Non-equilibrium quantum dynamics of many-body systems of ultracold atoms

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Collaboration with E. Altman, A. Burkov, V. Gritsev, B. Halperin, M. Lukin, A. Polkovnikov, experimental groups of I. Bloch (Mainz) and J. Schmiedmayer (Heidelberg/Vienna)

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Landau-Zener tunneling

Landau, Physics of the Soviet Union 3:46 (1932)

Probability of nonadiabatic transition

\[ P_{12} \propto e^{-2\pi \omega_{12} \tau_d} \]

\( \omega_{12} \) – Rabi frequency at crossing point
\( \tau_d \) – crossing time

Hysteresis loops of Fe8 molecular clusters

Wernsdorfer et al., cond-mat/9912123
Single two-level atom and a single mode field

\[ \mathcal{H} = \hbar \omega_1 a^{\dagger} a + \hbar \omega_2 \sigma_z + g \left( a \sigma_+ + a^{\dagger} \sigma_- \right) \]

Observation of collapse and revival in a one atom maser

Rempe, Walther, Klein,
PRL 58:353 (87)

See also solid state realizations
by R. Shoelkopf, S. Girvin

Jaynes and Cummings,
Proc. IEEE 51:89 (1963)
Superconductor to Insulator transition in thin films

Marcovic et al., PRL 81:5217 (1998)

Superconducting films of different thickness. Transition can also be tuned with a magnetic field.

Marcovic et al., PRL 81:5217 (1998)
Scaling near the superconductor to insulator transition

Yes at “higher” temperatures

No at lower” temperatures

Yazdani and Kapitulnik

Mason and Kapitulnik
Mechanism of scaling breakdown

New many-body state


Extended crossover

Dynamics of many-body quantum systems

Heavy Ion collisions at RHIC
Signatures of quark-gluon plasma?
Dynamics of many-body quantum systems

Big Bang and Inflation. Origin and manifestations of cosmological constant

Cosmic microwave background radiation. Manifestation of quantum fluctuations during inflation
Goal:
Create many-body systems with interesting collective properties
Keep them simple enough to be able to control and understand them
Non-equilibrium dynamics of many-body systems of ultracold atoms

Emphasis on enhanced role of fluctuations in low dimensional systems

Dynamical instability of strongly interacting bosons in optical lattices

Dynamics of coherently split condensates

Many-body decoherence and Ramsey interferometry
Dynamical Instability of strongly interacting bosons in optical lattices

Collaboration with E. Altman, B. Halperin, M. Lukin, A. Polkovnikov

References:

Atoms in optical lattices

Theory: Zoller et al. PRL (1998)

Experiment: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Phillips et al., J. Physics B (2002);
Esslinger et al., PRL (2004);
Ketterle et al., PRL (2006)
Equilibrium superfluid to insulator transition

Theory: Fisher et al. PRB (89), Jaksch et al. PRL (98)
Experiment: Greiner et al. Nature (01)
Moving condensate in an optical lattice. Dynamical instability

Theory: Niu et al. PRA (01), Smerzi et al. PRL (02)
Experiment: Fallani et al. PRL (04)

Related experiments by Eiermann et al, PRL (03)
Question: How to connect the dynamical instability (irreversible, classical) to the superfluid to Mott transition (equilibrium, quantum)
Dynamical instability

Classical limit of the Hubbard model. \( N t >> U \)

Discrete Gross-Pitaevskii equation

\[ i \frac{d \Psi_j}{dt} = -t \sum_{\langle k \rangle} \Psi_k + U |\Psi_j|^2 \Psi_j \]

Current carrying states \( \Psi_j \sim e^{ipx_j} \)

Linear stability analysis: States with \( p > \pi/2 \) are unstable

Amplification of density fluctuations
Dynamical instability for integer filling

Order parameter for a current carrying state  \( \Psi_j(p) = A(p) \ e^{ipx_j} \)

Current  \( J(p) = |A(p)|^2 \sin(p) \)

GP regime  \( A(p) = \sqrt{N} \). Maximum of the current for  \( p = \pi/2 \)

When we include quantum fluctuations, the amplitude of the order parameter is suppressed

\[
\frac{A(p = 0)}{\sqrt{N}} \approx 1 - \left( \frac{U}{Nt} \right)^{1/2}
\]

\( A(p) \) decreases with increasing phase gradient  \( p \)
Dynamical instability for integer filling

Vicinity of the SF-I quantum phase transition. Classical description applies for \( L > \xi \sim (U_c - U)^{-1/2} \)

Dynamical instability occurs for \( p \xi \approx \pi / 2 \)
Dynamical instability. Gutzwiller approximation

Wavefunction

\[ |\Psi(t)\rangle = \prod_j \left[ \sum_{n=0}^{\infty} f_{jn}(t) |n\rangle_j \right] \]

Time evolution

\[ -i \frac{df_{jn}}{dt} = -t \left( f_{jn-1} \phi_j + f_{jn+1} \phi_j^* \right) + \frac{U}{2} n (n-1) f_{jn} \]

\[ \phi_j(t) = \sum_{\langle i \rangle} \langle \Psi(t) | a_i | \Psi(t) \rangle \]

We look for stability against small fluctuations

Phase diagram. Integer filling

Altman et al., PRL 95:20402 (2005)
The first instability develops near the edges, where $N=1$. Optical lattice and parabolic trap. Gutzwiller approximation. Gutzwiller ansatz simulations (2D).
Phase diagram for a Bose-Einstein condensate moving in an optical lattice

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MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, and Department of Physics, MIT, Cambridge, Massachusetts 02139, USA.
Beyond semiclassical equations. Current decay by tunneling

Current carrying states are metastable. They can decay by thermal or quantum tunneling

Thermal activation  Quantum tunneling
Decay rate from a metastable state. Example

\[
S \equiv \int_0^{\tau_0} d\tau \left( \frac{1}{2m} \left( \frac{dx}{d\tau} \right)^2 + \varepsilon x^2 - bx^3 \right)
\]

Expansion in small \( \varepsilon \)

\[
\Gamma \sim e^{-S}
\]

\[
S \sim (p_c - p)^{5/2}
\]

Our small parameter of expansion: proximity to the classical dynamical instability

\[
\varepsilon \propto (p_c - p) \to 0
\]
Need to consider dynamics of many degrees of freedom to describe a phase slip

Strongly interacting regime. Vicinity of the SF-Mott transition

Decay of current by quantum tunneling

Action of a quantum phase slip in d=1,2,3

\[ S_d \sim \frac{1}{\xi^2} \left( 1 - \sqrt{3} p \xi \right)^{5/2-d} \]

\[ \xi \sim (U_c - U)^{-1/2} \]

Strongly interacting regime. Vicinity of the SF-Mott transition in d=1 and d=2

\[ S_{3d} \] is discontinuous at the transition. Phase slips are not important.

Sharp phase transition
Phase diagram for a Bose-Einstein condensate moving in an optical lattice

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(a)

![Graph showing phase diagram](image)

Similar results in NIST experiments, Fertig et al., PRL (2005)
Decay of current by thermal activation

Escape from metastable state by thermal activation

\[ \Gamma \sim e^{-\Delta E/T} \]
Thermally activated current decay. Weakly interacting regime

Activation energy in d=1,2,3

\[ \Delta E_1 = 1.3 \, J \, N \left( \frac{\pi}{2} - p \right)^3 \]
\[ \Delta E_2 = 10 \, J \, N \left( \frac{\pi}{2} - p \right)^{5/2} \]
\[ \Delta E_3 = 35 \, J \, N \left( \frac{\pi}{2} - p \right)^2 \]

Thermal fluctuations lead to rapid decay of currents

Crossover from thermal to quantum tunneling

\[ T_Q \sim \sqrt{N \, J \, U \, \times \left( \frac{\pi}{2} - p \right)} \]
Decay of current by thermal fluctuations

Unstable regimes for a Bose-Einstein condensate in an optical lattice

L. De Sarlo*, L. Fallani, J. E. Lye, M. Modugno†, R. Saers‡, C. Fort and M. Inguscio


FIG. 2: Absorption images of the condensate interacting for \( t = 15 \text{s} \) with a lattice with \( s = 0.2 \) for different values of quasimomentum ranging from 0 to \( 0.20 q_B \) and for respectively a condensed fraction of about 65% (top) and no detectable thermal component (bottom).

Also experiments by Brian DeMarco et al., arXiv 0708:3074
Dynamics of coherently split condensates. Interference experiments

Collaboration with A. Burkov and M. Lukin

Interference of independent condensates


Theory: Javanainen, Yoo, PRL 76:161 (1996)
and many more
Experiments with 2D Bose gas

Experiments with 1D Bose gas
Hofferberth et al. arXiv0710.1575
Interference of independent 1d condensates


\[ T_{\text{fit}} = 84 \pm 22 \text{ nK} \]

\[ n_{1d} = 60 \mu\text{m}^{-1} \]

\[ K = 47 \]
Studying dynamics using interference experiments

Prepare a system by splitting one condensate

Take to the regime of zero tunneling

Measure time evolution of fringe amplitudes
Finite temperature phase dynamics

\[ \mathcal{H}_0 = \int dx \left[ g n_1^2(x) + \rho (\partial_x \phi_1)^2 \right] + \int dx \left[ g n_2^2(x) + \rho (\partial_x \phi_2)^2 \right] \]

Temperature leads to phase fluctuations within individual condensates.

Interference experiments measure only the relative phase

\[ \phi_{\text{av}} = \frac{\phi_1 + \phi_2}{2} \]

\[ \phi = \phi_1 - \phi_2 \]
Relative phase dynamics

\[ \mathcal{H} = \int d^d r \left[ \frac{g}{2} (\Delta n)^2 + \frac{\rho}{2} (\nabla \phi)^2 \right] \]

Hamiltonian can be diagonalized in momentum space

A collection of harmonic oscillators with \( \omega_q = \sqrt{g \rho} \mid q \mid \)

Initial state \( \phi_q = 0 \)

Need to solve dynamics of harmonic oscillators at finite \( T \)

Conjugate variables

\[ \phi = \phi_1 - \phi_2 \]

\[ \Delta n = (n_1 - n_2) / 2 \]

Coherence

\[ \langle \Psi(t) \mid e^{i\phi} \mid \Psi(t) \rangle = e^{-\frac{1}{2} \sum_q \langle \phi_q^2(t) \rangle} \]
Relative phase dynamics

High energy modes, $\hbar \omega_{osc} > k_B T$, quantum dynamics
Low energy modes, $\hbar \omega_{osc} < k_B T$, classical dynamics

Combining all modes

\[ t < \frac{\hbar}{k_B T} \quad \text{Quantum dynamics} \]
\[ t > \frac{\hbar}{k_B T} \quad \text{Classical dynamics} \]

For studying dynamics it is important to know the initial width of the phase
Relative phase dynamics

\[ \frac{N}{2} \pm \delta N \quad \frac{N}{2} \mp \delta N \]

Naive estimate

\[ \delta N \sim \sqrt{N} \]

Long Phase Coherence Time and Number Squeezing of two Bose-Einstein Condensates on an Atom Chip

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Physics Department, Harvard University, Cambridge, MA 02138, USA
(Dated: August 27, 2006)
Relative phase dynamics

Separating condensates at finite rate

Instantaneous Josephson frequency

$$\omega_J = \sqrt{UJ}$$

Adiabatic regime

$$\dot{\omega}_J < \omega_J^2$$

Instantaneous separation regime

$$\dot{\omega}_J > \omega_J^2$$

Adiabaticity breaks down when

$$\omega_J \sim 1/\tau_s$$

Charge uncertainty at this moment

$$U \delta N^2 \sim \omega_J \sim 1/\tau_s$$

Squeezing factor

$$\frac{\delta N}{\sqrt{N}} \sim \sqrt{\frac{1}{NU\tau_s}} \sim \sqrt{\frac{1}{\mu \tau_s}}$$
Relative phase dynamics

Quantum regime

\[ \frac{h}{\mu} < t < \frac{h}{k_B T} \]

1D systems

\[ \langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} e^{-t/2\pi K\tau_s} \]

2D systems

\[ \langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} \left( \frac{t_0}{t} \right)^{1/16T_K T\tau_s} \]

Different from the earlier theoretical work based on a single mode approximation, e.g. Gardiner and Zoller, Leggett

Classical regime

\[ t > \frac{h}{k_B T} \]

1D systems

\[ \langle e^{i\phi(t)} \rangle \sim e^{-\left(\frac{t}{t_T}\right)^2/3} \]

\[ t_T \sim \frac{\mu K}{T^2} \]

2D systems

\[ \langle e^{i\phi(t)} \rangle \sim \left( \frac{t_0}{t} \right)^{T\over sT_K T} \]
1d BEC: Decay of coherence


\[ \Psi(t) \propto \exp\left[-\left(\frac{t}{t_0}\right)^{2/3}\right] \]

double logarithmic plot of the coherence factor

slopes: \(0.64 \pm 0.08\), \(0.67 \pm 0.1\), \(0.64 \pm 0.06\)

get \(t_0\) from fit with fixed slope \(2/3\) and calculate \(T\) from

\[ t_0 = \frac{2.61 \pi K}{T^2} \]

\(T_5 = 110 \pm 21\) nK
\(T_{10} = 130 \pm 25\) nK
\(T_{15} = 170 \pm 22\) nK
Many-body decoherence and Ramsey interferometry

Collaboration with A. Widera, S. Trotzky, P. Cheinet, S. Fölling, F. Gerbier, I. Bloch, V. Gritsev, M. Lukin

Preprint arXiv:0709.2094
Working with $N$ atoms improves the precision by $\sqrt{N}$. Need spin squeezed states to improve frequency spectroscopy.
Squeezed spin states for spectroscopy
Motivation: improved spectroscopy, e.g. Wineland et. al. PRA 50:67 (1994)

Generation of spin squeezing using interactions. Two component BEC. Single mode approximation

\[ H_s = \chi_s (S_{\text{tot}}^z)^2 \]

Kitagawa, Ueda, PRA 47:5138 (1993)

In the single mode approximation we can neglect kinetic energy terms

\[ H = \int dx \left[ g_c (n_1 + n_2)^2 + g_s (n_1 - n_2)^2 + \frac{|\nabla \Psi_1|^2}{2m} + \frac{|\nabla \Psi_2|^2}{2m} \right] \]

\[ H_{\text{SMA}} = \frac{g_s}{V} (N_1 - N_2)^2 \]
Interaction induced collapse of Ramsey fringes

Ramsey fringe visibility

\[ t_{\text{collapse}} \sim \frac{1}{\chi \sqrt{N}} \]

\[ t_{\text{revival}} \sim \frac{1}{\chi} \]

Experiments in 1d tubes:
A. Widera, I. Bloch et al.
Spin echo. Time reversal experiments

Single mode approximation

\[ \mathcal{H}_{SMA} = \frac{g_s}{V} (N_1 - N_2)^2 \]

\[ g_s = \frac{g_{11} - g_{12}}{2} \]

The Hamiltonian can be reversed by changing \( a_{12} \)

\[ a_s \rightarrow -a_s \]

\[ \mathcal{H}_{SMA} \rightarrow -\mathcal{H}_{SMA} \]

Predicts perfect spin echo
Spin echo. Time reversal experiments

Expts: A. Widera, I. Bloch et al.

No revival?

Experiments done in array of tubes. Strong fluctuations in 1d systems. Single mode approximation does not apply. Need to analyze the full model.

\[
\mathcal{H} = \int dx \left[ g_c (n_1 + n_2)^2 + g_s (n_1 - n_2)^2 + \frac{|\nabla \Psi_1|^2}{2m} + \frac{|\nabla \Psi_2|^2}{2m} \right]
\]
Interaction induced collapse of Ramsey fringes. Multimode analysis

Low energy effective theory: Luttinger liquid approach

Luttinger model

\[ S^+(x, t) \sim e^{i\phi_s(x, t)} \quad \quad [S^z(x), \phi_s(x') ] = -i\delta(x - x') \]

\[ \mathcal{H}_s = \int_0^L dx \left[ g_s(S^z)^2 + \frac{\rho}{2m} (\nabla \phi_s)^2 \right] \]

Changing the sign of the interaction reverses the interaction part of the Hamiltonian but not the kinetic energy

\[ \mathcal{H}_s = \sum_q \left[ g_s(t) S^z_q S_q^{z*} + \frac{\rho q^2}{m} \phi_{sq} \phi_{sq}^{*} \right] \]

\[ [S^z_{q'}, \phi_{sq}] = -i\delta_{qq'} \]

Time dependent harmonic oscillators can be analyzed exactly
Time-dependent harmonic oscillator

\[ \mathcal{H} = \frac{p^2}{2m(t)} + k \frac{q^2}{2} \]

Explicit quantum mechanical wavefunction can be found

\[ \psi(p, t) = \frac{\Phi\left(\frac{p}{c(t)}\right)}{\sqrt{c(t)}} e^{i\alpha(t)p^2 + i\gamma(t)} \]

From the solution of classical problem

\[ \ddot{c} + \omega^2(t) c = \frac{\omega_0^2}{c^3} \]

We solve this problem for each momentum component

\[ \mathcal{H}_s = \sum_q \left[ g_s(t) S_q^z S_q^{z*} + \frac{\rho q^2}{m} \phi_{sq} \phi_{sq}^* \right] \]
Interaction induced collapse of Ramsey fringes in one dimensional systems

Only q=0 mode shows complete spin echo
Finite q modes continue decay

The net visibility is a result of competition between q=0 and other modes

Conceptually similar to experiments with dynamics of split condensates.
T. Schumm’s talk

Fundamental limit on Ramsey interferometry
Summary

Experiments with ultracold atoms can be used to study new phenomena in nonequilibrium dynamics of quantum many-body systems.

Today’s talk: strong manifestations of enhanced fluctuations in dynamics of low dimensional condensates

Thanks to: