Probing interacting systems of cold atoms using interference experiments

Vladimir Gritsev	Harvard
Adilet Imambekov	Harvard
Anton Burkov	Harvard
Robert Cherng	Harvard
Anatoli Polkovnikov	Boston University
Ehud Altman	Weizmann Institute
Mikhail Lukin	Harvard
Eugene Demler	Harvard

CUA

Harvard-MIT CUA

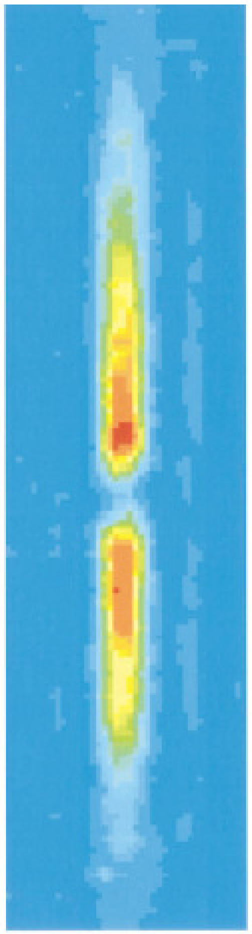
Outline

Measuring equilibrium correlation functions using interference experiments

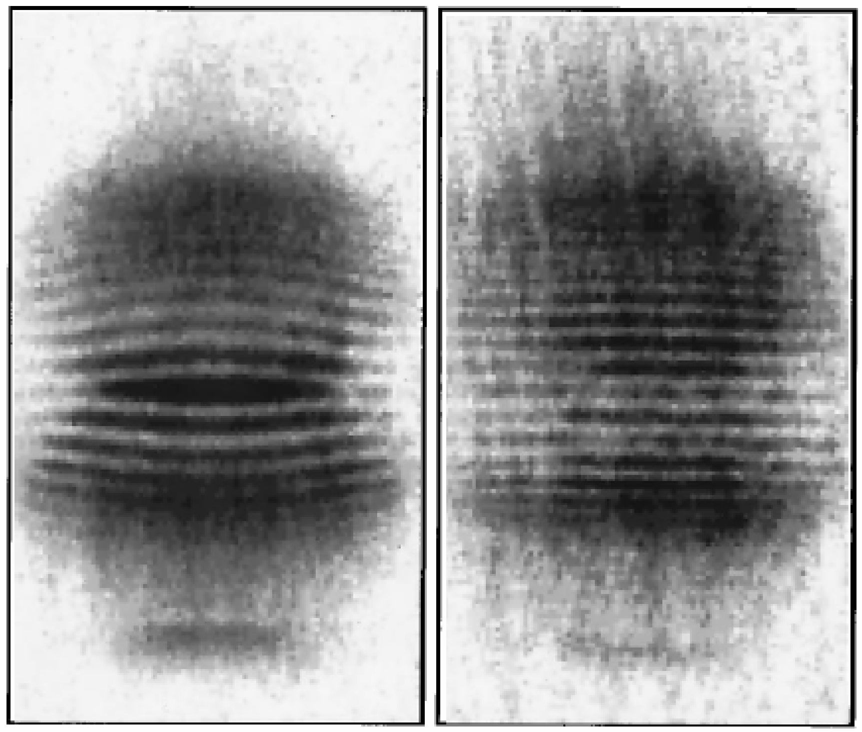
1. Interference of independent condensates
2. Interference of interacting 1D systems
3. Interference of 2D systems. Observation of the BKT transition
4. Full distribution function of fringe visibility in intereference experiments. Connection to quantum impurity problem

Studying non-equilibrium dynamics of interacting Bose systems in interference experiments

Interference of independent condensates



Experiments: Andrews et al., Science 275:637 (1997)



Theory: Javanainen, Yoo, PRL 76:161 (1996)
Cirac, Zoller, et al. PRA 54:R3714 (1996)
Castin, Dalibard, PRA 55:4330 (1997)
and many more

Nature 4877:255 (1963)

INTERFERENCE FRINGES PRODUCED BY SUPERPOSITION OF TWO INDEPENDENT MASER LIGHT BEAMS

By G. MAGYAR and Dr. L. MANDEL

Department of Physics, Imperial College of Science and Technology, London

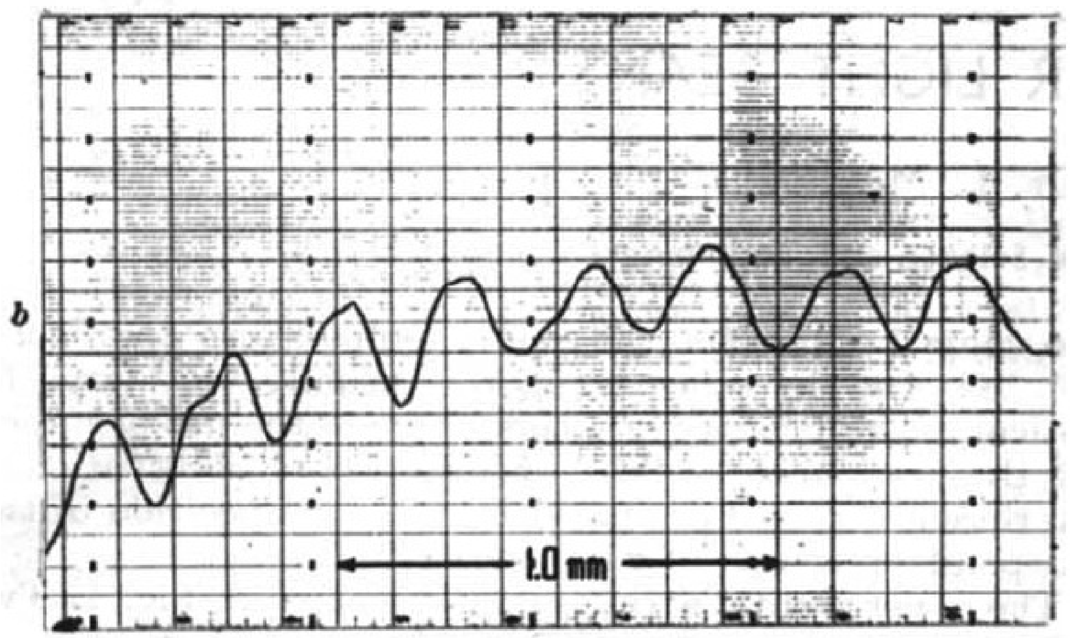
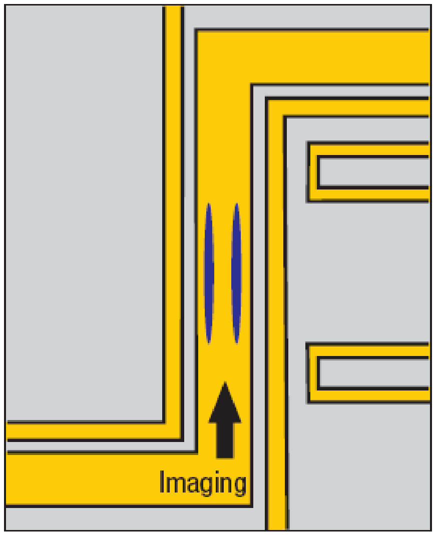
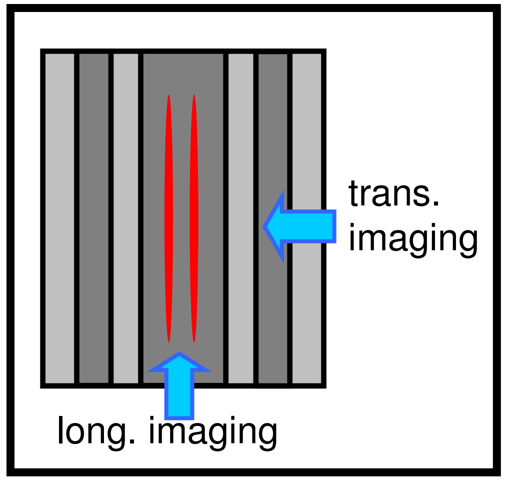


Fig. 2. An example of fringes recorded: (a) photograph; (b) microphotometer tracing

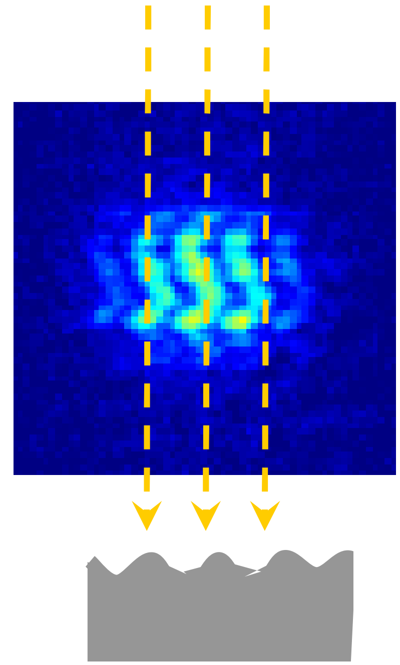
Interference of one dimensional condensates

Experiments: Schmiedmayer et al., Nature Physics (2005,2006)

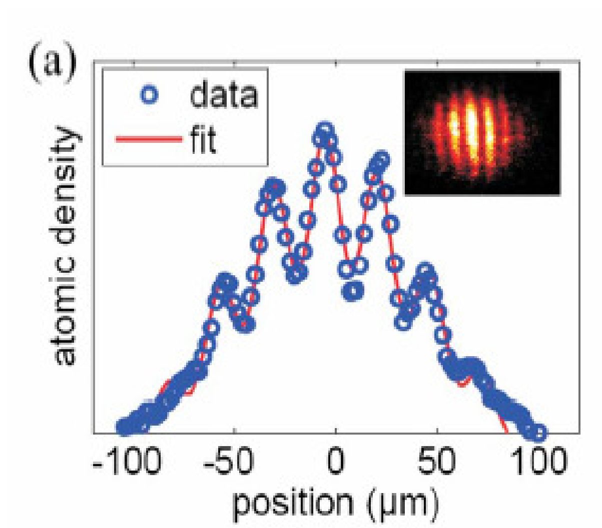




Transverse imaging



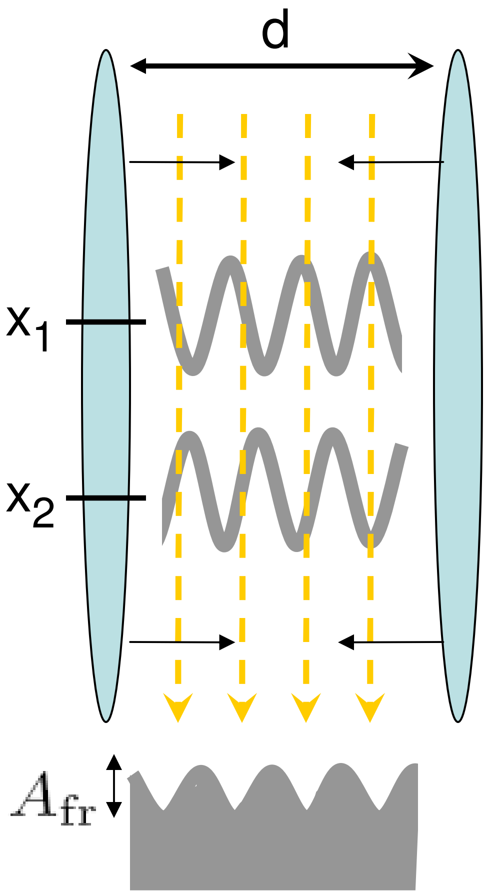
Longitudial
imaging



Figures courtesy of
J. Schmiedmayer

Interference of one dimensional condensates

Polkovnikov, Altman, Demler, PNAS 103:6125 (2006)



Amplitude of interference fringes, $A_{\rm fr}$

$$|A_{\rm fr}|\, e^{i\Delta\phi} = \int_0^L dx\; a_1^\dagger(x)\, a_2(x)$$

For independent condensates A_{fr} is finite but $\Delta\phi$ is random

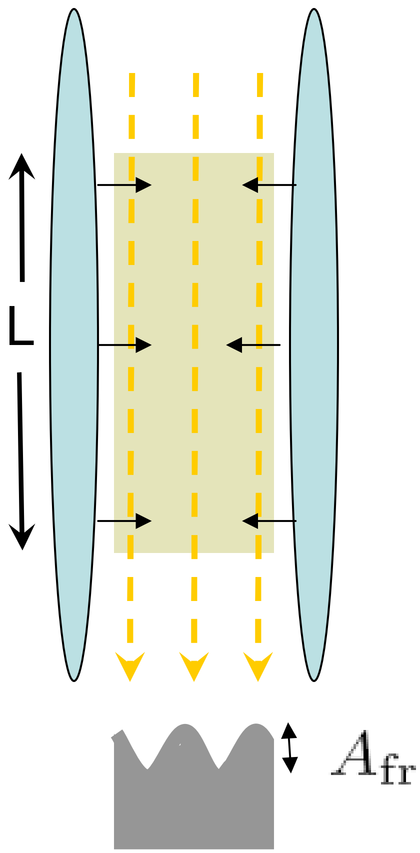
$$\langle |A_{\rm fr}|^2 \rangle = \int_0^L \int_0^L dx\, dy\, \langle a_1^\dagger(x)\, a_2(x)\, a_2^\dagger(y)\, a_1(y) \rangle$$

$$\simeq L \int_0^L dx\, \langle\, a_1(x)\, a_1^\dagger(0)\, \rangle \langle\, a_2(0) a_2^\dagger(x)\, \rangle$$

For identical condensates $\langle |A_{\rm fr}|^2 \rangle = L \int_0^L dx\, (\, G(x)\,)^2$

Instantaneous correlation function $G(x) = \langle\, a(x)\, a^\dagger(0)\, \rangle$

Interference between Luttinger liquids



Luttinger liquid at T=0

$$G(x) \sim \rho \left(\frac{\xi_h}{x} \right)^{1/2K}$$

K – Luttinger parameter

$$\langle |A_{\rm fr}|^2 \rangle \sim (\rho \xi_h)^{1/K} (L\rho)^{2-1/K}$$

For non-interacting bosons $K = \infty$ and $A_{\rm fr} \sim L$

For impenetrable bosons $K = 1$ and $A_{\rm fr} \sim \sqrt{L}$

Luttinger liquid at finite temperature

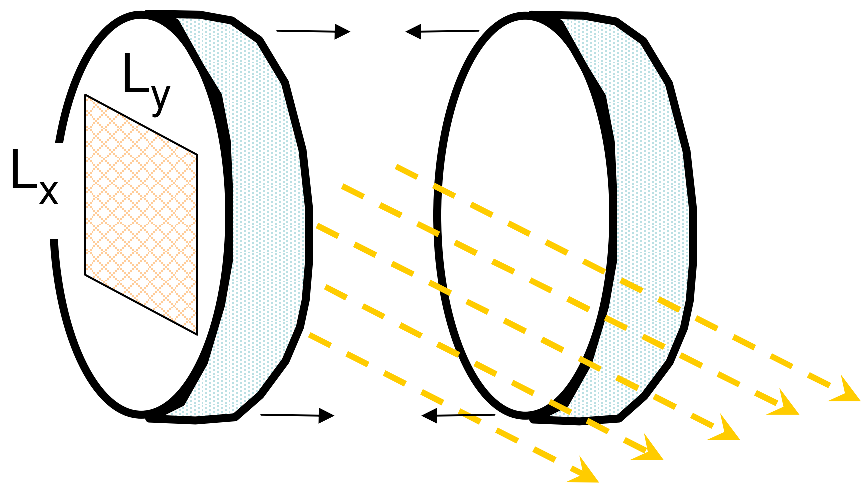
$$\langle |A_{\rm fr}|^2 \rangle \sim L \rho^2 \xi_h \left(\frac{\hbar^2}{m \xi_h^2} \frac{1}{T} \right)^{1-1/K}$$

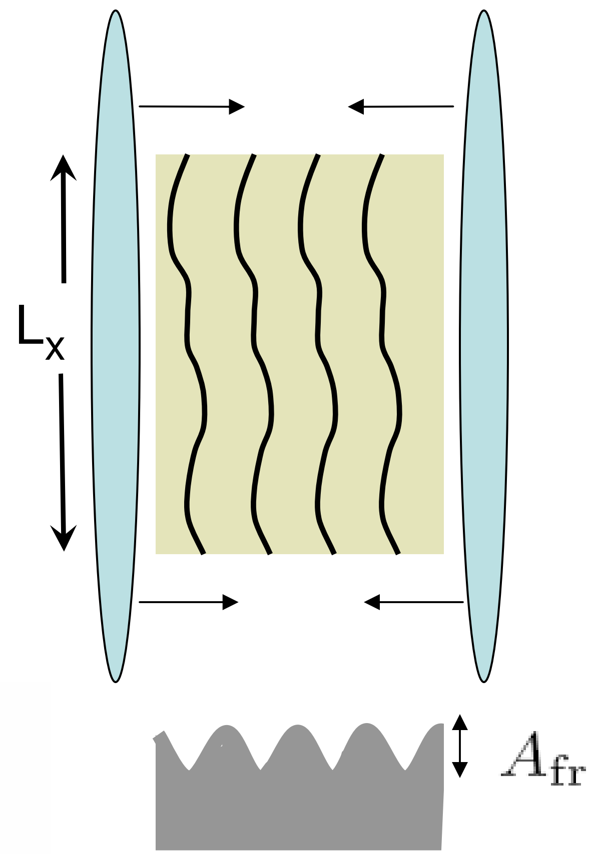
Analysis of $A_{\rm fr}$ can be used for thermometry

Interference of two dimensional condensates

Experiments: Hadzibabic et al. Nature (2006)

Gati et al., PRL (2006)



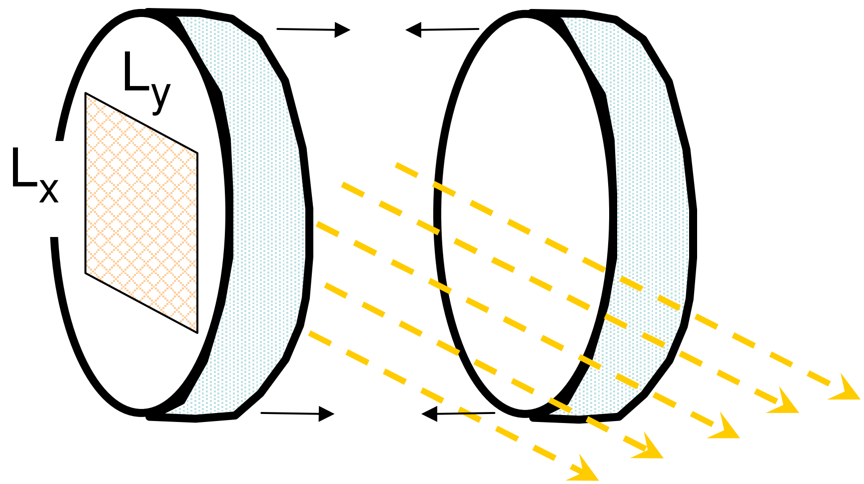


Probe beam parallel to the plane of the condensates

$$\langle\, |A_{\text{fr}}|^2 \rangle \,=\, L_x\, L_y \int_0^{L_x} \int_0^{L_y} d^2\vec{r}\, \left(\, G(\vec{r}\,)\,\right)^2$$

$$G(\vec{r}\,) \,=\, \langle\, a(\vec{r}\,)\, a^\dagger(0)\,\rangle$$

Interference of two dimensional condensates. Quasi long range order and the KT transition



Above KT transition

$$G(r) \sim e^{-r/\xi}$$

$$\langle |A_{\mathrm{fr}}|^2 \rangle \sim L_x L_y$$

$$\log \xi(T) \sim 1/\sqrt{T - T_{\mathrm{KT}}}$$

Below KT transition

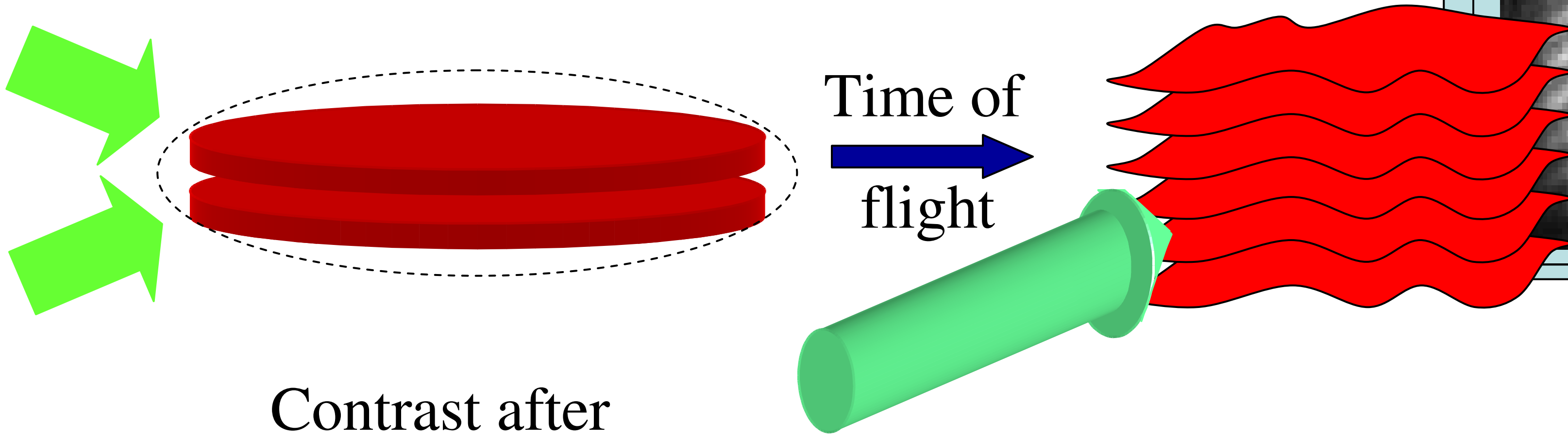
$$G(r) \sim \rho \left(\frac{\xi_h}{r} \right)^{\alpha}$$

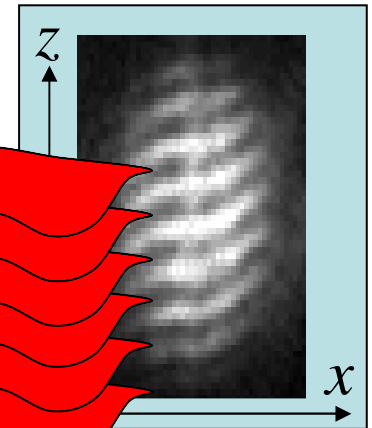
$$\alpha(T) = \frac{m T}{2 \pi \rho_s(T) \hbar^2}$$

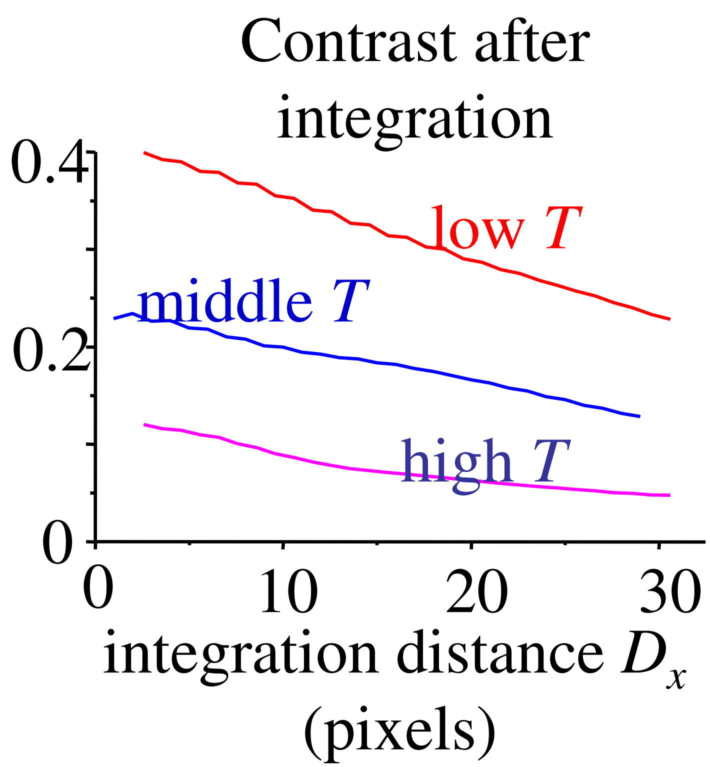
$$\langle |A_{\mathrm{fr}}|^2 \rangle \sim (L_x L_y)^{2-\alpha}$$

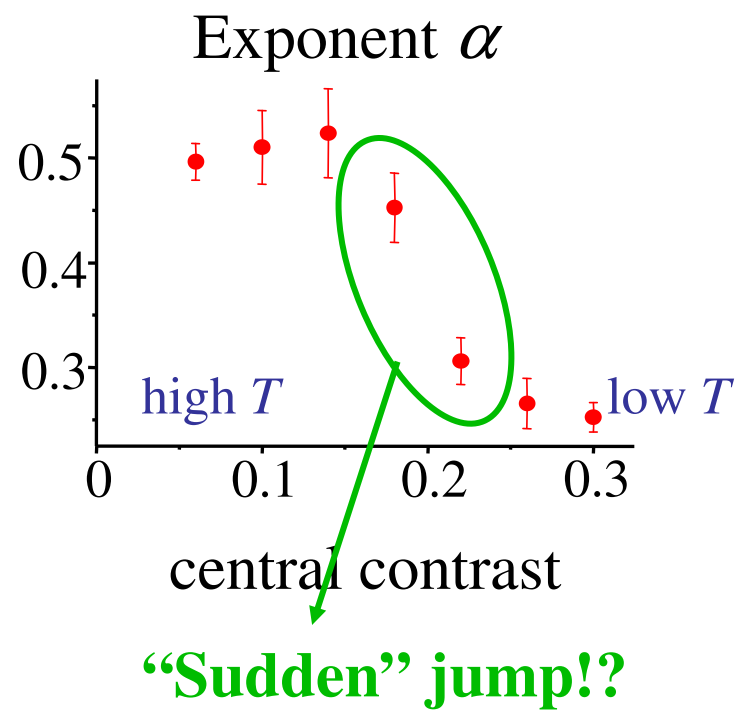
Experiments with 2D Bose gas

Hadzibabic, Dalibard et al., Nature 441:1118 (2006)





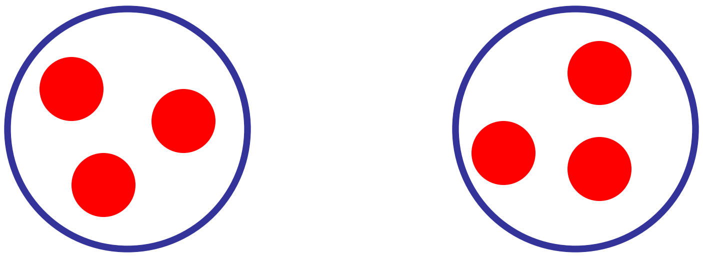




Fundamental noise in interference experiments

Amplitude of interference fringes is a quantum operator. The measured value of the amplitude will fluctuate from shot to shot. We want to characterize not only the average but the fluctuations as well.

Shot noise in interference experiments



Interference with a finite number of atoms.
How well can one measure the amplitude
of interference fringes in a single shot?

One atom: No
Very many atoms: Exactly
Finite number of atoms: ?

Consider higher moments of the interference fringe amplitude

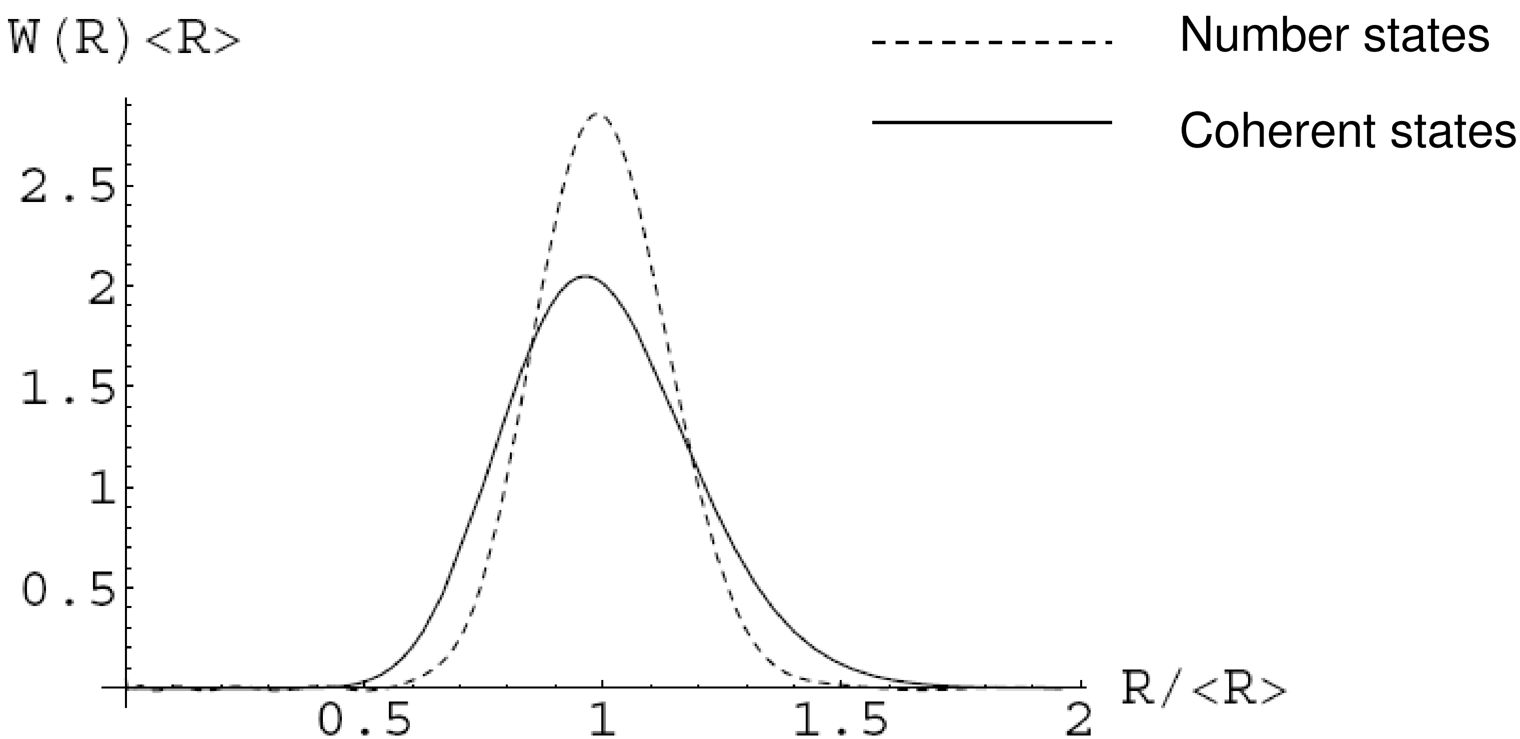
$\langle |A|^2 \rangle$, $\langle |A|^4 \rangle$, and so on

Obtain the entire distribution function of $|A|^2$

Shot noise in interference experiments

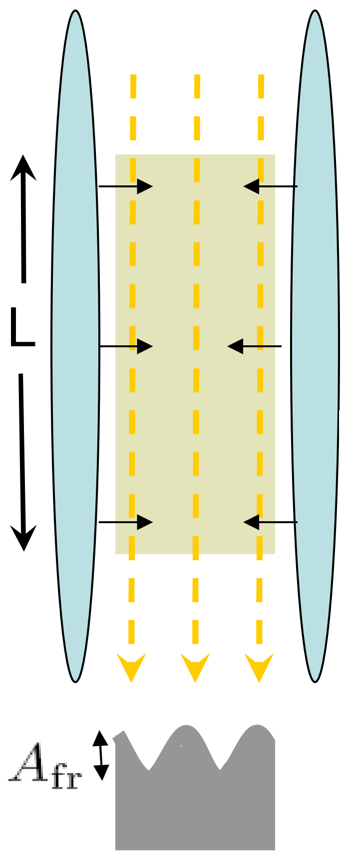
Polkovnikov, Europhys. Lett. 78:10006 (1997)
Imambekov, Gritsev, Demler, 2006 Varenna lecture notes

Interference of two condensates with 100 atoms in each cloud



Distribution function of fringe amplitudes for interference of fluctuating condensates

Gritsev, Altman, Demler, Polkovnikov, Nature Physics (2006)
Imambekov, Gritsev, Demler, cond-mat/0612011



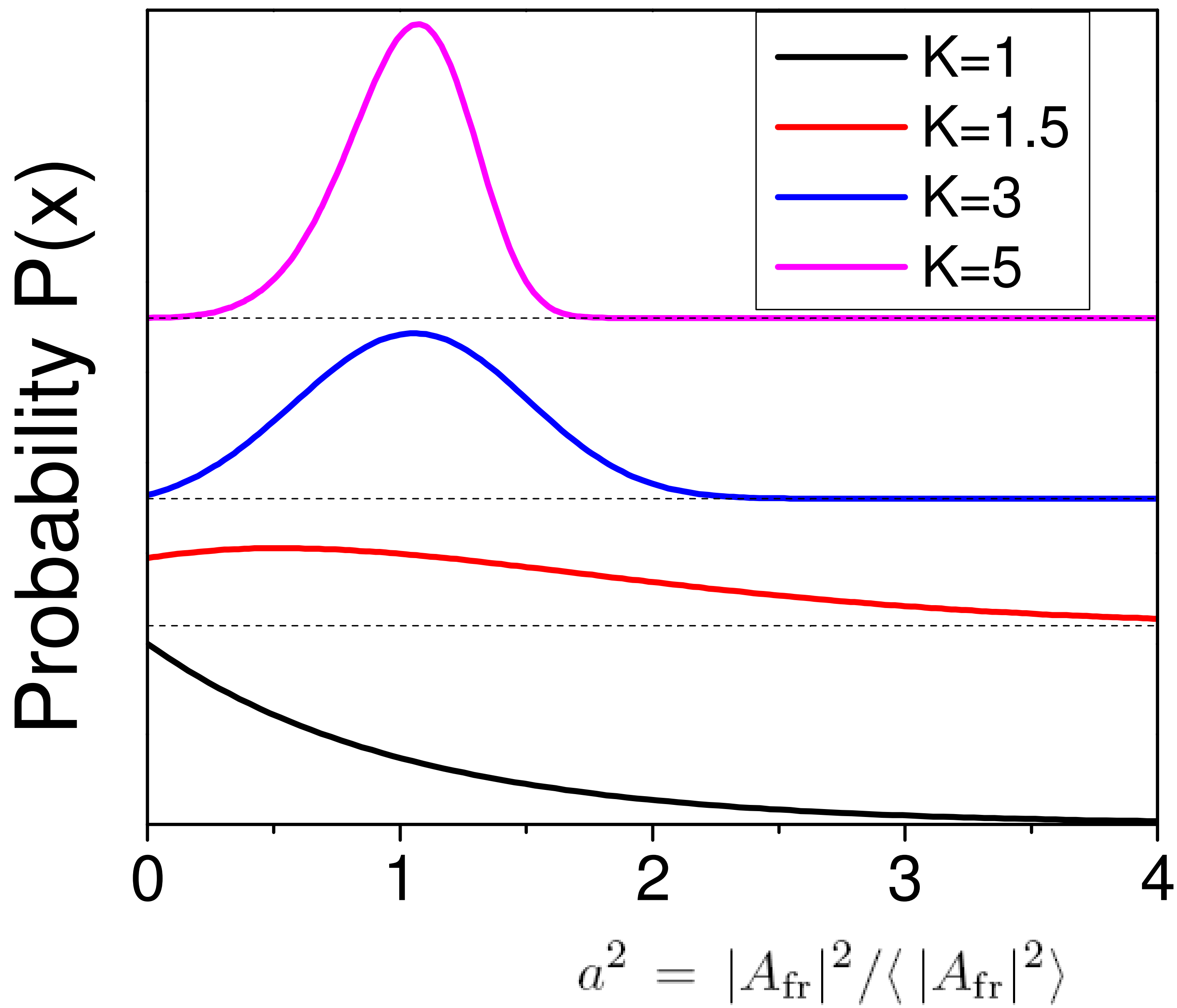
A_{fr} is a quantum operator. The measured value of $|A_{\mathrm{fr}}|$ will fluctuate from shot to shot.

$$\langle\, |A_{\mathrm{fr}}|^{2n} \rangle \;=\; \int_0^L dz_1 \,\ldots\, dz_n' \,|\langle\, a^\dagger(z_1)\,\ldots a^\dagger(z_n)\, a(z_1')\,\ldots a(z_n')\,\rangle|^2$$

Higher moments reflect higher order correlation functions

We need the full distribution function of $|A_{\mathrm{fr}}|$

Interference of 1d condensates at T=0. Distribution function of the fringe contrast



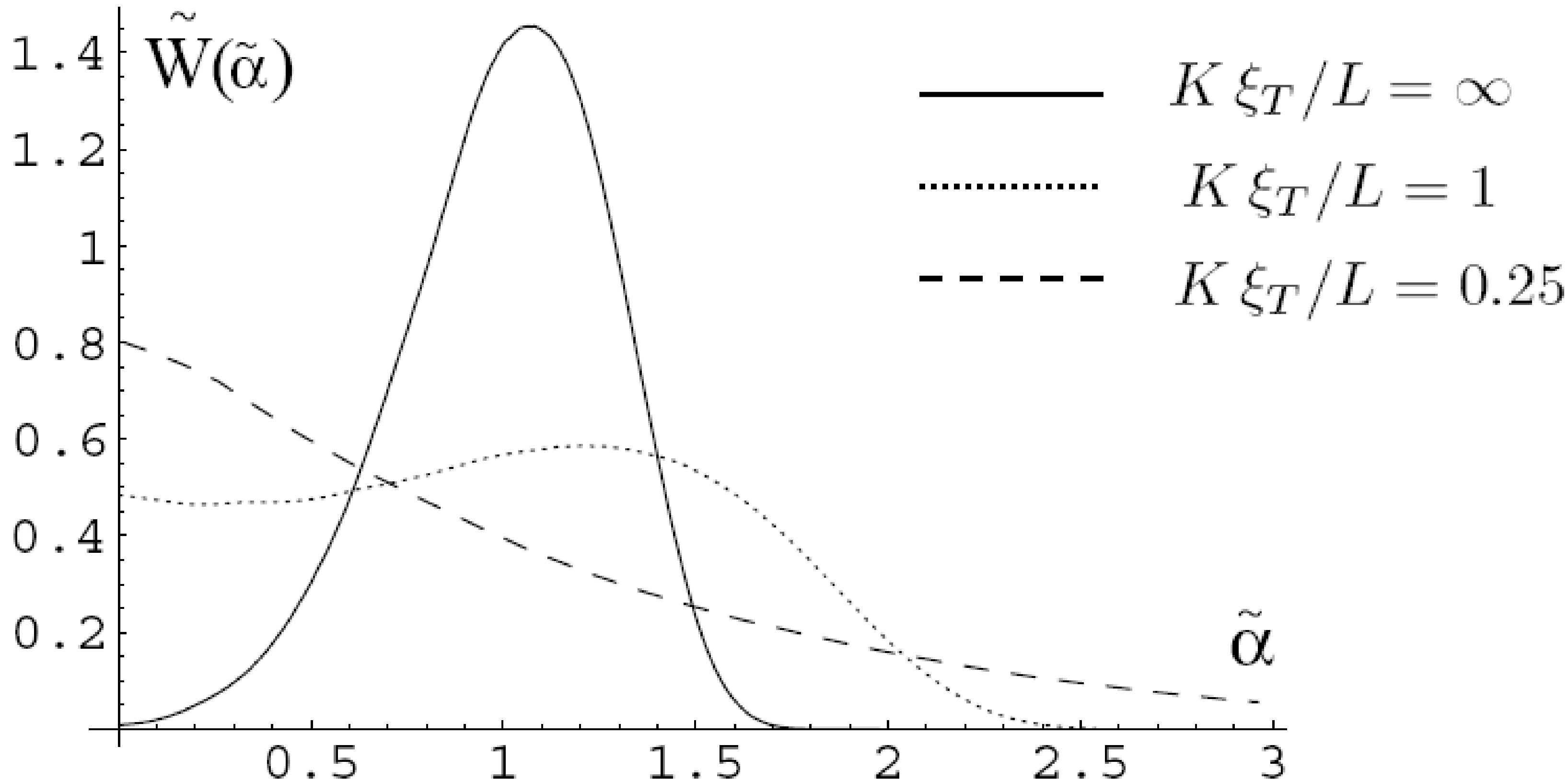
Narrow distribution for $K \to \infty$ Approaches Gumbel distribution.

Width $\sigma = \frac{\pi}{6K}$

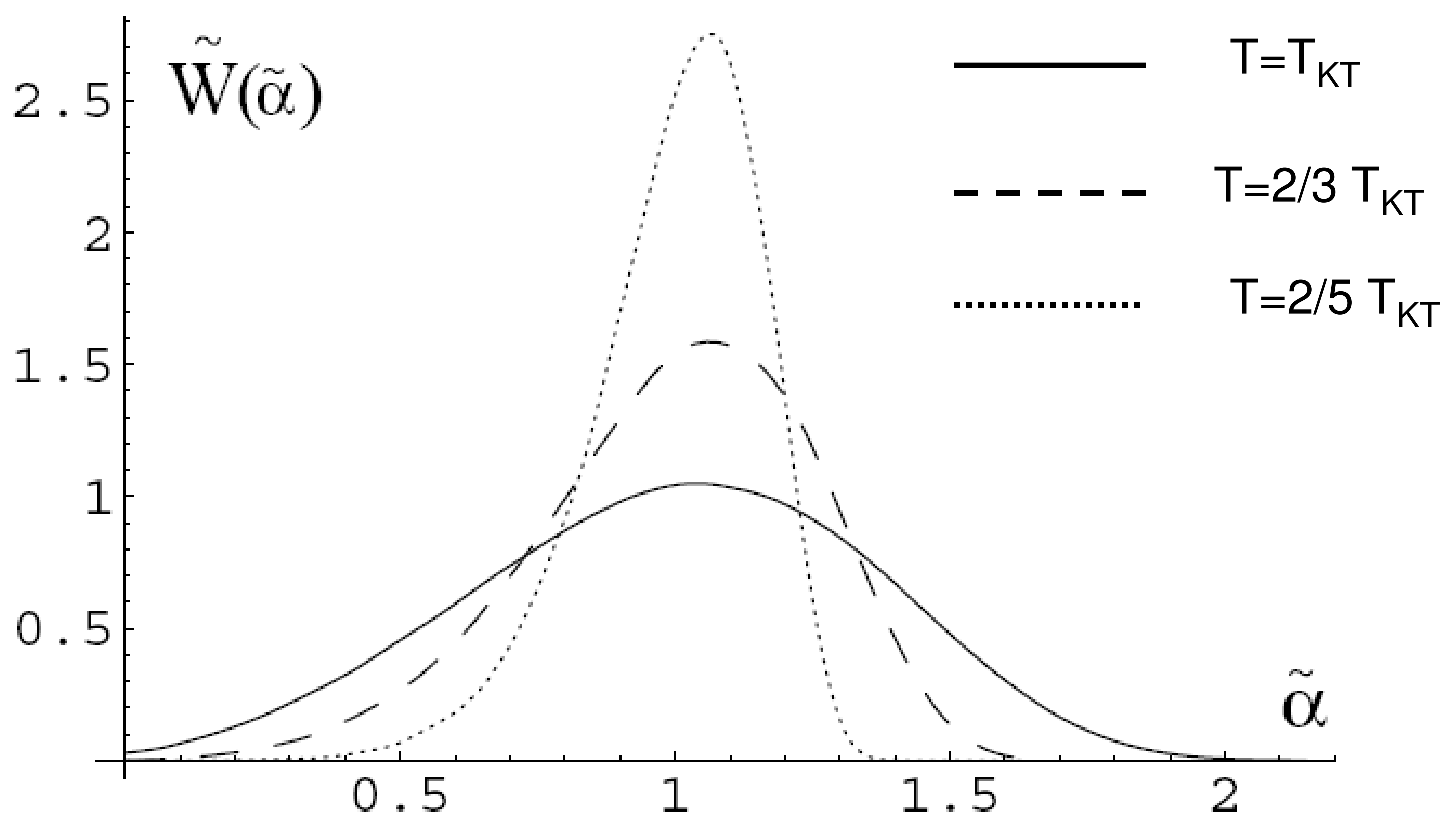
Wide Poissonian distribution for $K \to 1$

Interference of 1d condensates at finite temperature. Distribution function of the fringe contrast

Luttinger parameter *K=5*



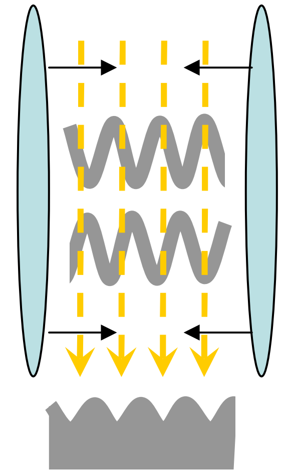
Interference of 2d condensates at finite temperature. Distribution function of the fringe contrast



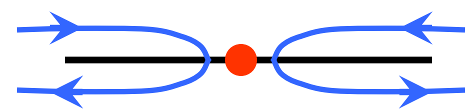
From visibility of interference fringes to other problems in physics

Interference between interacting 1d Bose liquids. Distribution function of the interference amplitude

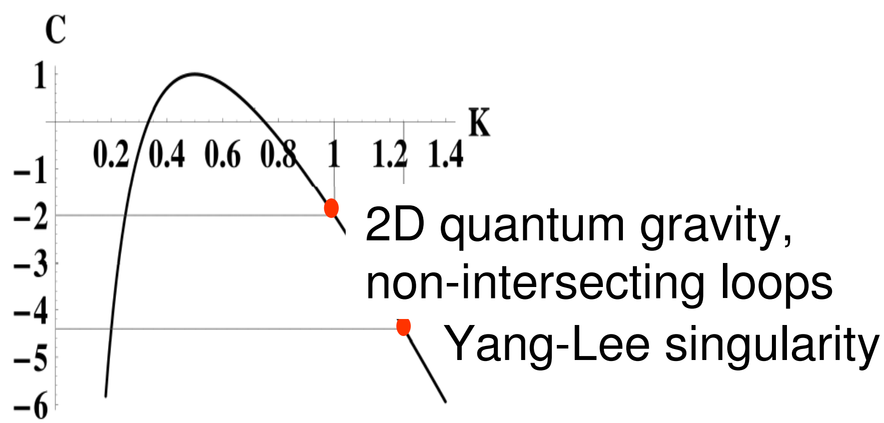
A_{fr} is a quantum operator. The measured value of $|A_{\mathrm{fr}}|$ will fluctuate from shot to shot.
How to predict the distribution function of $|A_{\mathrm{fr}}|$



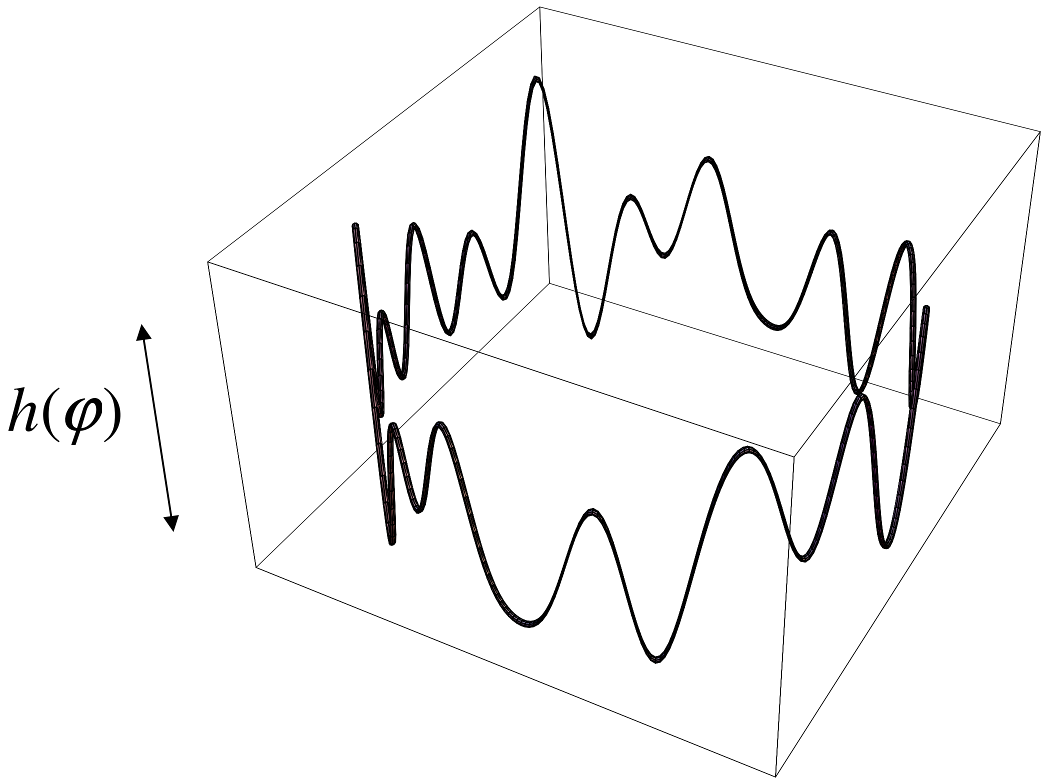
Quantum impurity problem: interacting one dimensional electrons scattered on an impurity



Conformal field theories with negative central charges: 2D quantum gravity, non-intersecting loop model, growth of random fractal stochastic interface, high energy limit of multicolor QCD, …



Fringe visibility and statistics of random surfaces



Fringe visibility

$\Updownarrow$

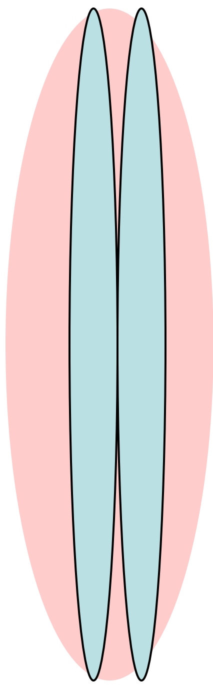
$$\text{Roughness} = \int h(\varphi)^2 d\varphi$$

Proof of the Gumbel distribution of interfernece fringe amplitude for 1d weakly interacting bosons relied on the known relation between 1/f Noise and Extreme Value Statistics

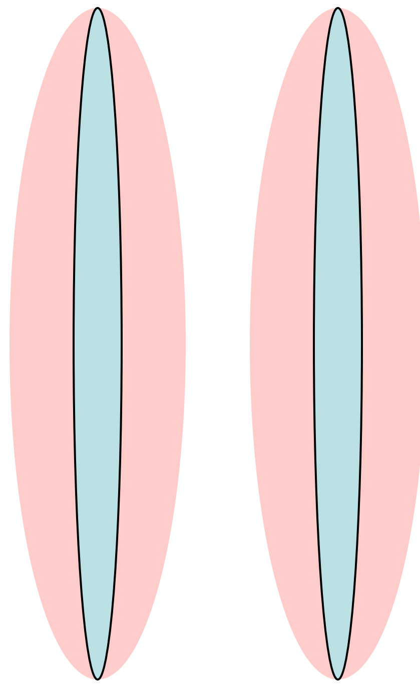
T.Antal et al. Phys.Rev.Lett. 87, 240601(2001)

Non-equilibrium coherent dynamics of low dimensional Bose gases probed in interference experiments

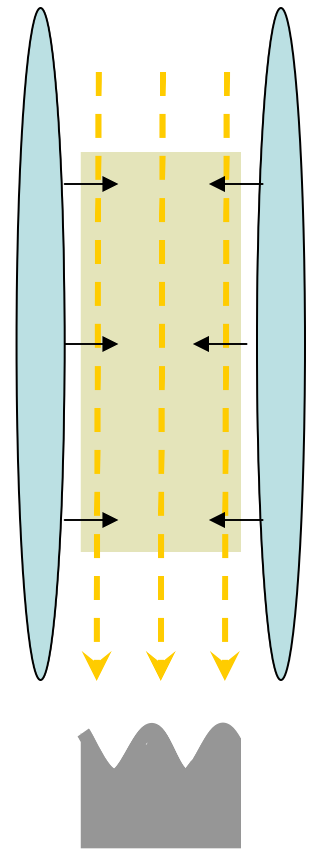
Studying dynamics using interference experiments. Thermal decoherence



Prepare a system by splitting one condensate



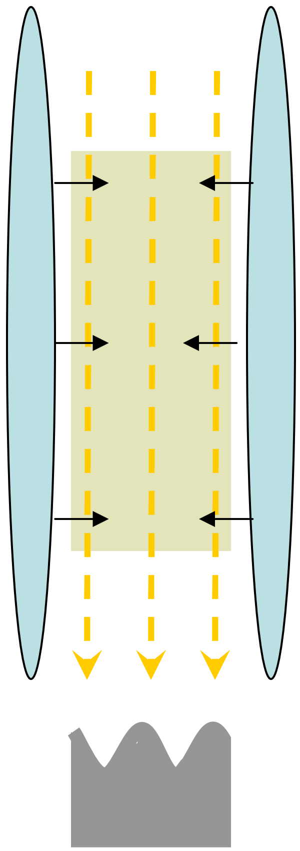
Take to the regime of zero tunneling



Measure time evolution of fringe amplitudes

Relative phase dynamics

Burkov, Lukin, Demler, cond-mat/0701058



Quantum regime $\frac{h}{\mu} < t < \frac{h}{k_B T}$

1D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} e^{-t/2\pi K \tau_s}$

2D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} \left(\frac{t_0}{t}\right)^{1/16 T_{KT} \tau_s}$

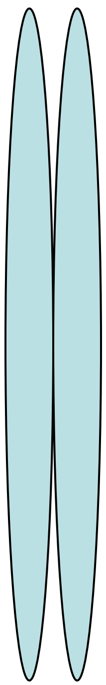
Different from the earlier theoretical work based on a single mode approximation, e.g. Gardiner and Zoller, Leggett

Classical regime $t > \frac{h}{k_B T}$

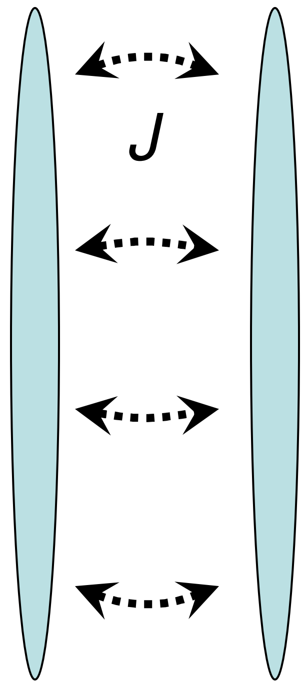
1D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\left(\frac{t}{t_T}\right)^{2/3}}$ $\quad t_T \sim \frac{\mu K}{T^2}$

2D systems $\langle e^{i\phi(t)} \rangle \sim \left(\frac{t_0}{t}\right)^{\frac{T}{8T_{KT}}}$

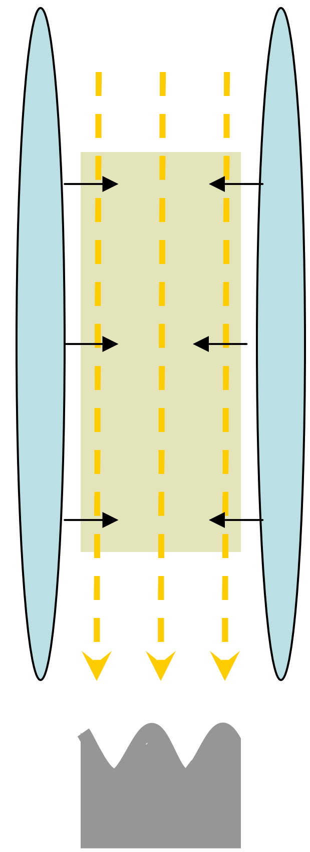
Quantum dynamics of coupled condensates. Studying Sine-Gordon model in interference experiments



Prepare a system by splitting one condensate



Take to the regime of finite tunneling. System described by the quantum Sine-Gordon model



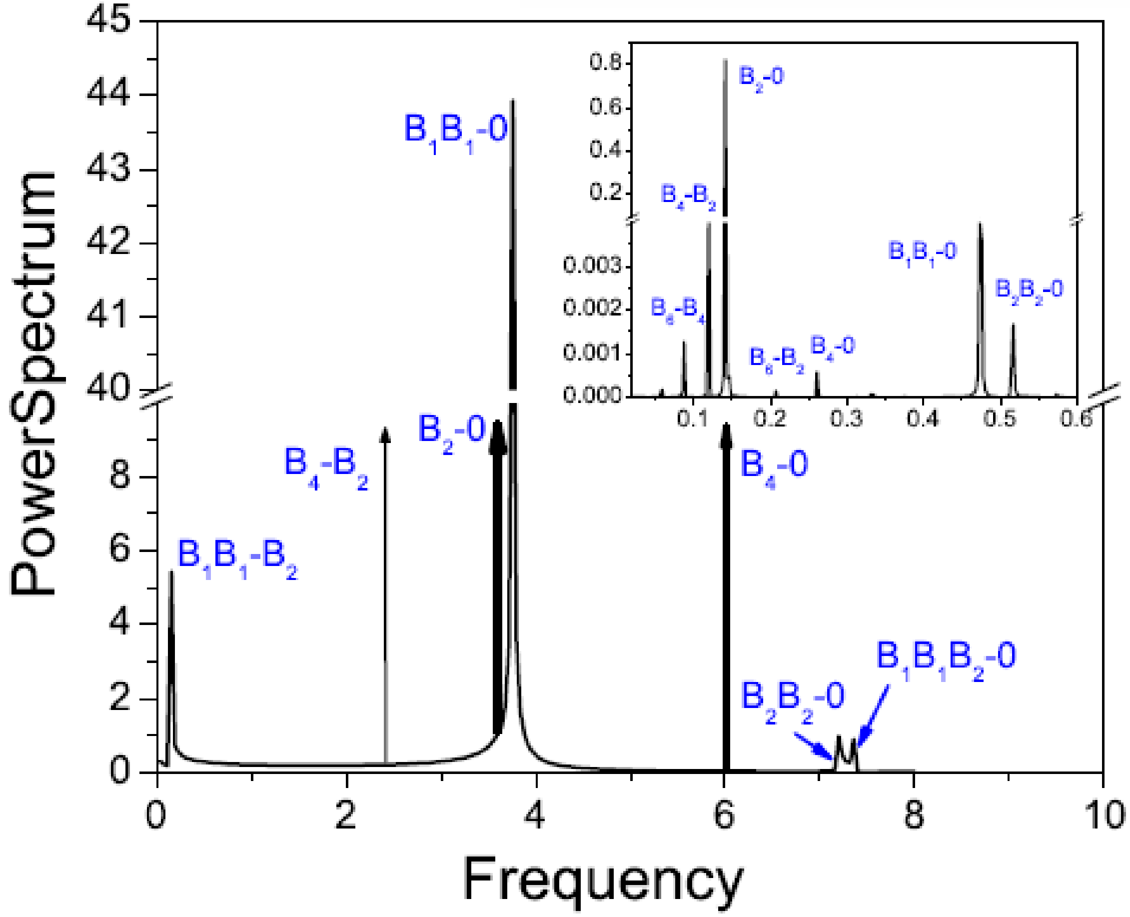
Measure time evolution of fringe amplitudes

Dynamics of quantum sine-Gordon model

Gritsev, Polkovnikov, Demler, cond-mat/0701421

Gritsev, Demler, Lukin, Polkovnikov, cond-mat/0702343

Power spectrum $P(\omega) = \left| \int_t e^{i\omega t} \langle \psi(t) \,|\, e^{i\phi} \,|\, \psi(t) \rangle \right|^2$



A combination of broad features and sharp peaks. Sharp peaks due to collective many-body excitations: breathers

Conclusions

Interference of extended condensates can be used to probe equilibrium correlation functions in one and two dimensional systems

Interference experiments can be used to study non-equilibrium dynamics of low dimensional superfluids and do spectroscopy of the quantum sine-Gordon model