Multicomponent systems of ultracold atoms

Eugene Demler Harvard University

Dynamical instability of spiral states

In collaboration with Robert Cherng, Vladimir Gritsev.
Thanks to Dan Stamper-Kurn

Many-body decoherence of Ramsey interference

In collaboration with Artur Widera, Stefan Trotzky, Patrick Chainet, Fabrice Gerbier, Simon Folling, Immanuel Bloch (Mainz) , Vladimir Gritsev, Mikhail Lukin

Bose-Fermi mixtures in optical lattices

In collaboration with Gil Refael (Caltech)

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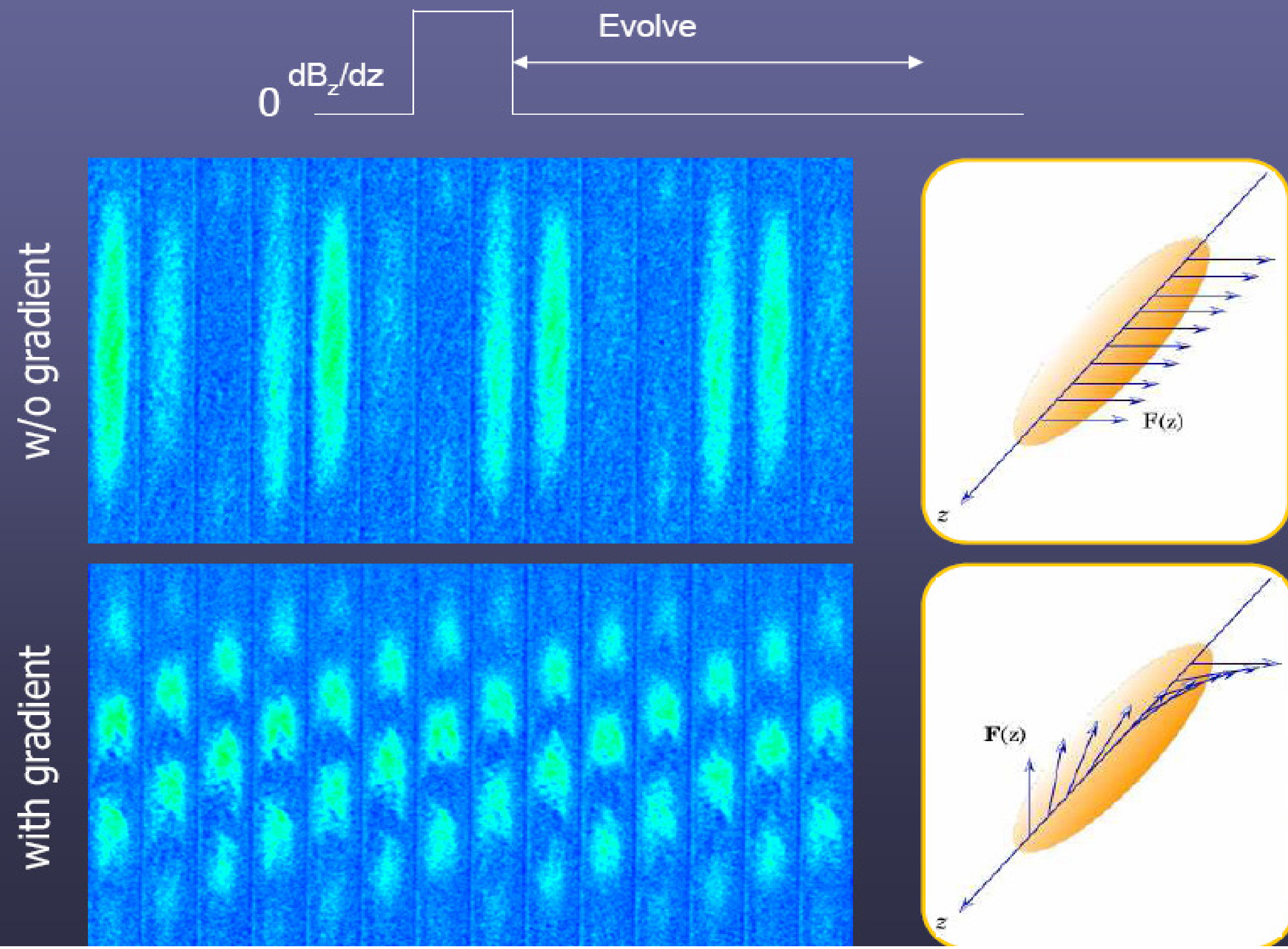
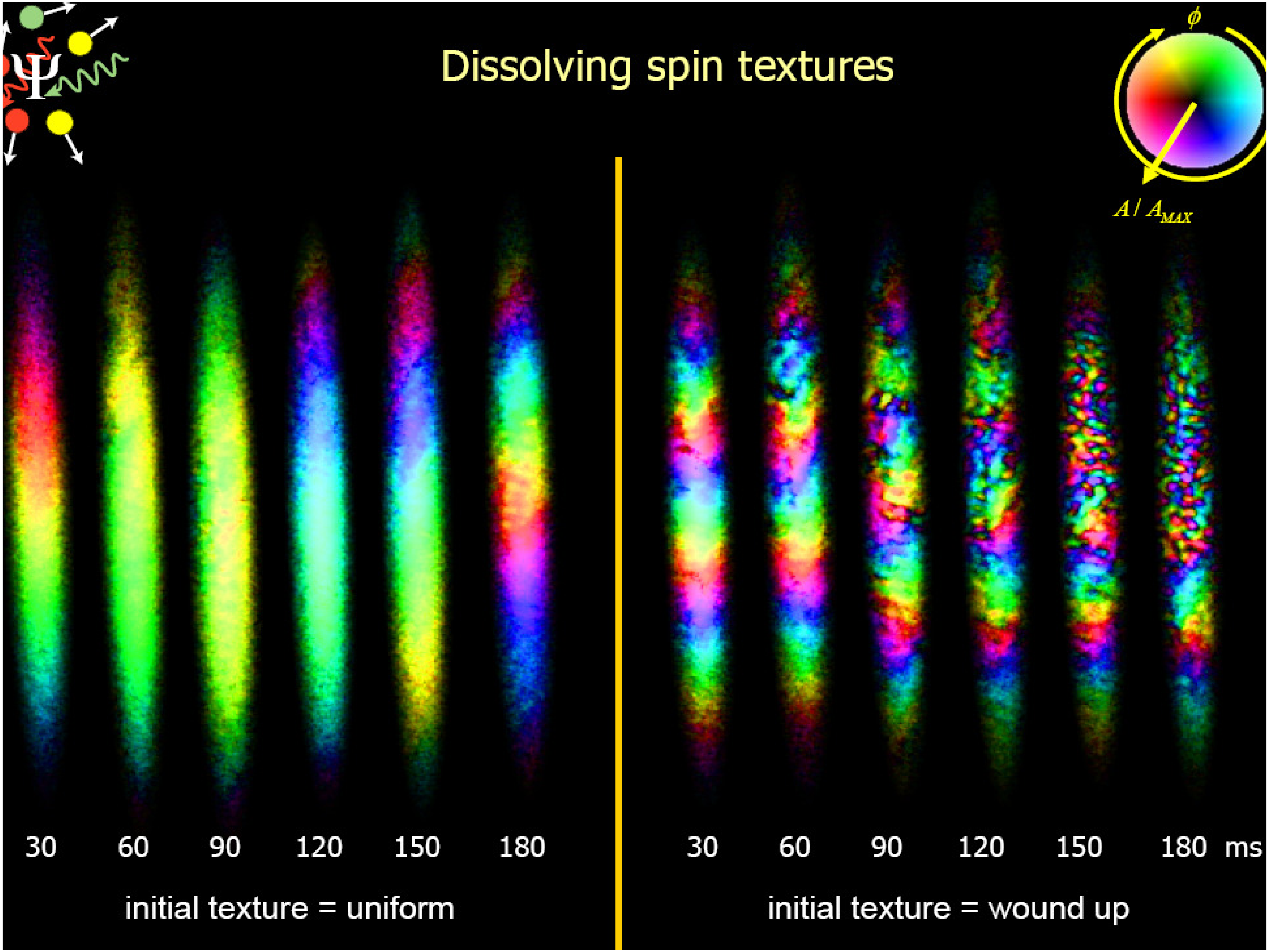
Dynamical Instability of the Spiral State of F=1 Ferromagnetic Condensate

Connections to spintronics in electron systems

Ferromagnetic spin textures created by D. Stamper-Kurn et al.

- generate helical spin pattern (uniform spin current) using inhomogeneous fie




Dissolving spin textures
Ψ
φ
A / A_MAX
30 60 90 120 150 180
initial texture = uniform
30 60 90 120 150 180 ms
initial texture = wound up

F=1 condensates

Spinor order parameter
Vector representation

$$\vec{\Psi} = \begin{pmatrix} \Psi_x \\ \Psi_y \\ \Psi_z \end{pmatrix}$$

Ferromagnetic State

$S_x = \cos\phi \qquad S_y = \sin\phi$

$$\vec{\Psi} = \frac{\Psi_0}{\sqrt{2}} \times \begin{pmatrix} i\ \sin\phi \\ i\ \cos\phi \\ 1 \end{pmatrix}$$

Polar (nematic) state

$$\vec{\Psi} = \Psi_0 \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathcal{H} = \frac{1}{2m} \nabla \Psi_\alpha^\dagger \nabla \Psi_\alpha + q F_z^2 + \frac{g_0}{2} \Psi_\alpha^\dagger \Psi_\beta^\dagger \Psi_\beta \Psi_\alpha + \frac{g_s}{2} \Psi_\alpha^\dagger \Psi_\alpha^\dagger \Psi_\beta \Psi_\beta$$

Ferromagnetic state realized for $g_s > 0$

Spiral Ferromagnetic State of F=1 condensate

Gross-Pitaevski equation

$$i\frac{\partial \Psi_\alpha}{\partial t} = -\frac{\nabla^2}{2m}\Psi_\alpha + g_0 \Psi^\dagger_\beta \Psi_\beta \Psi_\alpha + g_s \Psi^\dagger_\alpha \Psi_\beta \Psi_\beta$$

Mean-field spiral state

The nature of the mean-field state depends on the system preparation.

Sudden twisting

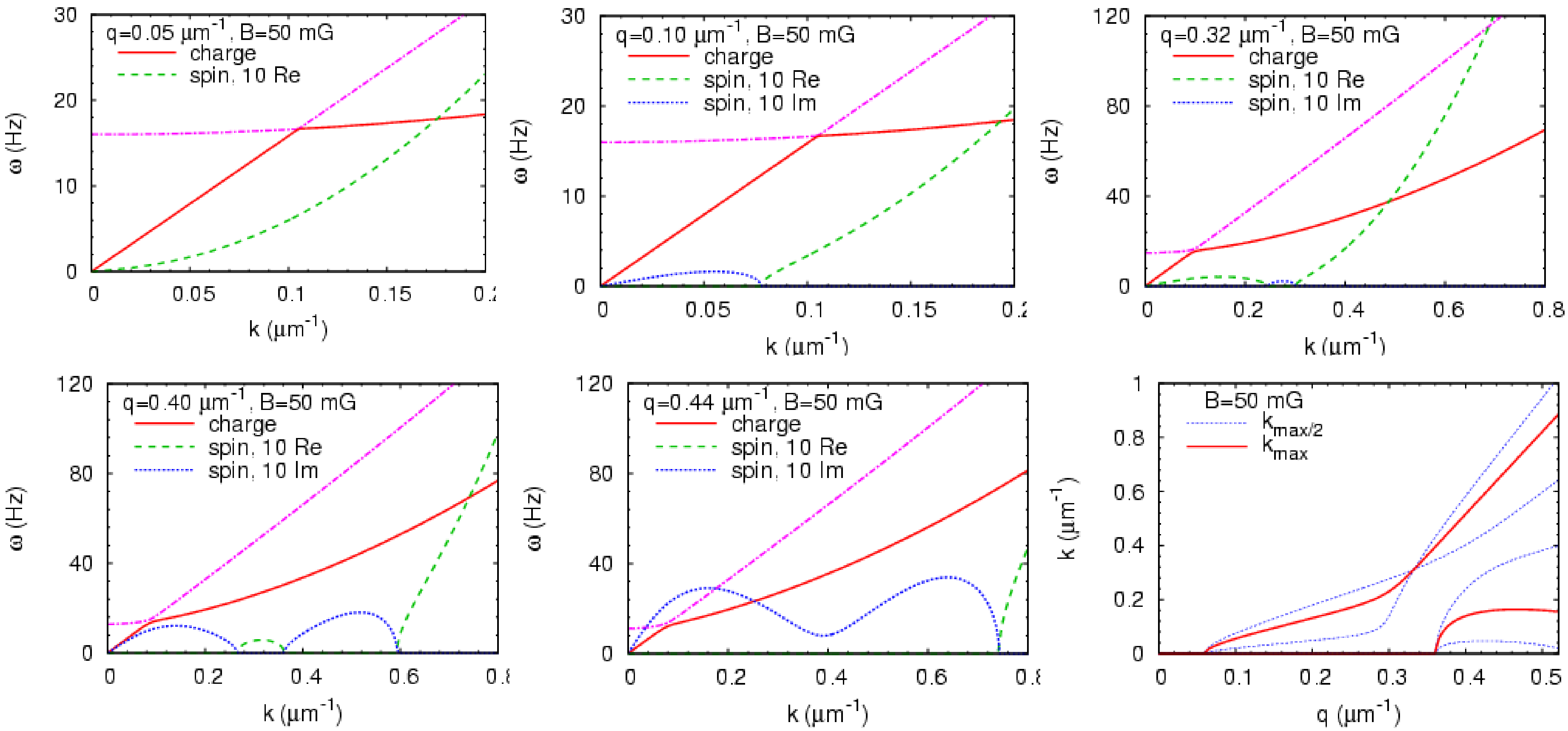
$$\Psi(x,t) = \Psi_0 \begin{pmatrix} i \cos qx \frac{1}{\sqrt{2}} e^{-i\omega_1 t} \\ i \sin qx \frac{1}{\sqrt{2}} e^{-i\omega_1 t} \\ \frac{1}{\sqrt{2}} e^{-i\omega_2 t} \end{pmatrix}$$

Adiabatic limit: θ determined from the condition of the stationary state.

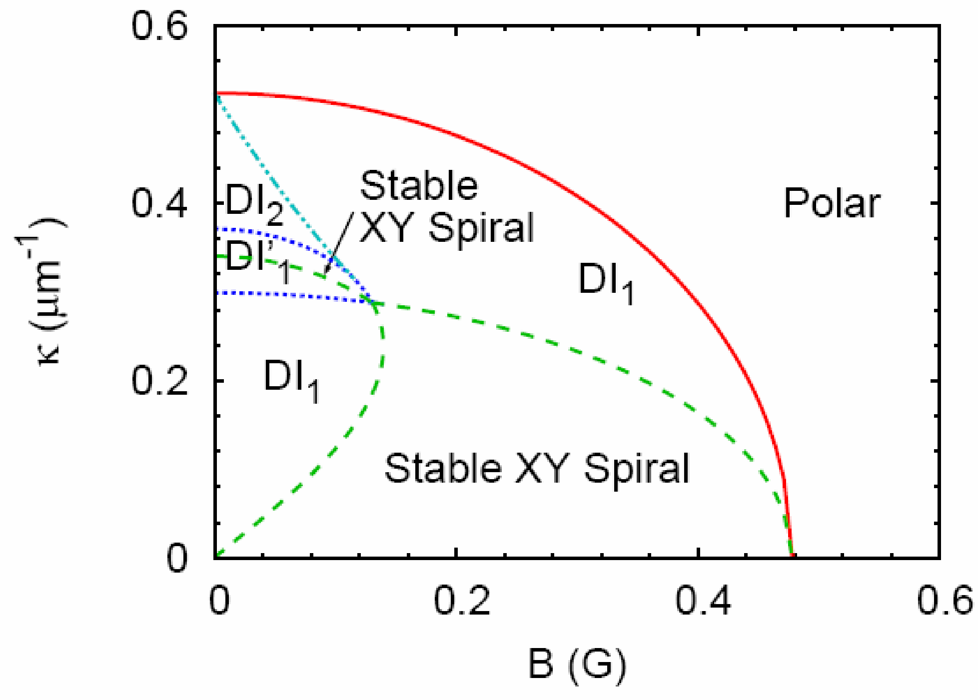
$$\vec{\Psi}(x) = \Psi_0 \begin{pmatrix} i \cos qx \cos\theta \\ i \sin qx \cos\theta \\ \sin\theta \end{pmatrix} e^{-i\mu t}$$

Instabillities can be obtained from the analysis of collective modes

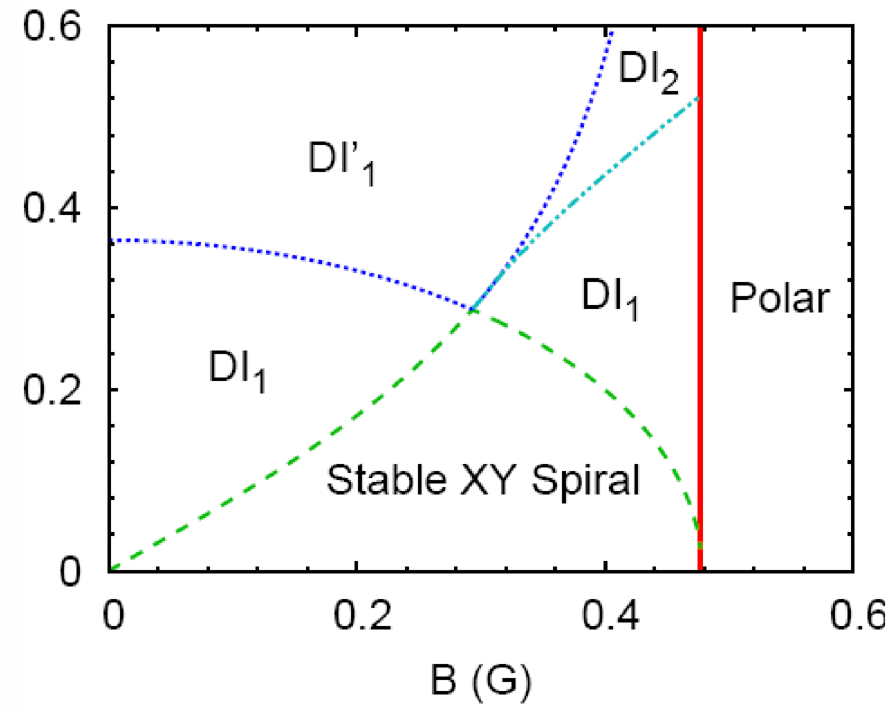
Collective modes



Instabilities of the spiral state

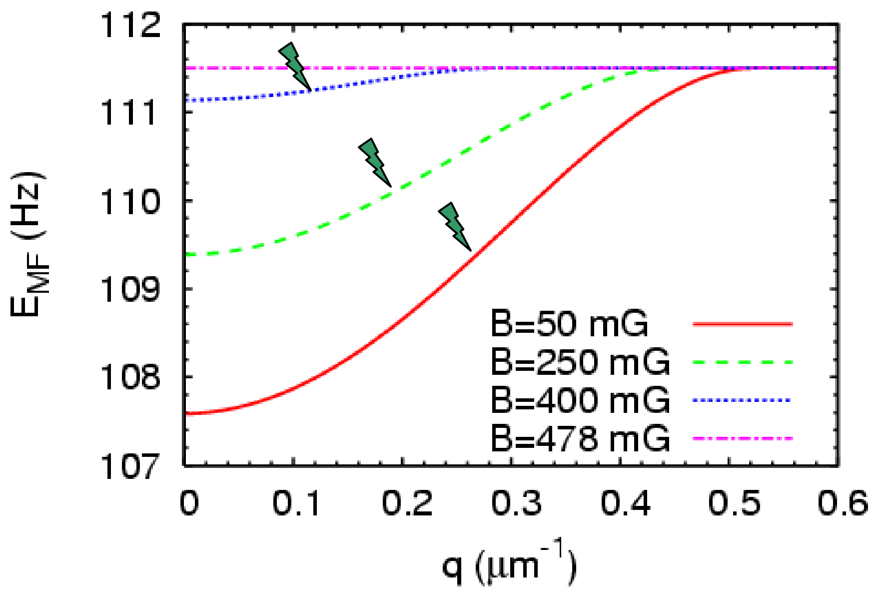


Adiabatic limit



Sudden limit

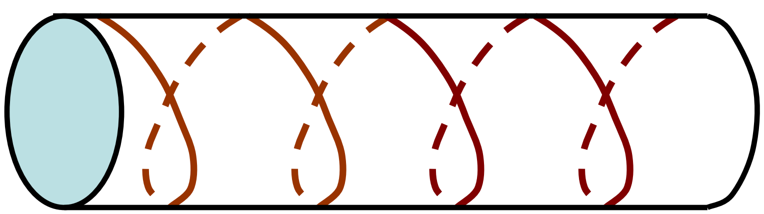
Mean-field energy



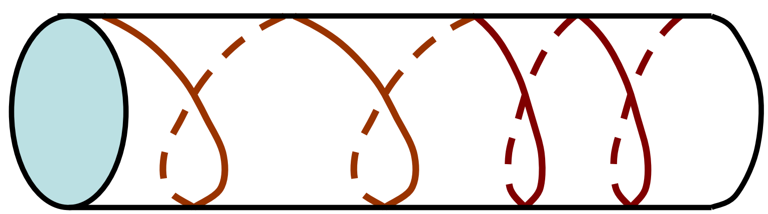
Inflection point suggests instability

Negative value of $\partial^2 E_{\mathrm{MF}}/\partial q^2$ shows that the system can lower its energy by making a non-uniform spiral winding

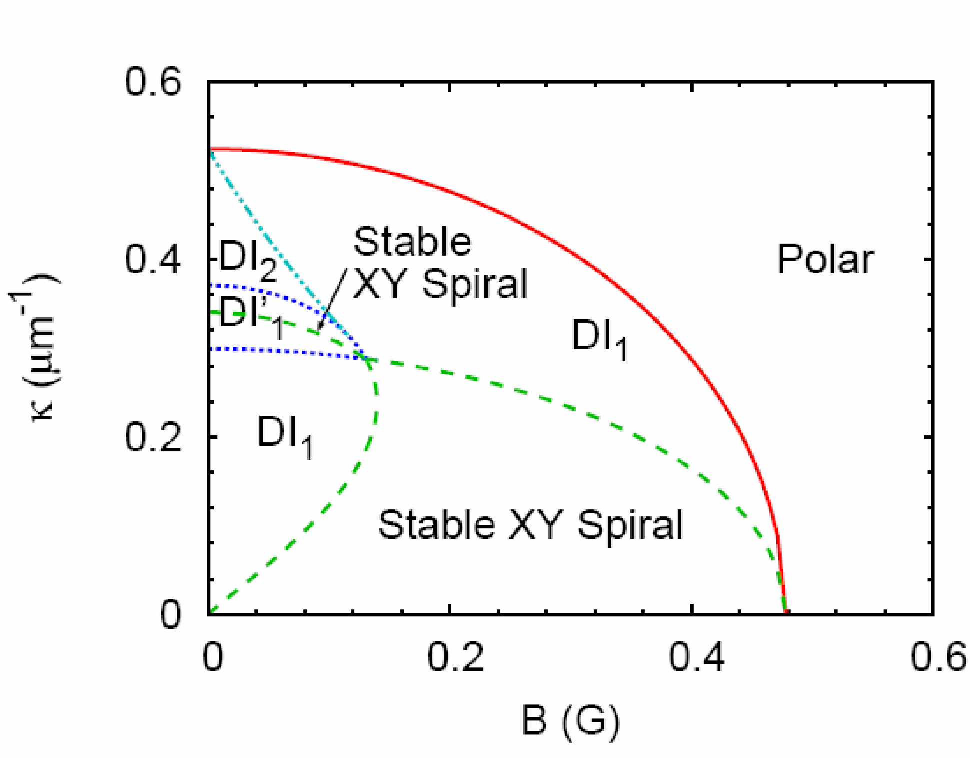
Uniform spiral



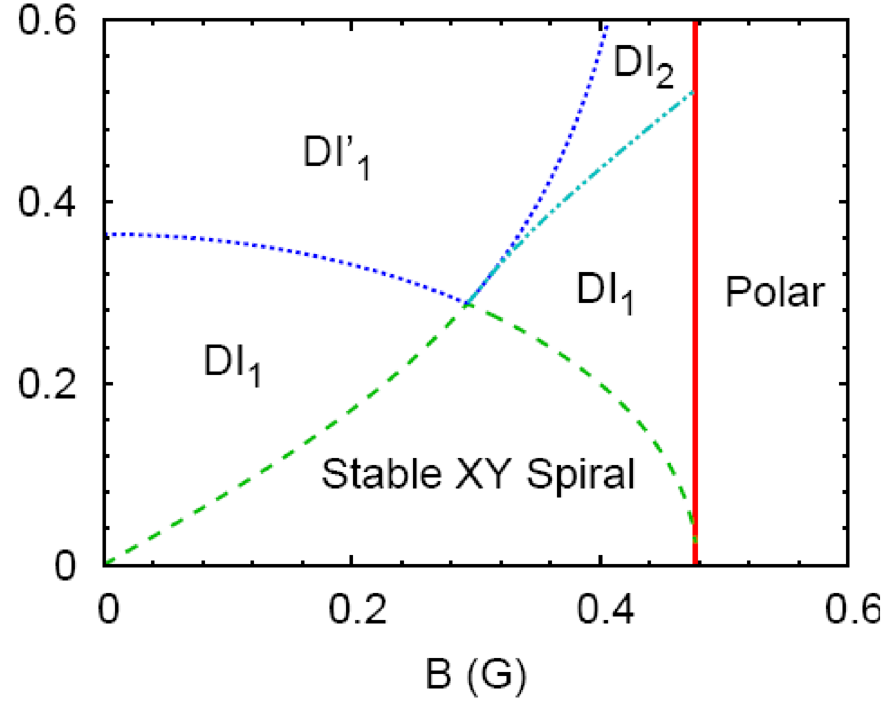
Non-uniform spiral



Instabilities of the spiral state



Adiabatic limit



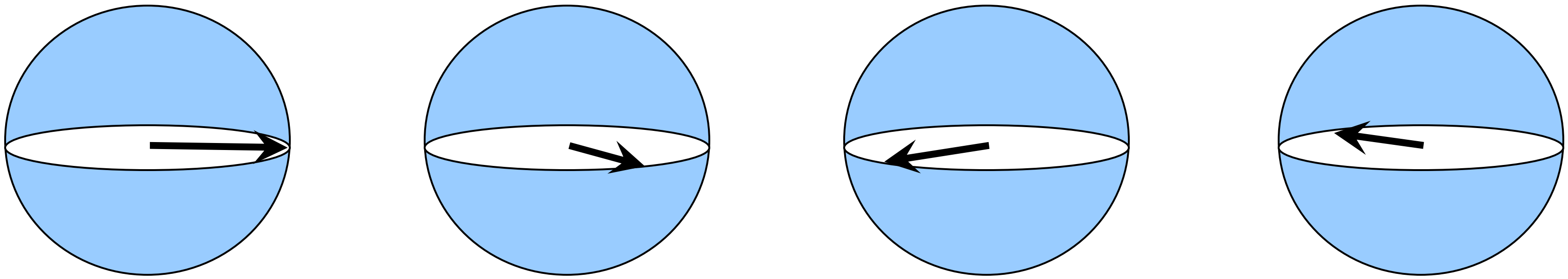
Sudden limit

Beyond mean-field: thermal and quantum phase slips?

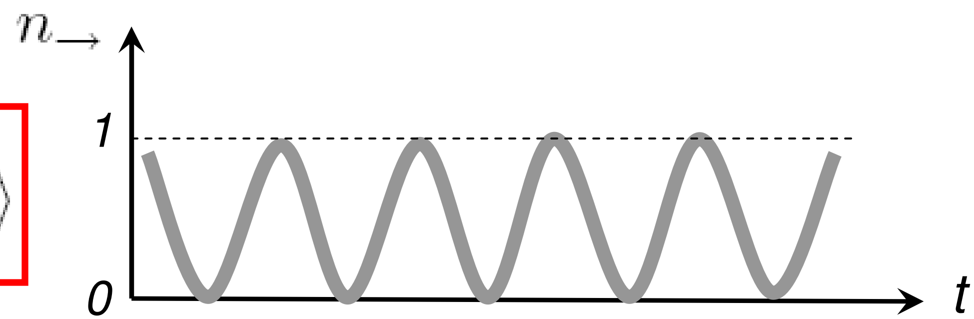
Many-body decoherence and Ramsey interferometry

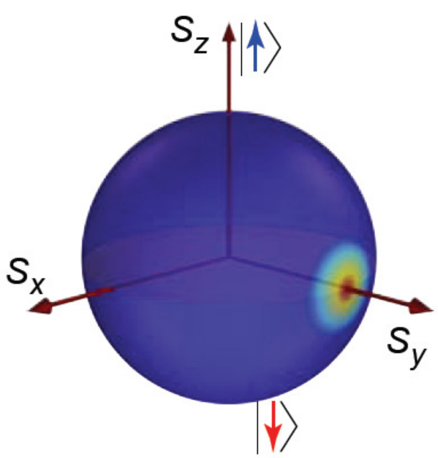
Connections to one dimensional electron systems.
New feature: application of Luttinger liquid model for describing non-equilibrium dynamics

Ramsey interference



$$|\Psi\rangle = e^{-iE_1 t}|\uparrow\rangle + e^{-iE_2 t}|\downarrow\rangle$$





Working with *N* atoms improves the precision by $\sqrt{N}$.
Need spin squeezed states to improve frequency spectroscopy

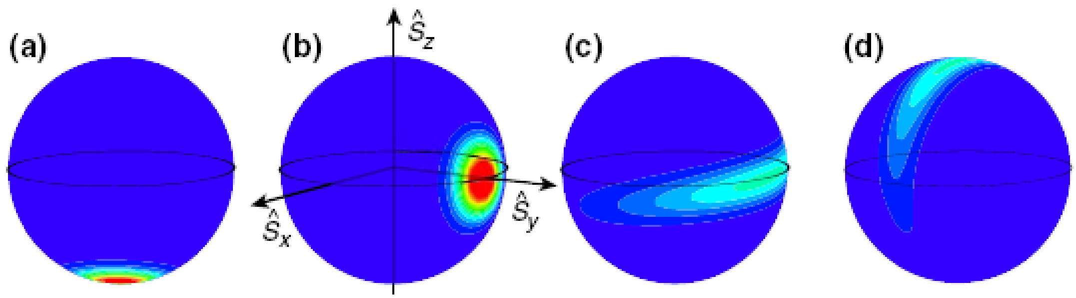
Squeezed spin states for spectroscopy

Motivation: improved spectroscopy, e.g. Wineland et. al. PRA 50:67 (1994)

Generation of spin squeezing using interactions.
Two component BEC. Single mode approximation

$$\mathcal{H} = \chi_s \left(S^z_{\mathrm{tot}}\right)^2$$

Kitagawa, Ueda, PRA 47:5138 (1993)



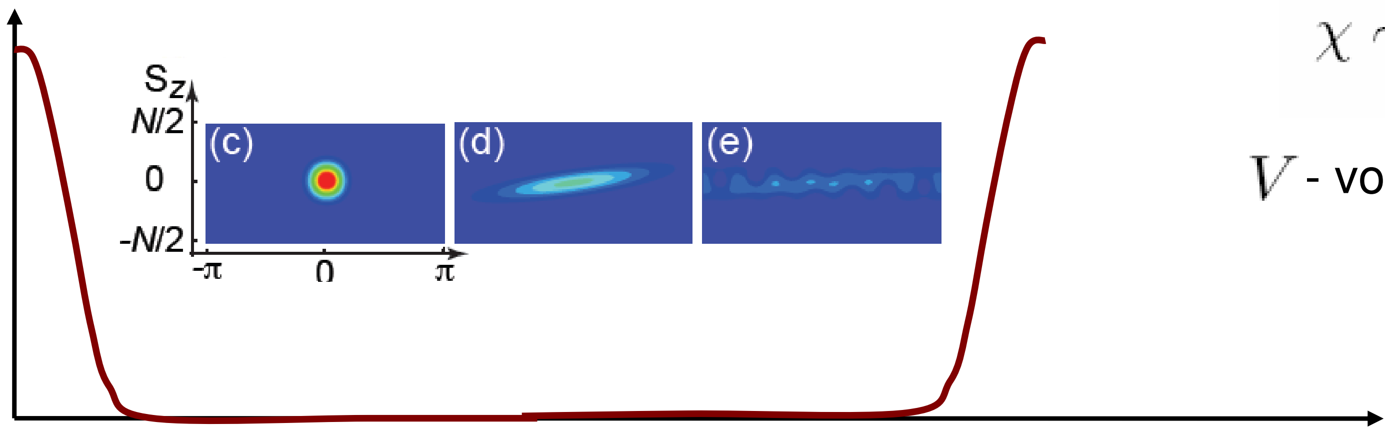
$$\mathcal{H} = \int dx \left[g_c \left(n_1 + n_2\right)^2 + g_s (n_1 - n_2)^2 + \frac{|\nabla \Psi_1|^2}{2m} + \frac{|\nabla \Psi_2|^2}{2m} \right]$$

In the single mode approximation we can neglect kinetic energy terms

$$\mathcal{H}_{\mathrm{SMA}} = \frac{g_s}{V} \left(N_1 - N_2\right)^2$$

Interaction induced collapse of Ramsey fringes

Ramsey fringe visibility



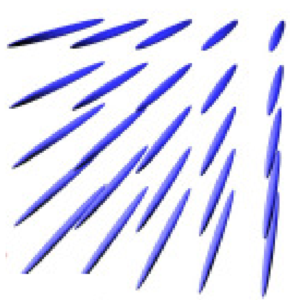
$$\chi \sim \frac{\hbar^2 a_s}{M V}$$

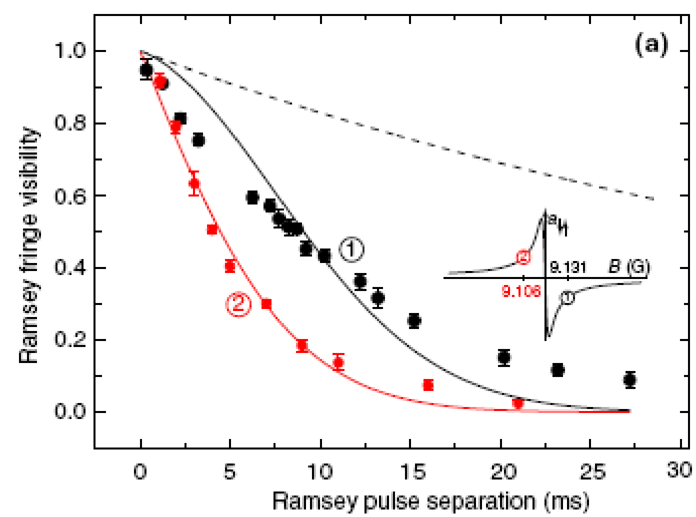
V - volume of the system

$$t_{\text{collapse}} \sim \frac{1}{\chi\sqrt{N}} \qquad t_{\text{revival}} \sim \frac{1}{\chi}$$

Experiments in 1d tubes:

A. Widera, I. Bloch et al.





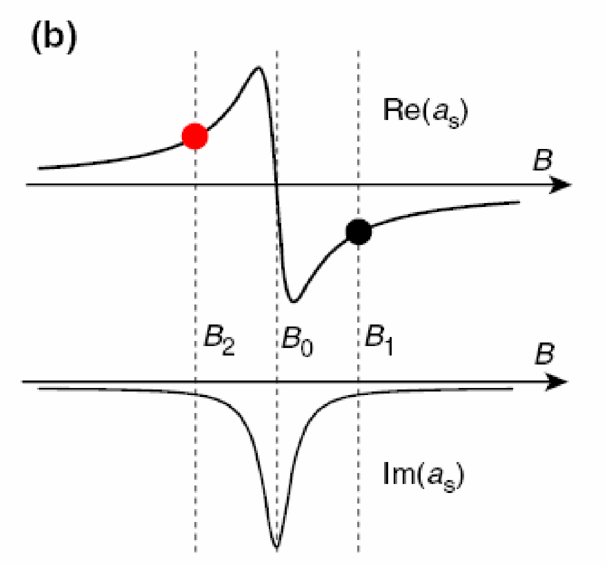
Spin echo. Time reversal experiments

Single mode approximation

$$\mathcal{H}_{\mathrm{SMA}} = \frac{g_s}{V}\left(N_1 - N_2\right)^2$$

$$g_s = \frac{g_{11} - g_{12}}{2}$$

The Hamiltonian can be reversed by changing a_{12}



$$a_s \rightarrow -a_s$$

$$\mathcal{H}_{\mathrm{SMA}} \rightarrow -\mathcal{H}_{\mathrm{SMA}}$$

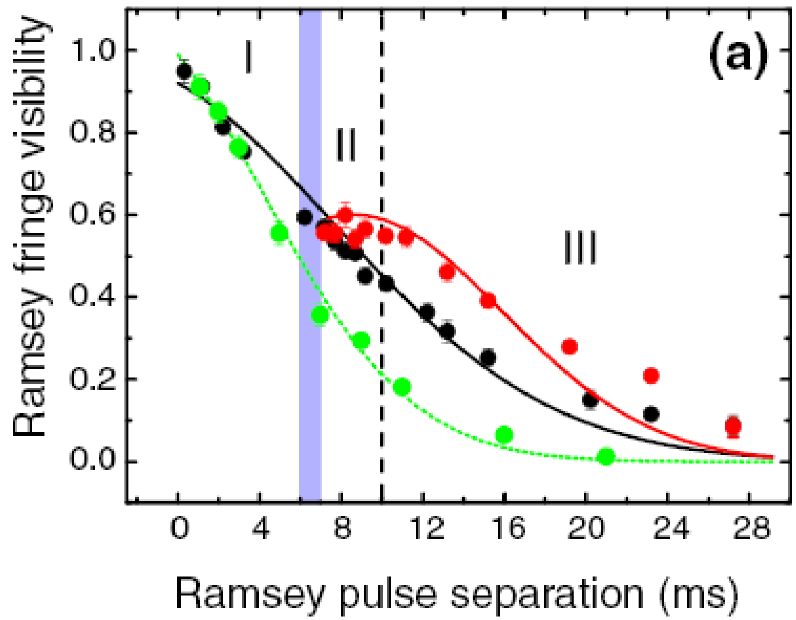
$$e^{i\int_T^{2T} \mathcal{H}_{\mathrm{SMA}}(t)dt} \times e^{i\int_0^{T} \mathcal{H}_{\mathrm{SMA}}(t)dt} = 1$$

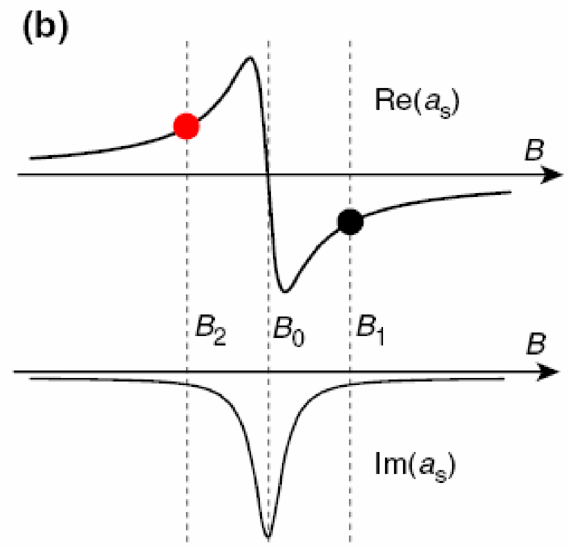
Predicts perfect spin echo

Spin echo. Time reversal experiments

Expts: A. Widera, I. Bloch et al.

$$a_s \longrightarrow -a_s$$





No revival?

Experiments done in array of tubes.
Strong fluctuations in 1d systems.
Single mode approximation does not apply.
Need to analyze the full model

$$\mathcal{H} = \int dx \left[g_c \, (n_1 + n_2)^2 \, + \, g_s (n_1 - n_2)^2 \, + \, \frac{|\nabla \Psi_1|^2}{2m} \, + \, \frac{|\nabla \Psi_2|^2}{2m} \right]$$

Interaction induced collapse of Ramsey fringes. Multimode analysis

Low energy effective theory: Luttinger liquid approach

Luttinger model

$$S^+(x,t) \sim e^{i\phi_s(x,t)} \qquad [S^z(x), \phi_s(x')] = -i\delta(x-x')$$

$$\mathcal{H}_s = \int_0^L dx \left[g_s (S^z)^2 + \frac{\rho}{2m} (\nabla \phi_s)^2 \right]$$

Changing the sign of the interaction reverses the interaction part of the Hamiltonian but not the kinetic energy

$$\mathcal{H}_s = \sum_q \left[g_s(t) S_q^z S_q^{z*} + \frac{\rho\, q^2}{m} \phi_{sq} \phi_{sq}^* \right]$$

$$[S_{q'}^z, \phi_{sq}] = -i\delta_{qq'}$$

Time dependent harmonic oscillators can be analyzed exactly

Time-dependent harmonic oscillator

$$\mathcal{H} = \frac{p^2}{2m(t)} + k\,\frac{q^2}{2}$$

See e.g. Lewis, Riesengeld (1969)
Malkin, Man'ko (1970)

Explicit quantum mechanical wavefunction can be found

$$\psi(p,t) = \frac{\Phi\left(\frac{p}{c(t)}\right)}{\sqrt{c(t)}}\, e^{i\alpha(t)p^2 + i\gamma(t)}$$

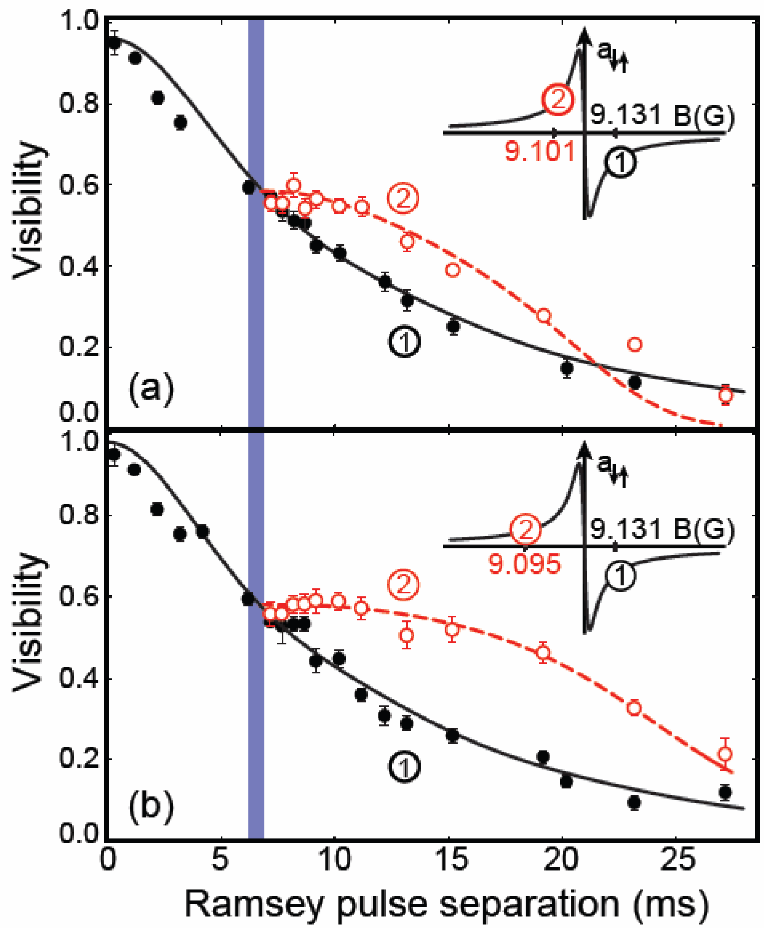
From the solution of classical problem

$$\ddot{c} + \omega^2(t)\, c = \frac{\omega_0^2}{c^3}$$

We solve this problem for each momentum component

$$\mathcal{H}_s = \sum_q \left[g_s(t) S_q^z S_q^{z*} + \frac{\rho\, q^2}{m} \phi_{sq} \phi_{sq}^* \right]$$

Interaction induced collapse of Ramsey fringes in one dimensional systems



Only q=0 mode shows complete spin echo
Finite q modes continue decay

The net visibility is a result of competition between q=0 and other modes

Conceptually similar to experiments with dynamics of split condensates.
T. Schumm's talk

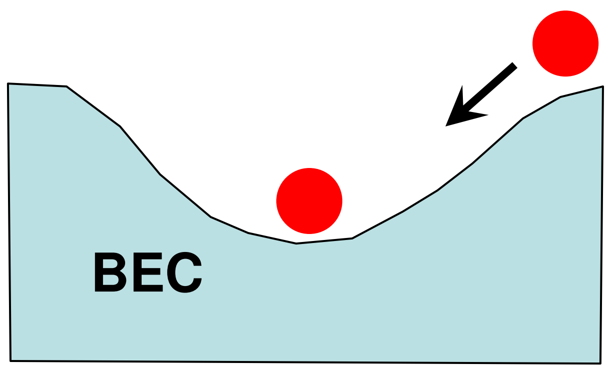
Fundamental limit on Ramsey interferometry

Boson Fermion mixtures

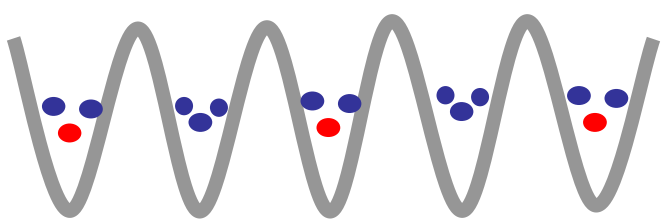
Connections to polaronic effects in electron-phonon systems.
New feature: quantum regime of bosons

Boson Fermion mixtures

Experiments: ENS, Florence, JILA, MIT, ETH, Hamburg, Rice, Duke, Mainz, …

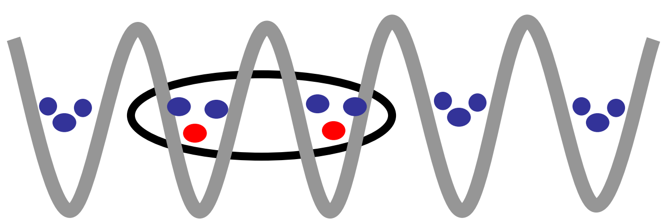


Bosons provide cooling for fermions and mediate interactions. They create non-local attraction between fermions



Charge Density Wave Phase

Periodic arrangement of atoms



Non-local Fermion Pairing

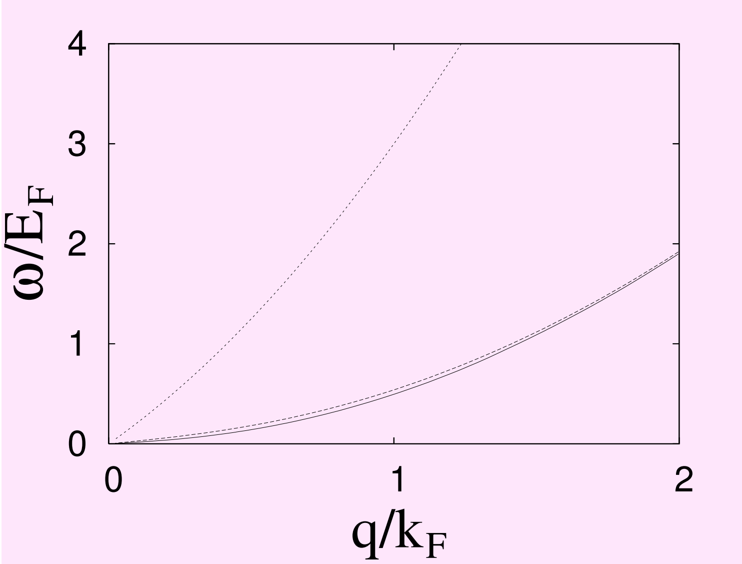
P-wave, D-wave, …

Theory: Pu, Illuminati, Efremov, Das, Wang, Matera, Lewenstein, Buchler, …

Boson Fermion mixtures

$$\begin{aligned}
\mathcal{H} &= \mathcal{H}_{bb} + \mathcal{H}_{\rm ff} + \mathcal{H}_{bf} \\
\mathcal{H}_{bb} &= -t_b \sum_{\langle ij \rangle} b_i^\dagger b_j + U_{bb} \sum_i n_{bi}(n_{bi} - 1) \\
\mathcal{H}_{ff} &= -t_f \sum_{\langle ij \rangle} f_i^\dagger f_j \\
\mathcal{H}_{bf} &= U_{bf} \sum_i n_{bi} n_{fi}
\end{aligned}$$

"Phonons" :
Bogoliubov (phase) mode



Effective fermion-"phonon" interaction

$$\begin{aligned}
\tilde{\mathcal{H}}_{bb} &= \sum_q \omega_q \beta_q^\dagger \beta_q \\
\tilde{\mathcal{H}}_{bf} &= \sum_{kq} g_q \left(\beta_q + \beta_{-q}^\dagger\right) f_{k+q}^\dagger f_k
\end{aligned}$$

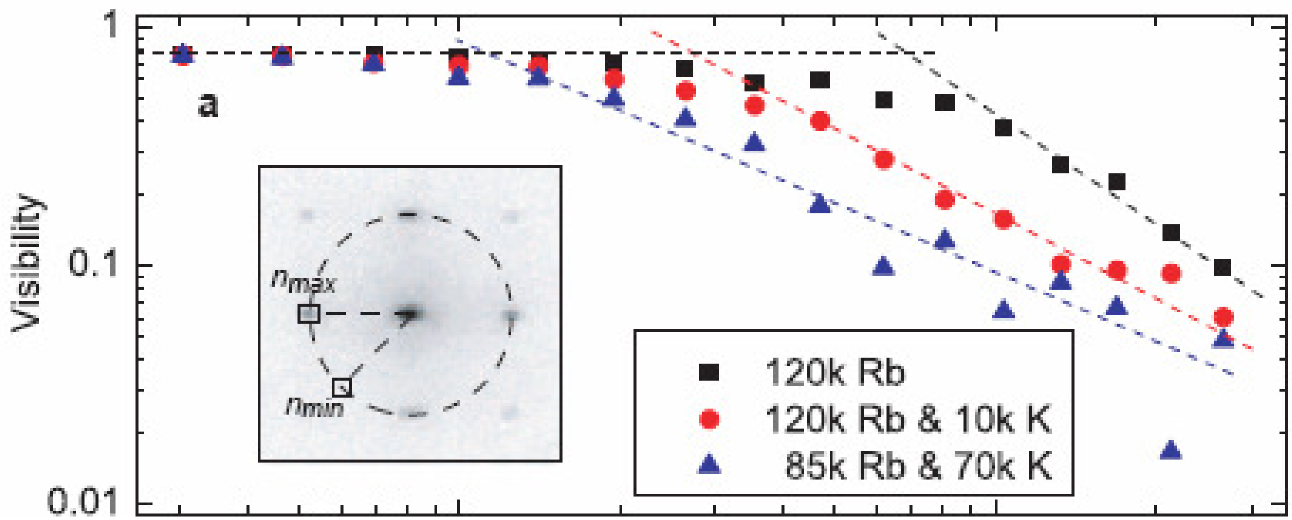
Fermion-"phonon" vertex $g_q \sim |q|$
Similar to electron-phonon systems

Bose-Fermi mixture in a three dimensional optical lattice

Gunter et al, PRL 96:180402 (2006)

See also Ospelkaus et al, PRL 96:180403 (2006)

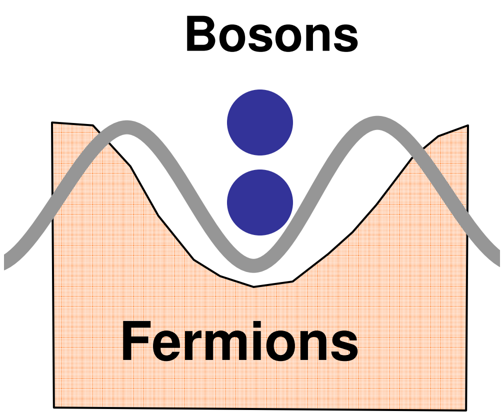
Suppression of superfluidity of bosons by fermions



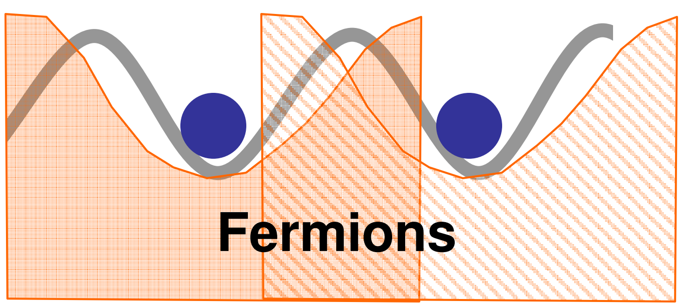
Similar observation for Bose-Bose mixtures,
see Catani et al., arXiv:0706.278

Issue of heating needs to be sorted out, see e.g. Pollet et al., cond-mat/0609604

Competing effects of fermions on bosons



Fermions provide screening.
Favors superfluid state of bosons

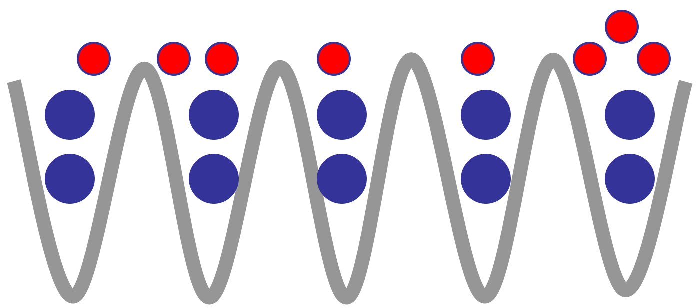


Orthogonality catastrophe due to fermions.
Polaronic dressing of bosons.
Favors Mott insulating state of bosons

Quantum regime of bosons

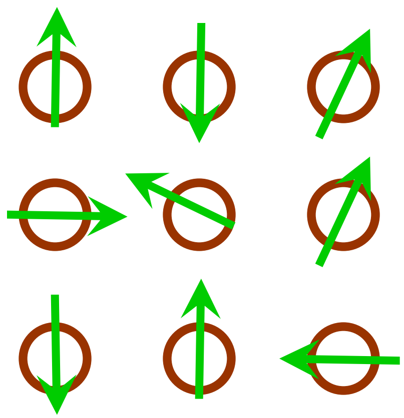
A better starting point:

Mott insulating state of bosons
Free Fermi sea



Theoretical approach: generalized Weiss theory

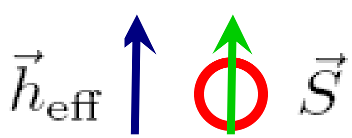
Weiss theory of magnetism



Heisenberg model

$$\mathcal{H} = \sum_{ij} J_{ij}\, \vec{S}_i\, \vec{S}_j$$

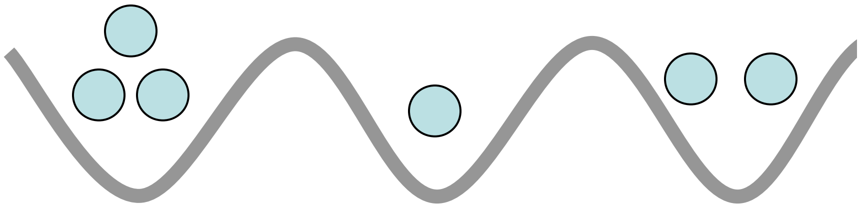
Mean field: a single spin in a self-consistent field



$$\mathcal{H} = \vec{h}_{\rm eff}\, \vec{S}$$

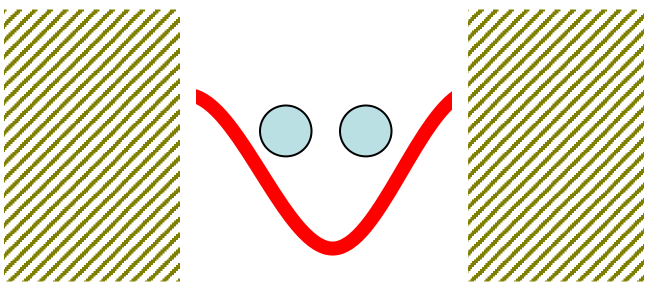
$$\vec{h}_{\rm eff} = \sum_{j} J_{ij}\, \langle\, \vec{S}_j\, \rangle$$

Weiss theory of the superfluid to Mott transition of bosons in an optical lattice



$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1) - \mu \sum_i n_i$$

Mean-field: a single site in a self-consistent field



$$\mathcal{H}_{\rm eff} = -zt(r^* b^\dagger + rb) + U\,n(n-1) - \mu n$$

$$r = \langle b \rangle$$

Weiss theory: quantum action

O(2) rotor: $b = \sqrt{\bar{n}}\, e^{i\phi}$

Conjugate variables $[n\,,\,\phi] = -i$

$$\mathcal{H}_{\rm eff} = -\,z\,t\,r\,\cos\phi + U\,n\,(\,n - 1\,) - \mu\,n$$

$$S_{\rm eff} = \int_0^\beta d\tau\,\left[i\dot{\phi}n - z\,t\,r\,\cos\phi + U\,n\,(\,n - 1\,) - \mu\,n\right]$$

Self-consistency condition $r = \bar{n}\,\langle\,\cos\phi\,\rangle$

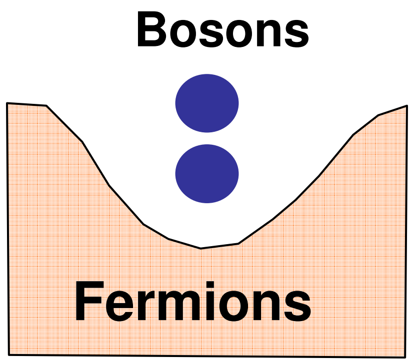
SF-Mott transition: expansion for small *r*

$$r = z\,t\,\bar{n}\,r \int \mathcal{D}\phi \sum_{n(\tau)} \int_0^\beta d\tau_1\,\cos\phi(0)\,\cos\phi(\tau_1)\,e^{-S_0}$$

$$S_0 = \int_0^\beta d\tau\,\left[i\,\dot{\phi}\,n + U\,n\,(\,n - 1\,) - \mu\,n\right]$$

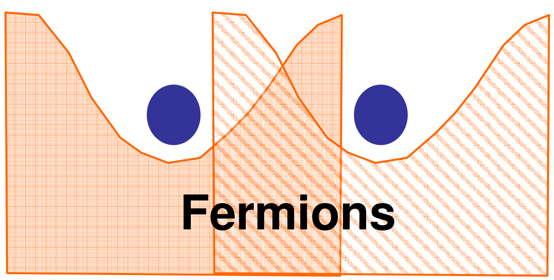
Adding fermions

Screening



$$\Delta S^{(1)}_{\rm ferm} = -\frac{1}{2} U^2_{BF}\, \rho_F(0) \int_0^\beta d\tau\, n^2$$

Orthogonality catastrophy



$$\Delta S^{(1)}_{\rm ferm} = \int_0^\beta d\tau_1 \int_0^\beta d\tau_2\, \alpha(\tau_1 - \tau_2)\, (\, n(\tau_1) - n(\tau_2)\,)^2$$

$$\alpha(\omega_\nu) = 2\,\pi\, \rho_F^2(0)\, U^2_{BF}\, |\omega_\nu|$$

X-Ray edge singularity: Roule, Gavoret, Nozieres (1969)
Ohmic dissipation: Caldeira, Leggett (1983)

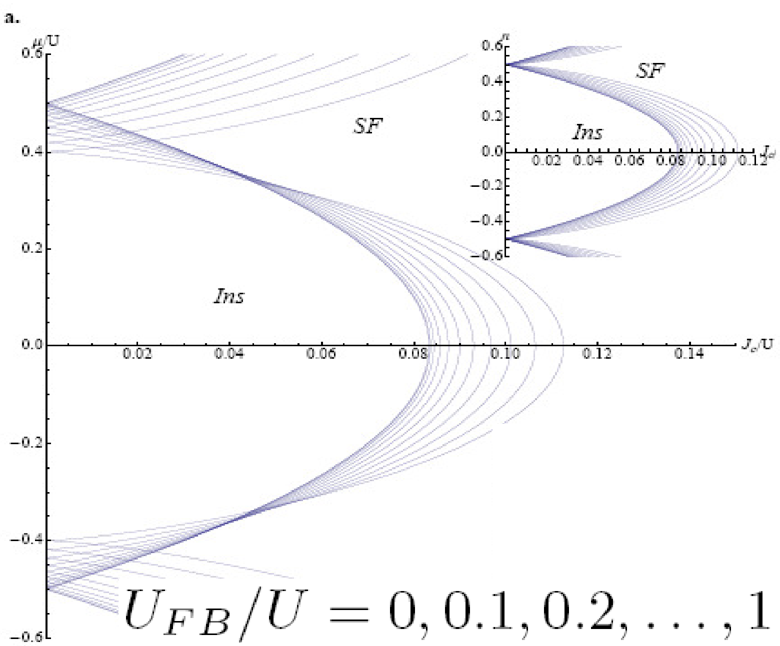
SF-Mott transition in the presence of fermions

$$r = z\, t\, \bar{n}\, r \int \mathcal{D}\phi \sum_{n(\tau)} \int_0^{\beta} d\tau_1 \,\cos\phi(0)\, \cos\phi(\tau_1)\, e^{-S_0}$$

Competition of screening and orthogonality catastrophy

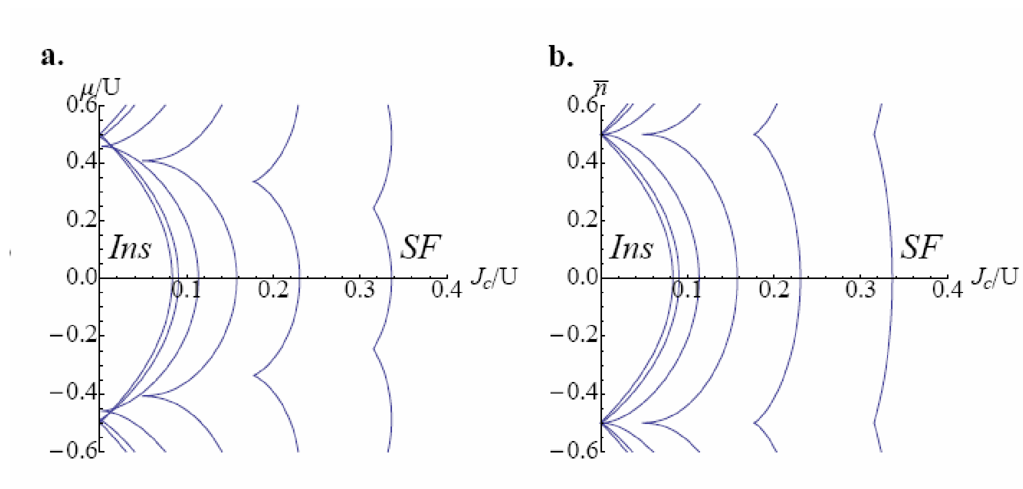
$$S_0 = \int_0^{\beta} d\tau \left[i\,\dot{\phi}\, n + \tilde{U}\, n\,(n-1) - \mu\, n \right] + \int_0^{\beta} d\tau_2\, \alpha(\tau_1 - \tau_2)\,(\,n(\tau_1) - n(\tau_2)\,)^2$$

Effect of fast fermions $t_F/U=5$



$U_{FB}/U = 0, 0.1, 0.2, \ldots, 1$

Effect of slow fermions $t_F/U=0.7$



$U_{FB}/U = 0, 0.1, 0.2, \ldots, 0.5$

Summary

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Thanks to Dan Stamper-Kurn

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