Non-equilibrium quantum dynamics of many-body systems of ultracold atoms

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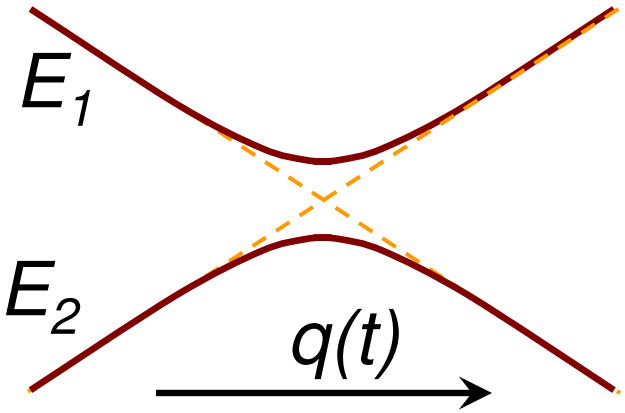
Collaboration with E. Altman, A. Burkov, V. Gritsev,
B. Halperin, M. Lukin, A. Polkovnikov,
experimental groups of I. Bloch (Mainz) and
J. Schmiedmayer (Heidelberg/Vienna)

Funded by NSF, Harvard-MIT CUA, AFOSR, DARPA, MURI

Landau-Zener tunneling



Landau, Physics of the Soviet Union 3:46 (1932)
Zener, Poc. Royal Soc. A 137:692 (1932)

Probability of nonadiabatic transition

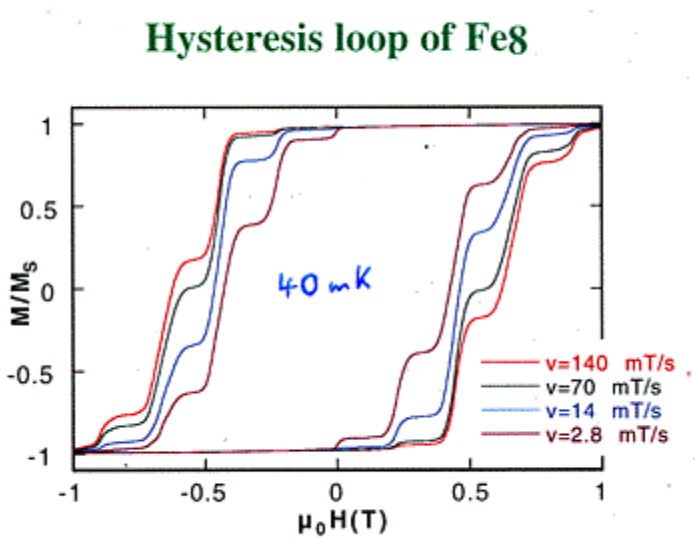
$$P_{12} \propto e^{-2\pi\omega_{12}\tau_d}$$

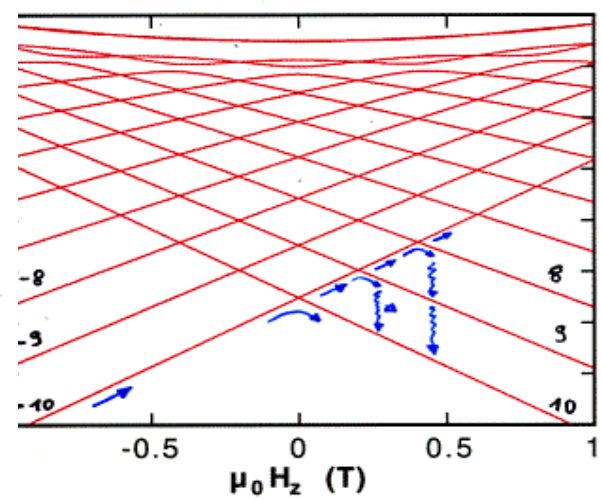
ω_{12} – Rabi frequency at crossing point

τ_d – crossing time

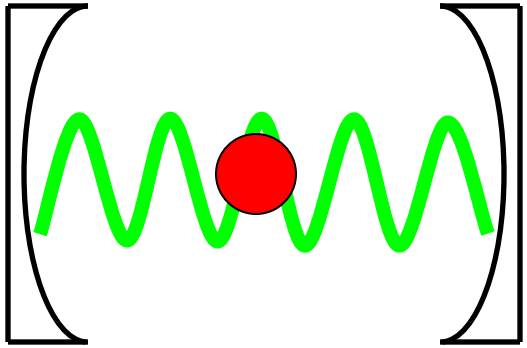
Hysteresis loops of Fe8 molecular clusters

Wernsdorfer et al., cond-mat/9912123





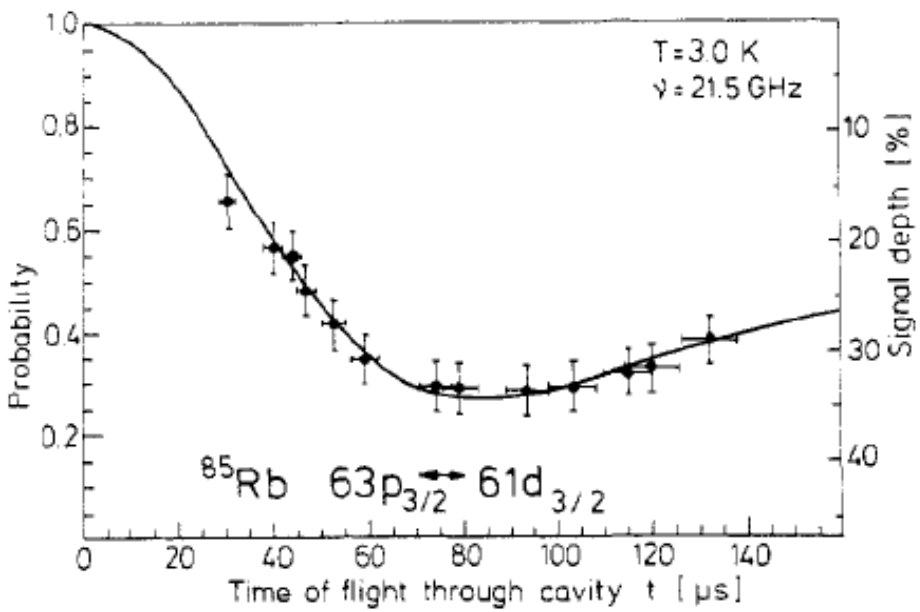
Single two-level atom and a single mode field



Jaynes and Cummings,
Proc. IEEE 51:89 (1963)

$$\mathcal{H} = \hbar\omega_1 a^\dagger a + \hbar\omega_2 \sigma_z + g\,(a\,\sigma_+ + a^\dagger\,\sigma_-)$$

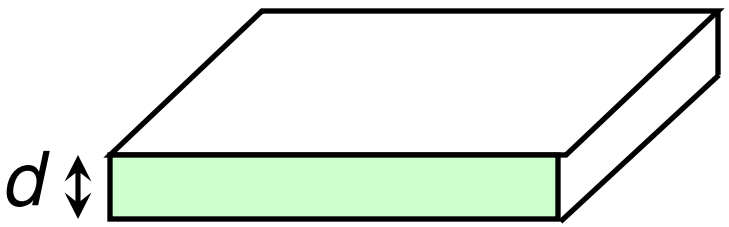
Observation of collapse and revival in a one atom maser



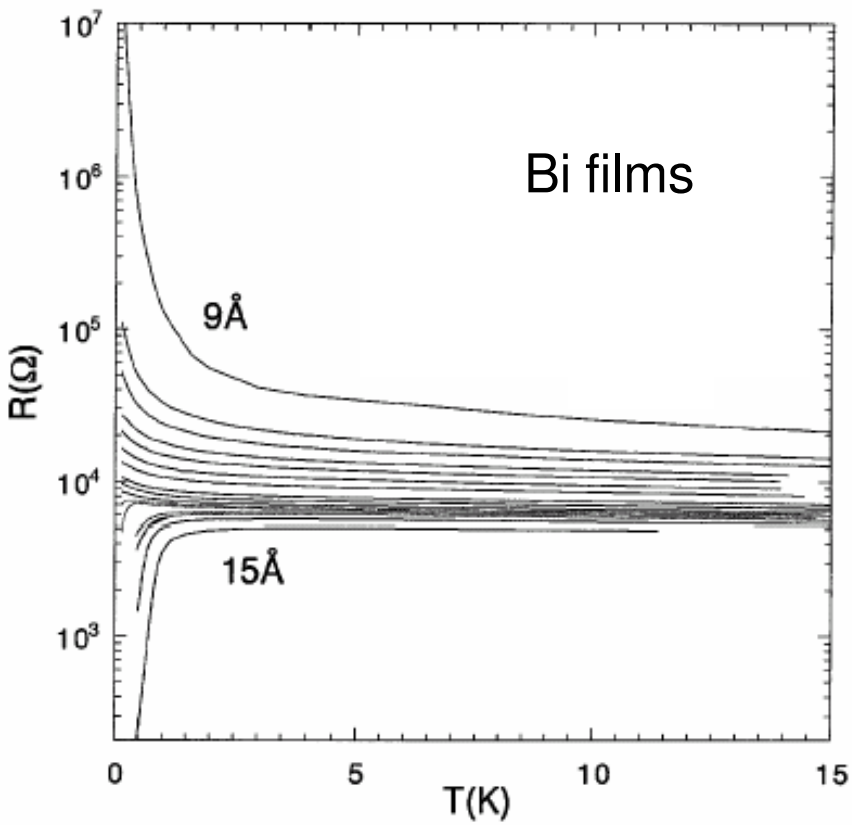
Rempe, Walther, Klein,
PRL 58:353 (87)

See also solid state realizations
by R. Shoelkopf, S. Girvin

Superconductor to Insulator transition in thin films



Superconducting films of different thickness. Transition can also be tuned with a magnetic field

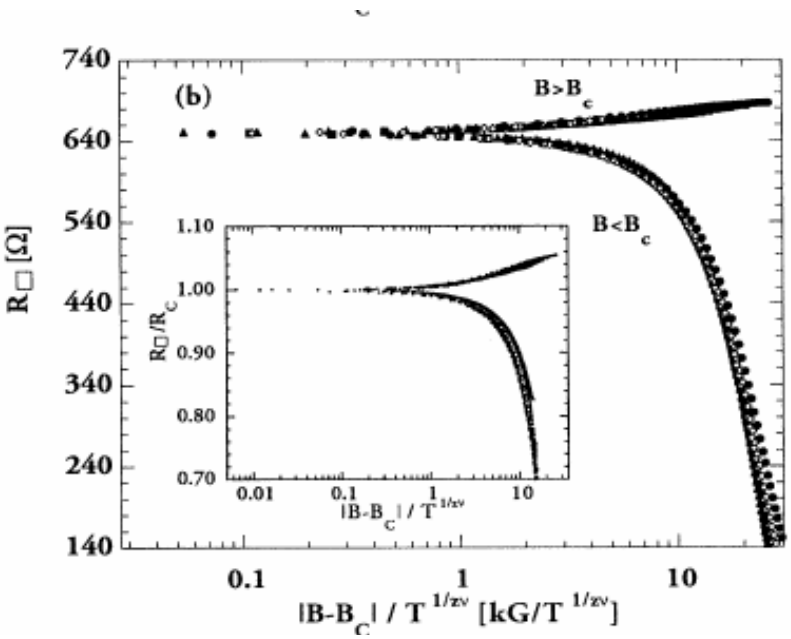


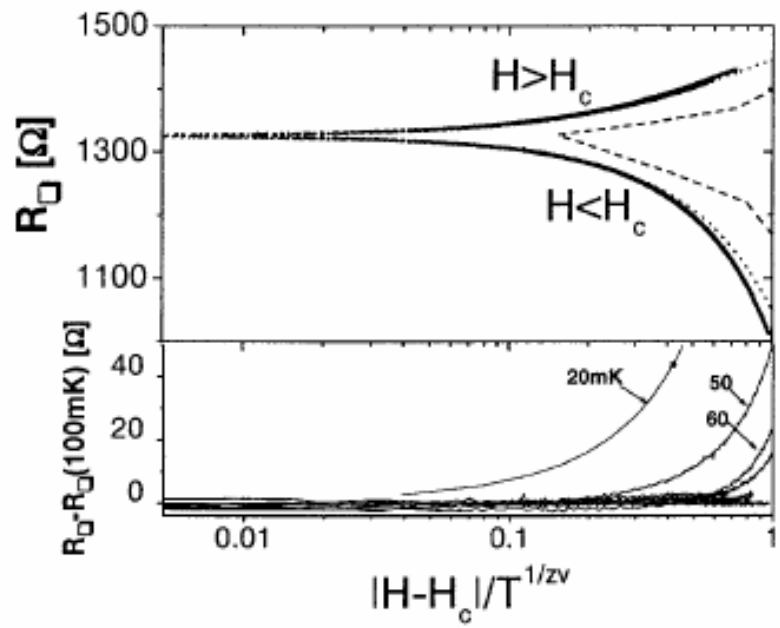
Marcovic et al., PRL 81:5217 (1998)

Scaling near the superconductor to insulator transition

Yes at “higher” temperatures

No at lower” temperatures





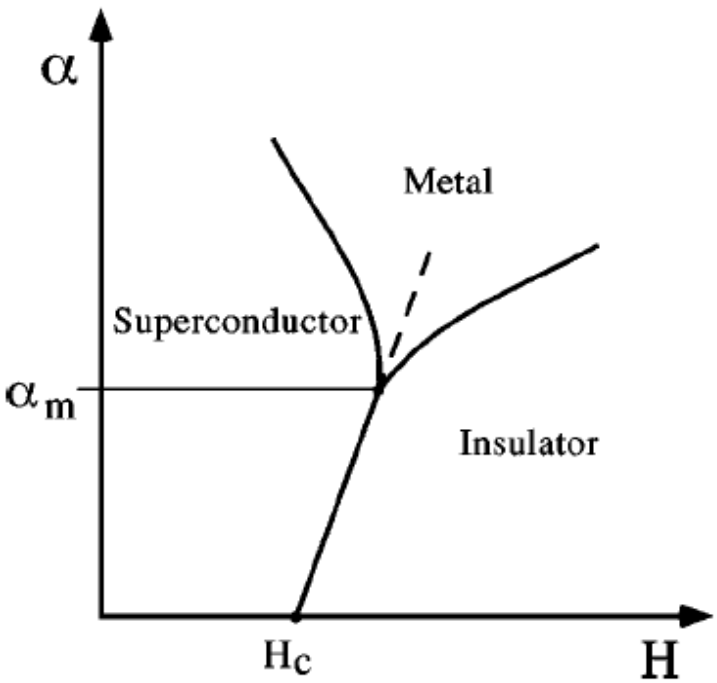
Yazdani and Kapitulnik
Phys.Rev.Lett. 74:3037 (1995)

Mason and Kapitulnik
Phys. Rev. Lett. 82:5341 (1999)

Mechanism of scaling breakdown

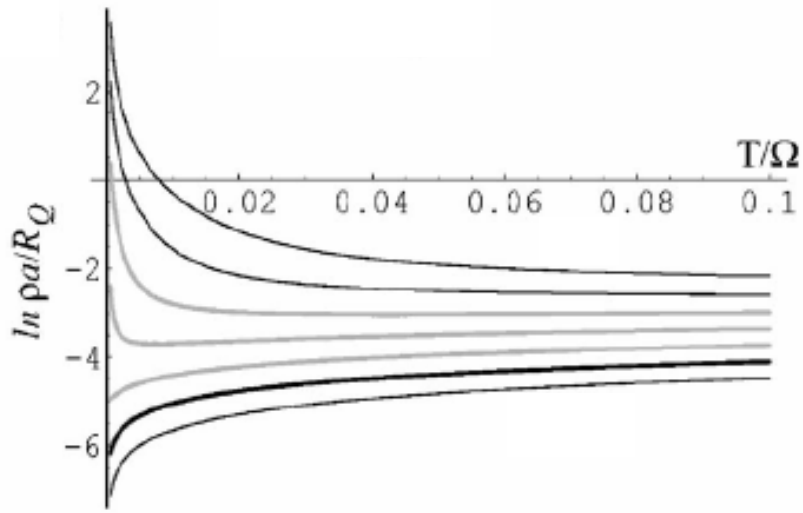
New many-body state

Kapitulnik, Mason, Kivelson, Chakravarty,
PRB 63:125322 (2001)



Extended crossover

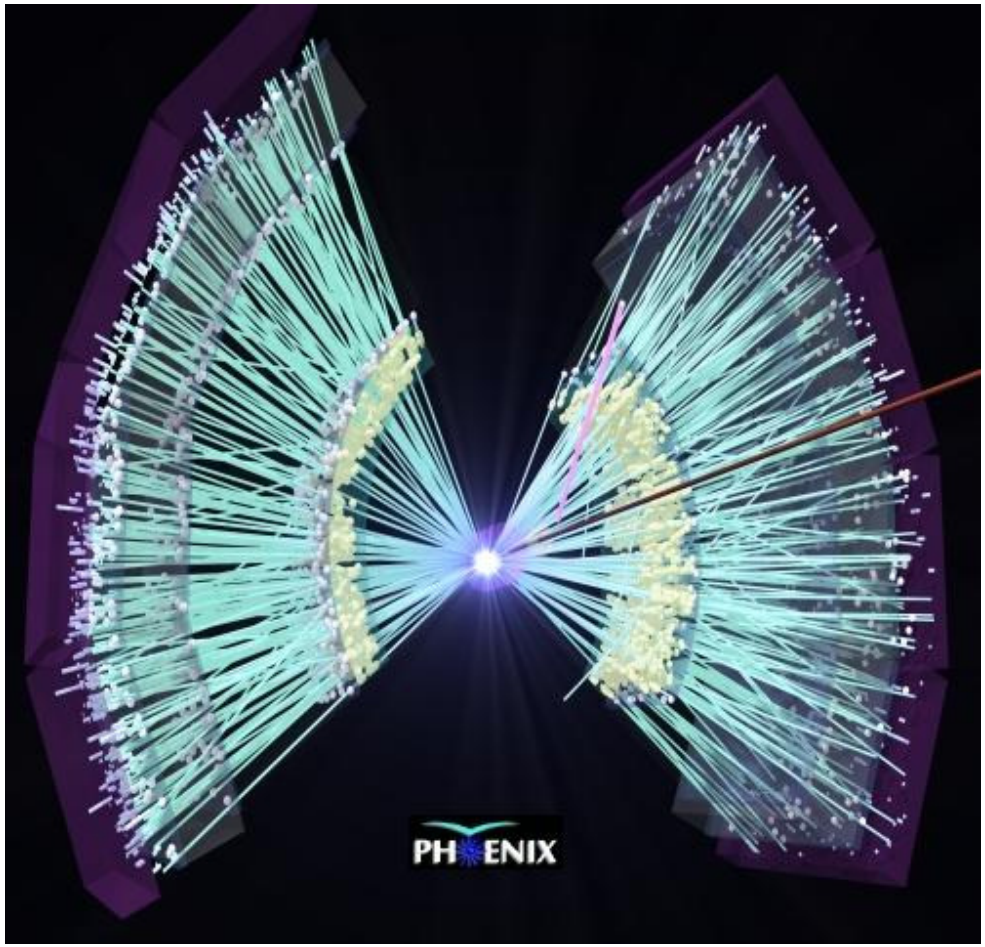
Refael, Demler, Oreg, Fisher
PRB 75:14522 (2007)



Dynamics of many-body quantum systems

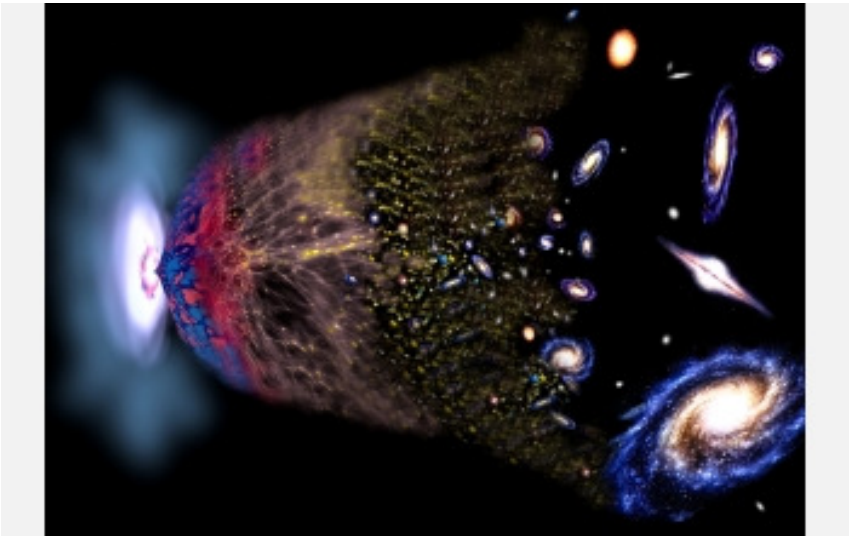
Heavy Ion collisions at RHIC

Signatures of quark-gluon plasma?

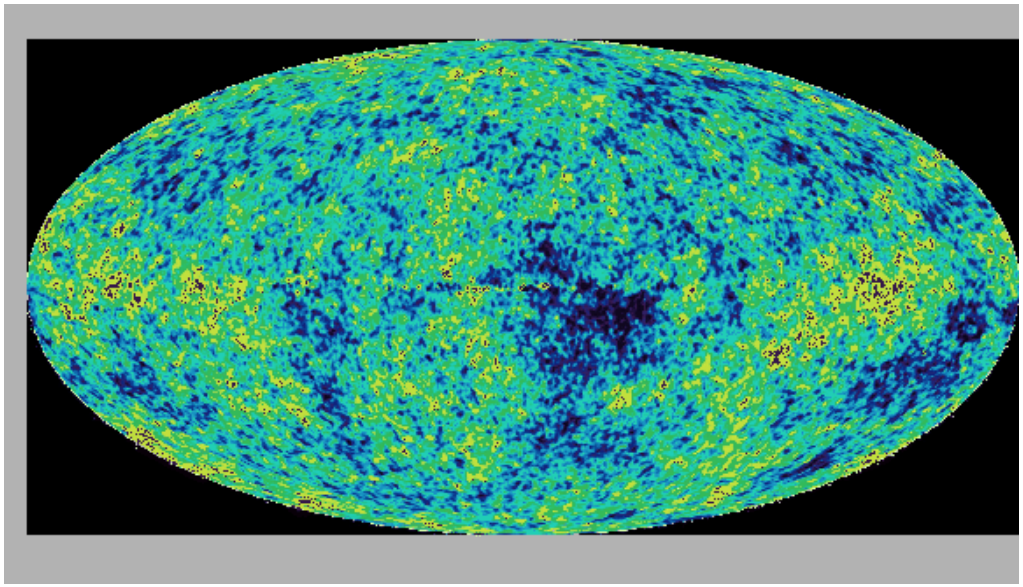


Dynamics of many-body quantum systems

Big Bang and Inflation.
Origin and manifestations
of cosmological constant



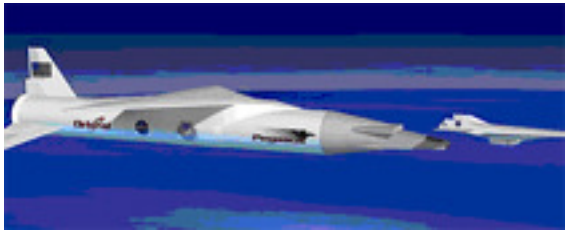
Cosmic microwave
background radiation.
Manifestation of
quantum fluctuations
during inflation

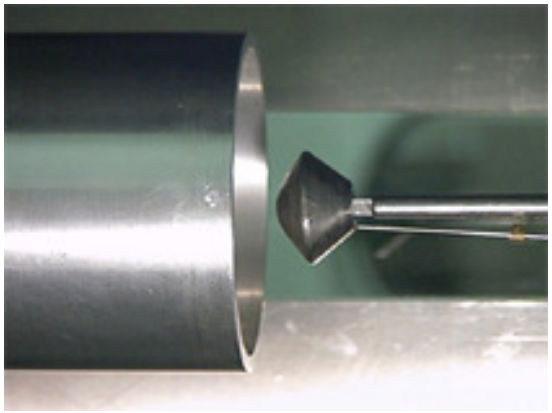


Goal:

Create many-body systems with interesting collective properties

Keep them simple enough to be able to control and understand them





Non-equilibrium dynamics of many-body systems of ultracold atoms

Emphasis on enhanced role of fluctuations in low dimensional systems

Dynamical instability of strongly interacting bosons in optical lattices

Dynamics of coherently split condensates

Many-body decoherence and Ramsey interferometry

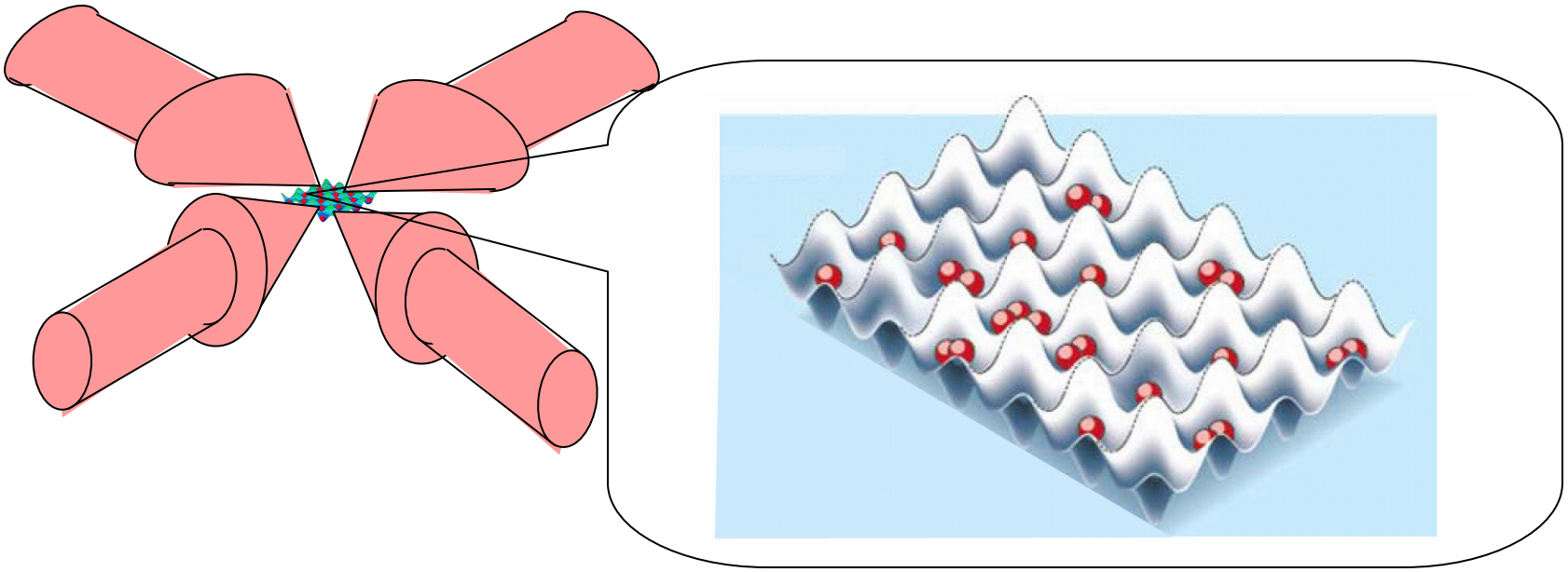
Dynamical Instability of strongly interacting bosons in optical lattices

Collaboration with E. Altman, B. Halperin, M. Lukin, A. Polkovnikov

References:

J. Superconductivity 17:577 (2004)
Phys. Rev. Lett. 95:20402 (2005)
Phys. Rev. A 71:63613 (2005)

Atoms in optical lattices

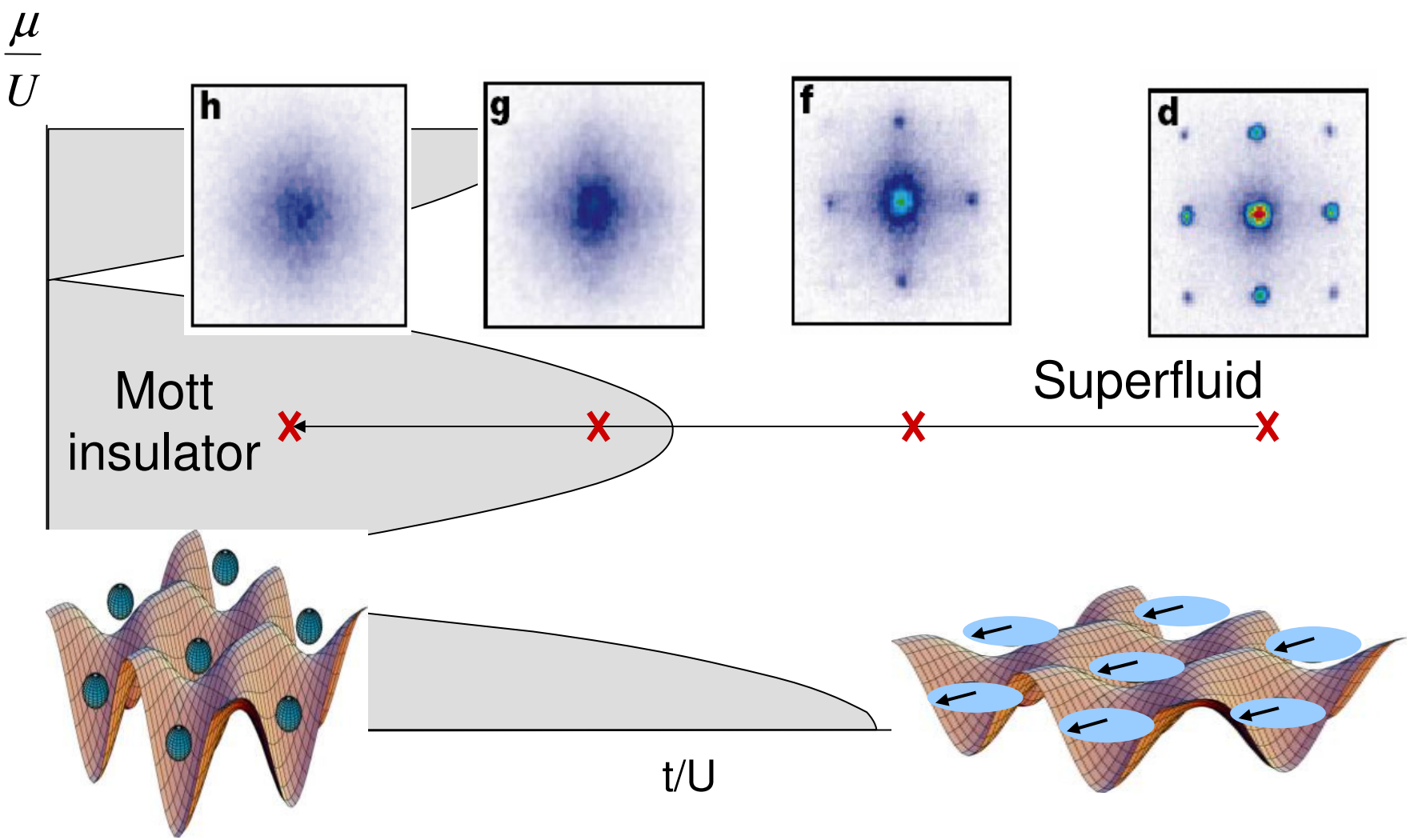


Theory: Zoller et al. PRL (1998)

Experiment: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Phillips et al., J. Physics B (2002)
Esslinger et al., PRL (2004);
Ketterle et al., PRL (2006)

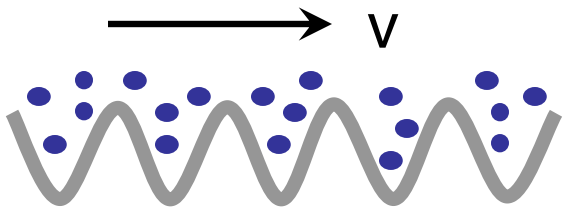
Equilibrium superfluid to insulator transition

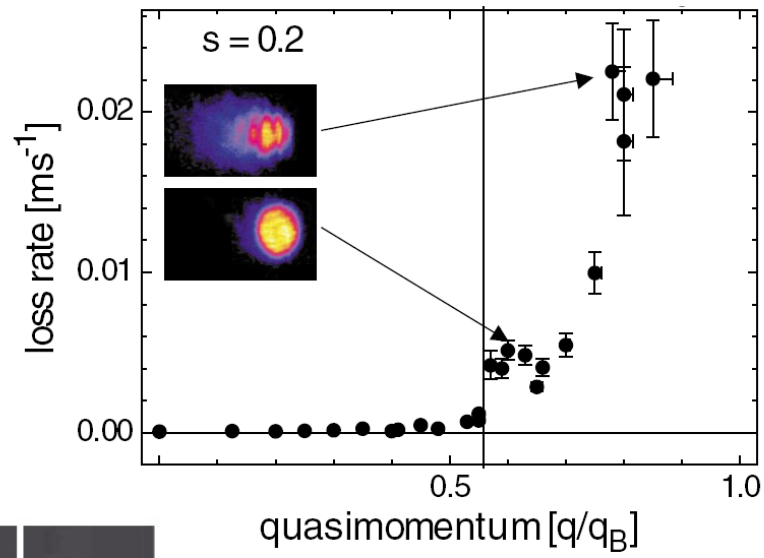
Theory: Fisher et al. PRB (89), Jaksch et al. PRL (98)
Experiment: Greiner et al. Nature (01)

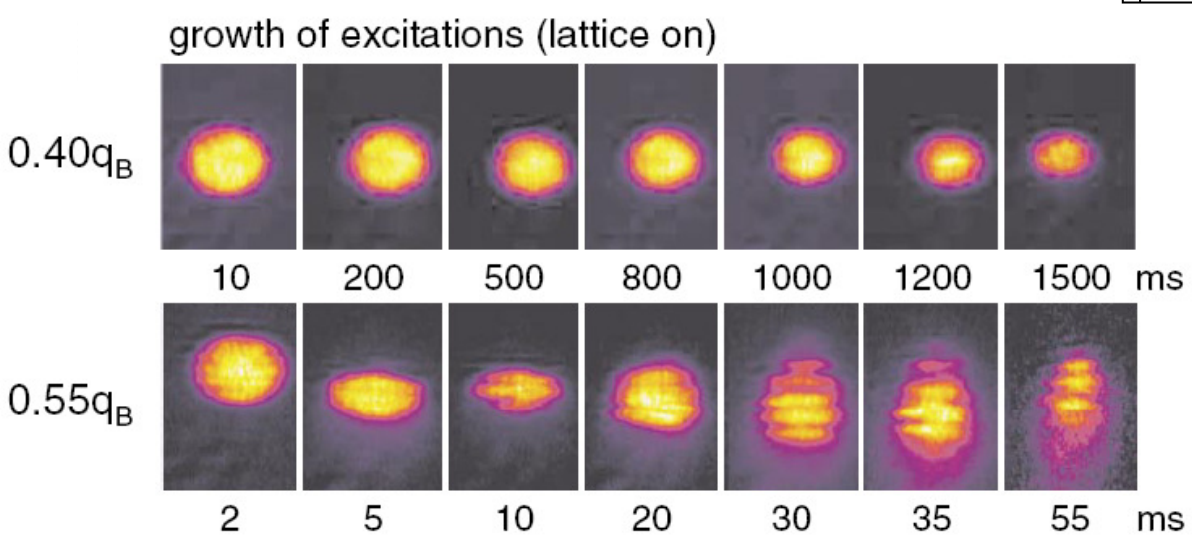


Moving condensate in an optical lattice. Dynamical instability

Theory: Niu et al. PRA (01), Smerzi et al. PRL (02)
Experiment: Fallani et al. PRL (04)

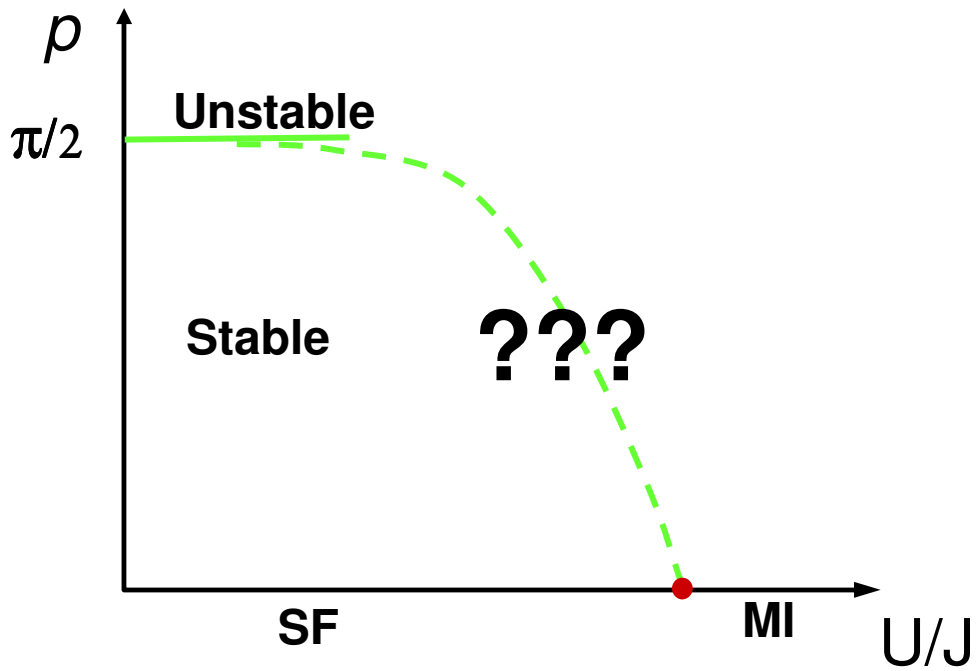




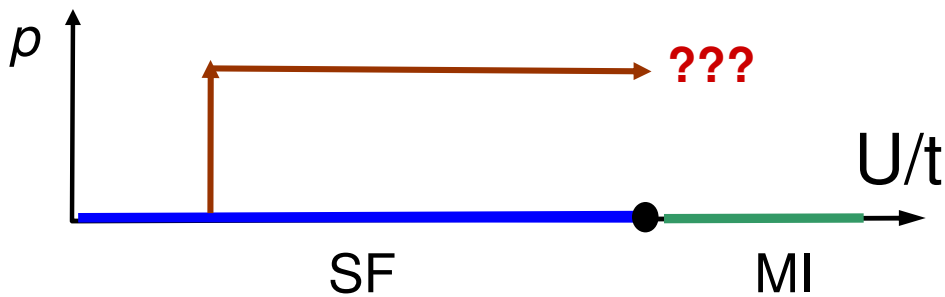


Related experiments by Eiermann et al, PRL (03)

Question: How to connect
the dynamical instability (irreversible, classical)
to the superfluid to Mott transition (equilibrium, quantum)



Possible experimental sequence:



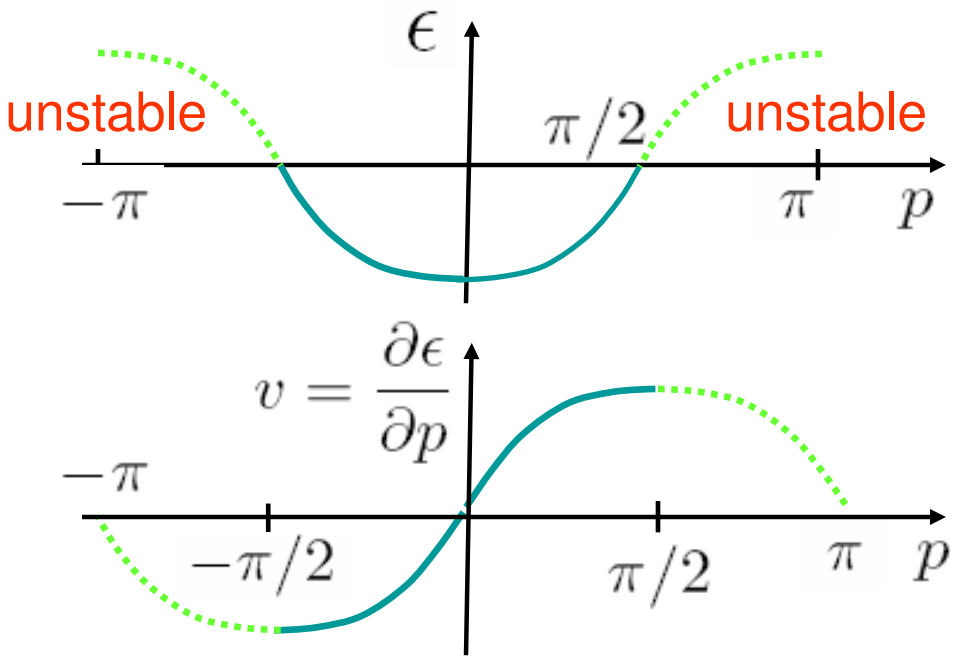
Dynamical instability

Classical limit of the Hubbard model. $N\,t >> U$
Discreet Gross-Pitaevskii equation

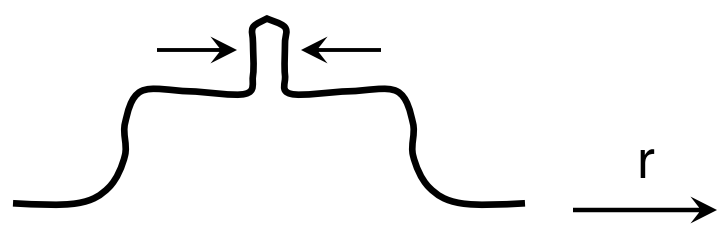
$$i\,\frac{d\,\Psi_j}{d\,t} = -t \sum_{\langle k \rangle} \Psi_k + U\,|\Psi_j|^2\,\Psi_j$$

Current carrying states $\Psi_j \sim e^{ipx_j}$

Linear stability analysis: States with p>$\pi/2$ are unstable



Amplification of density fluctuations



Dynamical instability for integer filling

Order parameter for a current carrying state $\Psi_j(p) = A(p)\ e^{ipx_j}$

Current $J(p) = |A(p)|^2 \sin(p)$

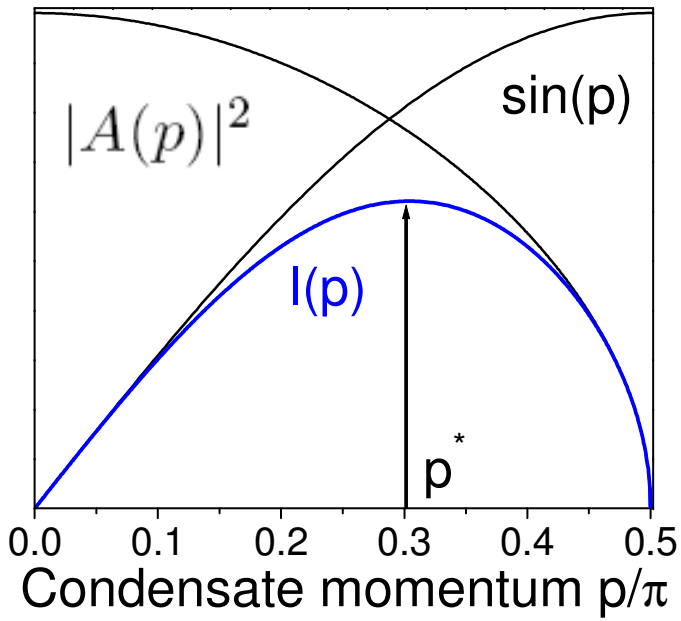
GP regime $A(p) = \sqrt{N}$. Maximum of the current for $p = \pi/2$

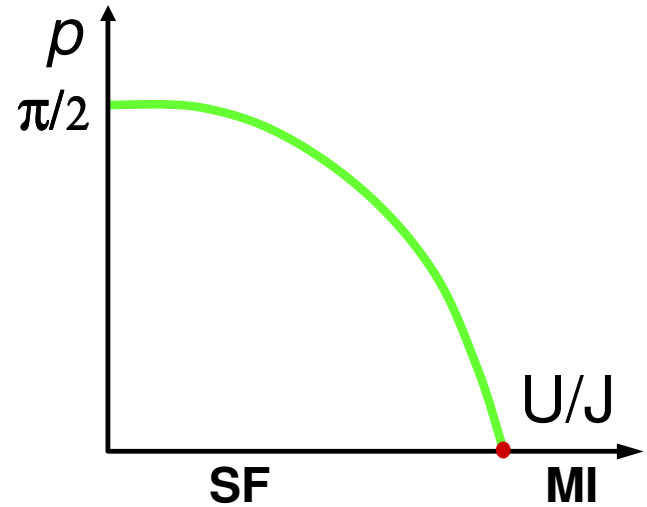
When we include quantum fluctuations, the amplitude of the order parameter is suppressed

$$\frac{A(p=0)}{\sqrt{N}} \approx 1 - \left(\frac{U}{Nt}\right)^{1/2}$$

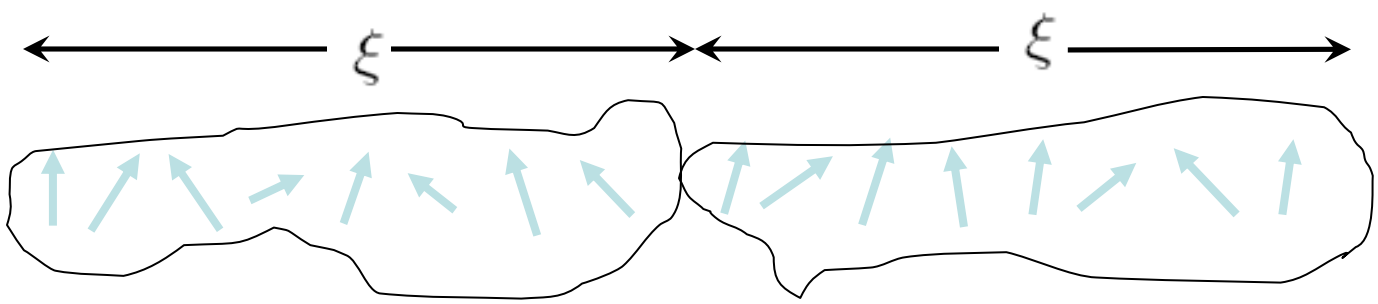
$A(p)$ decreases with increasing phase gradient p

Dynamical instability for integer filling





Vicinity of the SF-I quantum phase transition.
Classical description applies for $L > \xi \sim (U_c - U)^{-1/2}$



Dynamical instability occurs for $p\,\xi \approx \pi/2$

Dynamical instability. Gutzwiller approximation

Wavefunction

$$|\Psi(t)\rangle = \prod_j \left[\sum_{n=0}^{\infty} f_{jn}(t)\,|\,n\,\rangle_j \right]$$

Time evolution

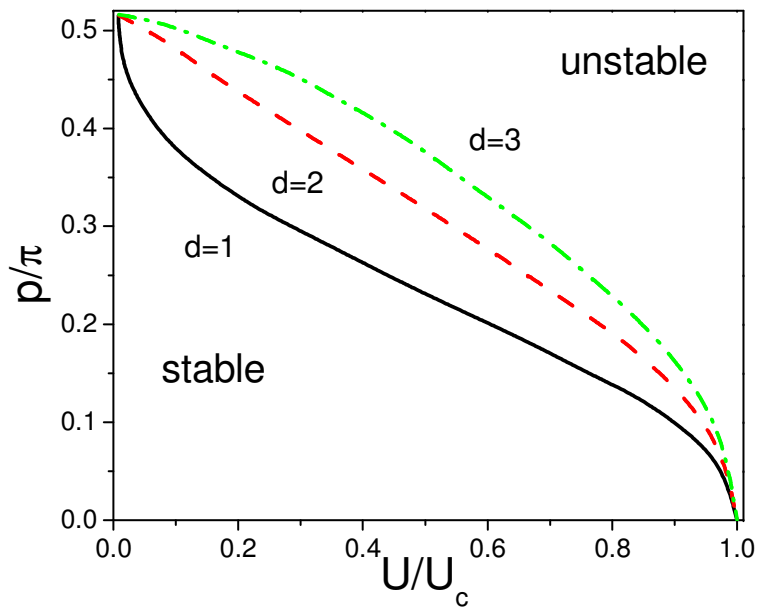
$$-i\frac{df_{jn}}{dt} = -t\,(\,f_{jn-1}\,\phi_j\,+\,f_{jn+1}\,\phi_j^*\,)\,+\,\frac{U}{2}\,n\,(n-1)\,f_{jn}$$

$$\phi_j(t) = \sum_{\langle i \rangle} \langle\,\Psi(t)\,|\,a_i\,|\,\Psi(t)\,\rangle$$

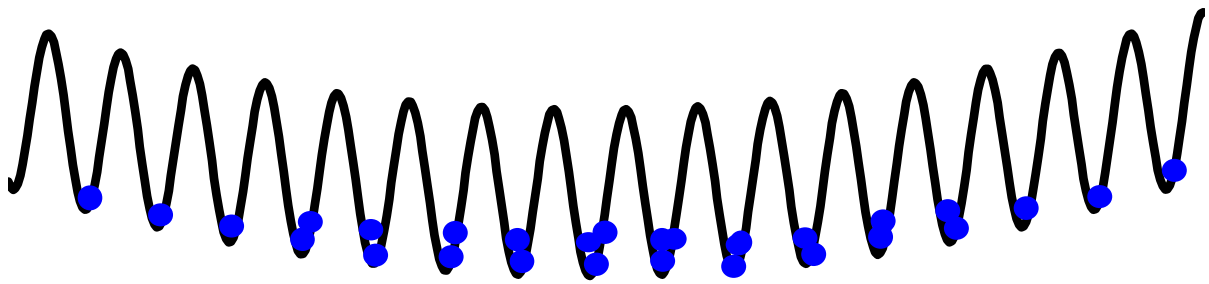
We look for stability against small fluctuations

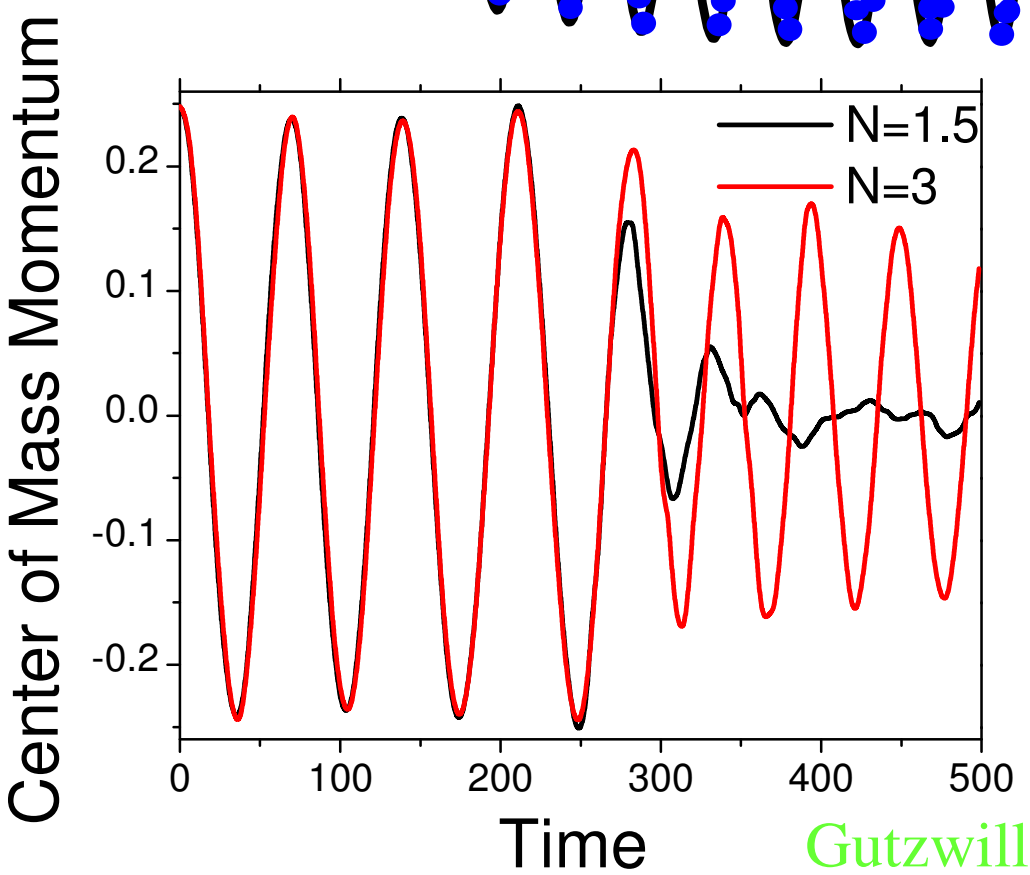
Phase diagram. Integer filling

Altman et al., PRL 95:20402 (2005)



Optical lattice and parabolic trap. Gutzwiller approximation





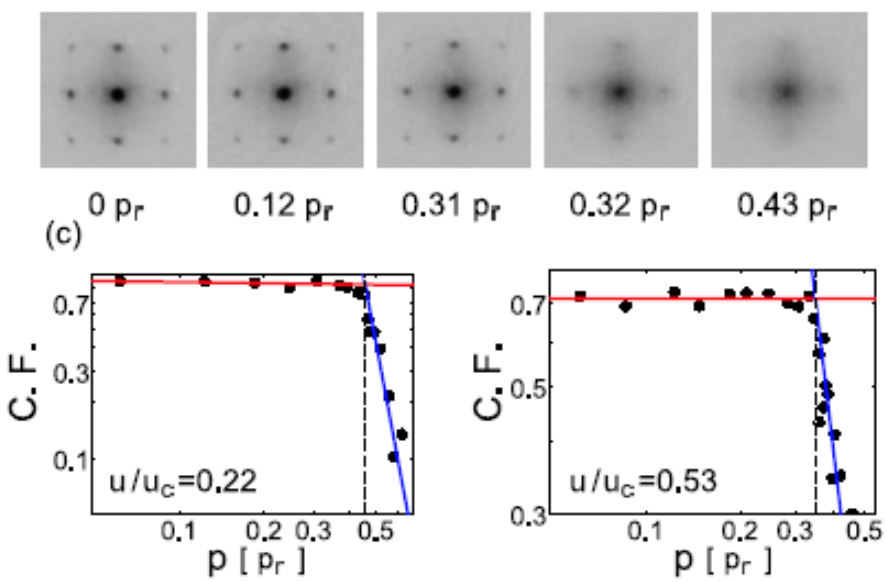
The first instability develops near the edges, where N=1

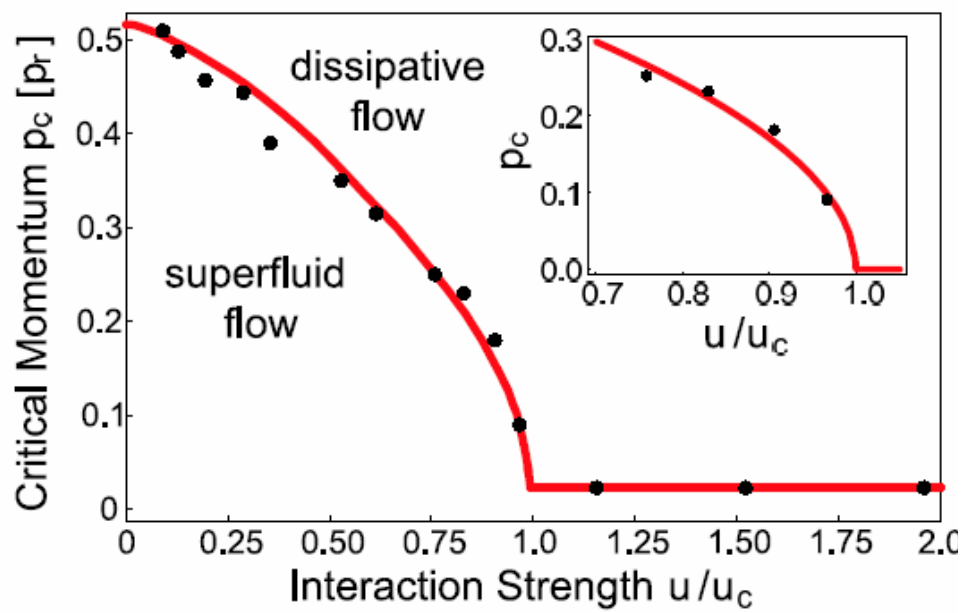
U=0.01 t
J=1/4

Gutzwiller ansatz simulations (2D)

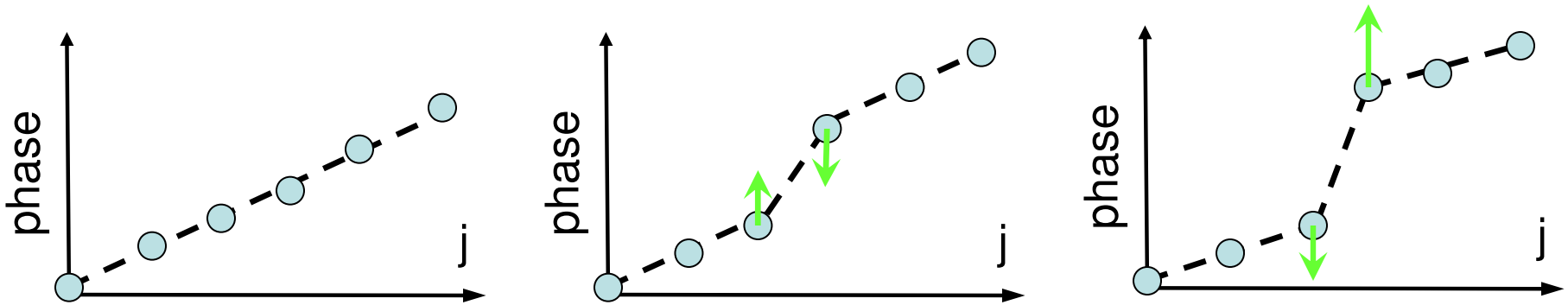
Phase diagram for a Bose-Einstein condensate moving in an optical lattice

Jongchul Mun, Patrick Medley, Gretchen K. Campbell,* Luis G. Marcassa,† David E. Pritchard, and Wolfgang Ketterle
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, and Department of Physics, MIT, Cambridge, Massachusetts 02139, USA.

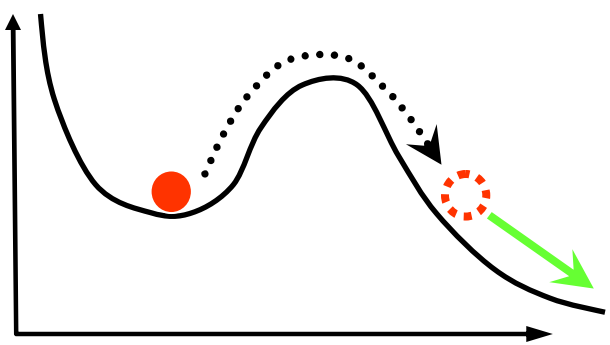




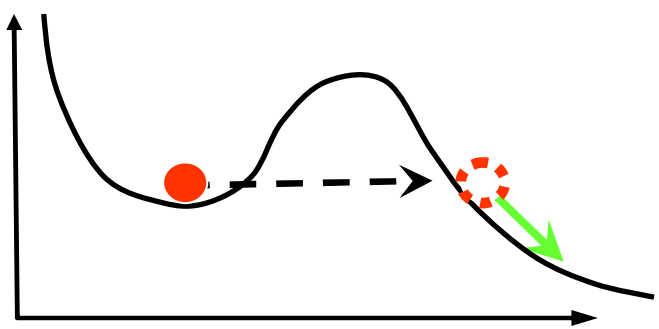
Beyond semiclassical equations. Current decay by tunneling



Current carrying states are metastable.
They can decay by thermal or quantum tunneling

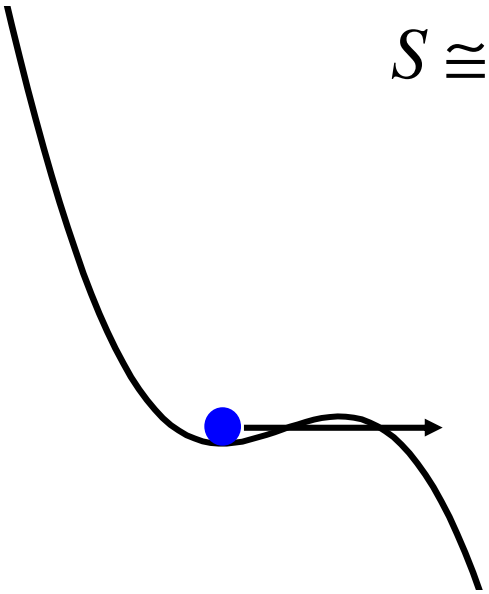


Thermal activation



Quantum tunneling

Decay rate from a metastable state. Example



$$S \cong \int_0^{\tau_0} d\tau \left(\frac{1}{2m} \left(\frac{dx}{d\tau} \right)^2 + \varepsilon x^2 - b x^3 \right)$$

Expansion in small ϵ

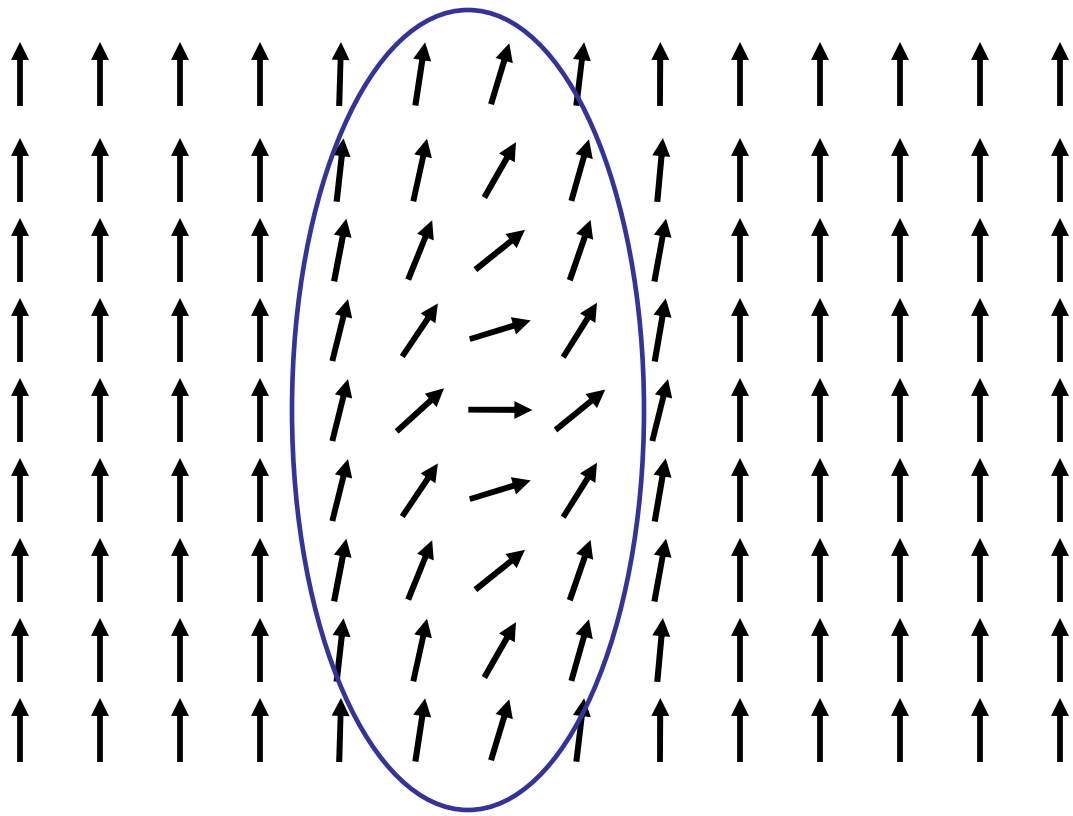
$$\Gamma \sim e^{-S}$$

$$S \sim (p_c - p)^{5/2}$$

Our small parameter of expansion:
proximity to the classical dynamical instability

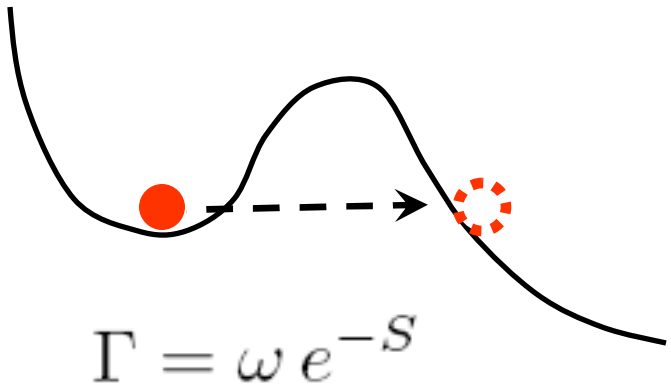
$$\varepsilon \propto (p_c - p) \to 0$$

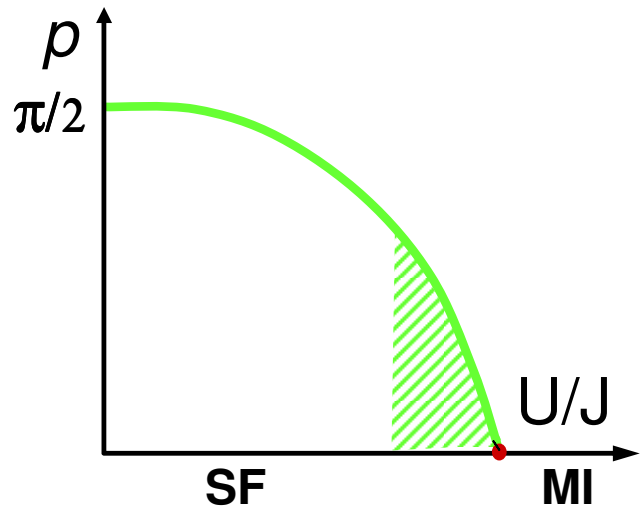
Need to consider dynamics of many degrees of freedom to describe a phase slip



A. Polkovnikov et al., Phys. Rev. A 71:063613 (2005)

Strongly interacting regime. Vicinity of the SF-Mott transition
Decay of current by quantum tunneling





Action of a quantum phase slip in d=1,2,3

$$S_d \sim \frac{1}{\xi^2}\,(1 - \sqrt{3}\,p\,\xi)^{5/2-d}$$

ξ - correlation length

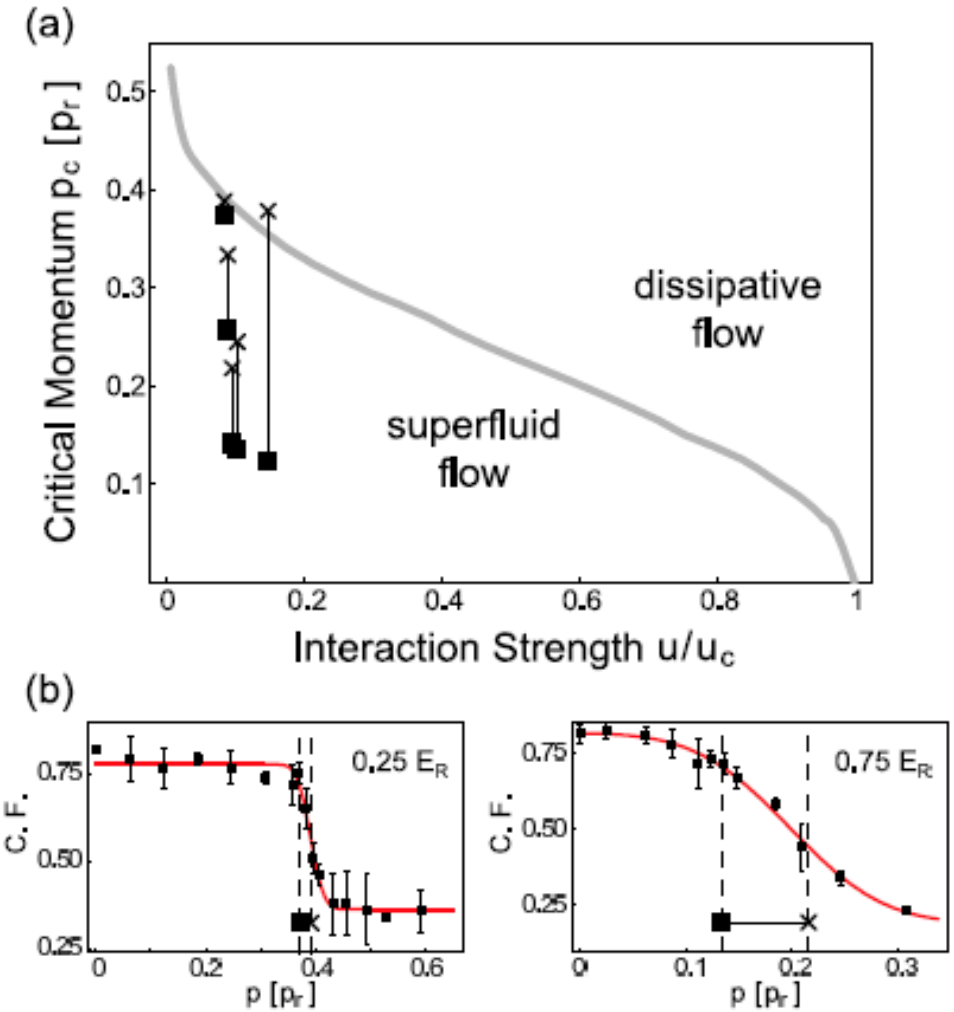
$$\xi \sim (U_c - U)^{-1/2}$$

Strong broadening of the phase transition in d=1 and d=2

S_{3d} is discontinuous at the transition. Phase slips are not important.
Sharp phase transition

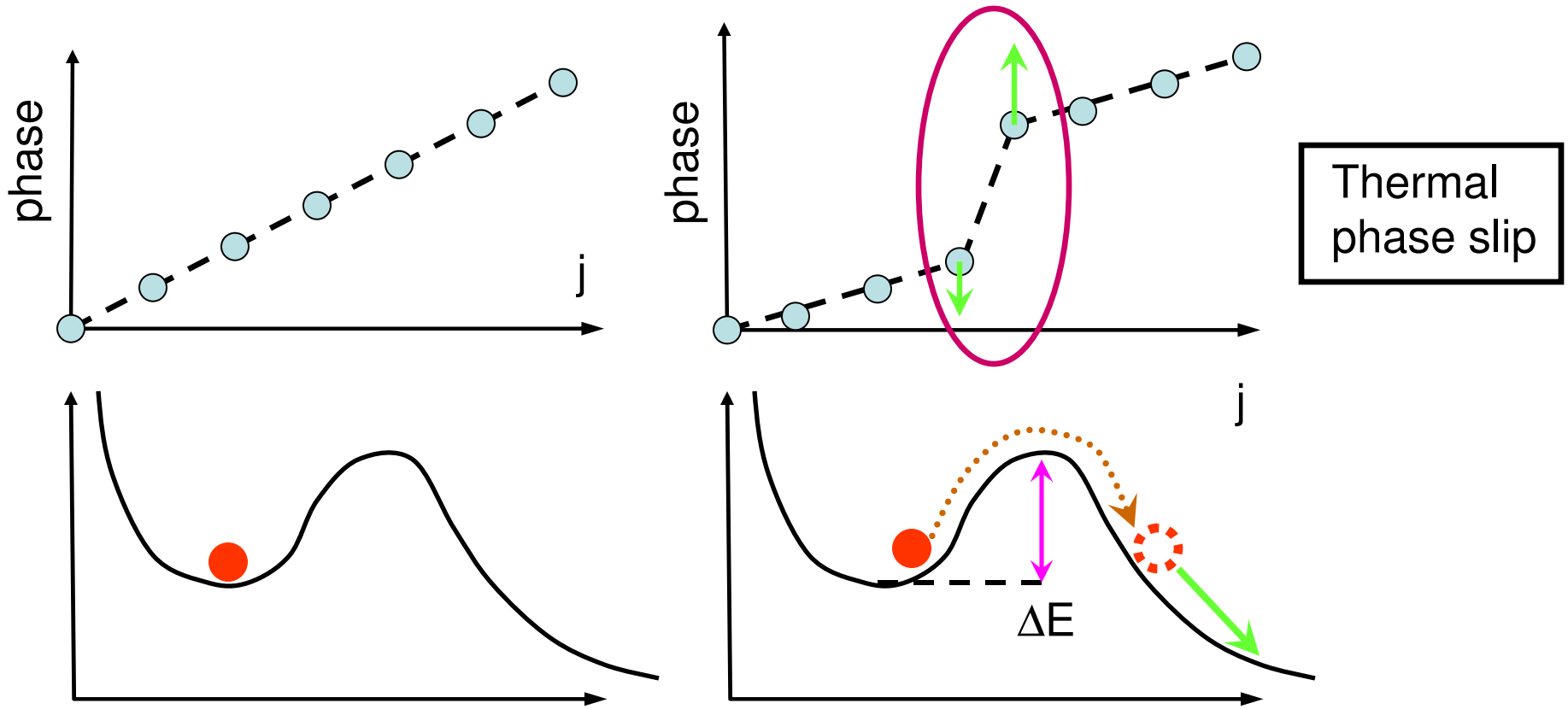
Phase diagram for a Bose-Einstein condensate moving in an optical lattice

Jongchul Mun, Patrick Medley, Gretchen K. Campbell,* Luis G. Marcassa,† David E. Pritchard, and Wolfgang Ketterle
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, and Department of Physics, MIT, Cambridge, Massachusetts 02139, USA.



Similar results
in NIST experiments,
Fertig et al., PRI (2005)

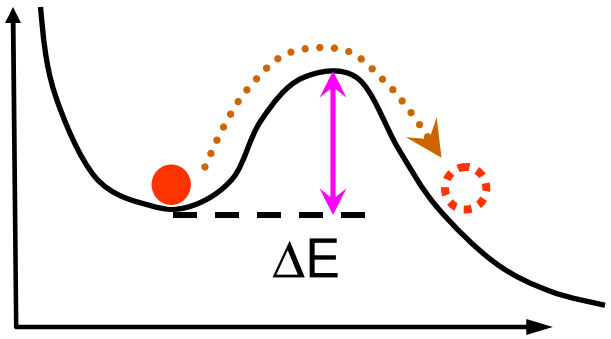
Decay of current by thermal activation



Escape from metastable state by thermal activation

$$\Gamma \sim e^{-\Delta E/T}$$

Thermally activated current decay. Weakly interacting regime



Thermal phase slip

$$\Gamma \sim e^{-\Delta E/T}$$

Activation energy in d=1,2,3

$$\Delta E_1 = 1.3\, J\, N\, (\frac{\pi}{2} - p)^3$$

$$\Delta E_2 = 10\, J\, N\, (\frac{\pi}{2} - p)^{5/2}$$

$$\Delta E_3 = 35\, J\, N\, (\frac{\pi}{2} - p)^2$$

Thermal fluctuations lead to rapid decay of currents

Crossover from thermal to quantum tunneling

$$T_Q \sim \sqrt{N\, J\, U \times (\frac{\pi}{2} - p)}$$

Decay of current by thermal fluctuations

Unstable regimes for a Bose-Einstein condensate in an optical lattice

L. De Sarlo*, L. Fallani, J. E. Lye, M. Modugno[1], R. Saers[†], C. Fort and M. Inguscio

Phys. Rev. Lett. (2004)

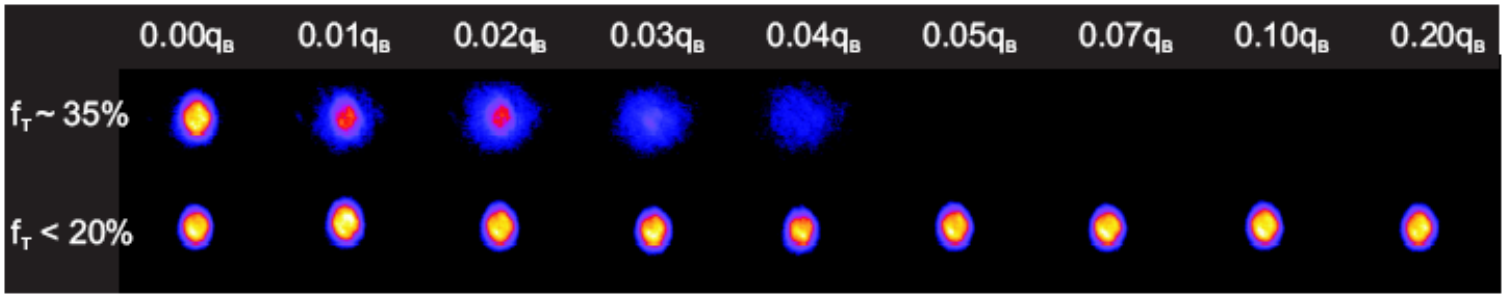


FIG. 2: Absorption images of the condensate interacting for $t = 15$s with a lattice with $s = 0.2$ for different values of quasimomentum ranging from 0 to $0.20\,q_B$ and for respectively a condensed fraction of about 65% (*top*) and no detectable thermal component (*bottom*).

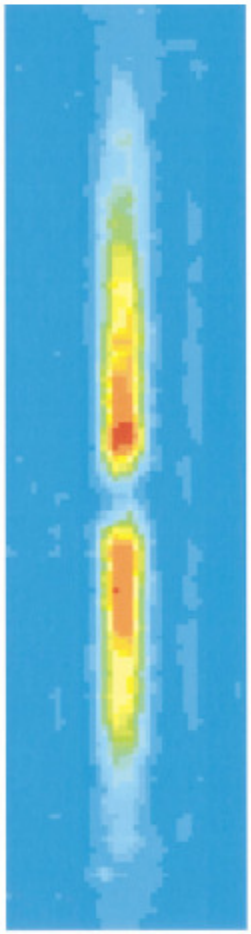
Also experiments by Brian DeMarco et al., arXiv 0708:3074

Dynamics of coherently split condensates. Interference experiments

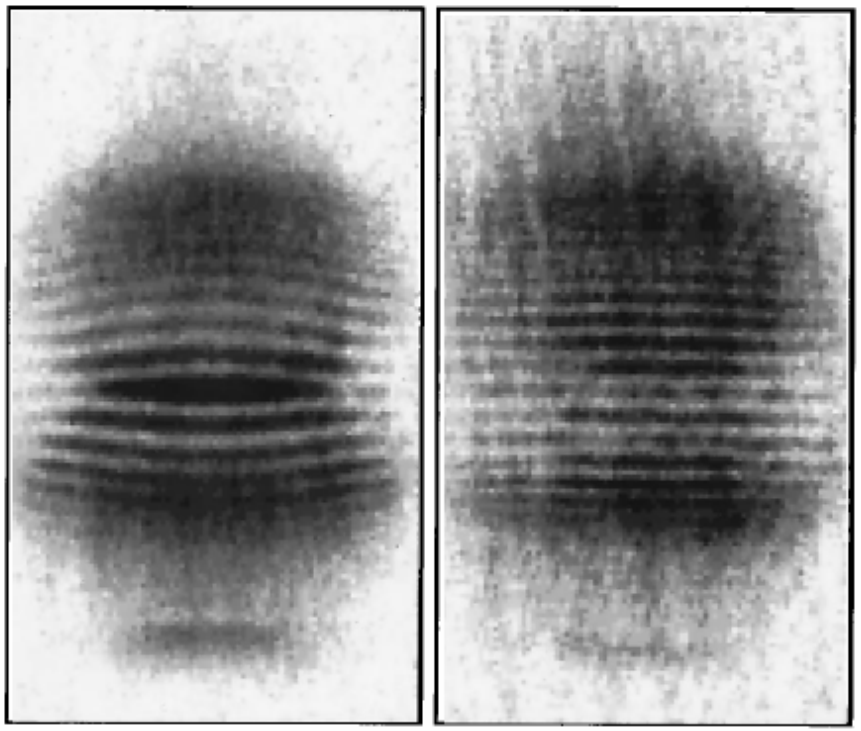
Collaboration with A. Burkov and M. Lukin

Phys. Rev. Lett. 98:200404 (2007)

Interference of independent condensates



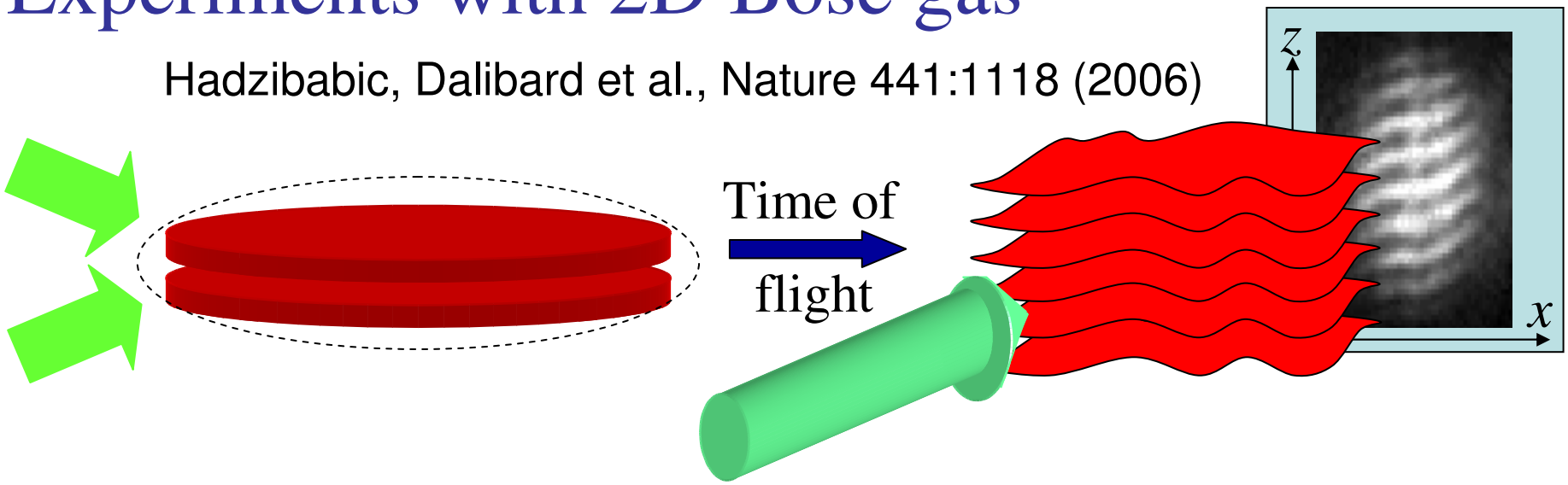
Experiments: Andrews et al., Science 275:637 (1997)



Theory: Javanainen, Yoo, PRL 76:161 (1996)
Cirac, Zoller, et al. PRA 54:R3714 (1996)
Castin, Dalibard, PRA 55:4330 (1997)
and many more

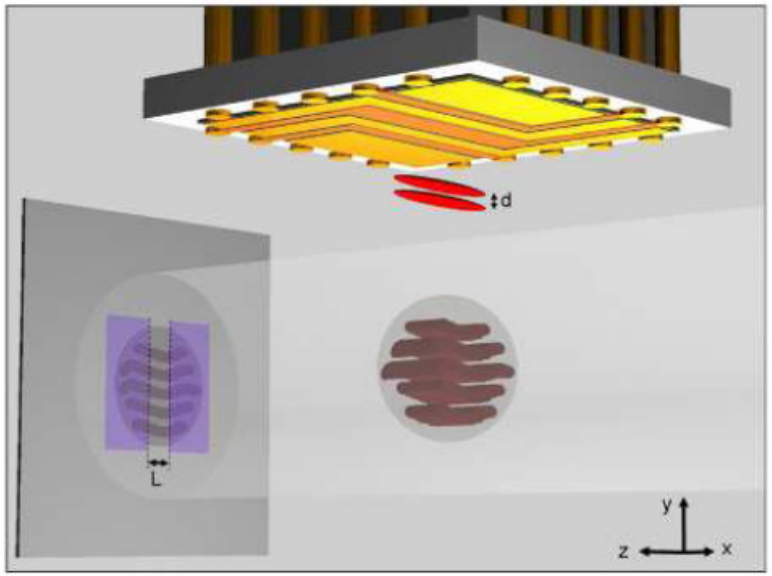
Experiments with 2D Bose gas

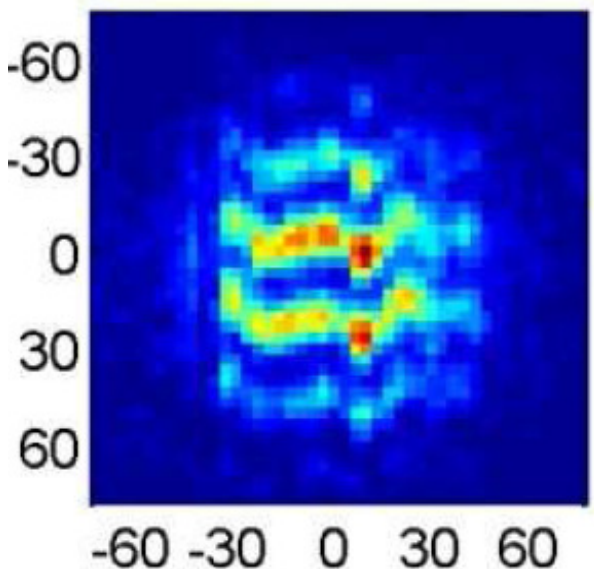
Hadzibabic, Dalibard et al., Nature 441:1118 (2006)



Experiments with 1D Bose gas

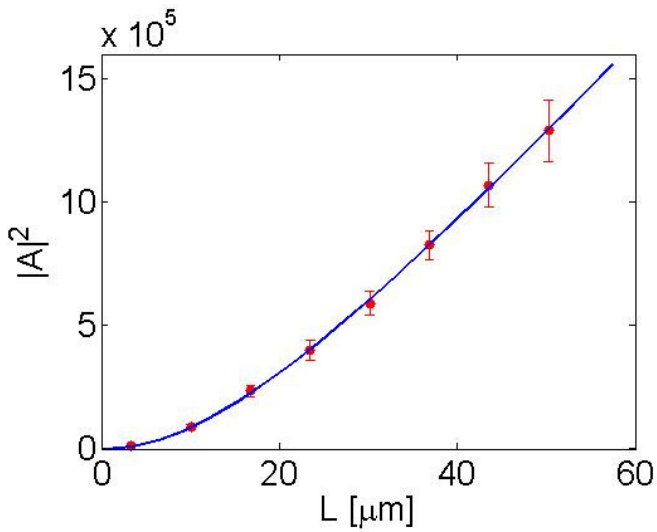
Hofferberth et al. arXiv0710.1575

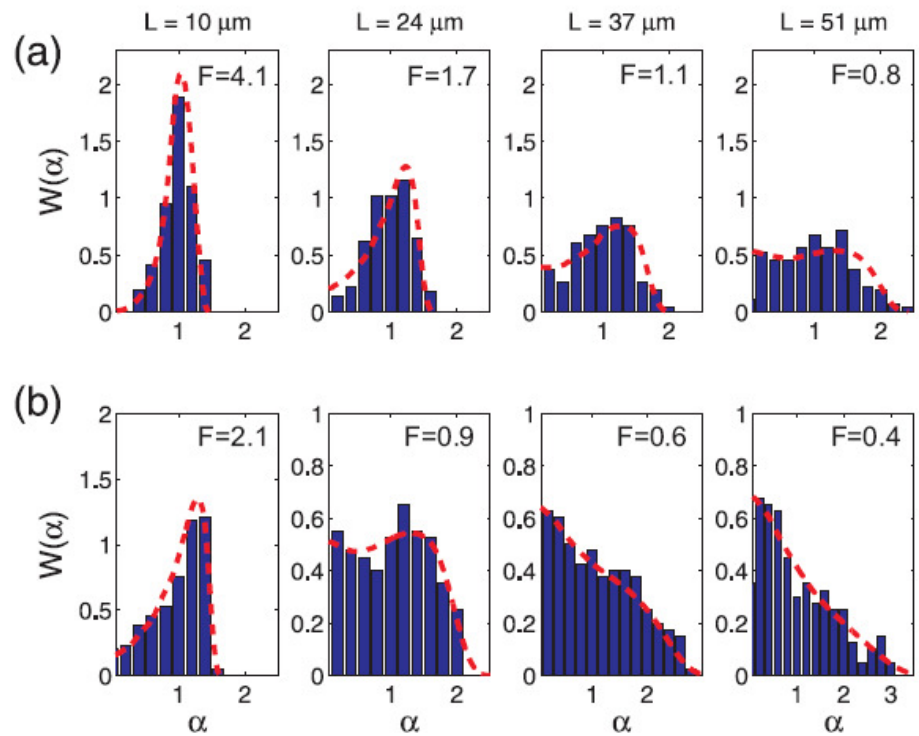




Interference of independent 1d condensates

S. Hofferberth, I. Lesanovsky, T. Schumm, J. Schmiedmayer,
A. Imambekov, V. Gritsev, E. Demler, arXiv:0710.1575



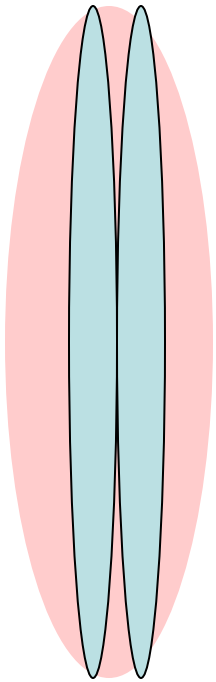


$$T_{fit} = 84 \pm 22 \text{ nK}$$

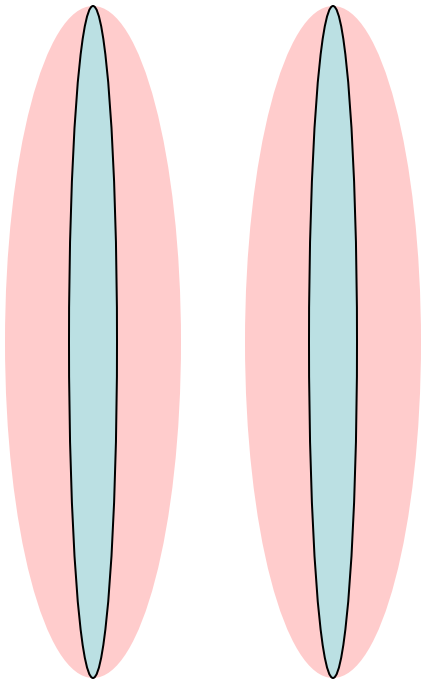
$$n_{1d} = 60 \mu\text{m}^{-1}$$

$$K = 47$$

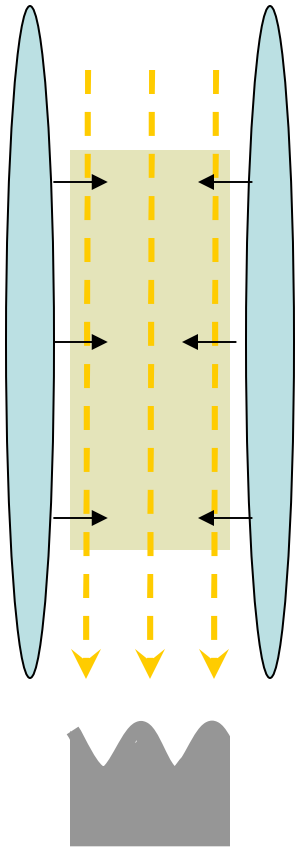
Studying dynamics using interference experiments



Prepare a system by splitting one condensate

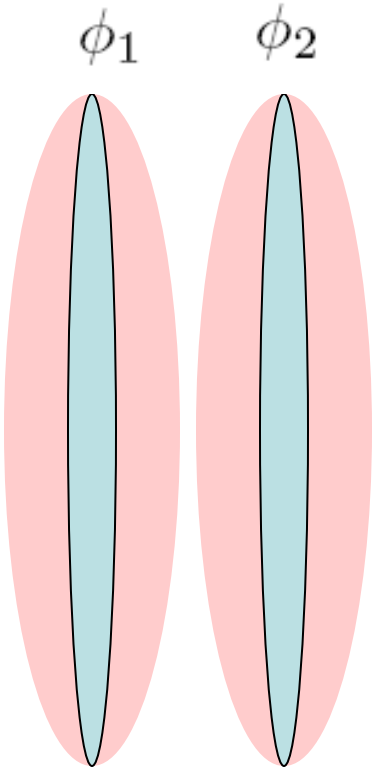


Take to the regime of zero tunneling



Measure time evolution of fringe amplitudes

Finite temperature phase dynamics



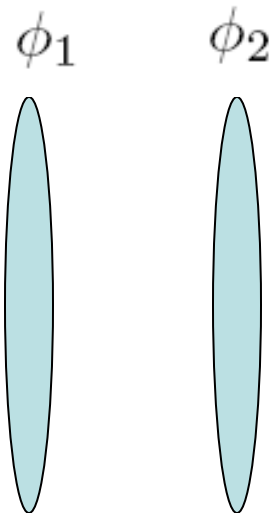
$$\mathcal{H}_0 = \int dx \, \left[g\, n_1^2(x) + \rho\, (\partial_x \phi_1)^2\right] + \int dx \, \left[g\, n_2^2(x) + \rho\, (\partial_x \phi_2)^2\right]$$

Temperature leads to phase fluctuations within individual condensates

Interference experiments measure only the relative phase

$$\phi_{av} = \frac{\phi_1 + \phi_2}{2} \qquad \phi = \phi_1 - \phi_2$$

Relative phase dynamics



$$\phi = \phi_1 - \phi_2$$

$$\Delta n = (n_1 - n_2)/2$$

Conjugate variables

$$\boxed{\mathcal{H} = \int d^d r \, [\, \frac{g}{2} (\Delta n)^2 + \frac{\rho}{2} (\nabla \phi)^2 \,]}$$

Hamiltonian can be diagonalized in momentum space

A collection of harmonic oscillators with $\omega_q = \sqrt{g\rho}\,|\,q\,|$

Initial state $\phi_q = 0$

Need to solve dynamics of harmonic oscillators at finite T

Coherence $\langle\, \Psi(t) \,|\, e^{i\phi} \,|\, \Psi(t) \,\rangle = e^{-\frac{1}{2}\sum_q \langle \phi_q^2(t) \rangle}$

Relative phase dynamics

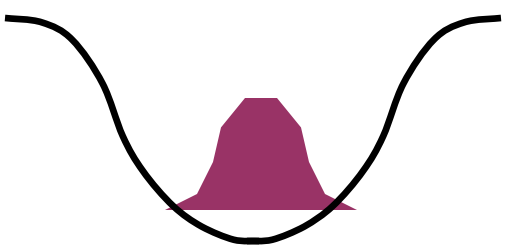
High energy modes, $\hbar\omega_{\rm osc} > k_{\rm B}T$, quantum dynamics

Low energy modes, $\hbar\omega_{\rm osc} < k_{\rm B}T$, classical dynamics

Combining all modes

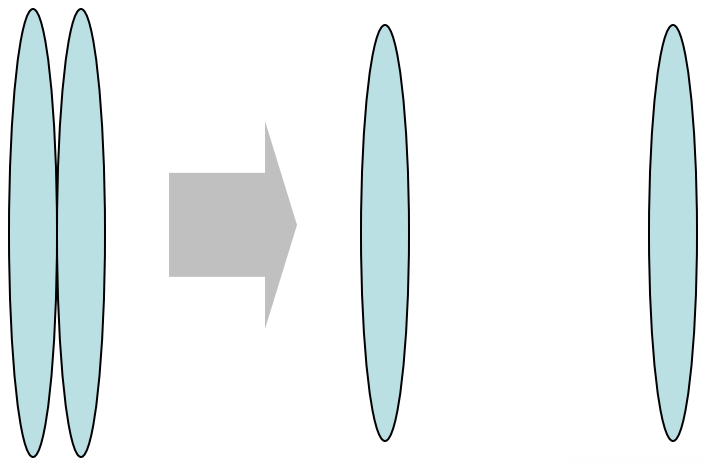
$$t < \frac{h}{k_{\rm B}T}$$ Quantum dynamics

$$t > \frac{h}{k_{\rm B}T}$$ Classical dynamics



For studying dynamics it is important to know the initial width of the phase

Relative phase dynamics



$$\frac{N}{2} \pm \delta N \qquad \frac{N}{2} \mp \delta N$$

Naive estimate

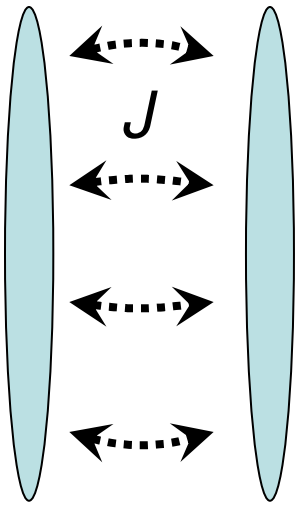
$$\delta N \sim \sqrt{N}$$

Long Phase Coherence Time and Number Squeezing of two Bose-Einstein Condensates on an Atom Chip

G.-B. Jo, Y. Shin, S. Will, T. A. Pasquini, M. Saba, W. Ketterle, and D. E. Pritchard*
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

M. Vengalattore, M. Prentiss
MIT-Harvard Center for Ultracold Atoms, Jefferson Laboratory, Physics Department, Harvard University, Cambridge, MA 02138, USA
(Dated: August 27, 2006)

Relative phase dynamics



$$\mathcal{H} = \frac{U}{2}(\Delta n)^2 - J\cos\phi$$

$$J(t) = J_0\, e^{-t/\tau_s}$$

Separating condensates at finite rate

Instantaneous Josephson frequency

$$\omega_J = \sqrt{U\,J}$$

Adiabatic regime $\dot{\omega}_J < \omega_J^2$

Instantaneous separation regime $\dot{\omega}_J > \omega_J^2$

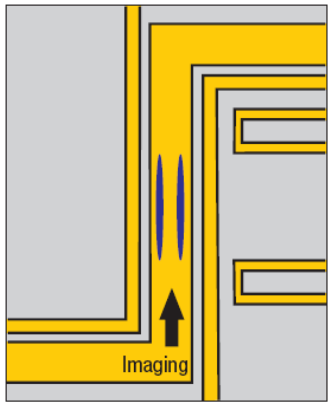
Adiabaticity breaks down when $\omega_J \sim 1/\tau_s$

Charge uncertainty at this moment

$$U\,\delta N^2 \sim \omega_J \sim 1/\tau_s$$

Squeezing factor $$\frac{\delta N}{\sqrt{N}} \sim \sqrt{\frac{1}{N\,U\,\tau_s}} \sim \sqrt{\frac{1}{\mu\,\tau_s}}$$

Relative phase dynamics

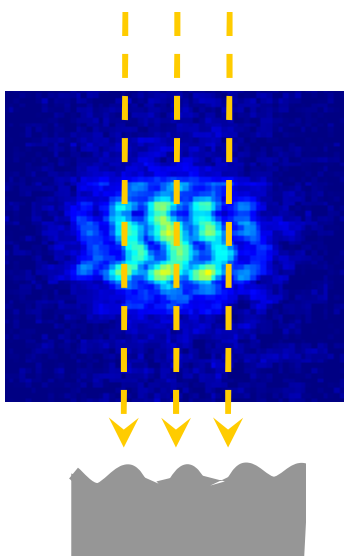


Quantum regime $\frac{h}{\mu} < t < \frac{h}{k_{\rm B}\,T}$

1D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_{\rm s}}}\, e^{-t/2\pi K\tau_{\rm s}}$

2D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_{\rm s}}} \left(\frac{t_0}{t}\right)^{1/16T_{KT}\tau_{\rm s}}$

Different from the earlier theoretical work based on a single mode approximation, e.g. Gardiner and Zoller, Leggett



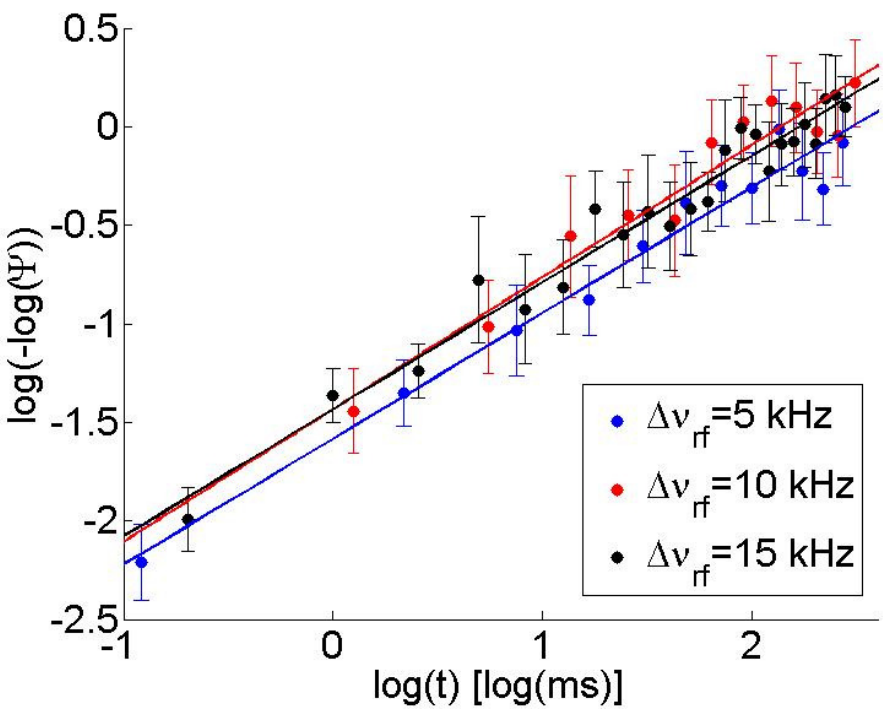
Classical regime $t > \frac{h}{k_{\rm B}\,T}$

1D systems $\langle e^{i\phi(t)} \rangle \sim e^{-(\frac{t}{t_{\rm T}})^{2/3}}$ $\quad t_{\rm T} \sim \frac{\mu K}{T^2}$

2D systems $\langle e^{i\phi(t)} \rangle \sim \left(\frac{t_0}{t}\right)^{\frac{T}{8T_{KT}}}$

1d BEC: Decay of coherence

Experiments: Hofferberth, Schumm, Schmiedmayer, Nature (2007)



$$\Psi(t) \propto \exp[-(t/t_0)^{2/3}]$$

double logarithmic plot of the coherence factor

slopes:

0.64 ± 0.08
0.67 ± 0.1
0.64 ± 0.06

get t_0 from fit with fixed slope 2/3 and calculate T from

$$t_0 = 2.61\pi K/T^2$$

T_5 = 110 ± 21 nK

T_{10} = 130 ± 25 nK

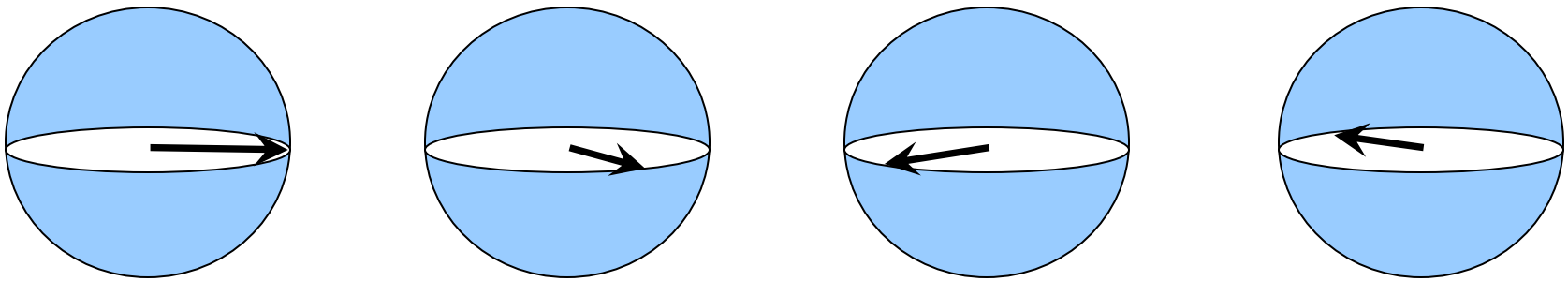
T_{15} = 170 ± 22 nK

Many-body decoherence and Ramsey interferometry

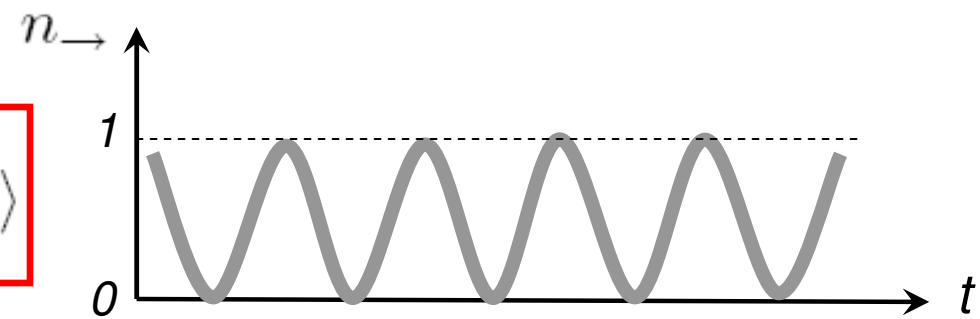
Collaboration with A. Widera, S. Trotzky, P. Cheinet, S. Fölling, F. Gerbier, I. Bloch, V. Gritsev, M. Lukin

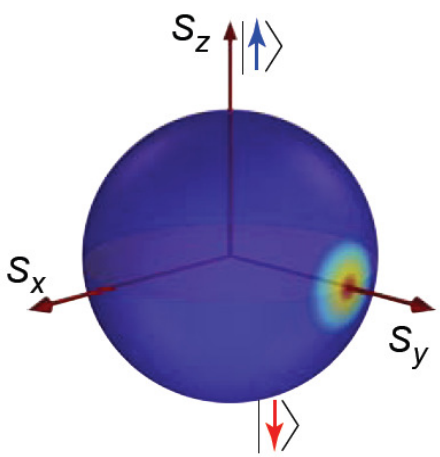
Preprint arXiv:0709.2094

Ramsey interference



$$|\Psi\rangle = e^{-iE_1 t}|\uparrow\rangle + e^{-iE_2 t}|\downarrow\rangle$$





Working with *N* atoms improves the precision by $\sqrt{N}$.
Need spin squeezed states to improve frequency spectroscopy

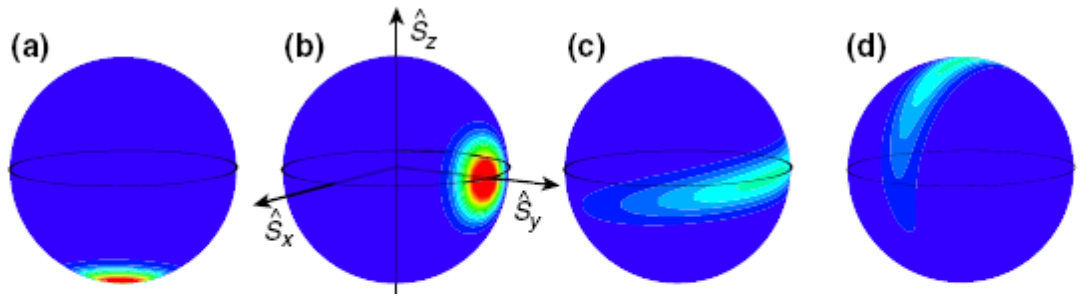
Squeezed spin states for spectroscopy

Motivation: improved spectroscopy, e.g. Wineland et. al. PRA 50:67 (1994)

Generation of spin squeezing using interactions.
Two component BEC. Single mode approximation

$$\mathcal{H} = \chi_s \left(S^z_{\rm tot}\right)^2$$

Kitagawa, Ueda, PRA 47:5138 (1993)



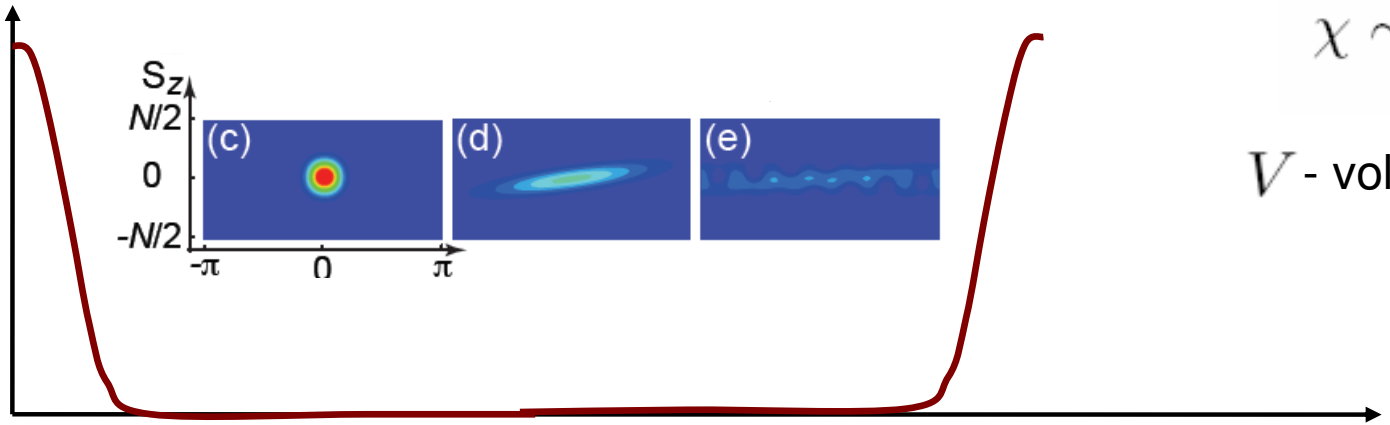
$$\mathcal{H} = \int dx \left[g_c \left(n_1 + n_2\right)^2 + g_s (n_1 - n_2)^2 + \frac{|\nabla \Psi_1|^2}{2m} + \frac{|\nabla \Psi_2|^2}{2m} \right]$$

In the single mode approximation we can neglect kinetic energy terms

$$\mathcal{H}_{\rm SMA} = \frac{g_s}{V} \left(N_1 - N_2\right)^2$$

Interaction induced collapse of Ramsey fringes

Ramsey fringe visibility



$$\chi \sim \frac{\hbar^2 a_s}{M V}$$

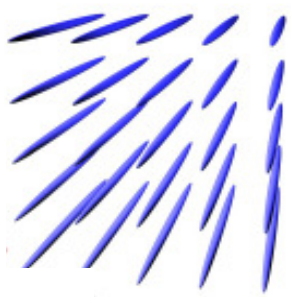
V - volume of the system

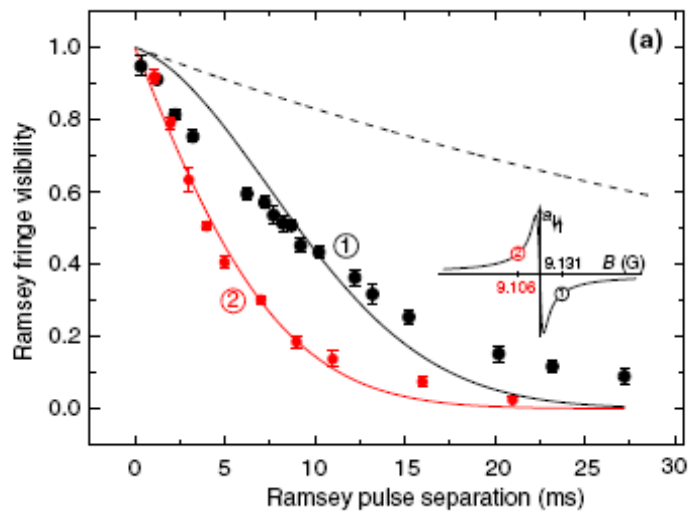
time

$$t_{\mathrm{collapse}} \sim \frac{1}{\chi\sqrt{N}}$$

$$t_{\mathrm{revival}} \sim \frac{1}{\chi}$$

Experiments in 1d tubes:
A. Widera, I. Bloch et al.





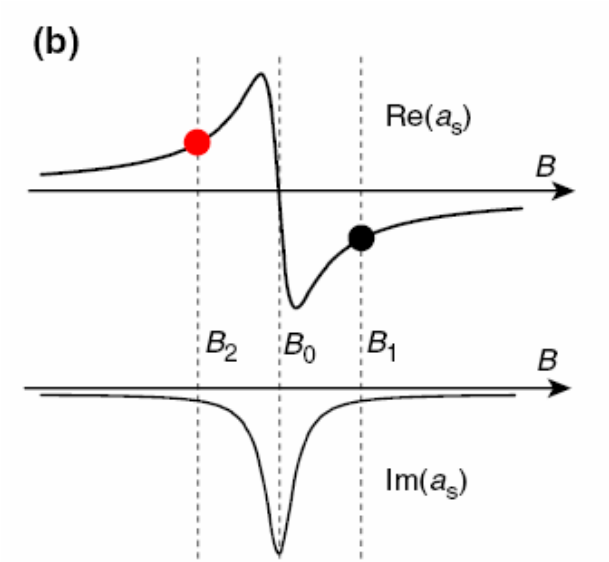
Spin echo. Time reversal experiments

Single mode approximation

$$\mathcal{H}_{\mathrm{SMA}} = \frac{g_s}{V}\left(N_1 - N_2\right)^2$$

$$g_s = \frac{g_{11} - g_{12}}{2}$$

The Hamiltonian can be reversed by changing a_{12}



$$a_s \rightarrow -a_s$$

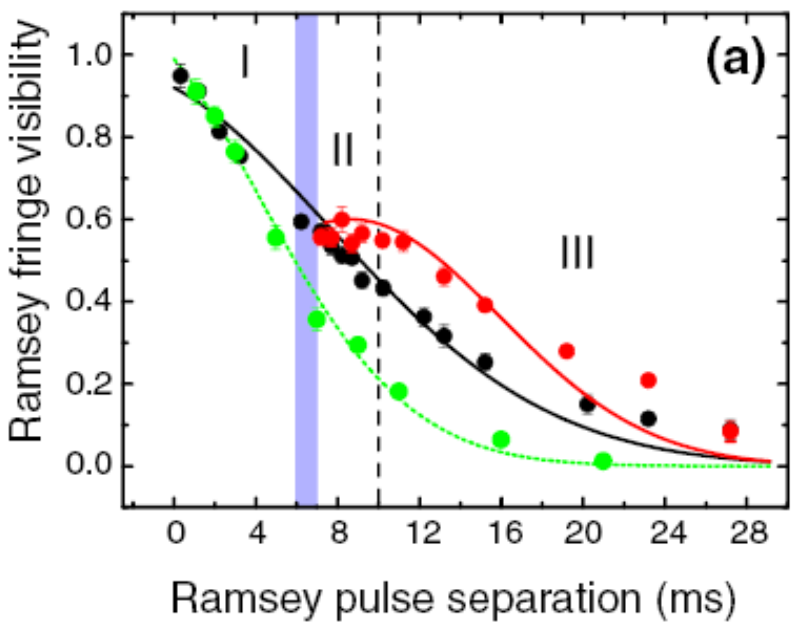
$$\mathcal{H}_{\mathrm{SMA}} \rightarrow -\mathcal{H}_{\mathrm{SMA}}$$

$$e^{i\int_T^{2T}\mathcal{H}_{\mathrm{SMA}}(t)dt} \times e^{i\int_0^{T}\mathcal{H}_{\mathrm{SMA}}(t)dt} = 1$$

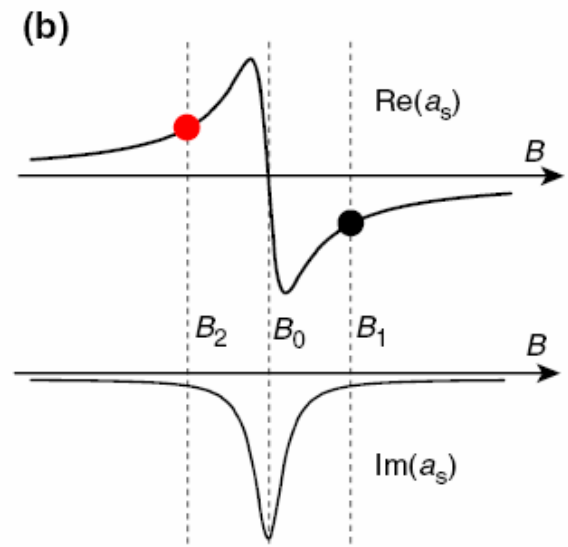
Predicts perfect spin echo

Spin echo. Time reversal experiments

Expts: A. Widera, I. Bloch et al.



$$a_s \rightarrow -a_s$$



No revival?

Experiments done in array of tubes.
Strong fluctuations in 1d systems.
Single mode approximation does not apply.
Need to analyze the full model

$$\mathcal{H} = \int dx \left[g_c (n_1 + n_2)^2 + g_s (n_1 - n_2)^2 + \frac{|\nabla \Psi_1|^2}{2m} + \frac{|\nabla \Psi_2|^2}{2m} \right]$$

Interaction induced collapse of Ramsey fringes. Multimode analysis

Low energy effective theory: Luttinger liquid approach

Luttinger model

$$S^+(x,t) \sim e^{i\phi_s(x,t)} \qquad [S^z(x), \phi_s(x')] = -i\delta(x-x')$$

$$\mathcal{H}_s = \int_0^L dx \left[g_s (S^z)^2 + \frac{\rho}{2m} (\nabla \phi_s)^2 \right]$$

Changing the sign of the interaction reverses the interaction part of the Hamiltonian but not the kinetic energy

$$\mathcal{H}_s = \sum_q \left[g_s(t) S^z_q S^{z*}_q + \frac{\rho\, q^2}{m} \phi_{sq} \phi^*_{sq} \right]$$

$$[S^z_{q'}, \phi_{sq}] = -i\delta_{qq'}$$

Time dependent harmonic oscillators can be analyzed exactly

Time-dependent harmonic oscillator

$$\mathcal{H} = \frac{p^2}{2m(t)} + k\,\frac{q^2}{2}$$

See e.g. Lewis, Riesengeld (1969)
Malkin, Man'ko (1970)

Explicit quantum mechanical wavefunction can be found

$$\psi(p,t) = \frac{\Phi\left(\frac{p}{c(t)}\right)}{\sqrt{c(t)}}\, e^{i\alpha(t)p^2 + i\gamma(t)}$$

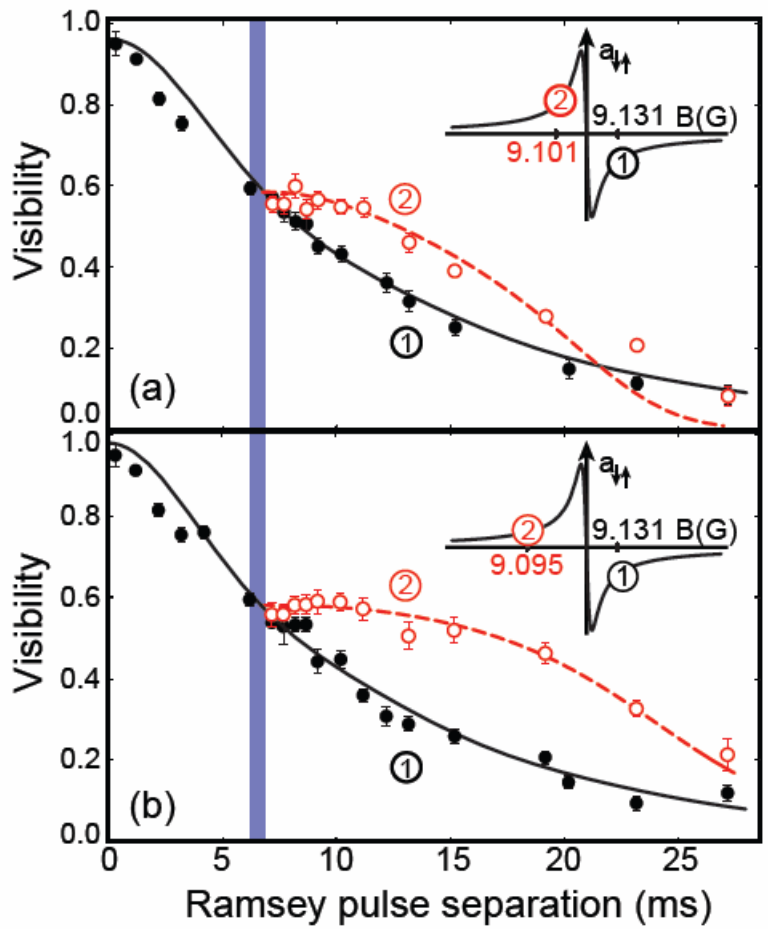
From the solution of classical problem

$$\ddot{c} + \omega^2(t)\, c = \frac{\omega_0^2}{c^3}$$

We solve this problem for each momentum component

$$\mathcal{H}_s = \sum_q \left[g_s(t) S_q^z S_q^{z*} + \frac{\rho\, q^2}{m} \phi_{sq} \phi_{sq}^* \right]$$

Interaction induced collapse of Ramsey fringes in one dimensional systems



Only q=0 mode shows complete spin echo
Finite q modes continue decay

The net visibility is a result of competition between q=0 and other modes

Conceptually similar to experiments with dynamics of split condensates.
T. Schumm's talk

Fundamental limit on Ramsey interferometry

Summary

Experiments with ultracold atoms can be used to study new phenomena in nonequilibrium dynamics of quantum many-body systems.

Today's talk: strong manifestations of enhanced fluctuations in dynamics of low dimensional condensates

Thanks to:

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