Quantum systems of ultracold atoms
New probes of many-body correlations

Analysis of quantum noise

Interference of fluctuating condensates and correlation functions

From quantum noise to high order correlation functions

Analysis of magnetization fluctuations in lattice models
Interference of two independent condensates

Experiments with 2D Bose gas

Experiments with 1D Bose gas S. Hofferberth et al. arXiv0710.1575
Interference of fluctuating condensates

Amplitude of interference fringes, $A_{fr}$

\[ |A_{fr}| e^{i\Delta \phi} = \int_0^L dx \, a_1^\dagger(x) a_2(x) \]

For independent condensates $A_{fr}$ is finite but $\Delta \phi$ is random

\[ \langle |A_{fr}|^2 \rangle = \int_0^L \int_0^L dx \, dy \, \langle a_1^\dagger(x) a_2(x) a_2^\dagger(y) a_1(y) \rangle \]

\[ \simeq L \int_0^L dx \, \langle a_1(x) a_1^\dagger(0) \rangle \langle a_2(0) a_2^\dagger(x) \rangle \]

For identical condensates

\[ \langle |A_{fr}|^2 \rangle = L \int_0^L dx \, (G(x))^2 \]

Instantaneous correlation function

\[ G(x) = \langle a(x) a^\dagger(0) \rangle \]

Polkovnikov, Altman, Demler, PNAS 103:6125(2006)
Interference between fluctuating condensates

2d: BKT transition, Hadzibabic et al, 2006

1d: Luttinger liquid, Hofferberth et al., 2007
Distribution function of fringe amplitudes for interference of fluctuating condensates

\( A_{fr} \) is a quantum operator. The measured value of \( |A_{fr}| \) will fluctuate from shot to shot.

\[
\langle |A_{fr}|^{2n} \rangle = \int_0^L dz_1 \ldots dz_n \langle a^\dagger(z_1) \ldots a^\dagger(z_n) a(z'_1) \ldots a(z'_n) \rangle^2
\]

Higher moments reflect higher order correlation functions

We need the full distribution function of \( |A_{fr}| \)
Distribution function of interference fringe contrast

Experiments: Hofferberth et al., arXiv0710.1575
Theory: Imambekov et al., cond-mat/0612011

Comparison of theory and experiments: no free parameters
Higher order correlation functions can be obtained

Quantum fluctuations dominate: asymetric Gumbel distribution
(low temp. $T$ or short length $L$)

Thermal fluctuations dominate: broad Poissonian distribution
(high temp. $T$ or long length $L$)

Intermediate regime:
double peak structure
Studying coherent dynamics of strongly interacting systems in interference experiments
Studying dynamics using interference experiments

Prepare a system by splitting one condensate

Take to the regime of zero tunneling

Measure time evolution of fringe amplitudes
Dynamics of split condensates

Theory: Borovk et al., PRL 2007
Experiment: Hofferberth et al., Nature 2007

\[ \langle e^{i\phi(t)} \rangle \sim e^{-\left(\frac{t}{t_T}\right)^{2/3}} \]

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Probing spin systems using distribution function of magnetization
Probing spin systems using distribution function of magnetization


Magnetization in a finite system

\[ M_{\text{tot}}^z = \sum_{i=1}^{L} M^z(i) \]

Average magnetization

\[ \langle M_{\text{tot}}^z \rangle = L \langle M^z \rangle \]

Higher moments of \( M_{\text{tot}}^z \) contain information about higher order correlation functions

\[ \langle (M_{\text{tot}}^z - \langle M_{\text{tot}}^z \rangle)^2 \rangle = L \sum_{i,j} \langle (M_z(i) - \langle M^z \rangle) (M^z(j) - \langle M^z \rangle) \rangle \]
Distribution Functions

\[ \mathcal{H} = -J \sum_i [2S^x(i)S^x(i + 1) + gS^z(i)] \]

x-Ferromagnet polarized

or

or

\[ P(m_x) \]

\[ P(m_z) \]

\[ m_z - \langle m_z \rangle \]

\[ m_z - \langle m_z \rangle \]

\[ m_z - \langle m_z \rangle \]
Using noise to detect spin liquids

Spin liquids have no broken symmetries
No sharp Bragg peaks

Algebraic spin liquids have long range spin correlations

\[ \langle S_i S_j \rangle = \frac{e^{i Q r_{ij}}}{|r_i - r_j|^{1+\eta}} \]

No static magnetization \( \langle S_A \rangle = 0 \)

Noise in magnetization exceeds shot noise

\[ \langle S_A^2 \rangle = \sum_{ij} \langle S_i S_j \rangle \sim A \int_A \frac{r \, dr}{|r|^{1+\eta}} \sim A^{1+\frac{1-\eta}{2}} \]
Work in progress on theoretical milestones for MURI

Investigate coherent quantum dynamics of strongly interacting spin systems and superfluids

Investigate experimental requirements for reaching and detecting the antiferromagnetic phase (Long lived doublon states)

Investigate theoretically new approaches to realizing and detecting of spin liquid states and anyon statistics (Measuring spin loop operators using coupling to cavity)
MURI quantum simulation project

Phase I
Validation and Verification
Simulate solvable Hamiltonians. Compare with calculations. Examples: low dimensional systems, precision study of Mott insulator phases, superexchange interactions, fermionic superfluidity in optical lattice.

New tools for detection and characterization of strongly correlated states
Optical addressability → Quantum gas microscope
Critical velocity in moving superfluids
Bragg spectroscopy in optical lattices
Quantum noise analysis

Phase II
Combine all tools and methods developed during phase I to tackle goals of this MURI project:
Quantum magnetism
Fermionic superfluidity in systems with repulsive interactions

Ultimate goal: use the results of quantum analogue simulations to identify new solid state systems with favorable properties
Quantum Simulations of Condensed Matter Systems using Ultracold Atomic Gases

Markus Greiner (principal investigator), Eugene Demler, John Doyle, Luming Duan, Mark Kasevich, Wolfgang Ketterle, Mikhail Lukin, Subir Sachdev, Martin Zwierlein, Joseph Thywissen, Immanuel Bloch, Peter Zoller

Collaborating Universities: Harvard, MIT, Stanford, Michigan, Toronto, University of Mainz, Germany, University of Innsbruck,