
Hanburry-Brown-Twiss experiments

Classical theory of the second order coherence



Hanbury Brown and Twiss,
Proc. Roy. Soc. (*London*),
A, 242, pp. 300-324

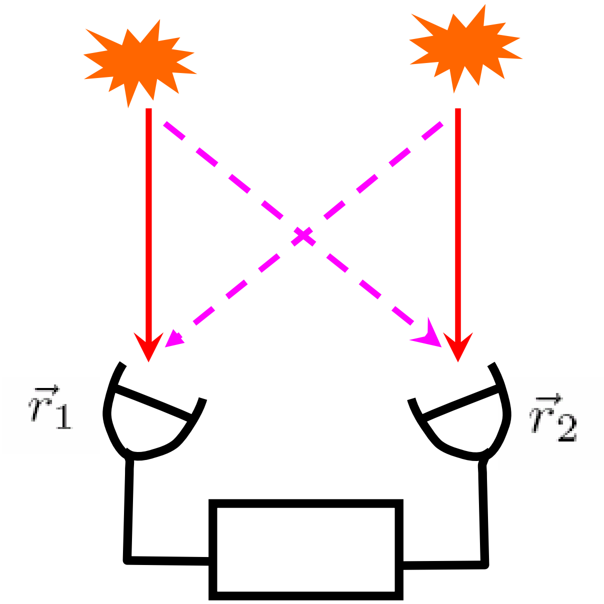




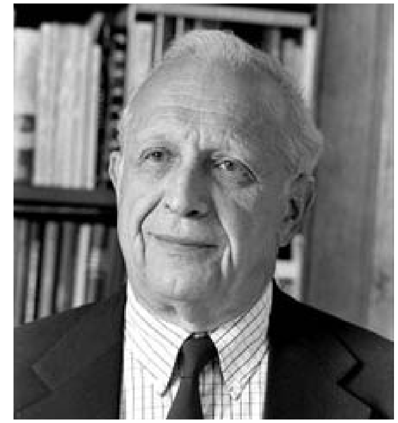
$$\langle I(\vec{r}_1)\ I(\vec{r}_2)\rangle = A + B\ \cos\left((\vec{k}-\vec{k}')\ (\vec{r}_1-\vec{r}_2)\right)$$

Measurements of the angular diameter of Sirius
Proc. Roy. Soc. (*London*), *A*, 248, pp. 222-237

Quantum theory of HBT experiments



Glauber,
Quantum Optics and Electronics (1965)



For bosons

$$A = A_1 + A_2$$

For fermions

$$A = A_1 - A_2$$

HBT experiments with matter

Experiments with neutrons
Ianuzzi et al., Phys Rev Lett (2006)

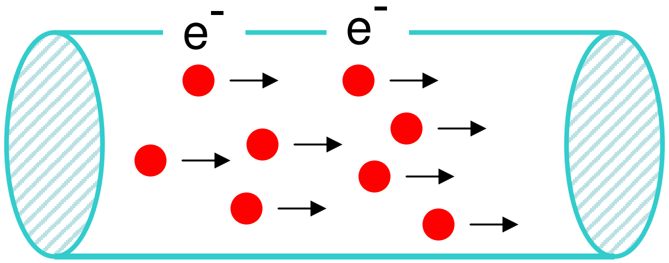
Experiments with electrons
Kiesel et al., Nature (2002)

Experiments with 4He, 3He
Westbrook et al., Nature (2007)

Experiments with ultracold atoms
Bloch et al., Nature (2005,2006)

Shot noise in electron transport

Proposed by Schottky to measure the electron charge in 1918



Spectral density of the current noise

$$S_\omega = \int \langle \{ \delta I(t), \delta I(0) \}_+ \rangle \, e^{i\omega t} \, dt$$

Related to variance of transmitted charge

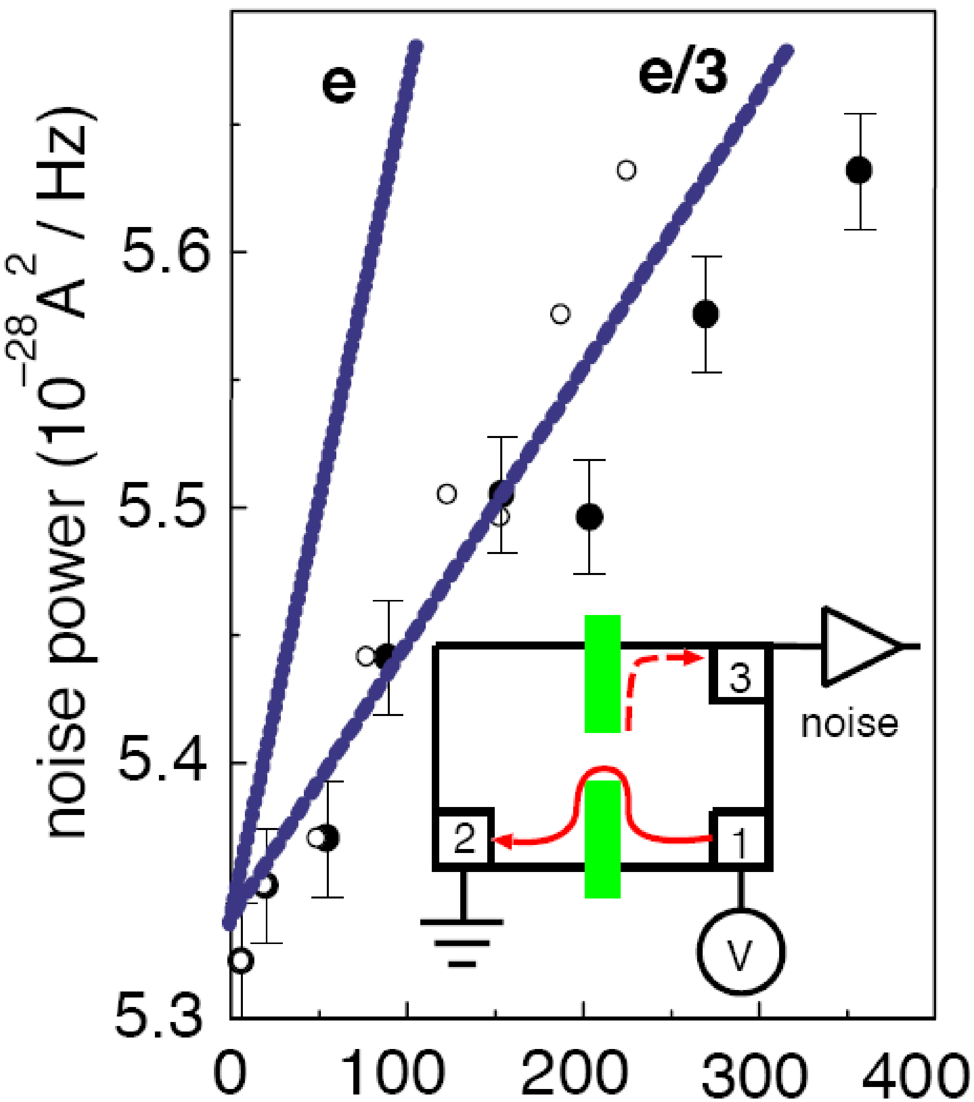
$$S_0 = \frac{2}{\tau} \langle \delta q^2(\tau) \rangle$$

When shot noise dominates over thermal noise

$$\boxed{S_0 = 2\, e\, I}$$

Poisson process of independent transmission of electrons

Shot noise in electron transport



Current noise for tunneling across a Hall bar on the 1/3 plateau of FQE

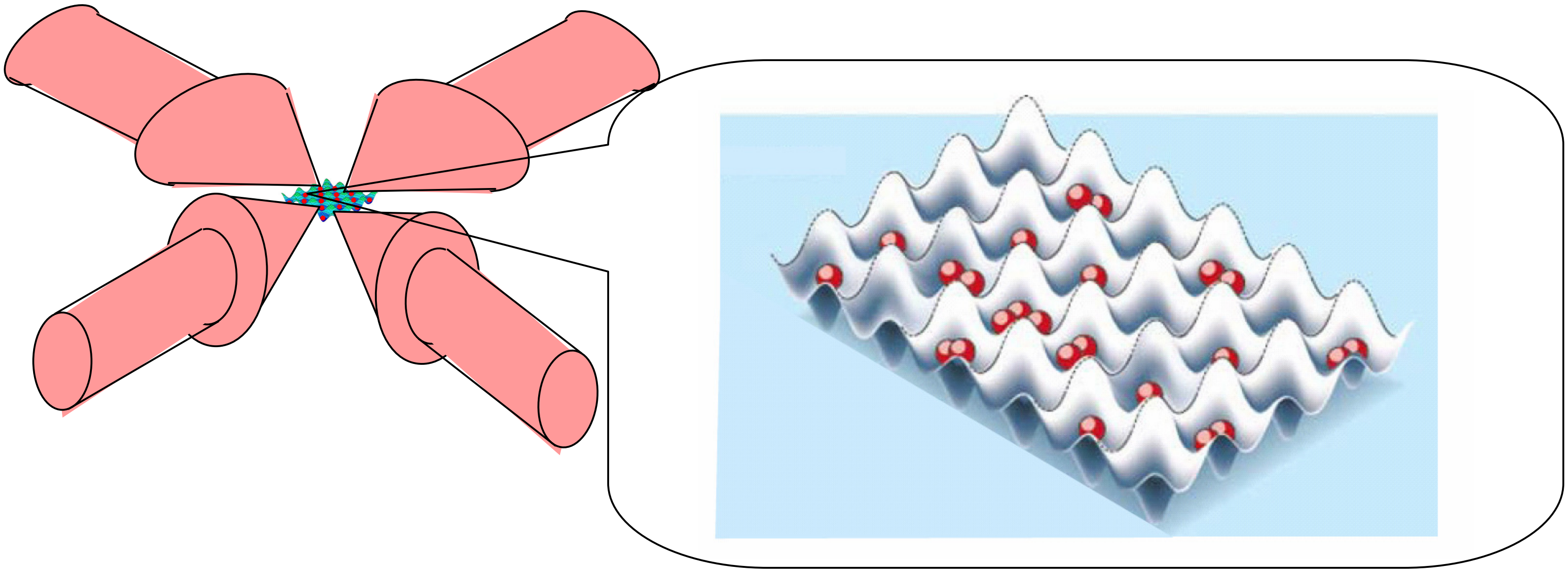
Etien et al. PRL 79:2526 (1997)
see also Heiblum et al. Nature (1997)

Hanburry-Brown-Twiss experiments with ultracold atoms in optical lattices

Theory: Altman, Demler, Lukin, PRA 70:13603 (2004)

Experiment: Folling et al., Nature 434:481 (2005);
Spielman et al., PRL 98:80404 (2007);
Tom et al. Nature 444:733 (2006)

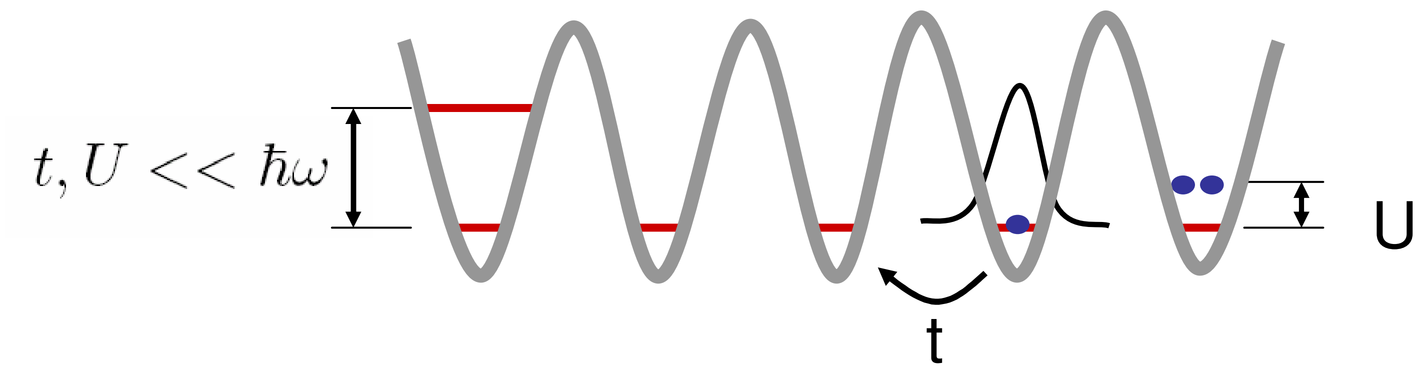
Atoms in optical lattices



Theory: Jaksch et al. PRL (1998)

Experiment: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Phillips et al., J. Physics B (2002)
Esslinger et al., PRL (2004);
Ketterle et al., PRL (2006)

Bose Hubbard model

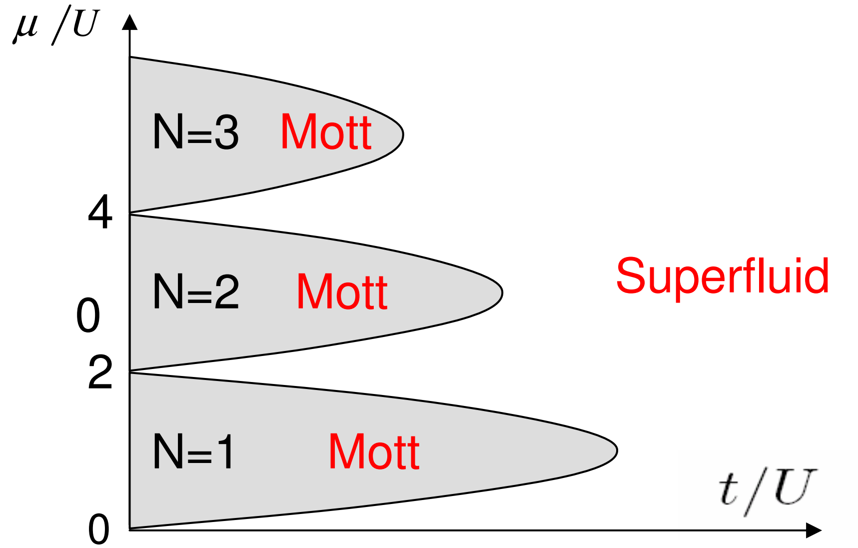


$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i \left(n_i - 1 \right) - \mu \sum_i n_i$$

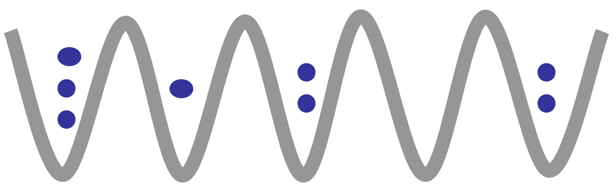
t – tunneling of atoms between neighboring wells

U – repulsion of atoms sitting in the same well

Bose Hubbard model

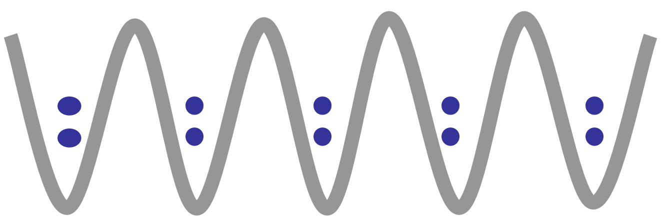


M.P.A. Fisher et al., PRB40:546 (1989)



$$U << Nt$$

Superfluid phase
Weak interactions

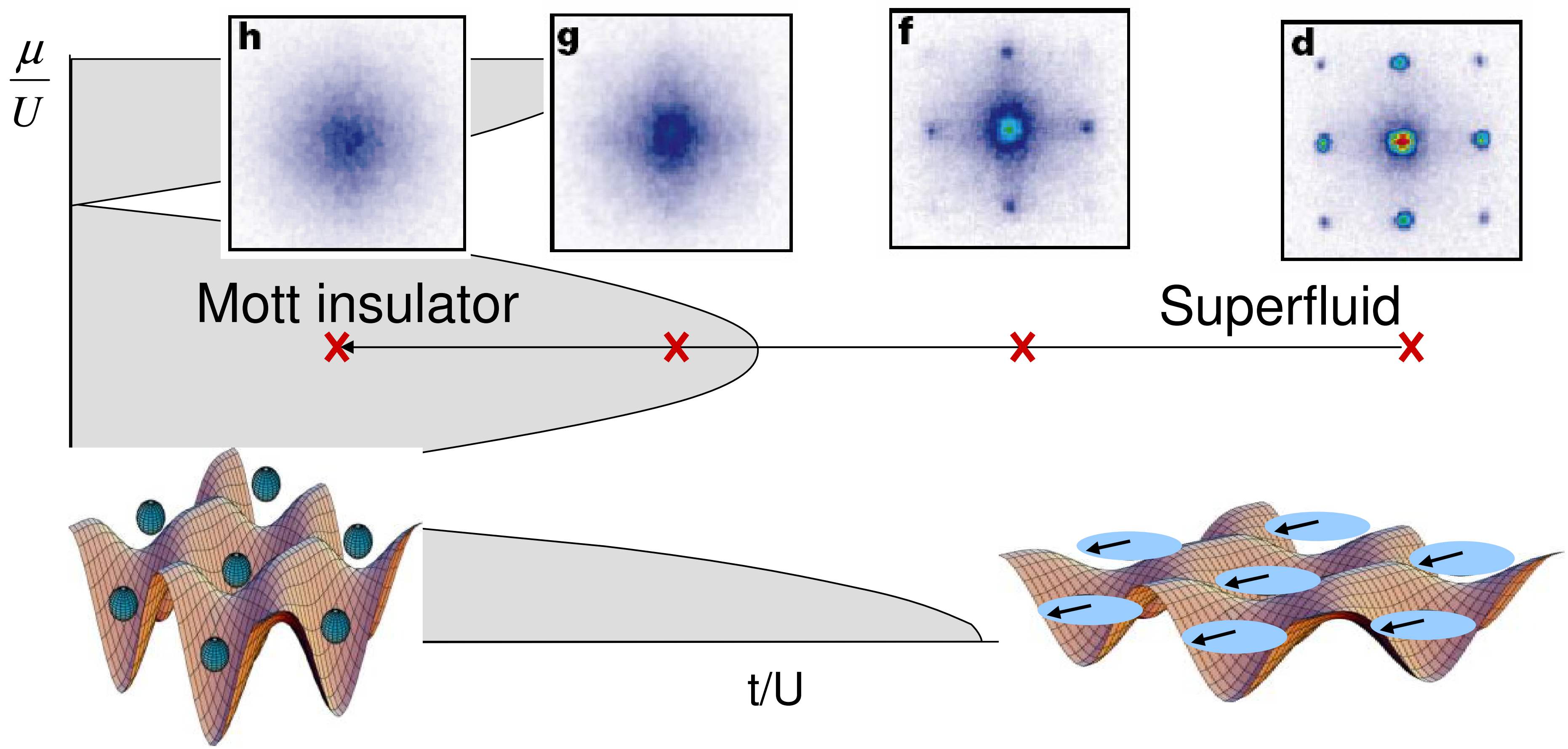


$$U >> Nt$$

Mott insulator phase
Strong interactions

Superfluid to insulator transition in an optical lattice

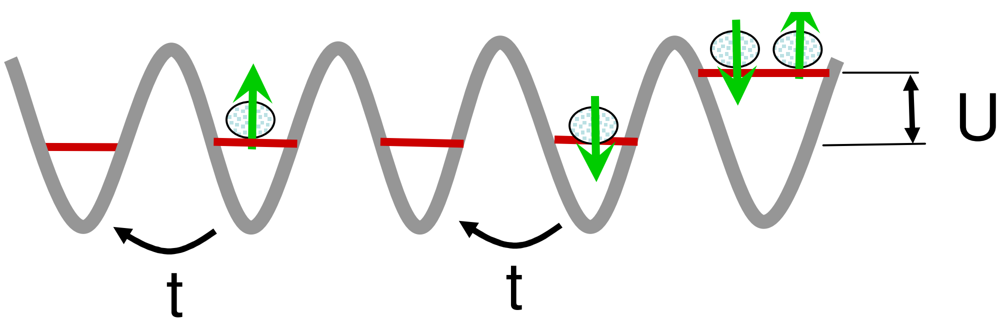
M. Greiner et al., Nature 415 (2002)



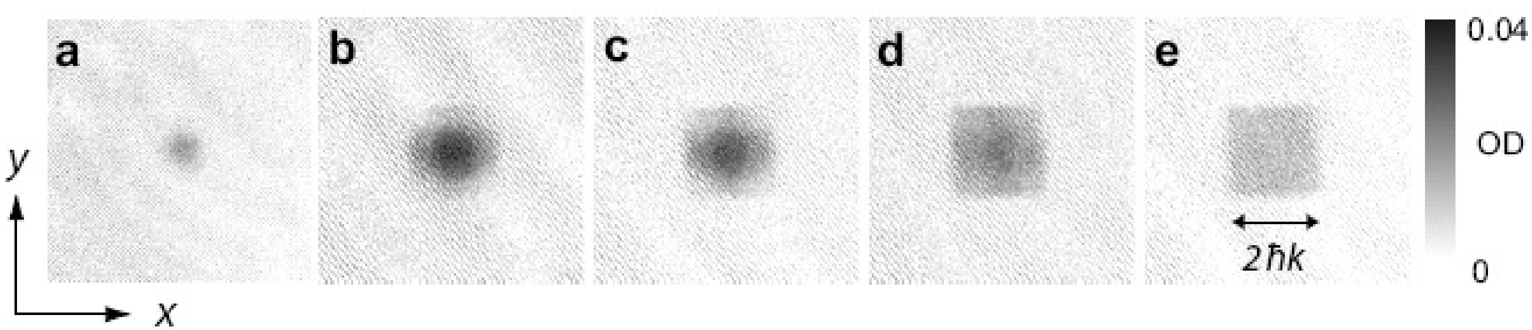
Why study ultracold atoms in optical lattices

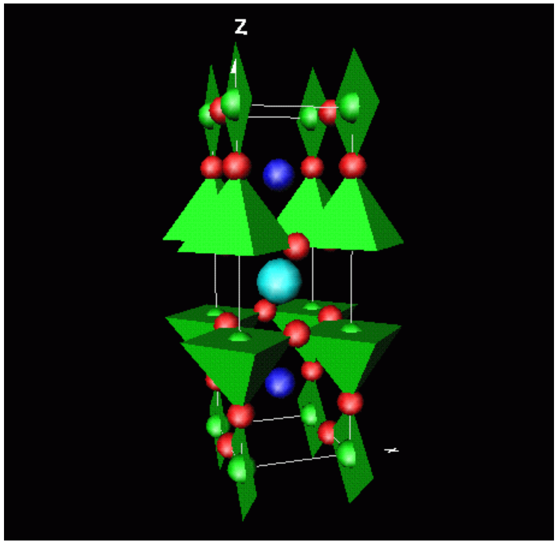
Fermionic atoms in optical lattices

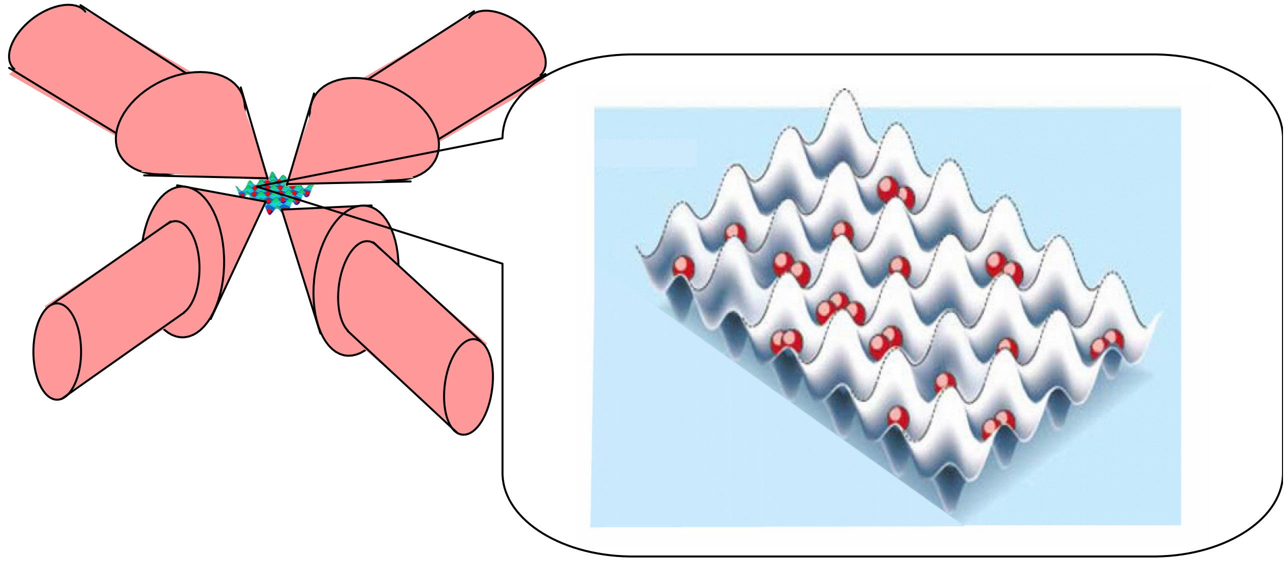
$$\mathcal{H} = -t \sum_{\langle ij \rangle\, \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_i$$



Experiments with fermions in optical lattice, Kohl et al., PRL 2005







$\mathrm{YBa_2Cu_3O_7}$

Atoms in optical lattice

Antiferromagnetic and
superconducting Tc
of the order of 100 K

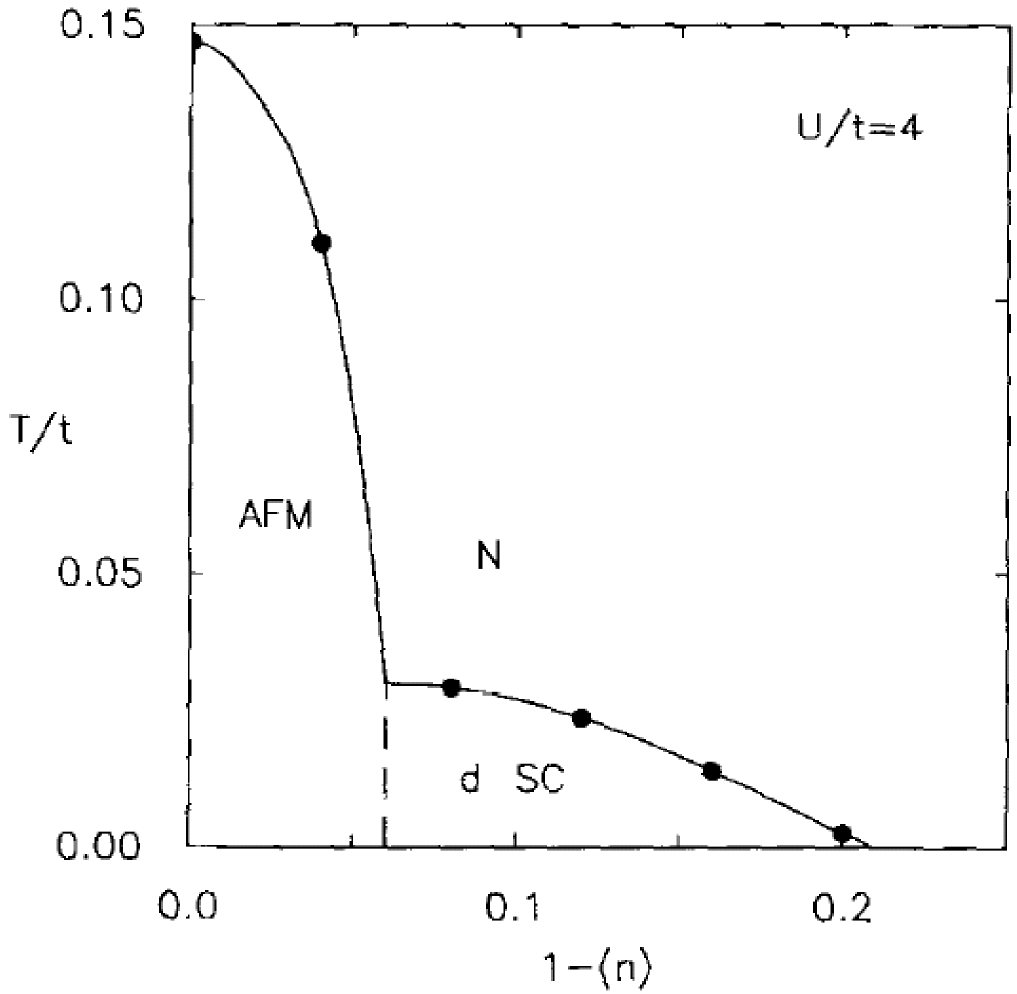
Antiferromagnetism and
pairing at sub-micro Kelvin
temperatures

Same microscopic model

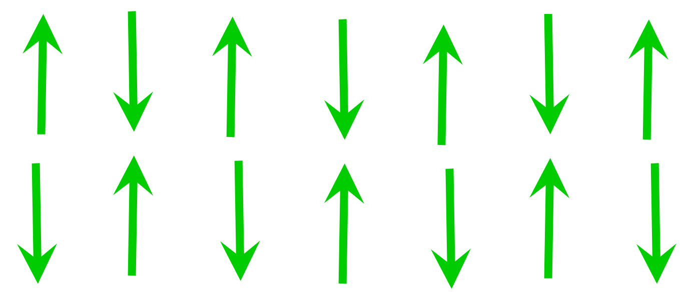
$$\mathcal{H} = -t \sum_{\langle ij \rangle \, \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i$$

Positive U Hubbard model

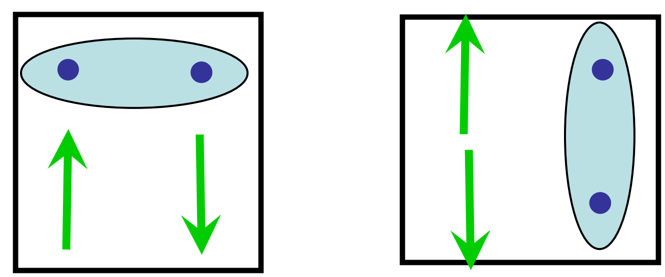
Possible phase diagram. Scalapino, Phys. Rep. 250:329 (1995)

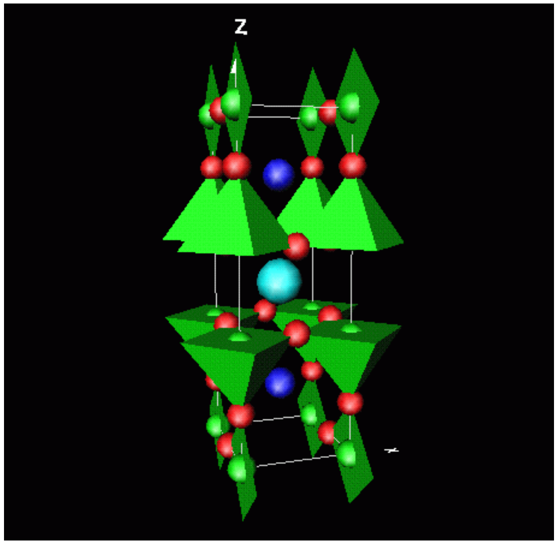


Antiferromagnetic insulator

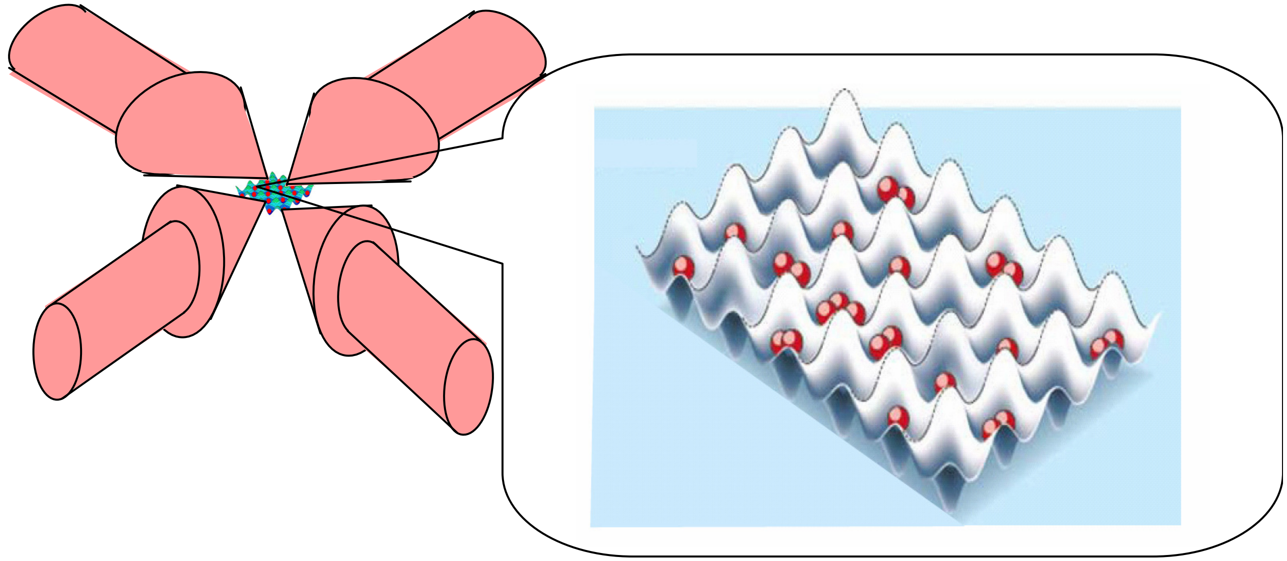


D-wave superconductor





$YBa_2Cu_3O_7$



Atoms in optical lattice

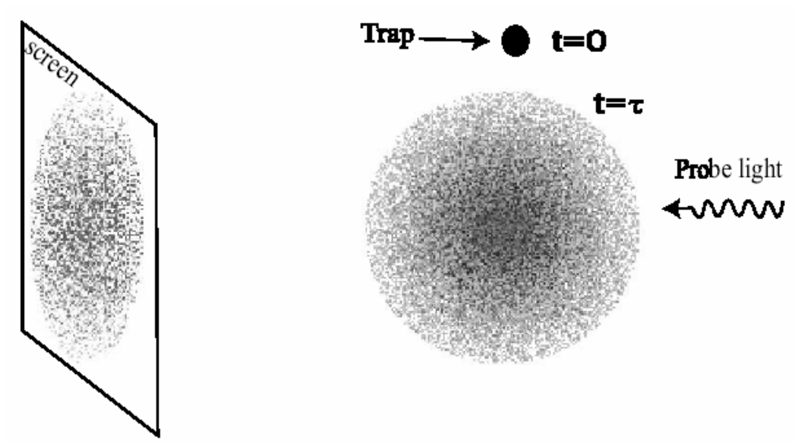
Same microscopic model

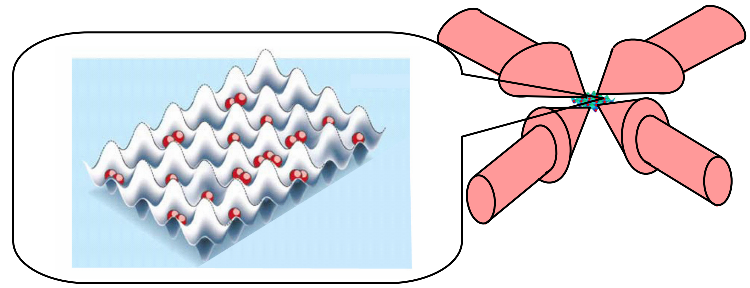
Quantum simulations of strongly correlated electron systems using ultracold atoms

Detection?

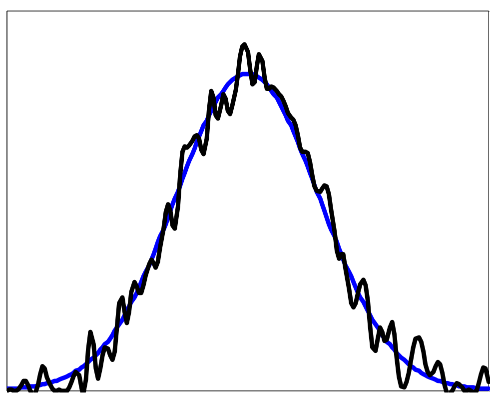
Quantum noise analysis as a probe of many-body states of ultracold atoms

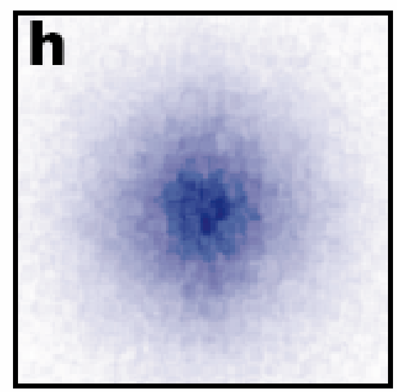
Time of flight experiments





Quantum noise interferometry of atoms in an optical lattice

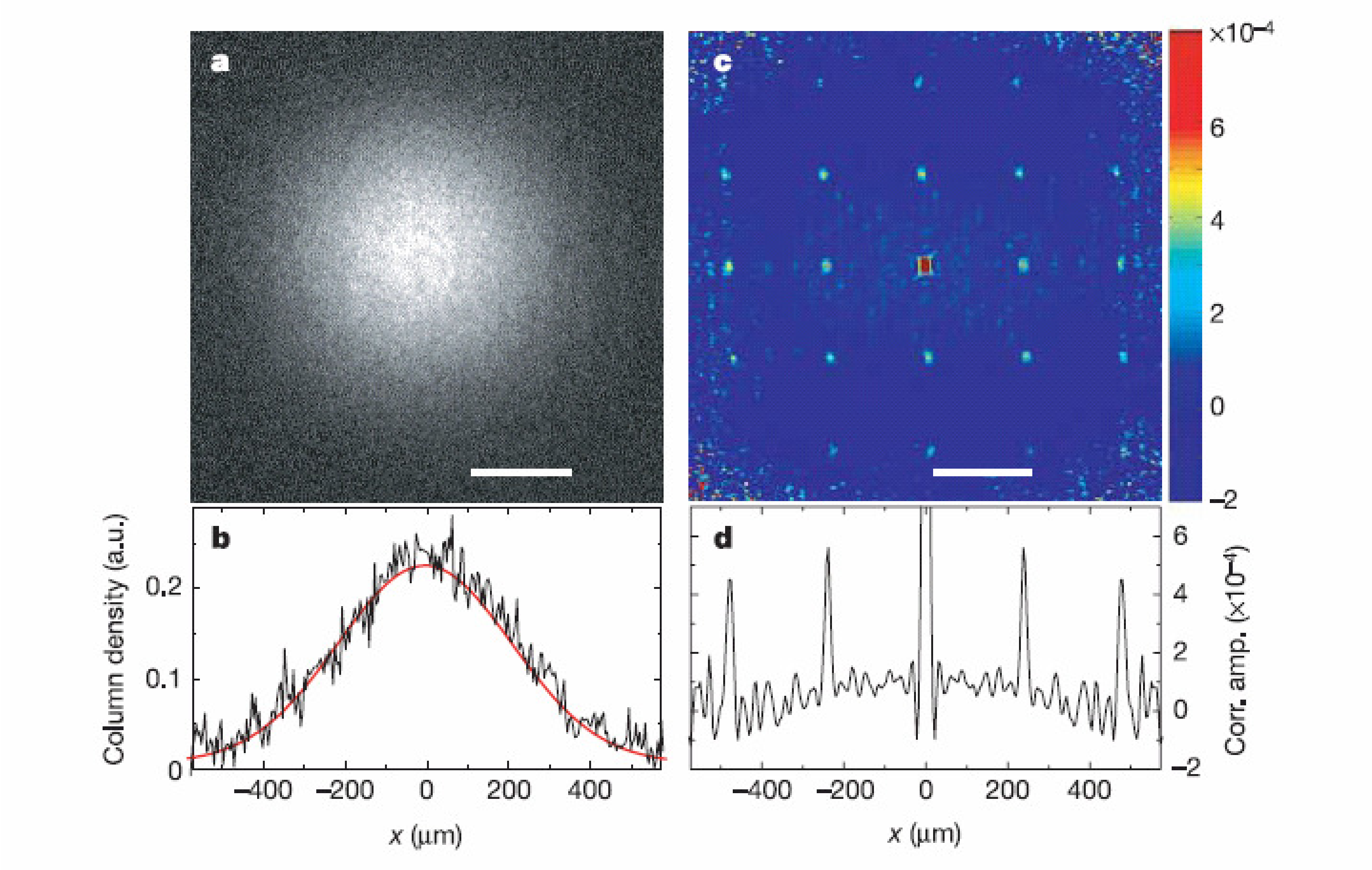




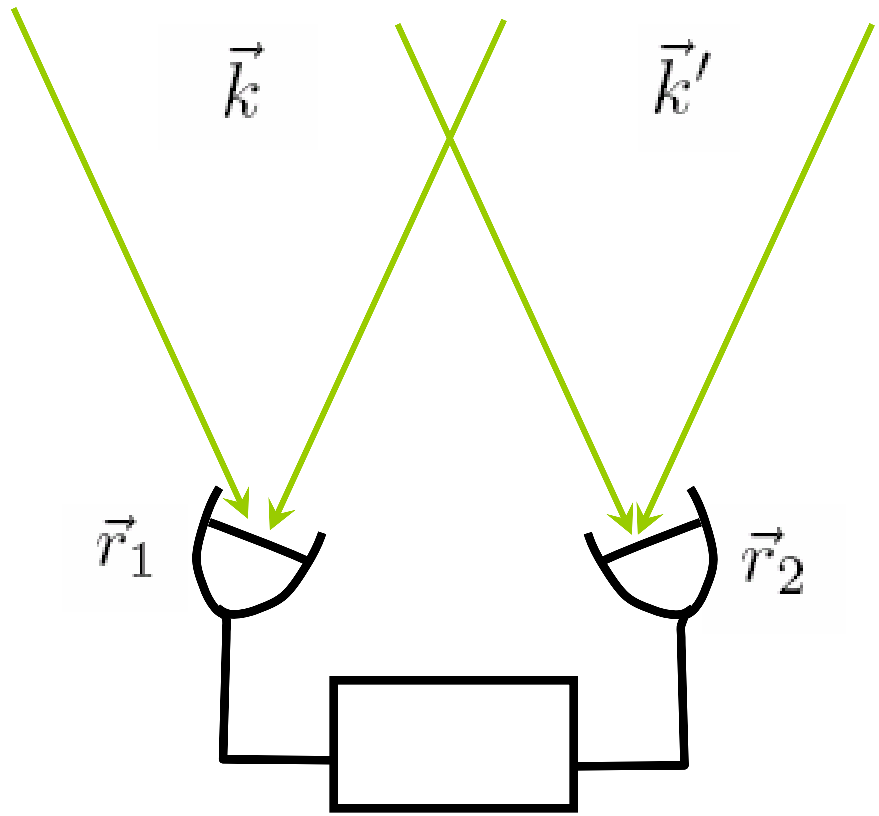
Second order coherence $G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$

Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

Experiment: Folling et al., Nature 434:481 (2005)

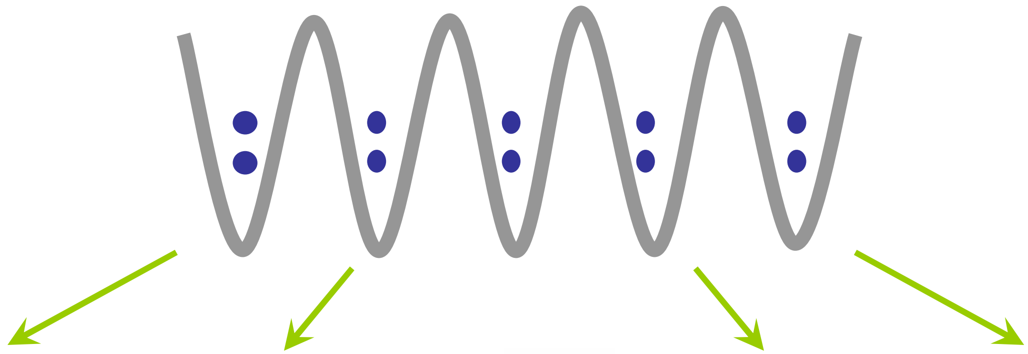


Hanburry-Brown-Twiss stellar interferometer



$$\langle I(\vec{r}_1)\; I(\vec{r}_2)\rangle = A + B\;\cos\Big((\vec{k}-\vec{k}')\;(\vec{r}_1-\vec{r}_2)\Big)$$

Second order coherence in the insulating state of bosons



Bosons at quasimomentum $\vec{k}$ expand as plane waves

with wavevectors $\vec{k}, \ \vec{k}+\vec{G}_1, \ \vec{k}+\vec{G}_2$

First order coherence: $\langle \ \rho(\vec{r}) \ \rangle$

Oscillations in density disappear after summing over $\vec{k}$

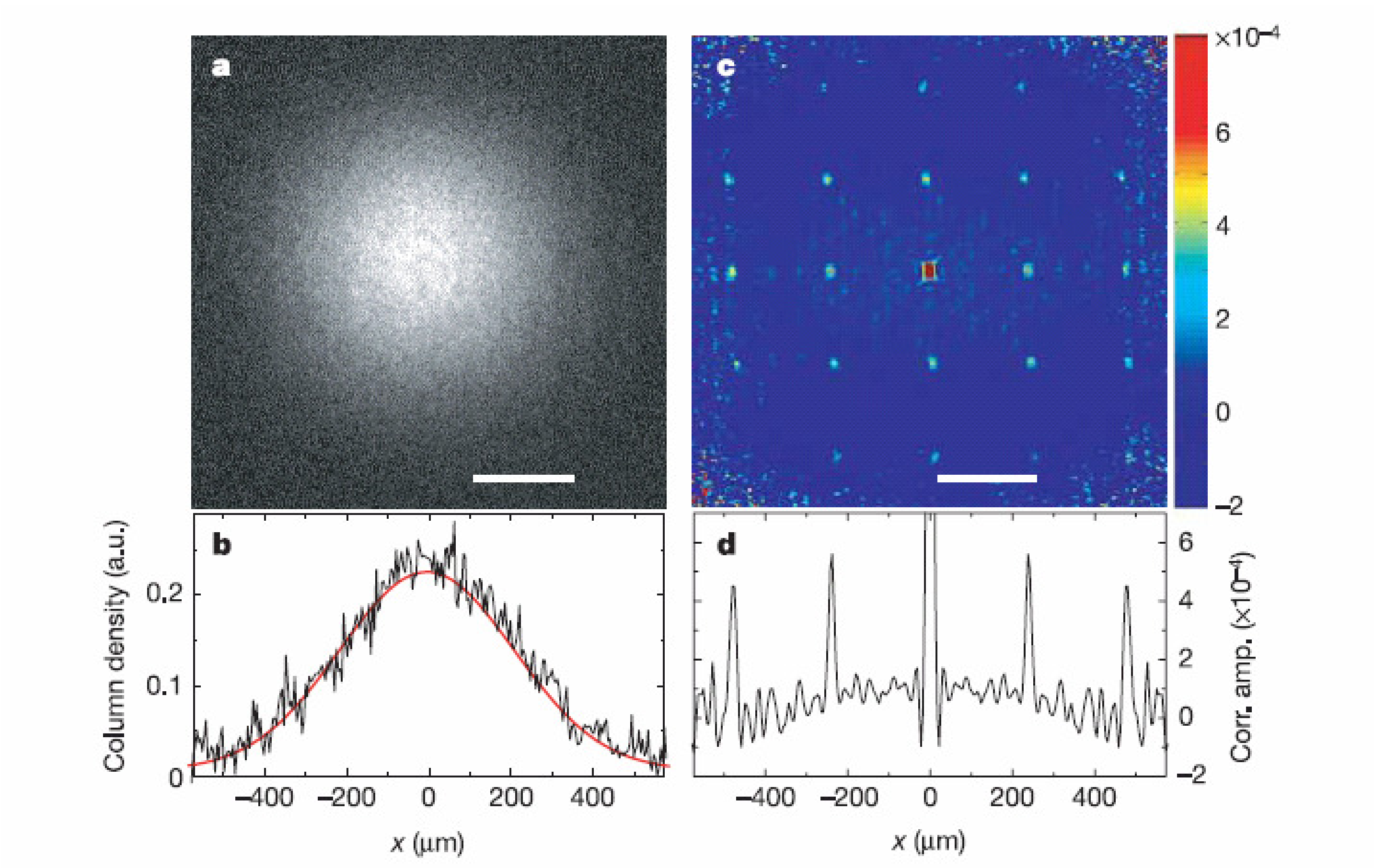
Second order coherence: $\langle \ \rho(\vec{r}_1) \ \rho(\vec{r}_2) \ \rangle$

Correlation function acquires oscillations at reciprocal lattice vectors

$$\langle \ \rho(\vec{r}_1) \ \rho(\vec{r}_2) \ \rangle = A_0 + A_1 \ \cos\left(\ \vec{G}_1(\vec{r}_1 - \vec{r}_2) \ \right) + A_2 \ \cos\left(\ \vec{G}_2(\vec{r}_1 - \vec{r}_2) \ \right) + \ldots$$

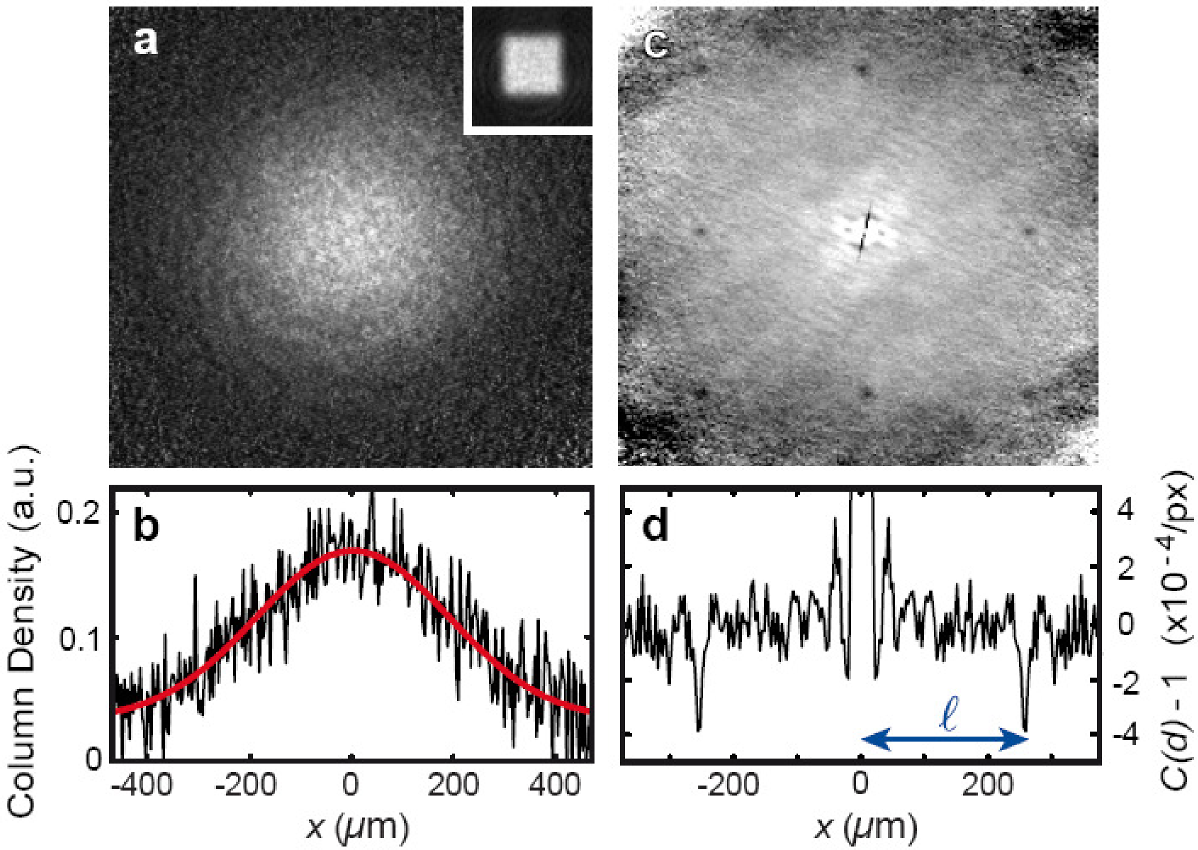
Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

Experiment: Folling et al., Nature 434:481 (2005)



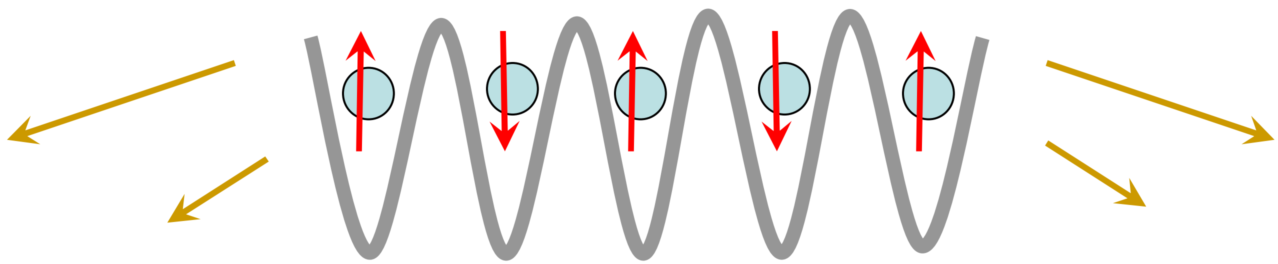
Second order coherence in the insulating state of fermions.
Hanburry-Brown-Twiss experiment

Experiment: Tom et al. Nature 444:733 (2006)



How to detect antiferromagnetism

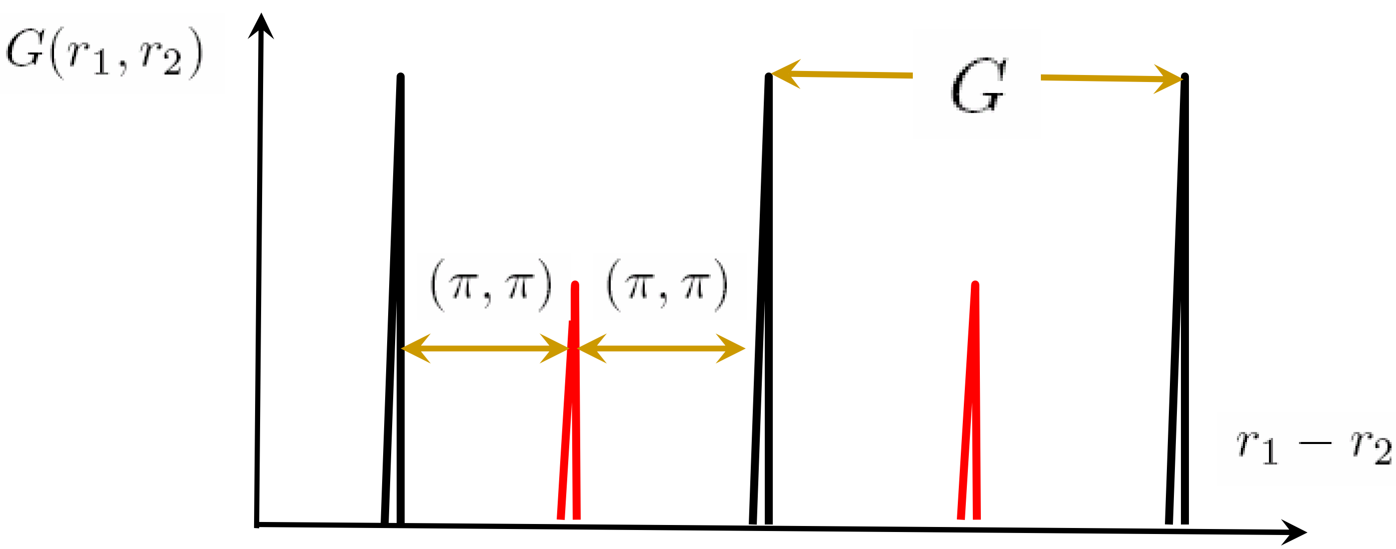
Probing spin order in optical lattices



$$r = \frac{\hbar \, k \, t}{m}$$

Correlation Function Measurements

$$\begin{aligned} G(r_1, r_2) &= \langle \, n(r_1) \; n(r_2) \, \rangle_{\mathrm{TOF}} - \langle \, n(r_1) \, \rangle_{\mathrm{TOF}} \; \langle \, n(r_2) \, \rangle_{\mathrm{TOF}} \\ &\sim \langle \, n(k_1) \; n(k_2) \, \rangle_{\mathrm{LAT}} - \langle \, n(k_1) \, \rangle_{\mathrm{LAT}} \; \langle \, n(k_2) \, \rangle_{\mathrm{LAT}} \end{aligned}$$



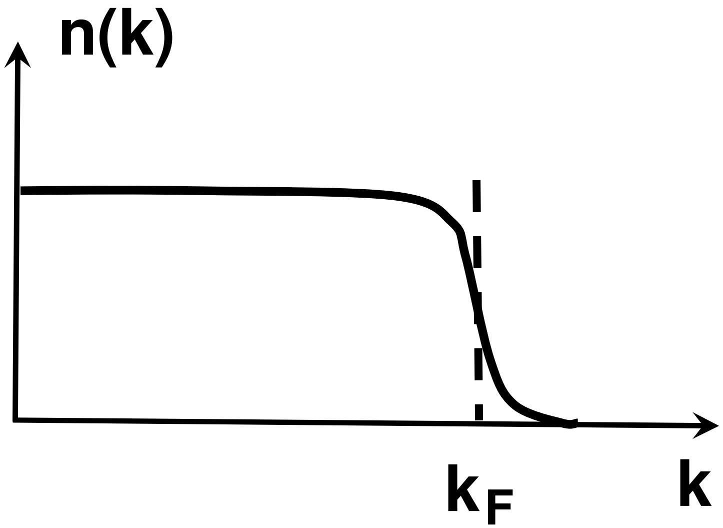
Extra Bragg peaks appear in the second order correlation function in the AF phase

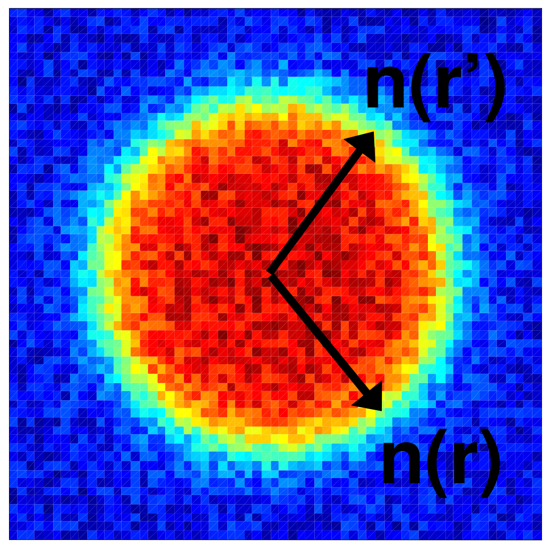
How to detect fermion pairing

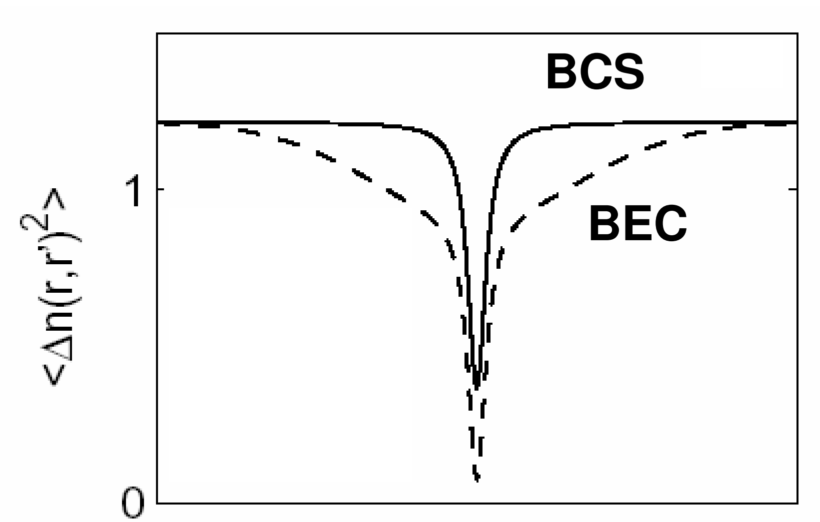
Quantum noise analysis of TOF images
is more than HBT interference

Second order interference from the BCS superfluid

Theory: Altman et al., PRA 70:13603 (2004)





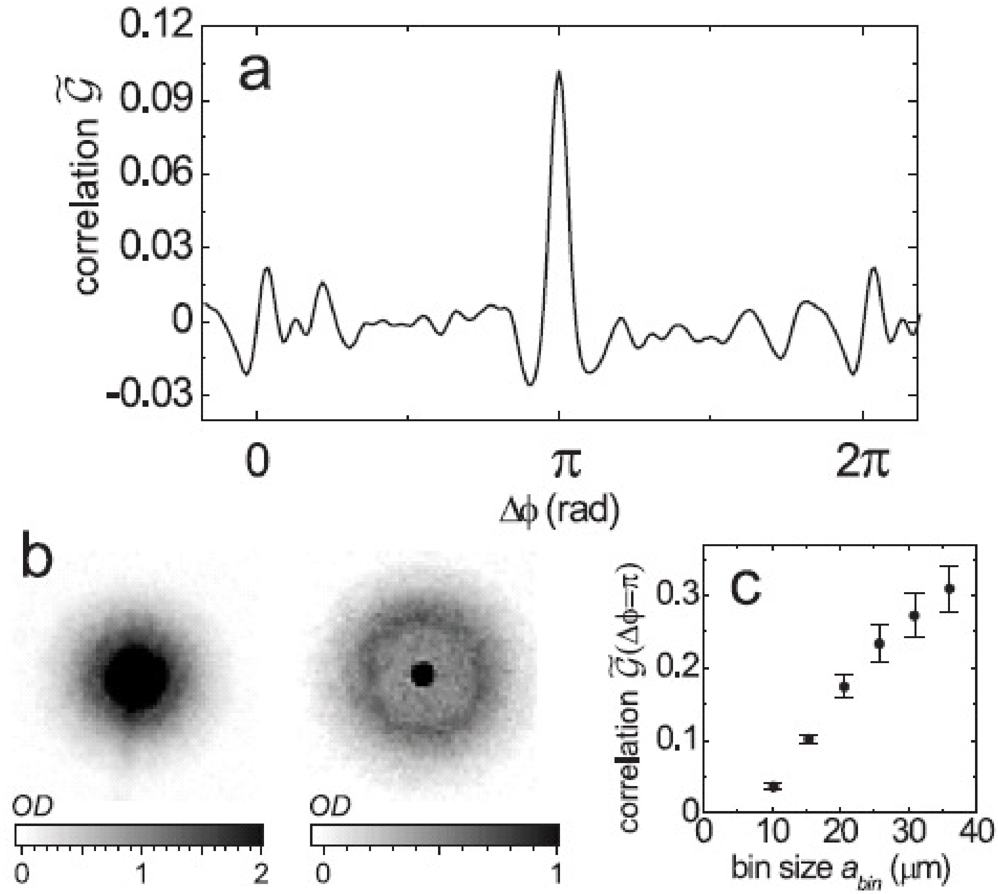


$$\Delta n(\mathbf{r},\mathbf{r}') \equiv n(\mathbf{r}) - n(\mathbf{r}')$$

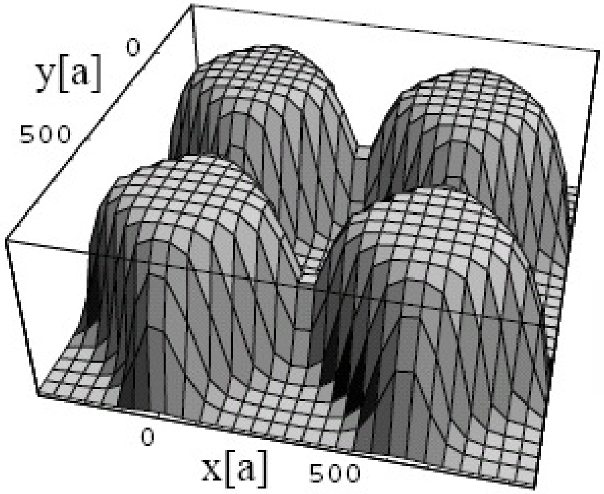
$$\Delta n(\mathbf{r},-\mathbf{r}) \left| \Psi_{BCS} \right\rangle = 0$$

Momentum correlations in paired fermions

Greiner et al., PRL 94:110401 (2005)



Fermion pairing in an optical lattice



Second Order Interference
In the TOF images

$$G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$$

Normal State

$$G_{\rm N}(r_1, r_2) = \delta(r_1 - r_2)\rho(r_1) - \rho^2(r_1) \sum_G \delta(r_1 - r_2 - \frac{G\hbar t}{m})$$

Superfluid State

$$G_{\rm S}(r_1, r_2) = G_{\rm N}(r_1, r_2) + \Psi(r_1) \sum_G \delta(r_1 + r_2 + \frac{G\hbar t}{m})$$

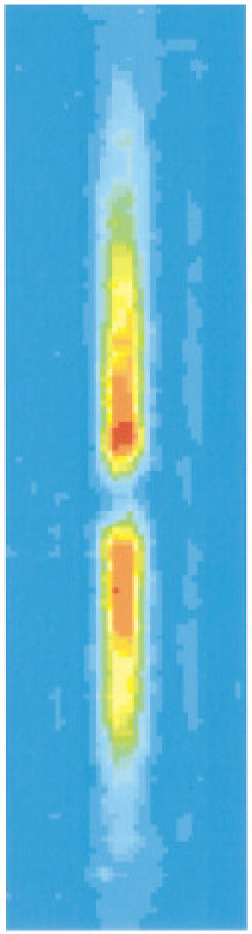
$\Psi(r) = |u(Q(r))v(Q(r))|^2$ **measures the Cooper pair wavefunction**

$$Q(r) = \frac{mr}{\hbar t}$$

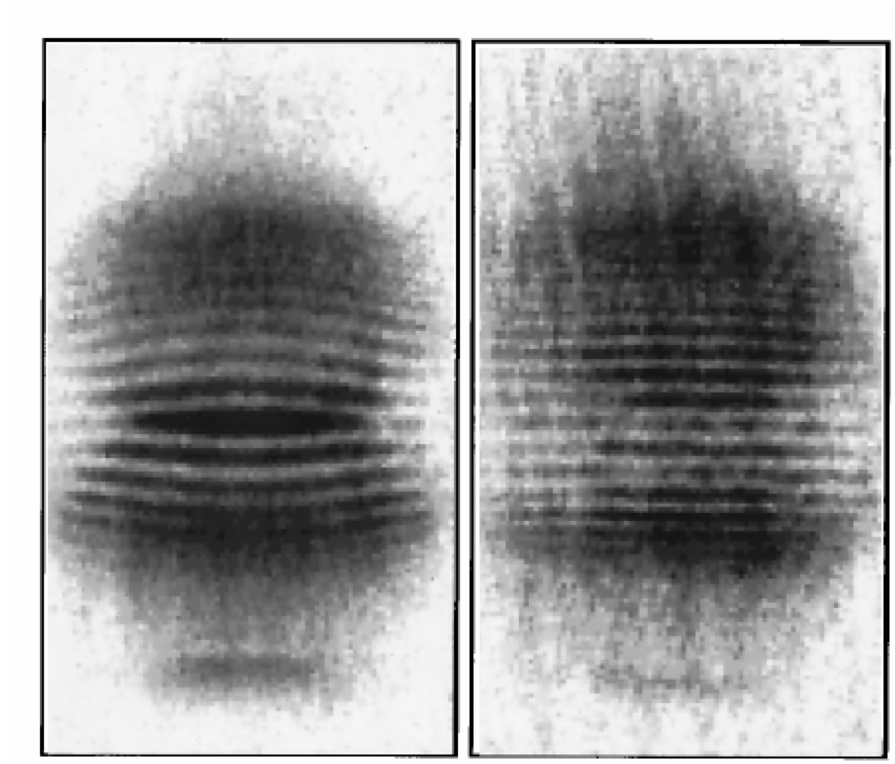
One can identify unconventional pairing

Interference experiments with cold atoms

Interference of independent condensates



Experiments: Andrews et al., Science 275:637 (1997)



Theory: Javanainen, Yoo, PRL 76:161 (1996)
Cirac, Zoller, et al. PRA 54:R3714 (1996)
Castin, Dalibard, PRA 55:4330 (1997)
and many more

Nature 4877:255 (1963)

INTERFERENCE FRINGES PRODUCED BY SUPERPOSITION OF TWO INDEPENDENT MASER LIGHT BEAMS

By G. MAGYAR and Dr. L. MANDEL
Department of Physics, Imperial College of Science and Technology, London

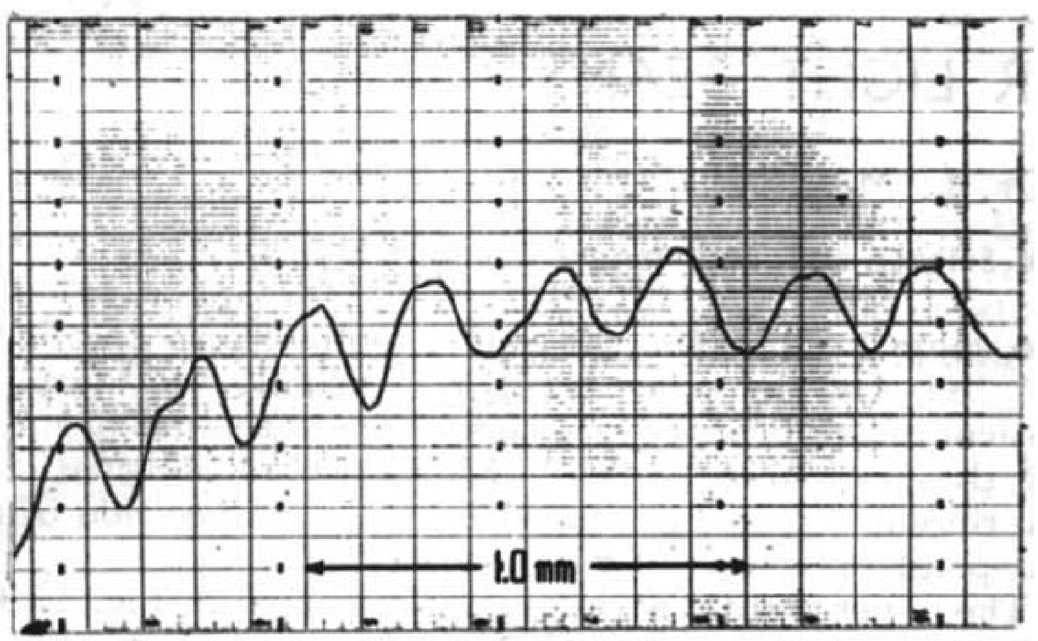
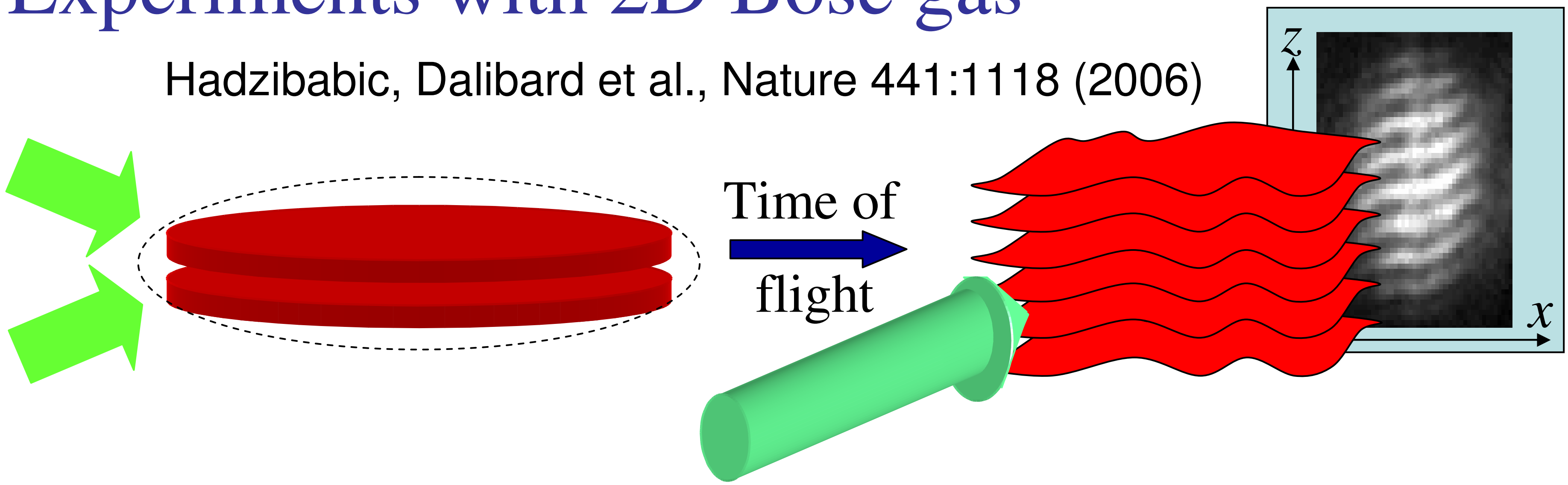
b


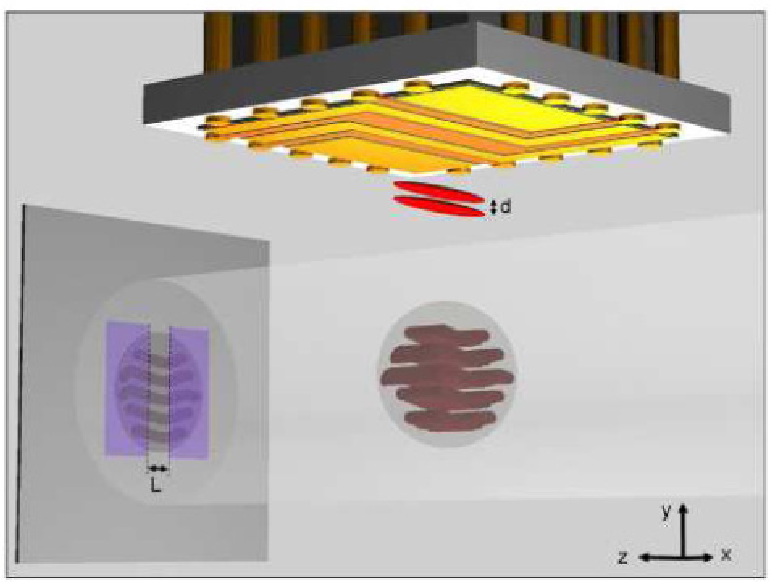
Fig. 2. An example of fringes recorded: (*a*) photograph; (*b*) microphotometer tracing

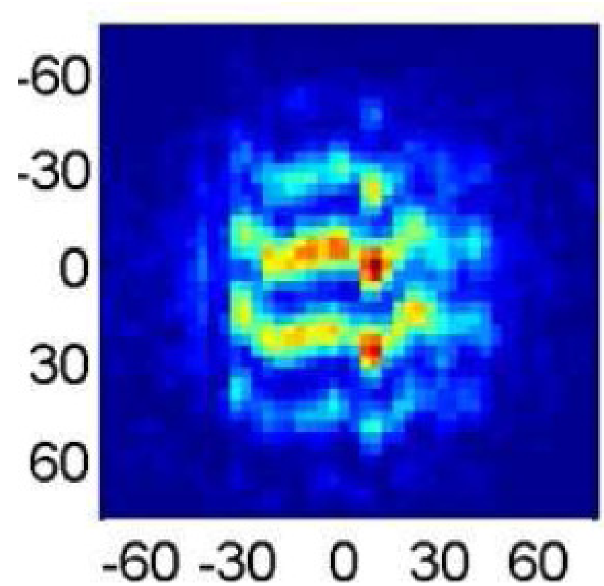
Experiments with 2D Bose gas

Hadzibabic, Dalibard et al., Nature 441:1118 (2006)

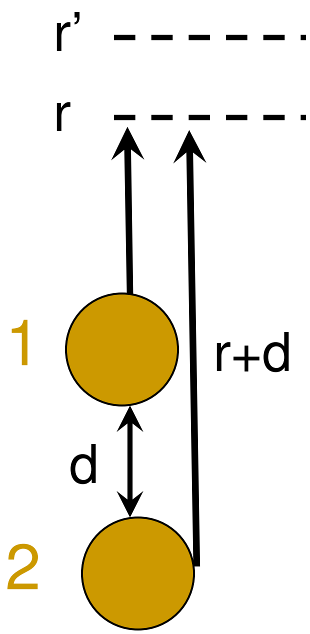


Experiments with 1D Bose gas S. Hofferberth et al. arXiv0710.1575





Interference of two independent condensates



$$\psi(r) = \psi_1(r) + \psi_2(r)$$

$$\rho_{\rm int}(r) = \psi_1^\dagger(r)\,\psi_2(r) + {\rm c.c.}$$

$$\psi_1(r) = e^{i\phi_1 + ik_1 r} \qquad k_1 = \frac{m\,v_1}{\hbar} = \frac{m\,r}{\hbar\,t}$$

$$\psi_2(r) = e^{i\phi_2 + ik_2 r} \qquad k_2 = \frac{m\,v_2}{\hbar} = \frac{m\,(r+d)}{\hbar\,t}$$

$$\rho_{\rm int}(r) = e^{i\,(k_2 - k_1)\,r}\, e^{i\,(\phi_2 - \phi_1)} + {\rm c.c.}$$

$$\boxed{\rho_{\rm int}(r) = e^{i\,\frac{m\,d\,r}{\hbar\,t}}\, e^{i\,(\phi_2 - \phi_1)} + {\rm c.c.}}$$

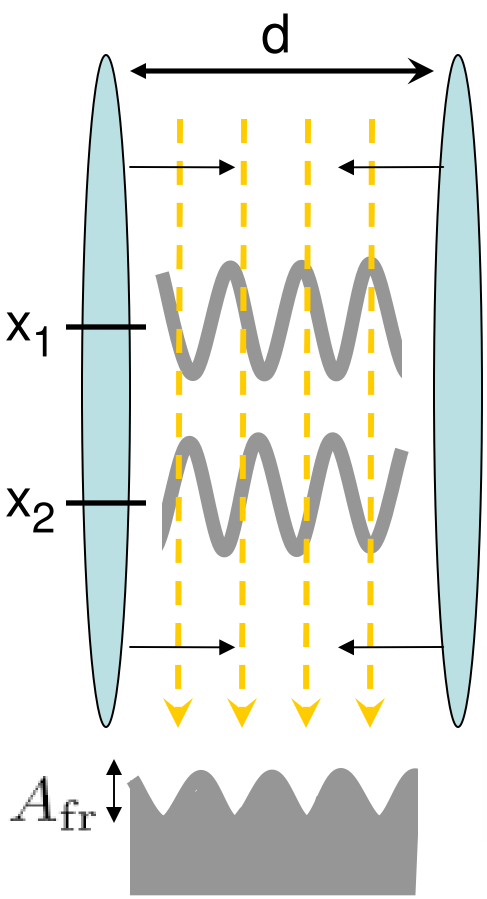
Clouds 1 and 2 do not have a well defined phase difference.
However each individual measurement shows an interference pattern

$$\langle\, \rho_{\rm int}(r)\,\rangle = 0$$

$$\langle\, \rho_{\rm int}(r)\, \rho_{\rm int}(r')\,\rangle = e^{i\,\frac{m\,d}{\hbar\,t}\,(r - r')} + {\rm c.c.}$$

Interference of fluctuating condensates

Polkovnikov, Altman, Demler, PNAS 103:6125(2006)



Amplitude of interference fringes, $A_{\rm fr}$

$$|A_{\rm fr}|\, e^{i\Delta\phi} = \int_0^L dx\, e^{i(\phi_1(x)-\phi_2(x))}$$

For independent condensates A_{fr} is finite but $\Delta\phi$ is random

$$\langle\, |A_{\rm fr}|^2 \,\rangle = \int_0^L dx_1 \int_0^L dx_2 \,\langle\, e^{i(\phi_1(x_1)-\phi_2(x_1))}\, e^{-i(\phi_1(x_2)-\phi_2(x_2))} \rangle$$

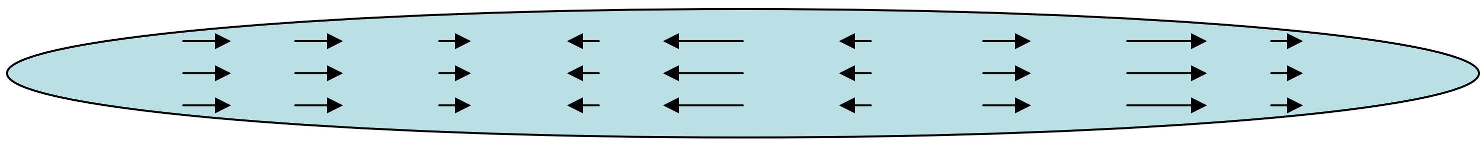
$$\langle\, |A_{\rm fr}|^2 \,\rangle \approx L \int_0^L dx \,\langle\, e^{i(\phi_1(x)-\phi_1(0))} \rangle \,\langle\, e^{-i(\phi_2(x)-\phi_2(0))} \,\rangle$$

For identical condensates $\langle |A_{\rm fr}|^2 \rangle = L \int_0^L dx \,(\, G(x)\,)^2$

Instantaneous correlation function $G(x) = \langle\, e^{i\phi(x)}\, e^{-\phi(0)} \,\rangle$

Fluctuations in 1d BEC

Thermal fluctuations

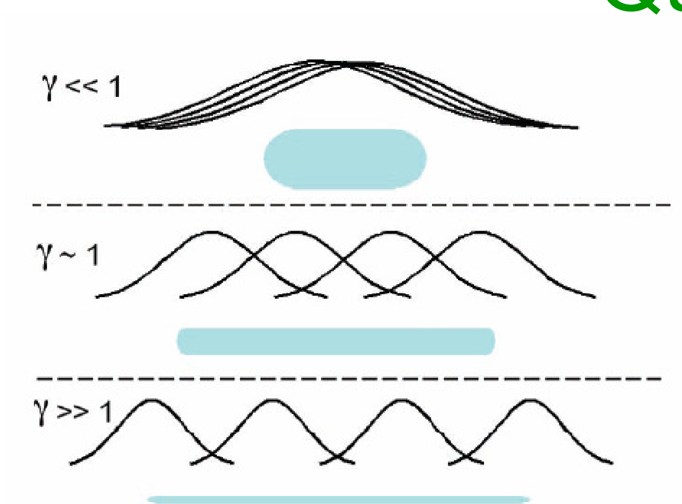


Thermally energy of the superflow velocity $\mathrm{v}_s = \nabla\phi(x)$

$$\langle\, e^{i\phi(x_1)}\, e^{i\phi(x_2)} \,\rangle \sim e^{-|x_1 - x_2|/\xi_T}$$

$$\xi_T = \sqrt{\frac{\hbar^2\, m}{T}}$$

Quantum fluctuations



$$\langle\, e^{i\phi(x_1)}\, e^{i\phi(x_2)} \,\rangle \sim \left(\frac{\xi_h}{|x_1 - x_2|} \right)^{1/2K}$$

$$K = \sqrt{\frac{n}{g\, m}}$$

Interference between Luttinger liquids

Luttinger liquid at T=0 $$G(x) \sim \rho \left(\frac{\xi_h}{x} \right)^{1/2K}$$

$$\langle |A_{\rm fr}|^2 \rangle \sim (\rho \xi_h)^{1/K} (L\rho)^{2-1/K}$$

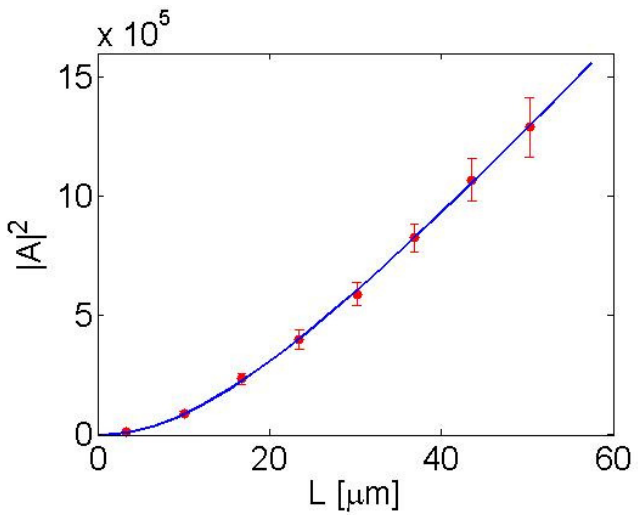
K – Luttinger parameter

For non-interacting bosons $K = \infty$ and $A_{\rm fr} \sim L$

For impenetrable bosons $K = 1$ and $A_{\rm fr} \sim \sqrt{L}$

Finite temperature

$$\langle |A_{\rm fr}|^2 \rangle \sim L \rho^2 \xi_h \left(\frac{\hbar^2}{m \xi_h^2} \frac{1}{T} \right)^{1-1/K}$$



Experiments: Hofferberth, Schumm, Schmiedmayer

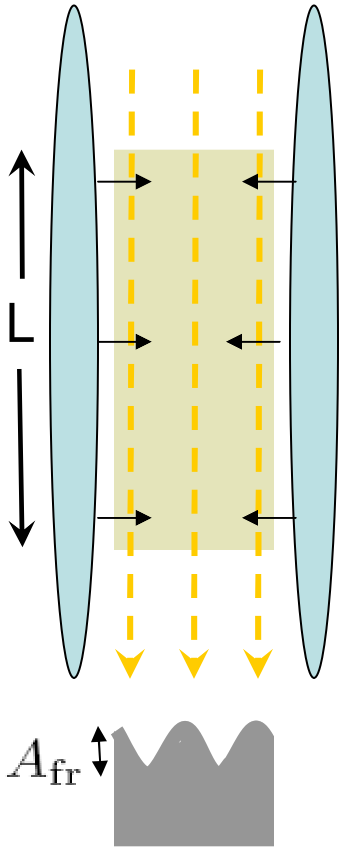
$$n_{1d} = 60 \mu\text{m}^{-1}$$

$$K = 47$$

$$T_{fit} = 84 \pm 22 \text{ nK}$$

Distribution function of fringe amplitudes for interference of fluctuating condensates

Gritsev, Altman, Demler, Polkovnikov, Nature Physics 2006
Imambekov, Gritsev, Demler, cond-mat/0612011



$A_{\rm fr}$ is a quantum operator. The measured value of $|A_{\rm fr}|$ will fluctuate from shot to shot.

$$\langle\, |A_{\rm fr}|^{2n} \,\rangle = \int_0^L dz_1 \dots dz'_n \,|\, \langle\, e^{i\phi(z_1)} \dots e^{i\phi(z_n)}\, e^{-i\phi(z'_1)} \dots e^{-i\phi(z'_n)}\, \rangle|^2$$

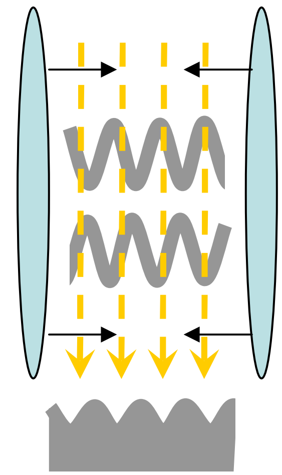
Higher moments reflect higher order correlation functions

We need the full distribution function of $|A_{\rm fr}|$

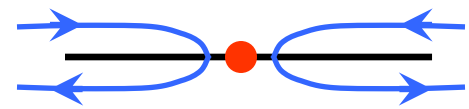
Interference between interacting 1d Bose liquids. Distribution function of the interference amplitude

Normalized amplitude of interference fringes $a^2 = |A_{\rm fr}|^2 / \langle\, |A_{\rm fr}|^2 \rangle$

Distribution function of fringe amplitudes $W(\,K,\, a^2\,)$

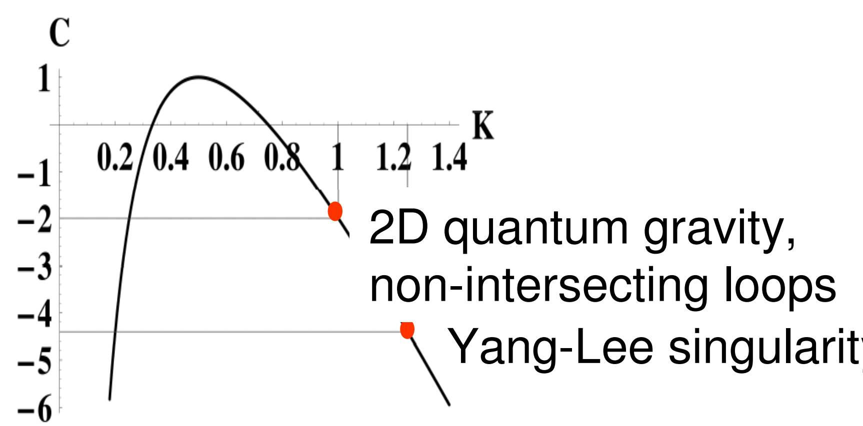


Quantum impurity problem. Need analytically continued partition function



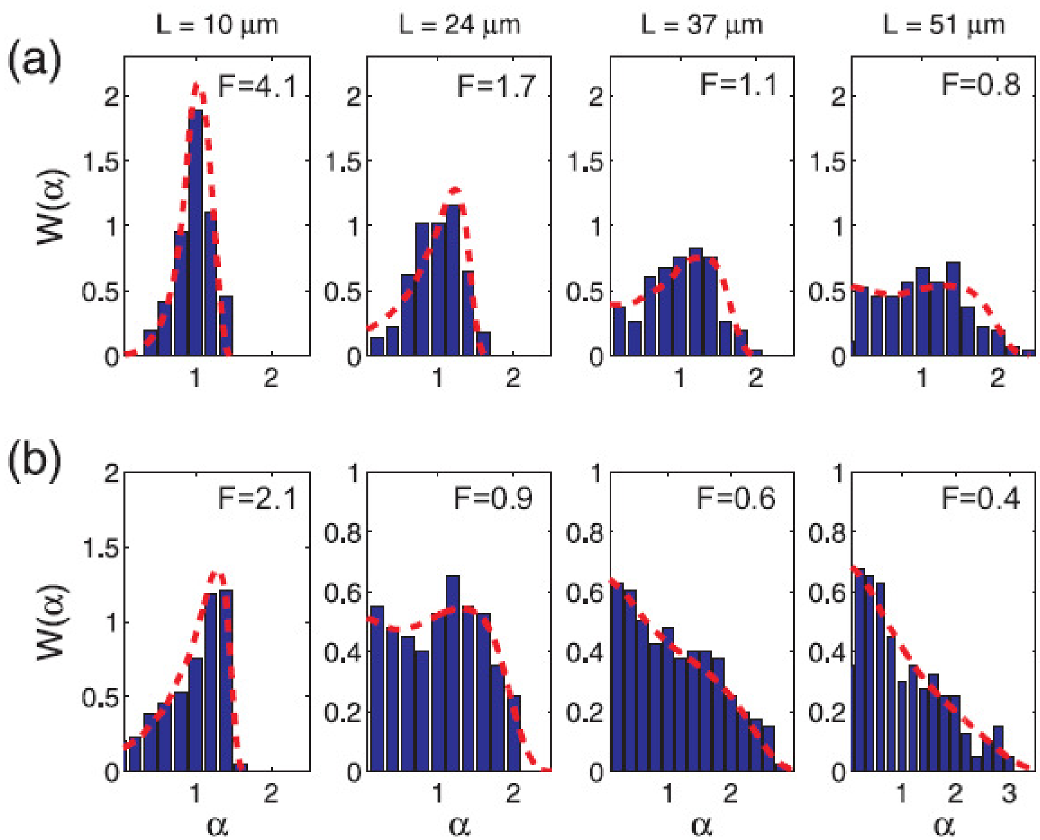
$$W(K,a^2) = 2 \int_0^\infty g\, dg\, Z_{\rm imp}(K, ig) J_0(2ga^2)$$

Conformal field theories with negative central charges: 2D quantum gravity, non-intersecting loop model, growth of random fractal stochastic interface,…



Distribution function of interference fringe contrast

Experiments: Hofferberth et al., arXiv0710.1575
Theory: Imambekov et al. , cond-mat/0612011



Quantum fluctuations dominate:
asymetric Gumbel distribution
(low temp. T or short length L)

Thermal fluctuations dominate:
broad Poissonian distribution
(high temp. T or long length L)

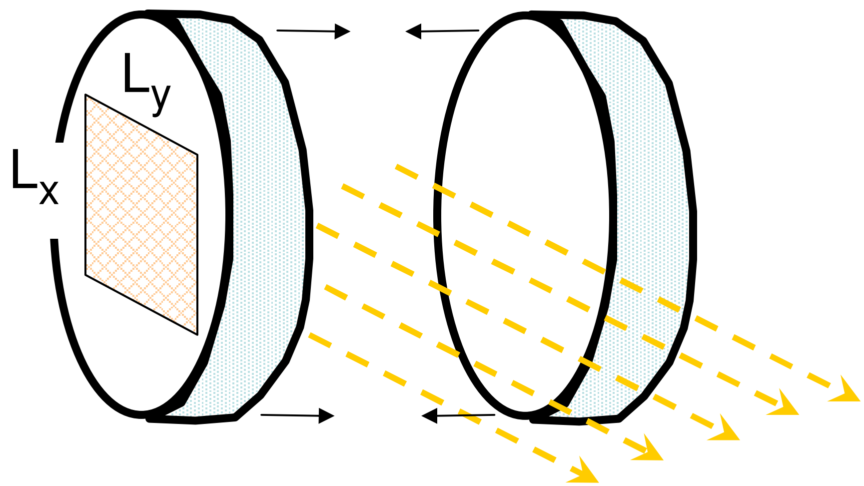
Intermediate regime:
double peak structure

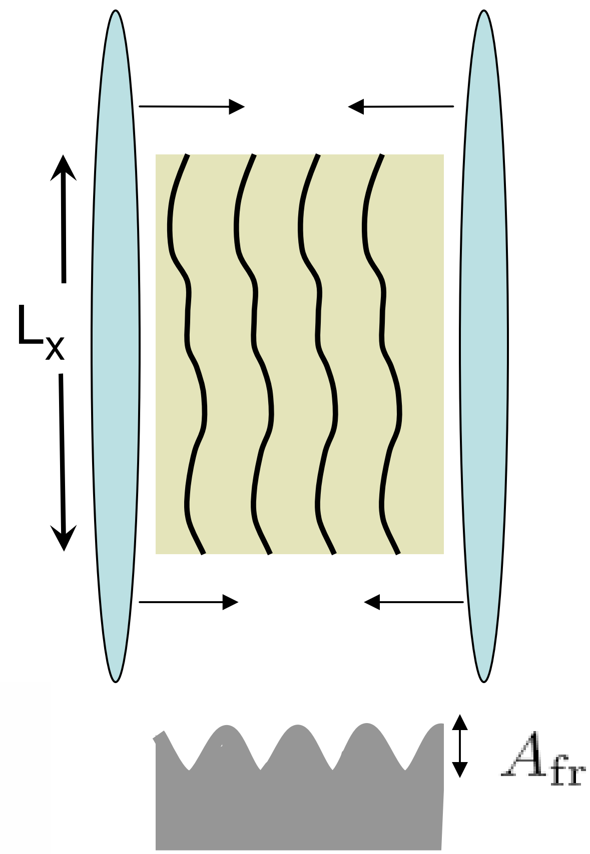
Comparison of theory and experiments: no free parameters
Higher order correlation functions can be obtained

Interference of two dimensional condensates

Experiments: Hadzibabic et al. Nature (2006)

Gati et al., PRL (2006)



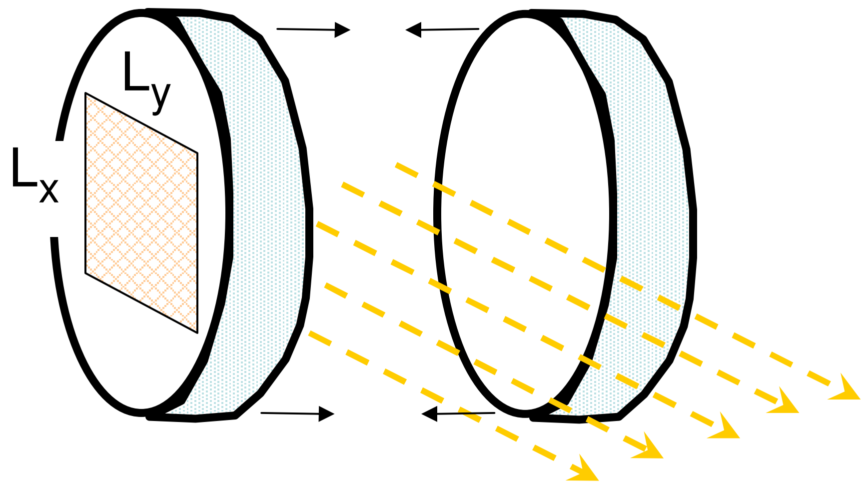


Probe beam parallel to the plane of the condensates

$$\langle\, |A_{\rm fr}|^2 \rangle \,=\, L_x\, L_y \int_0^{L_x} \int_0^{L_y} \, d^2\vec{r}\, \left(\, G(\vec{r}\,)\,\right)^2$$

$$G(\vec{r}\,) \,=\, \langle\, a(\vec{r}\,)\, a^\dagger(0)\,\rangle$$

Interference of two dimensional condensates. Quasi long range order and the KT transition



Above KT transition

$$G(r) \sim e^{-r/\xi}$$

$$\langle |A_{\rm fr}|^2 \rangle \sim L_x L_y$$

$$\log \xi(T) \sim 1/\sqrt{T - T_{\rm KT}}$$

Below KT transition

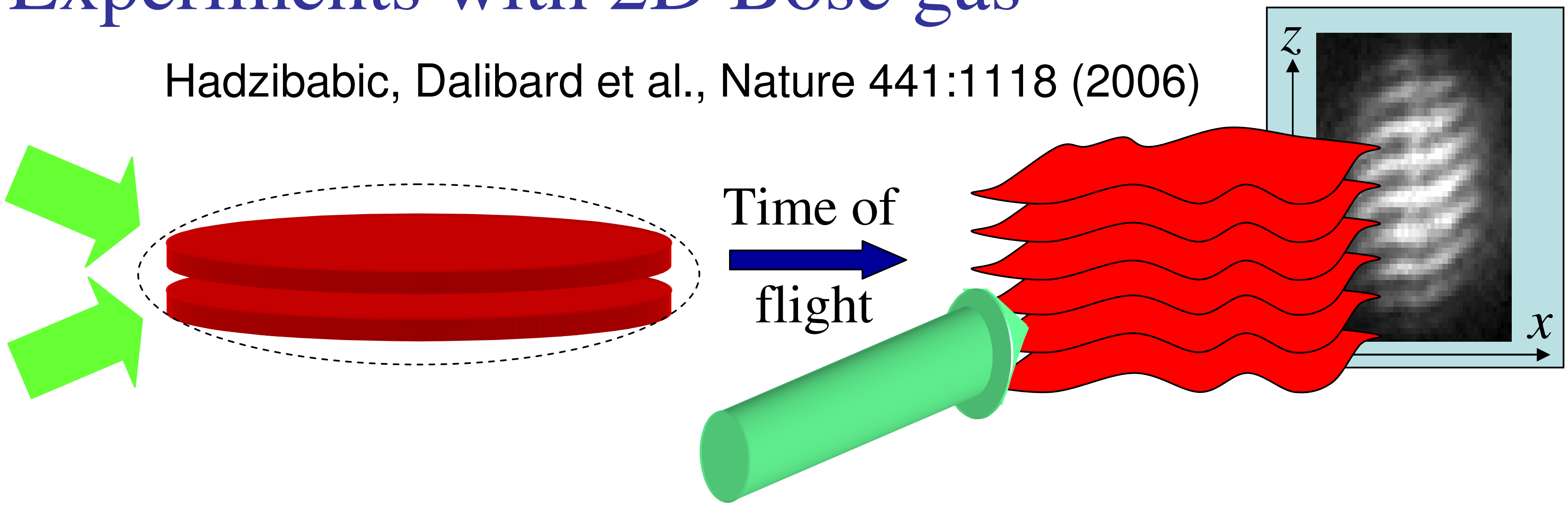
$$G(r) \sim \rho \left(\frac{\xi_h}{r} \right)^{\alpha}$$

$$\alpha(T) = \frac{mT}{2\pi \rho_s(T) \hbar^2}$$

$$\langle |A_{\rm fr}|^2 \rangle \sim (L_x L_y)^{2-\alpha}$$

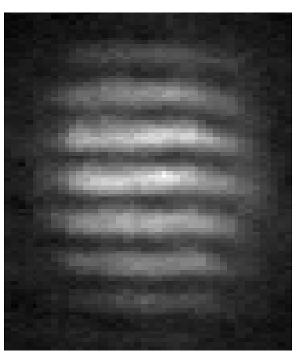
Experiments with 2D Bose gas

Hadzibabic, Dalibard et al., Nature 441:1118 (2006)

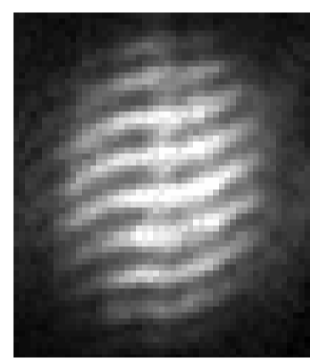


Typical interference patterns

low temperature

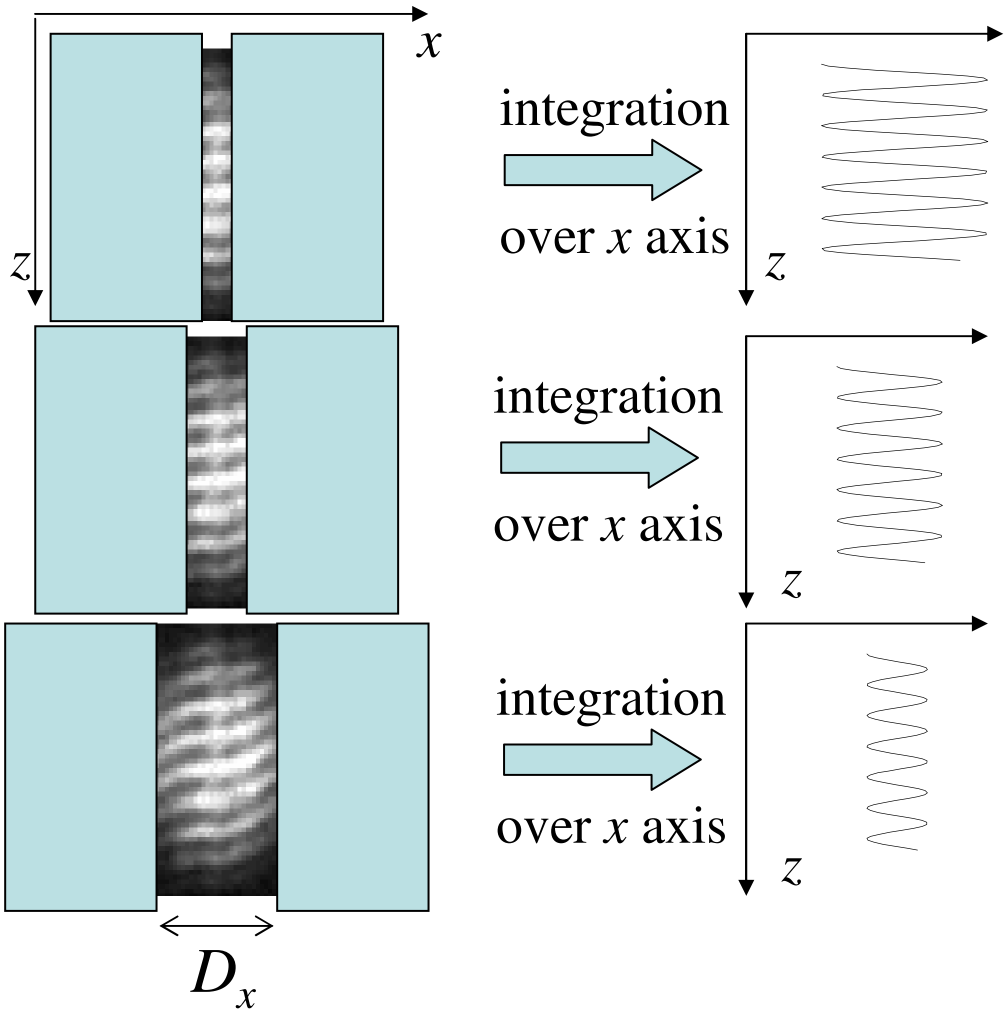


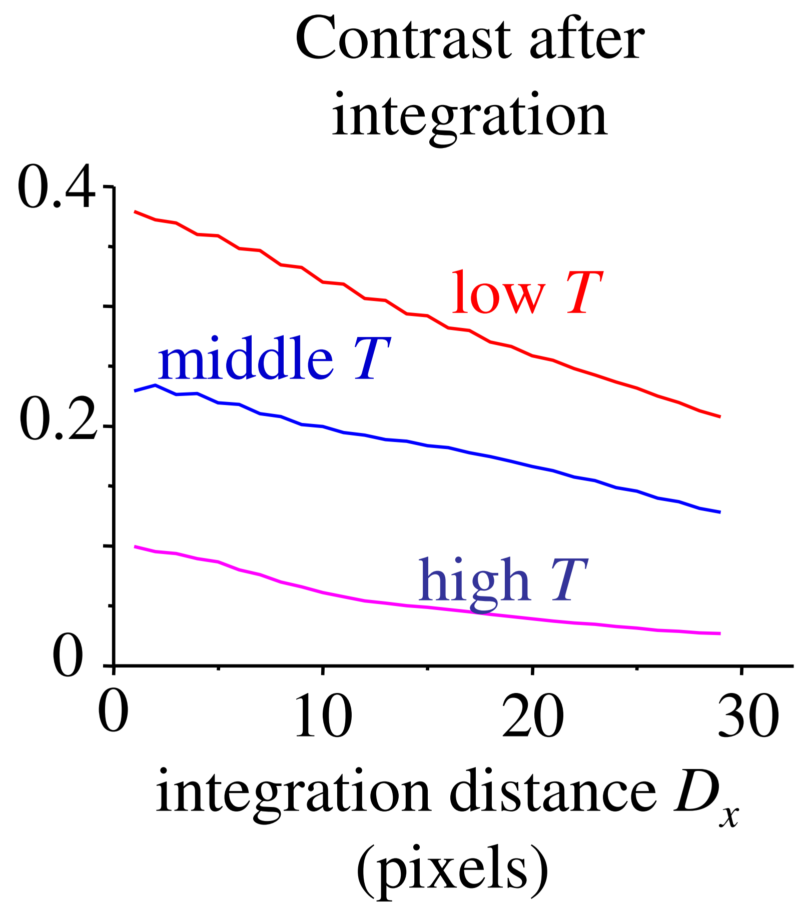
higher temperature



Experiments with 2D Bose gas

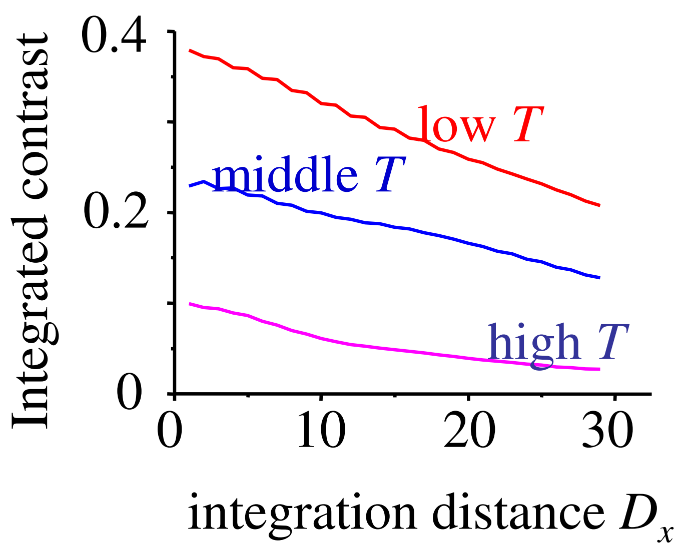
Hadzibabic et al., Nature 441:1118 (2006)





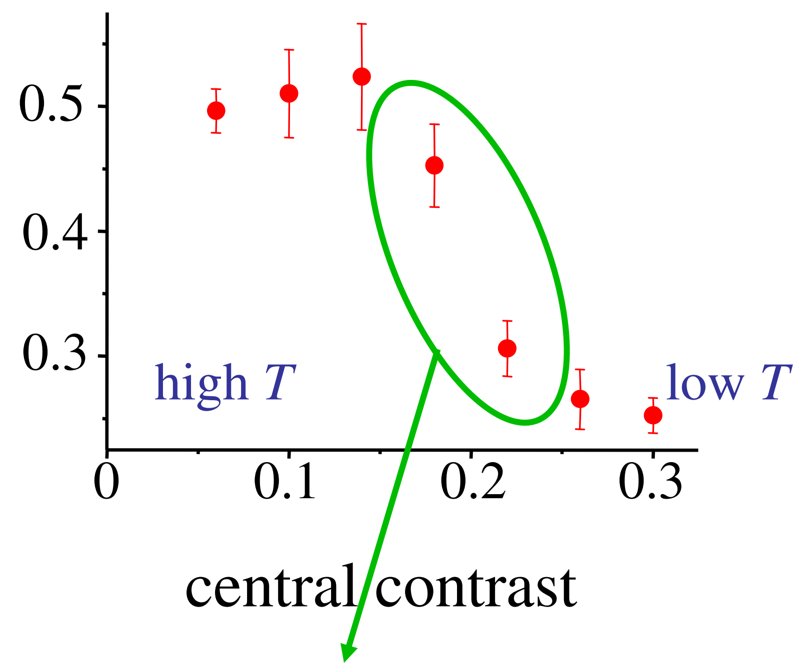
Experiments with 2D Bose gas

Hadzibabic et al., Nature 441:1118 (2006)



fit by: $$C^2 \sim \frac{1}{D_x}\int^{D_x}[g_1(0,x)]^2 dx \sim \left(\frac{1}{D_x}\right)^{2\alpha}$$

Exponent α



"Sudden" jump!?

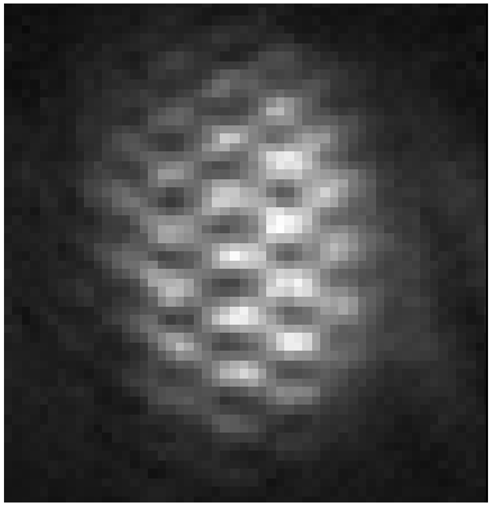
⇨ if $g_1(r)$ decays exponentially with $\ell_{\text{coh}} \ll D_x$: $\alpha = 1/2$

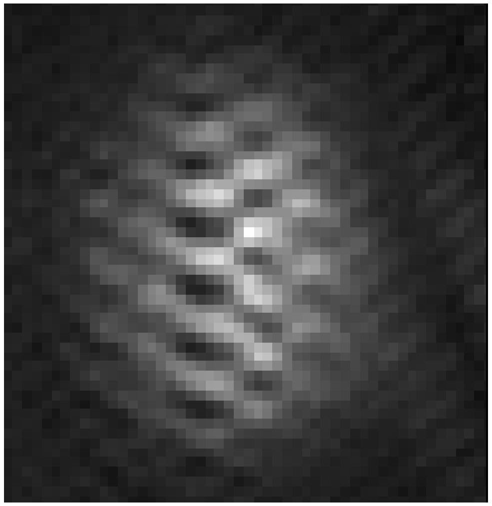
⇨ if $g_1(r)$ decays algebraically or exponentially with a large ℓ_{coh}:

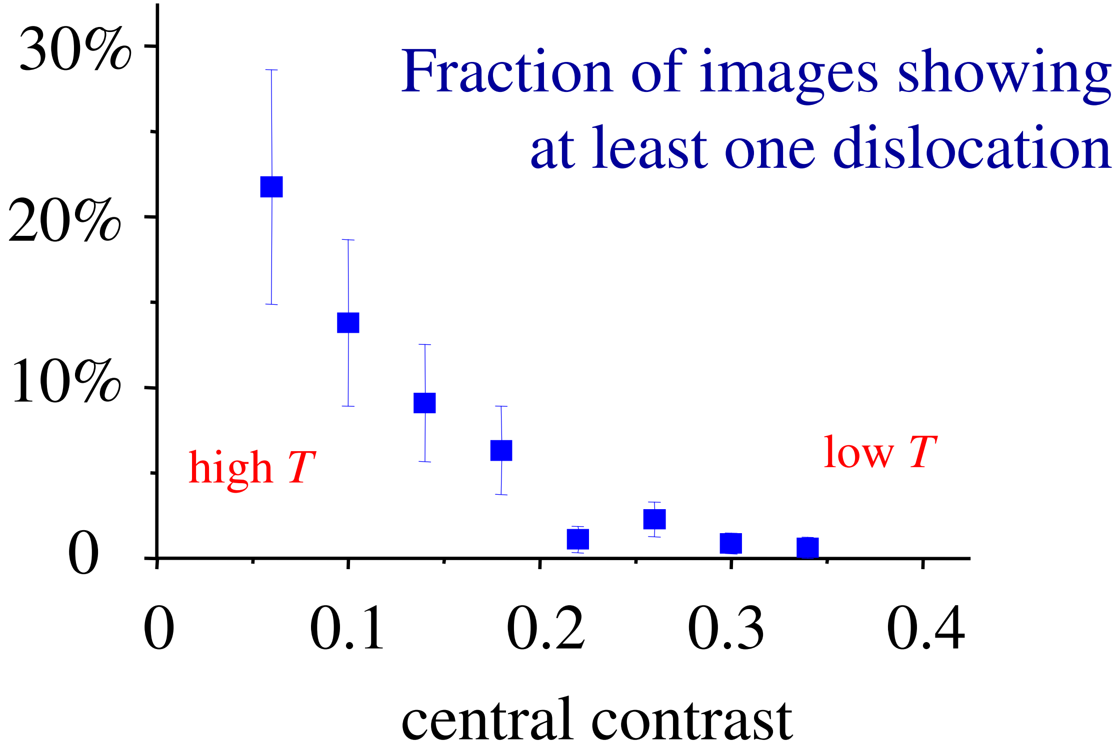
$$\alpha < 1/2$$

Experiments with 2D Bose gas. Proliferation of thermal vortices

Hadzibabic et al., Nature 441:1118 (2006)







The onset of proliferation coincides with α shifting to 0.5!

Summary

Experiments with ultracold atoms provide a new perspective on the physics of strongly correlated many-body systems. Quantum noise is a powerful tool for analyzing many body states of ultracold atoms

Thanks to:

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MURI Program in Optical Lattices

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