Optical lattice emulator

Strongly correlated systems:
from electronic materials
to ultracold atoms

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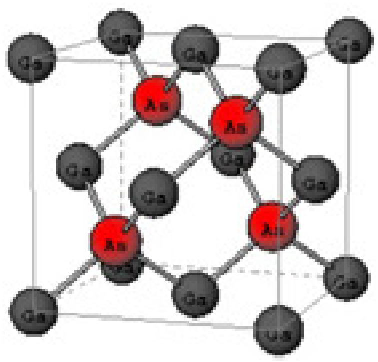
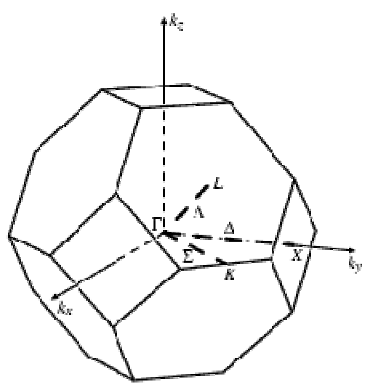
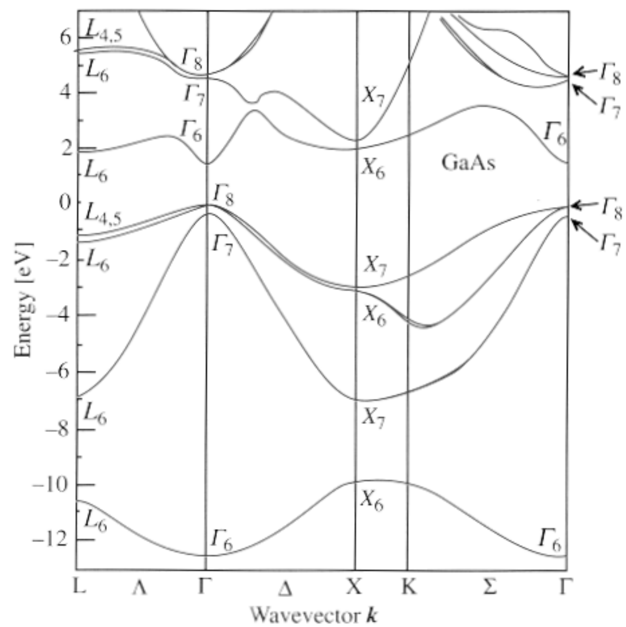
OLE OPTICAL LATTICE EMULATOR

“Conventional” solid state materials

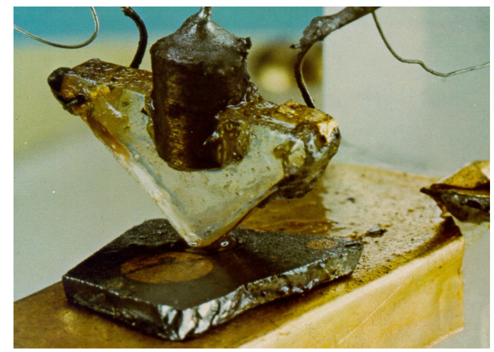
Description in terms of non-interacting electrons.
Band structure and Landau Fermi liquid theory



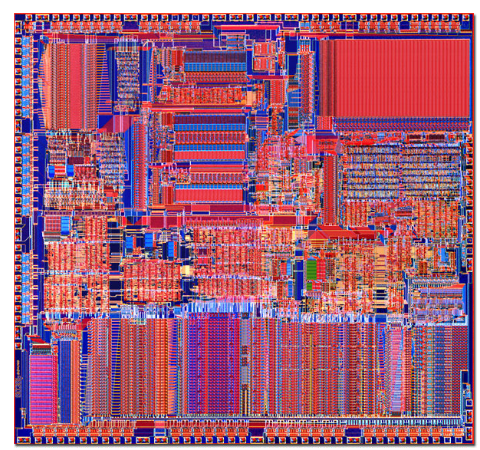




Electronic band structure of GaAs



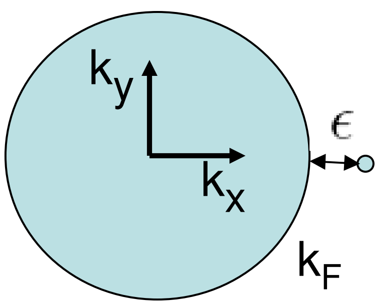
First semiconductor transistor



Intel 386DX microprocessor

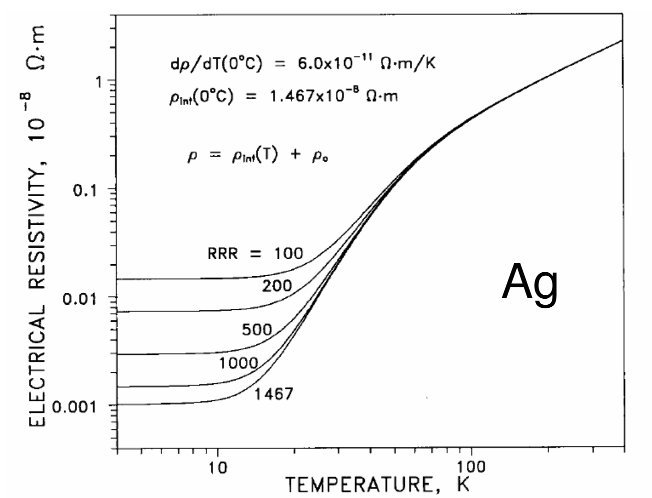
“Conventional” solid state materials

Electron-phonon and electron-electron interactions
are irrelevant at low temperatures

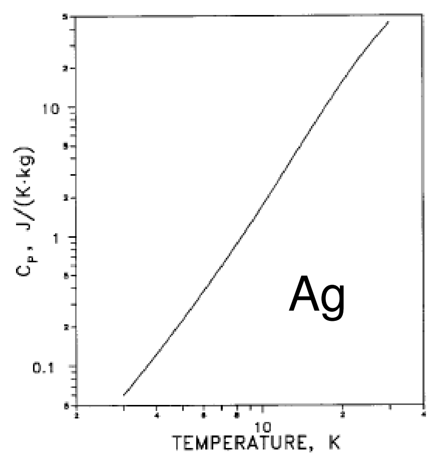


$$\frac{1}{\tau_{e-e}} \sim \epsilon^2 \qquad \frac{1}{\tau_{e-ph}} \sim \epsilon^3$$

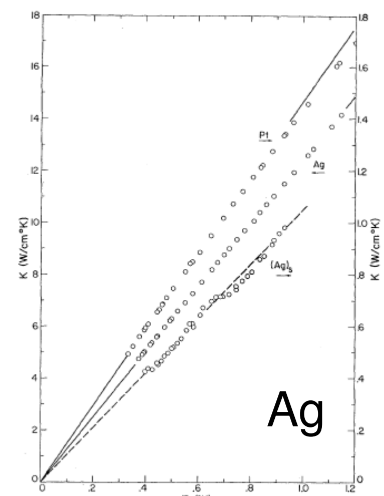
Landau Fermi liquid theory: when frequency and temperature are smaller than E_F electron systems are equivalent to systems of non-interacting fermions



$$\rho = \rho_0 + aT^2$$

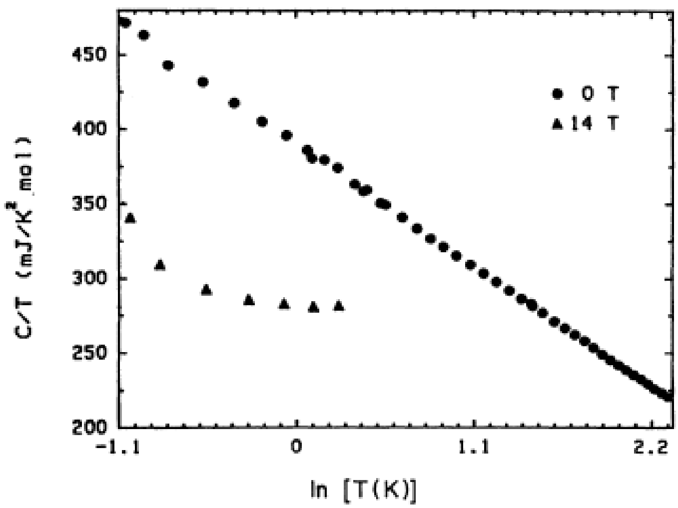


$$c/T = \text{const}$$



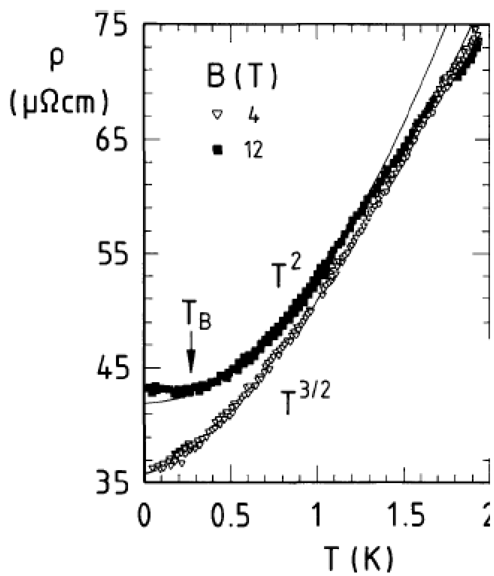
$$\kappa/T = \text{const}$$

Non Fermi liquid behavior in novel quantum materials

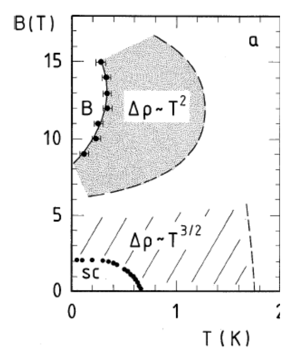


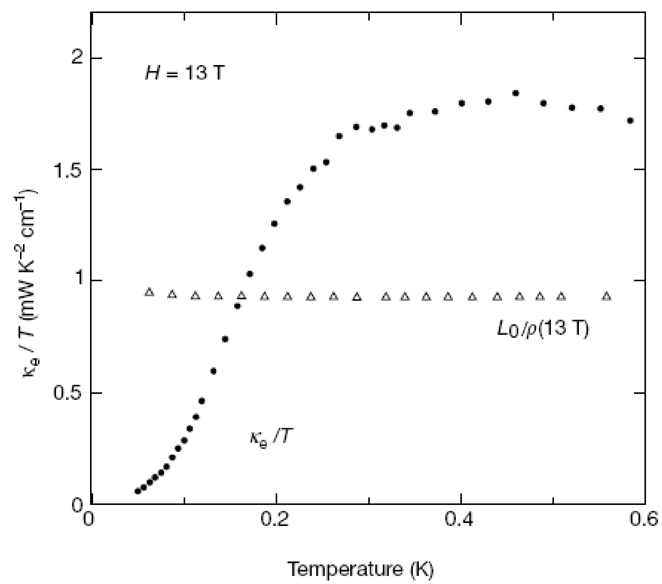
$UCu_{3.5}Pd_{1.5}$
Andraka, Stewart,
PRB 47:3208 (93)

Violation of the
Wiedemann-Franz law
in high Tc superconductors
Hill et al., Nature 414:711 (2001)



$CeCu_2Si_2$. Steglich et al.,
Z. Phys. B 103:235 (1997)



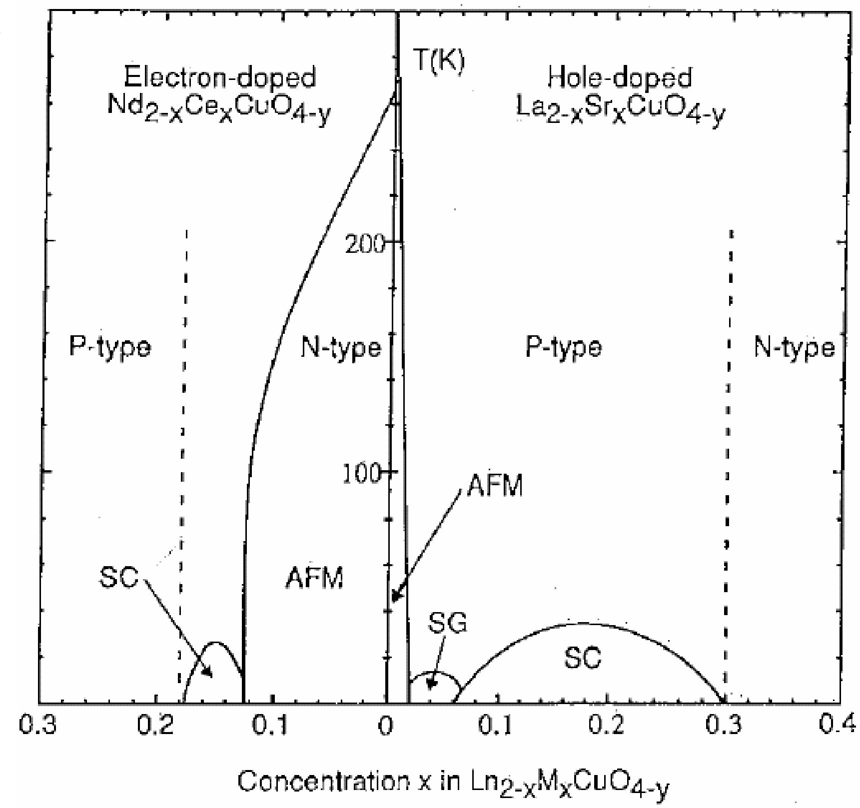


Puzzles of high temperature superconductors

Unusual "normal" state

Maple, JMMM 177:18 (1998)

Resistivity, opical conductivity,
Lack of sharply defined quasiparticles,
Signatures of AF, CDW, and SC fluctuations



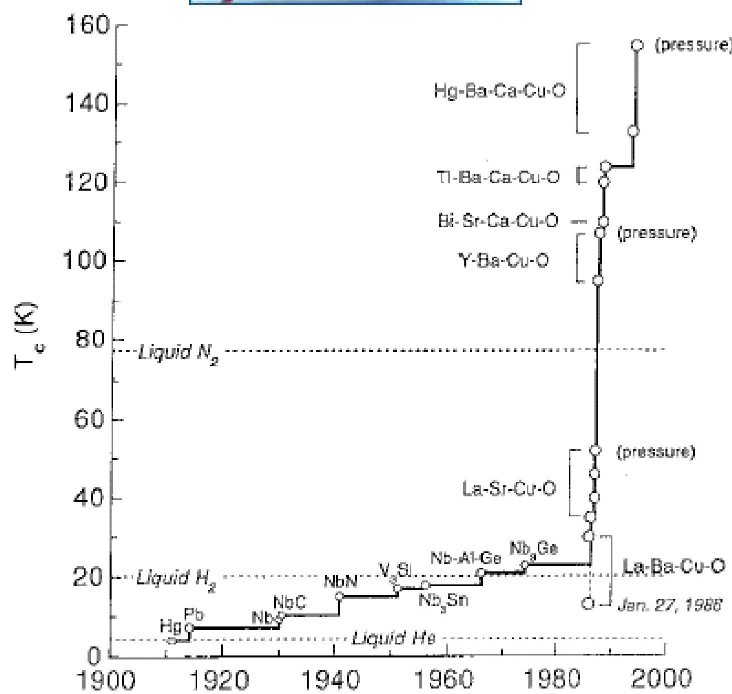
Mechanism of Superconductivity

High transition temperature,
retardation effect, isotope effect,
role of elecron-electron
and electron-phonon interactions

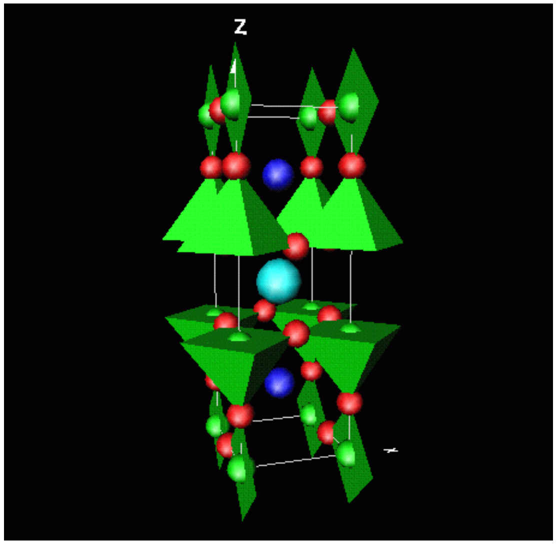
Competing orders

Role of magnetsim, stripes,
possible fractionalization

Applications of quantum materials: High Tc superconductors



High temperature superconductors



$\mathrm{YBa_2Cu_3O_7}$

Superconducting
Tc 93 K

Picture courtesy of UBC Superconductivity group

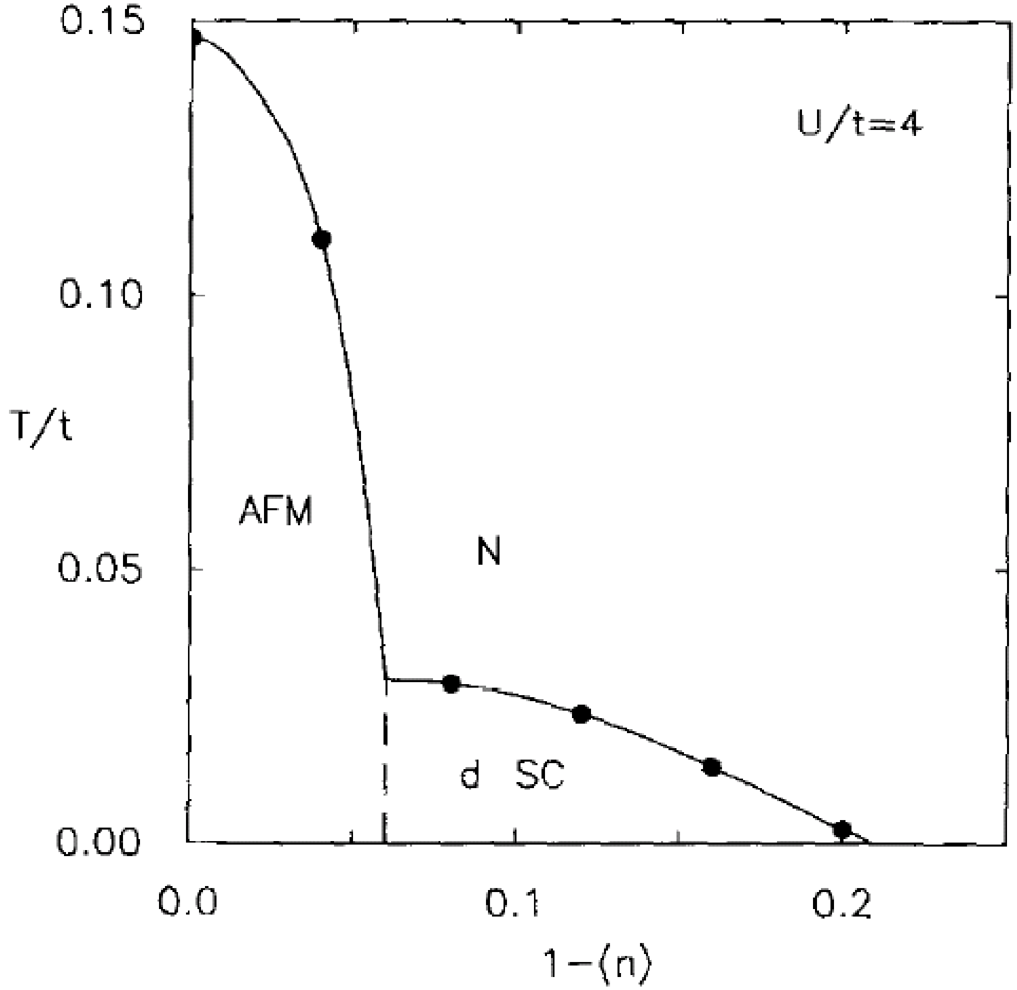
Hubbard model – minimal model for cuprate superconductors

P.W. Anderson. cond-mat/0201429

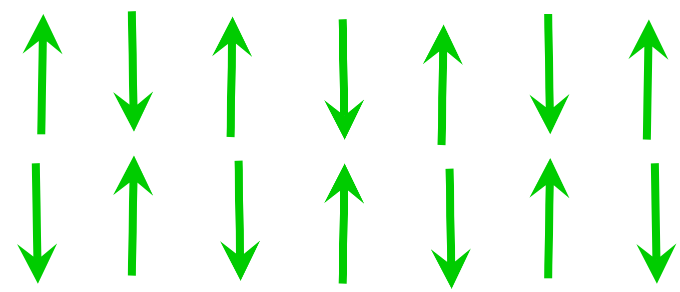
$$\mathcal{H} = -t \sum_{\langle ij \rangle\, \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_{i}$$

Positive U Hubbard model

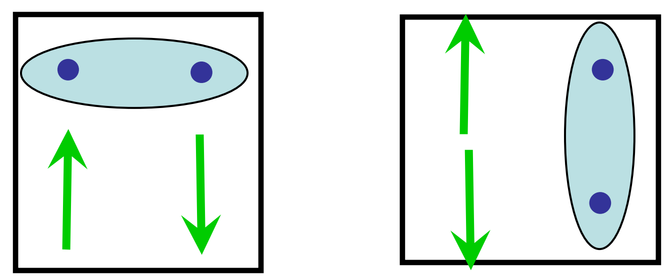
Possible phase diagram. Scalapino, Phys. Rep. 250:329 (1995)



Antiferromagnetic insulator

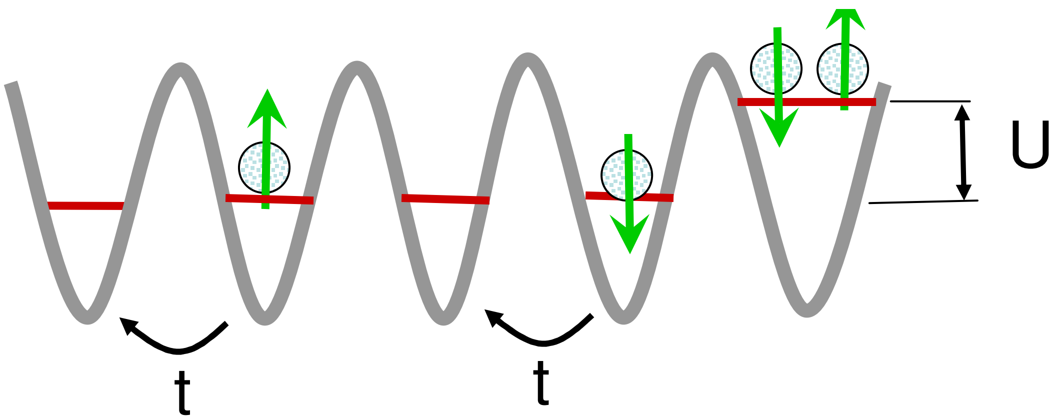


D-wave superconductor



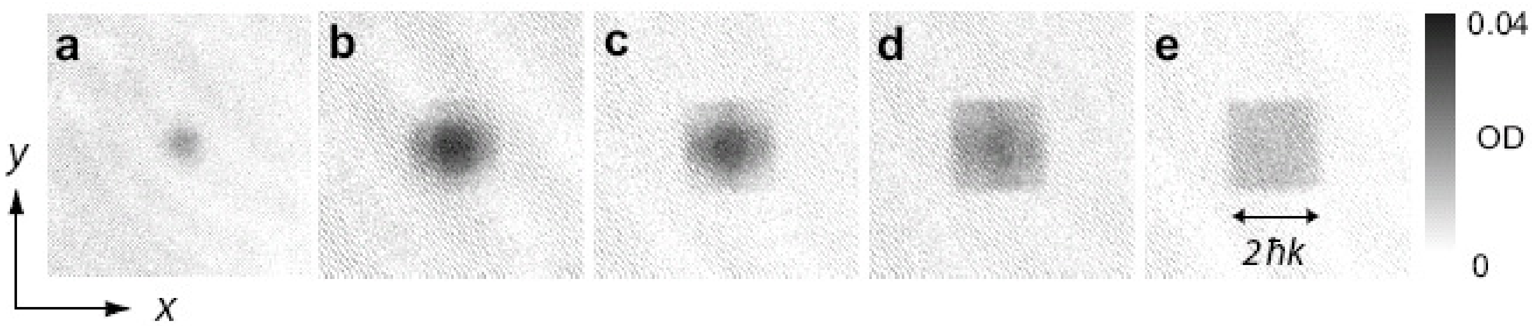
Fermionic atoms in optical lattices

$$\mathcal{H} = -t \sum_{\langle ij \rangle \, \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_{i}$$

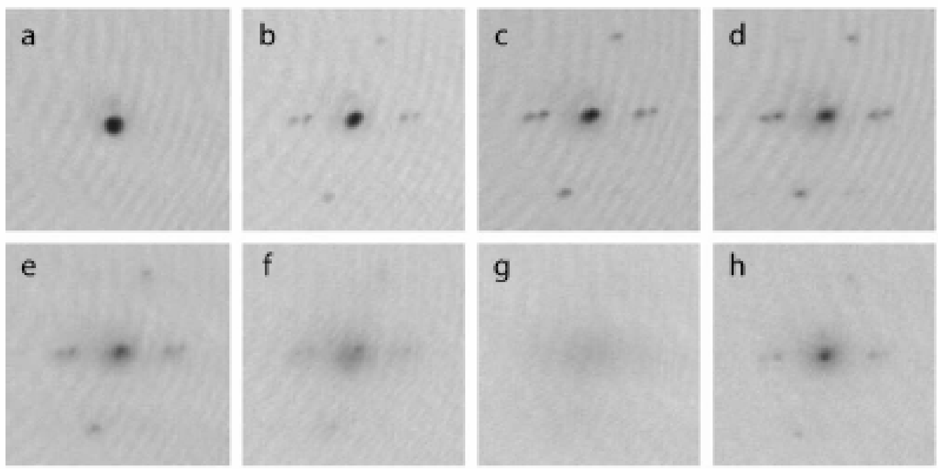


Quantum simulation of the fermionic Hubbard model using ultracold atoms in optical latices

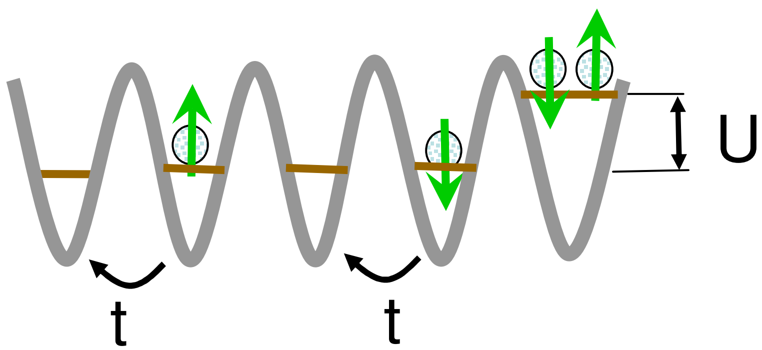
Fermions in a 3d optical lattice, Kohl et al., PRL 2005



Superfluidity of fermions in an optical lattice, Chin et al., Nature 2006



Simulation of condensed matter systems: Hubbard Model and high Tc superconductivity



Fermions with repulsive interactions in an optical lattice can be described by the same microscopic model as cuprate high temperature superconductors

Theory: Hofstetter et al., PRL 89:220407 (02)

Questions for future work:

- What is the ground state of the Hubbard model away from filling n=1

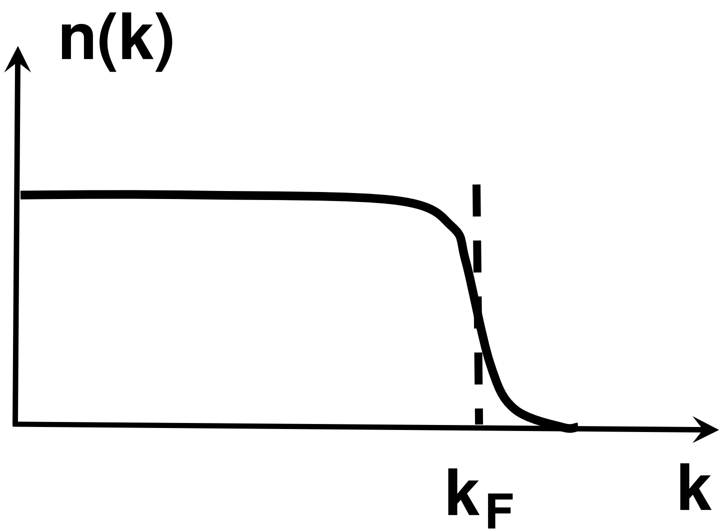
- Beyond "plain vanilla" Hubbard model
  a) Boson-Fermion mixtures: Hubbard model + phonons
  b) Inhomogeneous systems (stripes), role of disorder

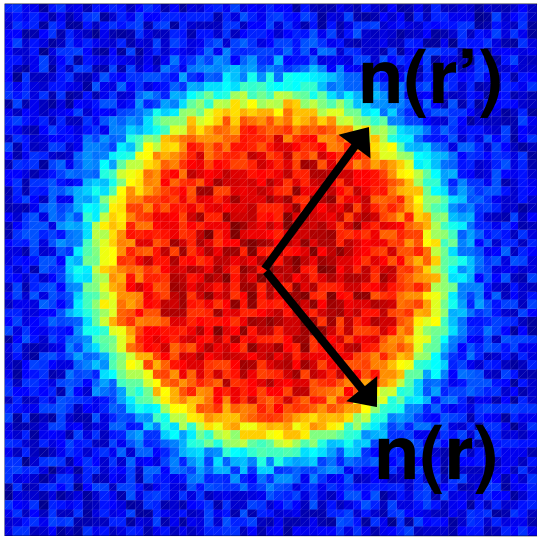
- Detection of many-body states
  (spin antiferromagnetisim, d-wave superconductivity , CDW, …)

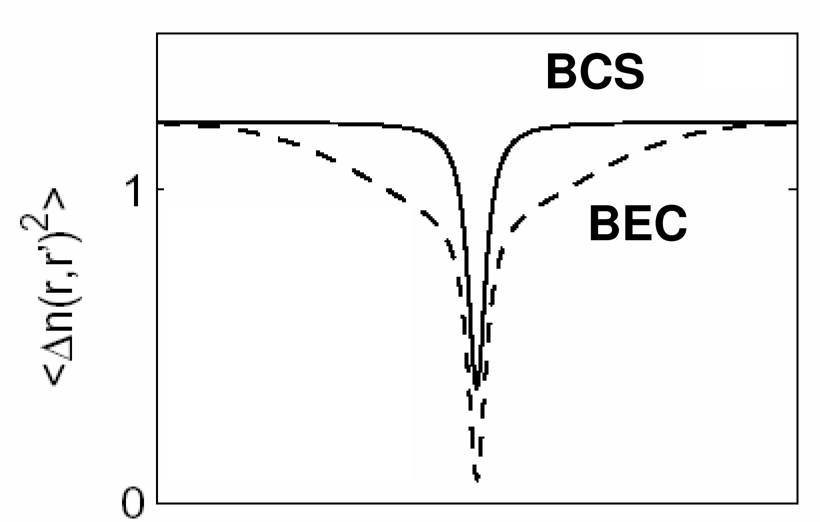
How to detect antiferromagnetic order and d-wave pairing in optical lattices?

Quantum noise ?!

Second order interference from the BCS superfluid





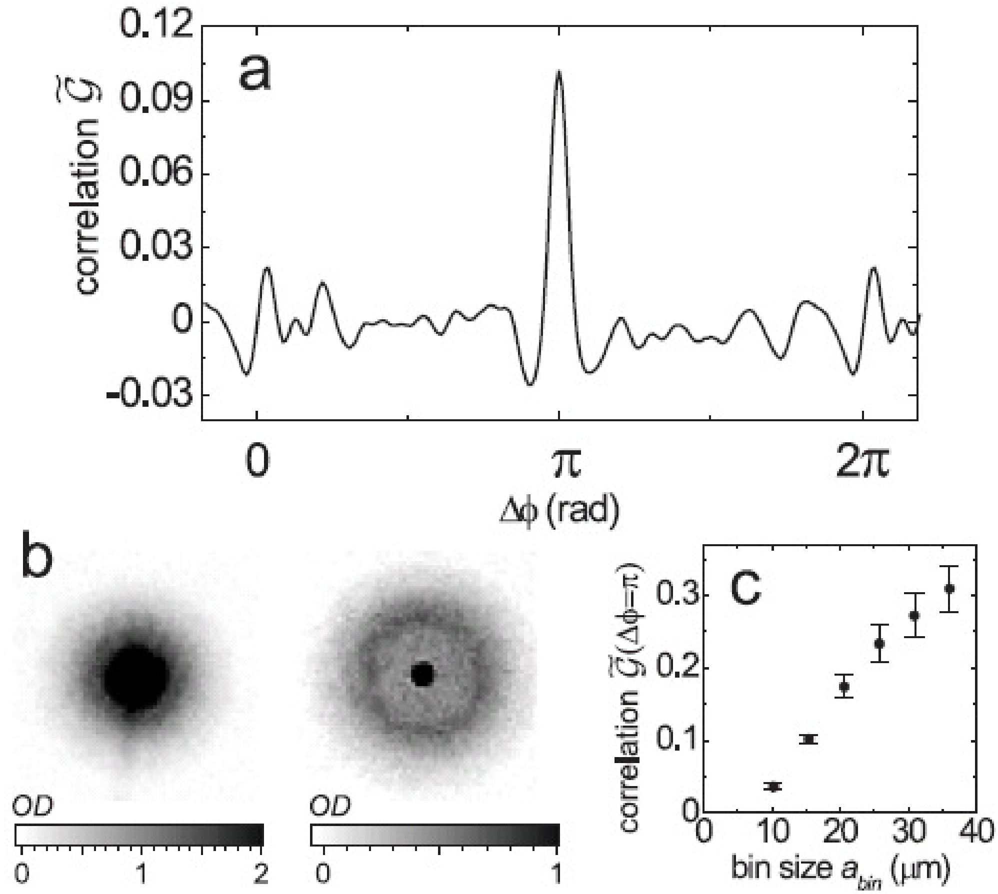


$$\Delta n(\mathbf{r},\mathbf{r}') \equiv n(\mathbf{r}) - n(\mathbf{r}')$$

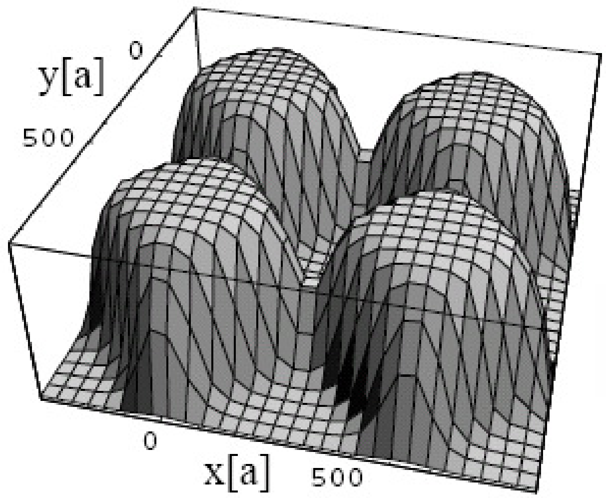
$$\Delta n(\mathbf{r},-\mathbf{r})\left|\Psi_{BCS}\right\rangle = 0$$

Momentum correlations in paired fermions

Greiner et al., PRL 94:110401 (2005)



Fermion pairing in an optical lattice



Second Order Interference
In the TOF images

$$G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$$

Normal State

$$G_{\mathrm{N}}(r_1, r_2) = \delta(r_1 - r_2)\rho(r_1) - \rho^2(r_1) \sum_G \delta(r_1 - r_2 - \frac{G\hbar t}{m})$$

Superfluid State

$$G_{\mathrm{S}}(r_1, r_2) = G_{\mathrm{N}}(r_1, r_2) + \Psi(r_1) \sum_G \delta(r_1 + r_2 + \frac{G\hbar t}{m})$$

$\Psi(r) = |u(Q(r))v(Q(r))|^2$ **measures the Cooper pair wavefunction**

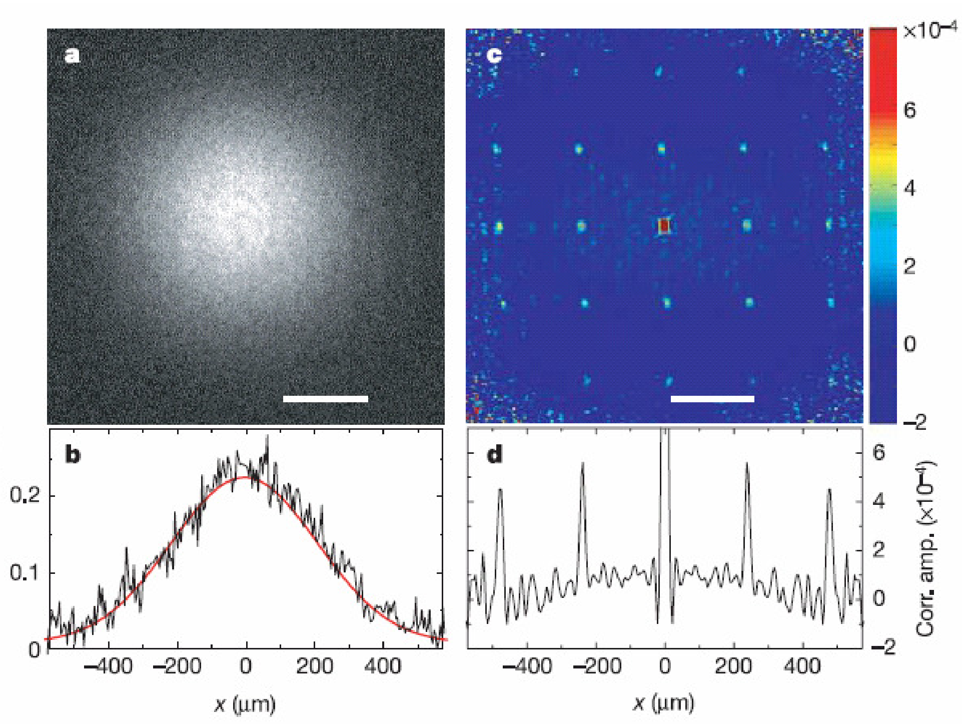
$$Q(r) = \frac{mr}{\hbar t}$$

One can identify unconventional pairing

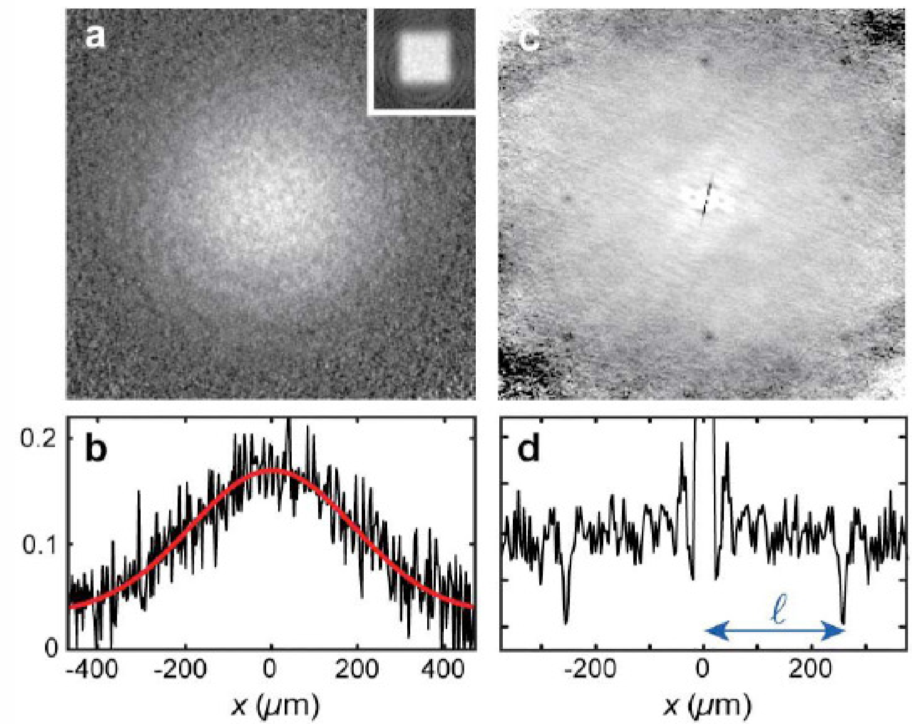
Second order coherence in the insulating state of bosons and fermions

Theory: Altman et al., PRA 70:13603 (2004)

Expt: Folling et al., Nature (2005); Spielman et al., PRL (2007); Rom et al., Nature (2006)



"Bosonic" bunching

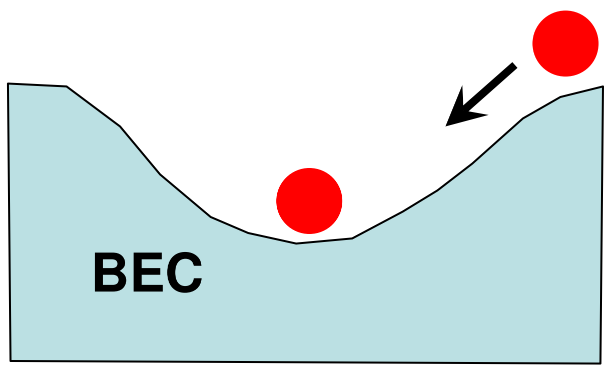


"Fermionic" antibunching

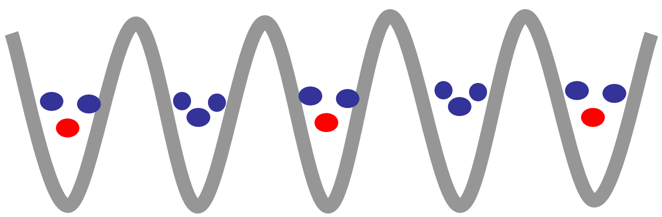
A powerful tool for detecting antiferromagnetic order

Boson Fermion mixtures

Experiments: ENS, Florence, JILA, MIT, ETH, Hamburg, Rice, Duke, Mainz, …

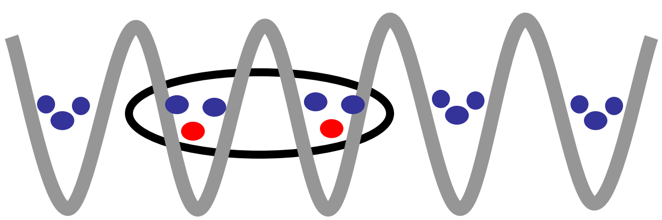


Bosons provide cooling for fermions and mediate interactions. They create non-local attraction between fermions



Charge Density Wave Phase

Periodic arrangement of atoms



Non-local Fermion Pairing

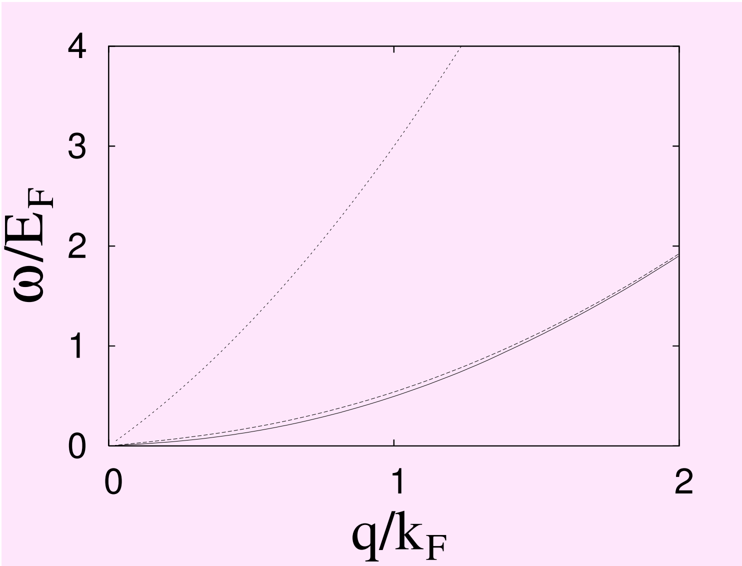
P-wave, D-wave, …

Theory: Pu, Illuminati, Efremov, Das, Wang, Matera, Lewenstein, Buchler, …

Boson Fermion mixtures

$$\mathcal{H} = \mathcal{H}_{bb} + \mathcal{H}_{\rm ff} + \mathcal{H}_{bf}$$
$$\mathcal{H}_{bb} = -t_b \sum_{\langle ij \rangle} b_i^\dagger b_j + U_{bb} \sum_i n_{bi}(n_{bi} - 1)$$
$$\mathcal{H}_{ff} = -t_f \sum_{\langle ij \rangle} f_i^\dagger f_j$$
$$\mathcal{H}_{bf} = U_{bf} \sum_i n_{bi} n_{fi}$$

“Phonons” :
Bogoliubov (phase) mode



Effective fermion-”phonon” interaction

$$\tilde{\mathcal{H}}_{bb} = \sum_q \omega_q \beta_q^\dagger \beta_q$$
$$\tilde{\mathcal{H}}_{bf} = \sum_{kq} g_q \left(\beta_q + \beta_{-q}^\dagger\right) f_{k+q}^\dagger f_k$$

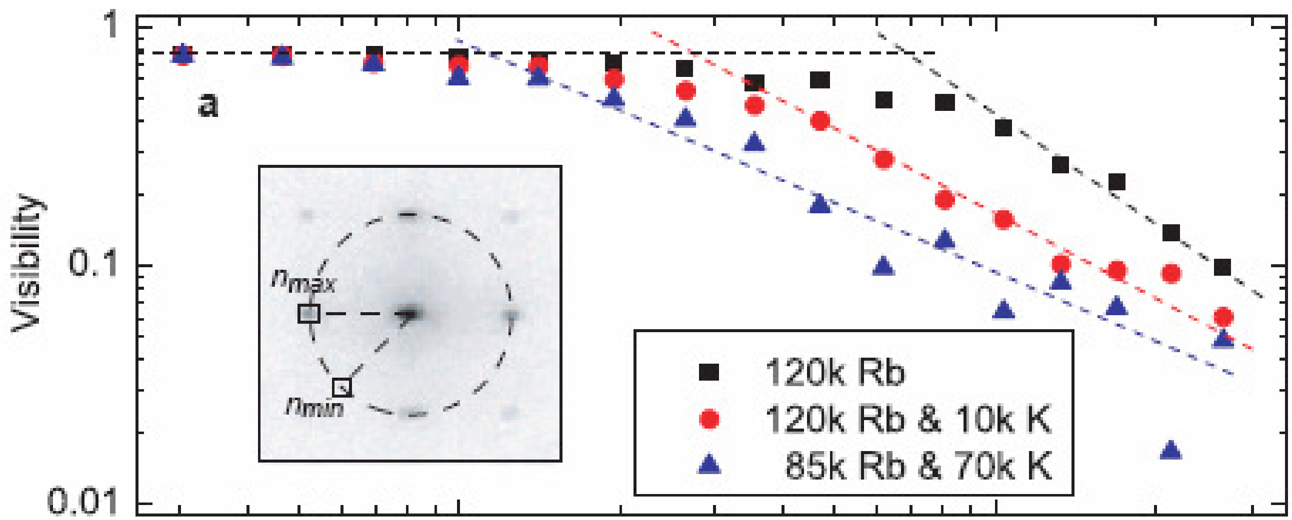
Fermion-”phonon” vertex $g_q \sim |q|$
Similar to electron-phonon systems

Bose-Fermi mixture in a three dimensional optical lattice

Gunter et al, PRL 96:180402 (2006)

See also Ospelkaus et al, PRL 96:180403 (2006)

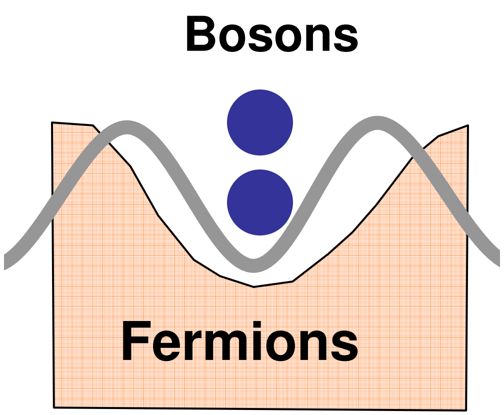
Suppression of superfluidity of bosons by fermions



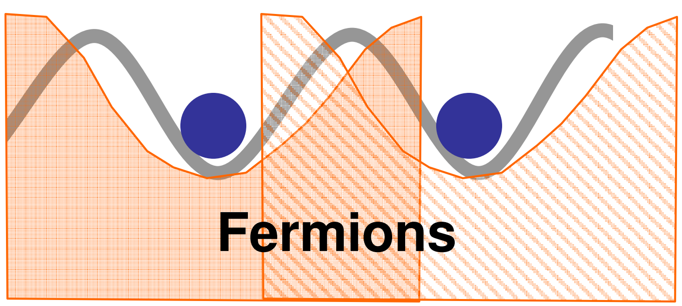
Similar observation for Bose-Bose mixtures,
see Catani et al., arXiv:0706.278

Issue of heating and density rearrangements need to be sorted out,
see e.g. Pollet et al., cond-mat/0609604

Competing effects of fermions on bosons



Fermions provide screening.
Favors superfluid state of bosons

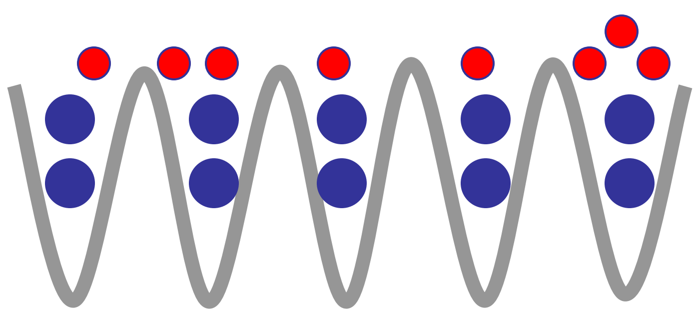


Orthogonality catastrophe due to fermions.
Polaronic dressing of bosons.
Favors Mott insulating state of bosons

Quantum regime of bosons

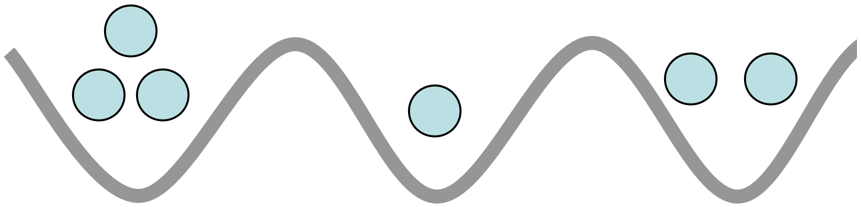
A better starting point:

Mott insulating state of bosons
Free Fermi sea



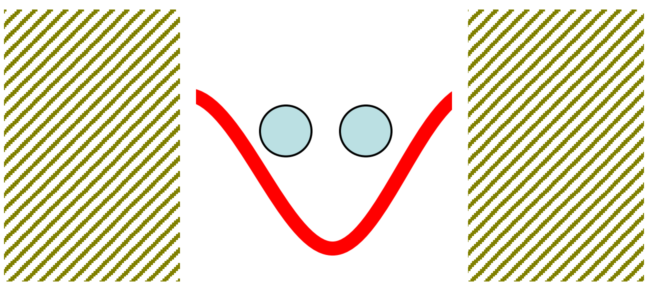
Theoretical approach: generalized Weiss theory

Weiss theory of the superfluid to Mott transition of bosons in an optical lattice



$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1) - \mu \sum_i n_i$$

Mean-field: a single site in a self-consistent field



$$\mathcal{H}_{\text{eff}} = -z\,t\,(r^* b^\dagger + r\,b) + U\,n\,(n-1) - \mu\,n$$

$$r = \langle b \rangle$$

Weiss theory: quantum action

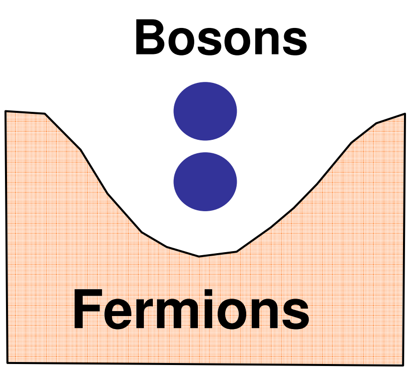
Conjugate variables $[\, n \,, \, \phi] \, = \, -i$

$$S_{\text{eff}} \, = \int_0^\beta d\tau \, \left[i\dot{\phi} n - \, z \, t \, r \, \cos\phi + \, U \, n\,(\,n \, - \, 1\,) - \mu \, n \right]$$

Self-consistency condition $r \, = \, \bar{n} \, \langle \, \cos\phi \, \rangle$

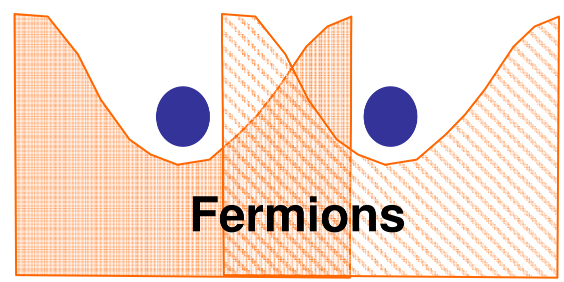
Adding fermions

Screening



$$\Delta S^{(1)}_{\text{ferm}} \, = \, -\frac{1}{2} \, U^2_{BF} \, \rho_F(0) \int_0^\beta d\tau \, n^2$$

Orthogonality catastrophy



$$\Delta S^{(1)}_{\text{ferm}} \, = \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \, \alpha(\tau_1 - \tau_2) \, (\, n(\tau_1) \, - \, n(\tau_2) \,)^2$$

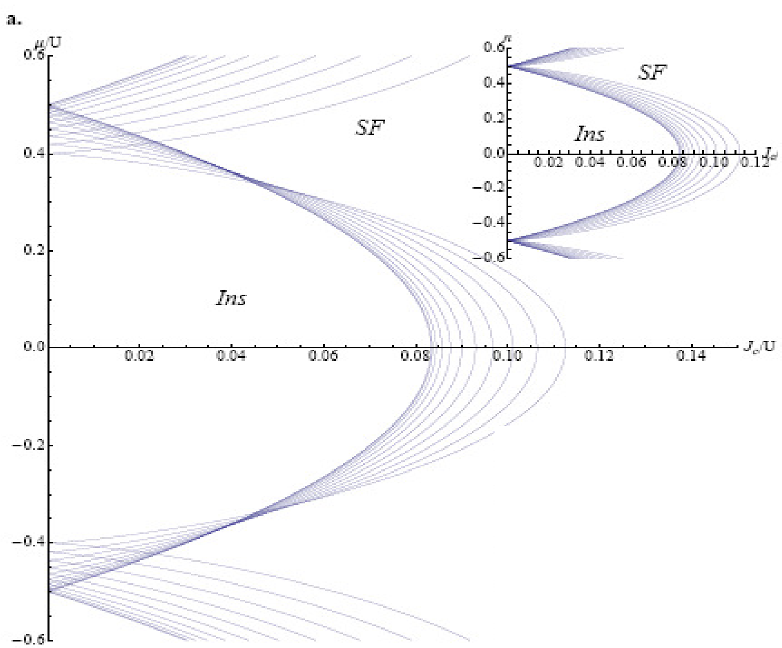
$$\alpha(\omega_\nu) \, = \, 2 \, \pi \, \rho^2_F(0) \, U^2_{BF} \, |\omega_\nu|$$

SF-Mott transition in the presence of fermions

Competition of screening and orthogonality catastrophy (G. Refael and ED)

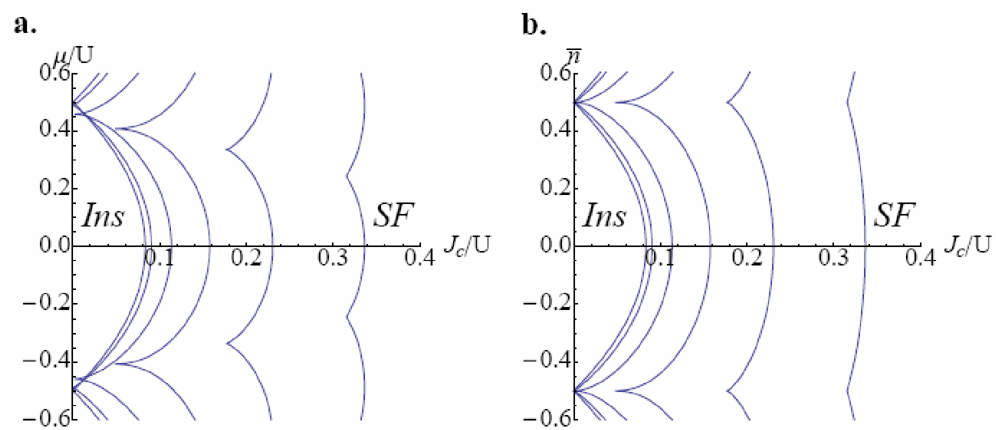
$$S_0 = \int_0^\beta d\tau \left[i\,\dot{\phi}\,n + \tilde{U}\,n\,(n-1) - \mu\,n \right] + \int_0^\beta d\tau_2\,\alpha(\tau_1 - \tau_2)\,(\,n(\tau_1) - n(\tau_2)\,)^2$$

Effect of fast fermions $t_F/U=5$

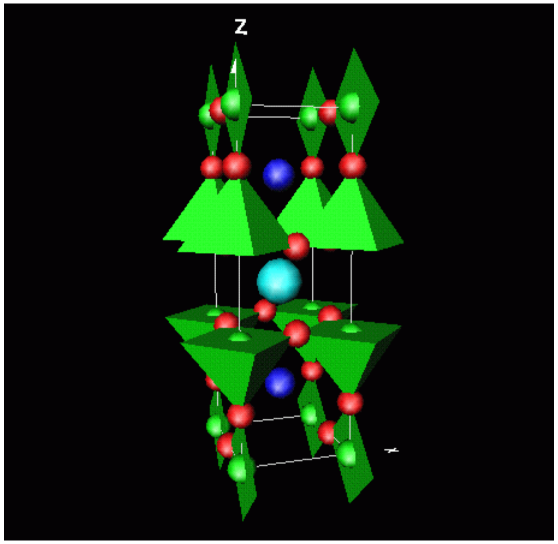


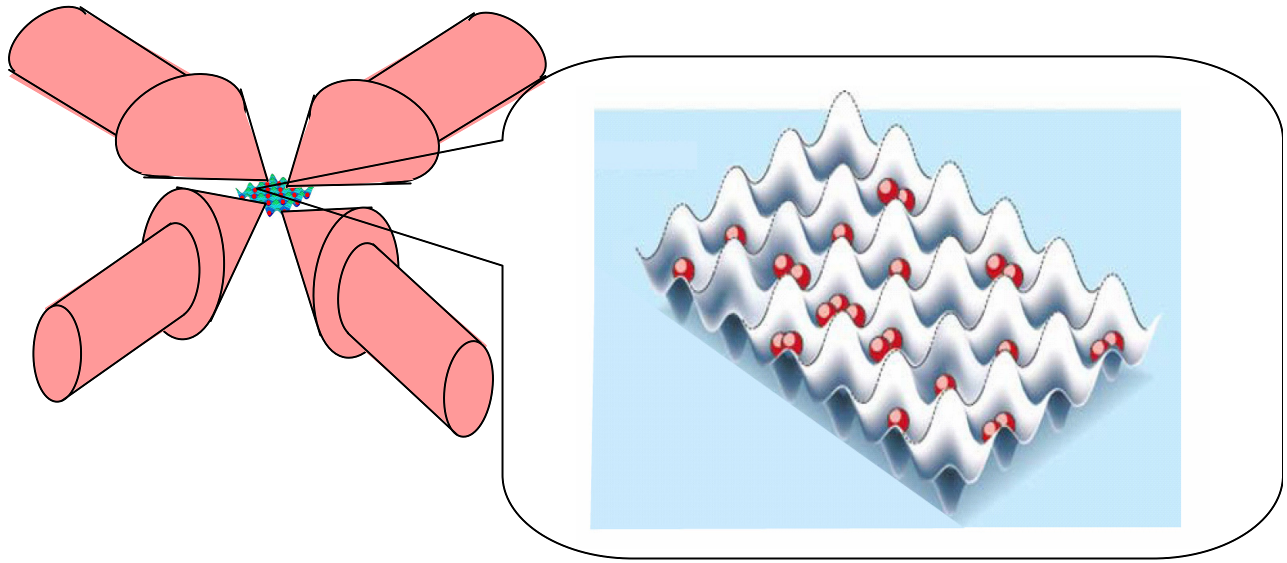
$$U_{FB}/U = 0, 0.1, 0.2, \ldots, 1$$

Effect of slow fermions $t_F/U=0.7$



$$U_{FB}/U = 0, 0.1, 0.2, \ldots, 0.5$$





$YBa_2Cu_3O_7$

Atoms in optical lattice

Antiferromagnetic and
superconducting Tc
of the order of 100 K

Antiferromagnetism and
pairing at sub-micro Kelvin
temperatures

Same microscopic model

$$\mathcal{H} = -t \sum_{\langle ij \rangle \, \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i$$