Probing interacting systems of cold atoms using interference experiments

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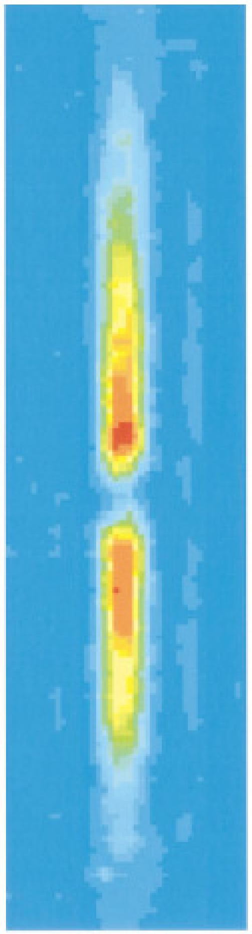
Robert Cherng, Anatoli Polkovnikov, Ehud Altman,
Mikhail Lukin, Eugene Demler

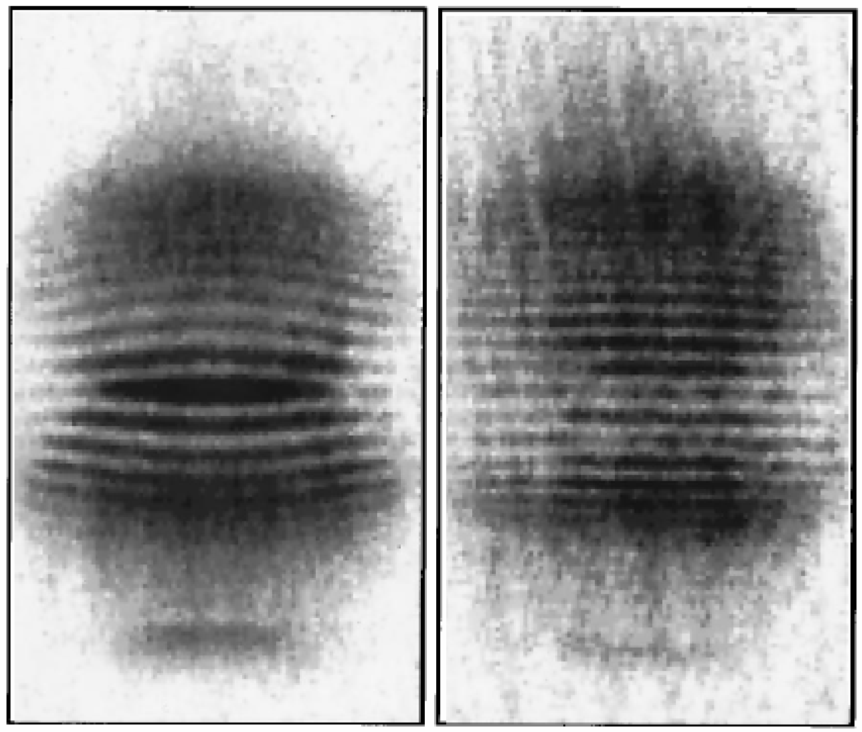
Measuring equilibrium correlation functions using interference experiments

Studying non-equilibrium dynamics of interacting Bose systems in interference experiments

Interference of independent condensates

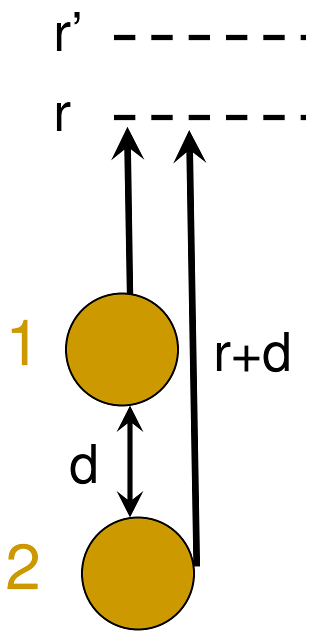


Experiments: Andrews et al., Science 275:637 (1997)



Theory: Javanainen, Yoo, PRL 76:161 (1996)
Cirac, Zoller, et al. PRA 54:R3714 (1996)
Castin, Dalibard, PRA 55:4330 (1997)
and many more

Interference of two independent condensates



$$\rho_{\mathrm{int}}(r) = a_1^\dagger(r)\, a_2(r) + \mathrm{c.c.}$$

$$a_1(r) = e^{i\,\phi_1 + i\,k_1 r} \qquad k_1 = \frac{m\,v_1}{\hbar} = \frac{m\,r}{\hbar\,t}$$

$$a_2(r) = e^{i\,\phi_2 + i\,k_2 r} \qquad k_2 = \frac{m\,v_2}{\hbar} = \frac{m\,(r+d)}{\hbar\,t}$$

$$\rho_{\mathrm{int}}(r) = e^{i\,(k_2 - k_1)\,r}\, e^{i\,(\phi_2 - \phi_1)} + \mathrm{c.c.}$$

$$\rho_{\mathrm{int}}(r) = e^{i\,\frac{m\,d\,r}{\hbar\,t}}\, e^{i\,(\phi_2 - \phi_1)} + \mathrm{c.c.}$$

Clouds 1 and 2 do not have a well defined phase difference.
However each individual measurement shows an interference pattern

$$\langle\, \rho_{\mathrm{int}}(r)\,\rangle = 0$$

$$\langle\, \rho_{\mathrm{int}}(r)\, \rho_{\mathrm{int}}(r')\,\rangle = e^{i\,\frac{m\,d}{\hbar\,t}\,(r - r')} + \mathrm{c.c.}$$

Nature 4877:255 (1963)

INTERFERENCE FRINGES PRODUCED BY SUPERPOSITION OF TWO INDEPENDENT MASER LIGHT BEAMS

By G. MAGYAR and DR. L. MANDEL
Department of Physics, Imperial College of Science and Technology, London

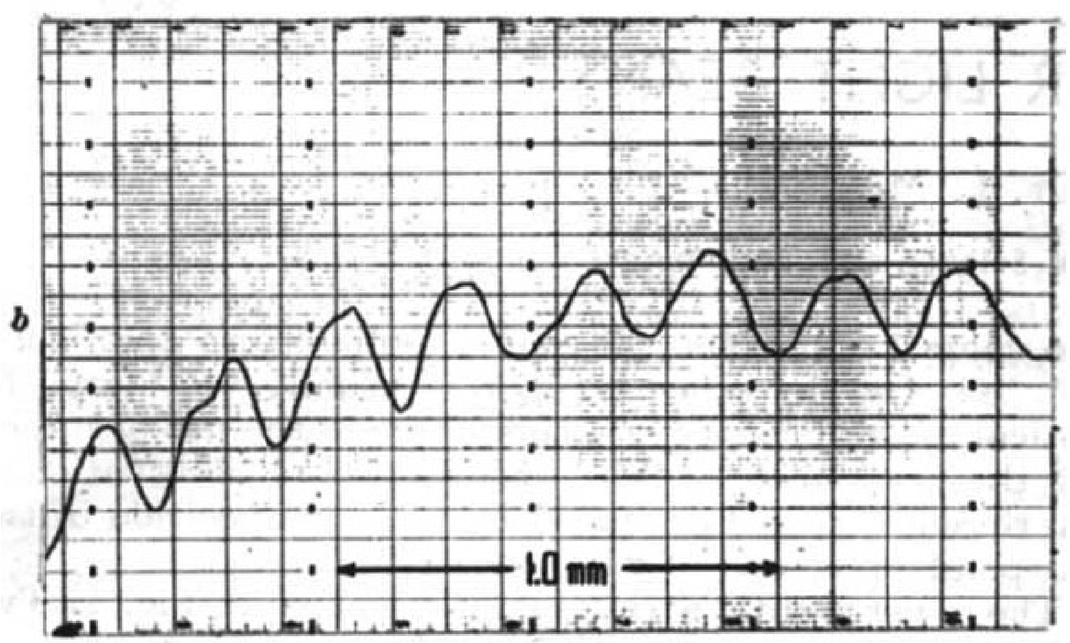
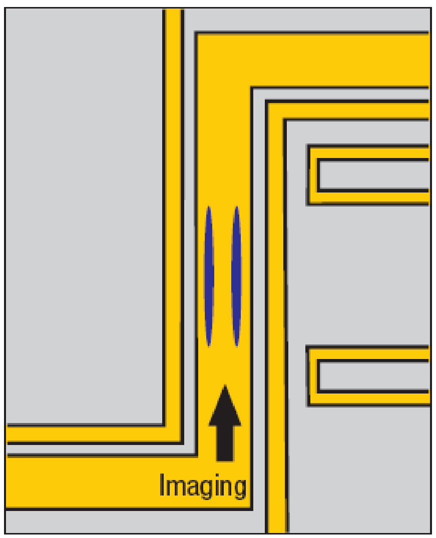


Fig. 2. An example of fringes recorded: (*a*) photograph; (*b*) microphotometer tracing

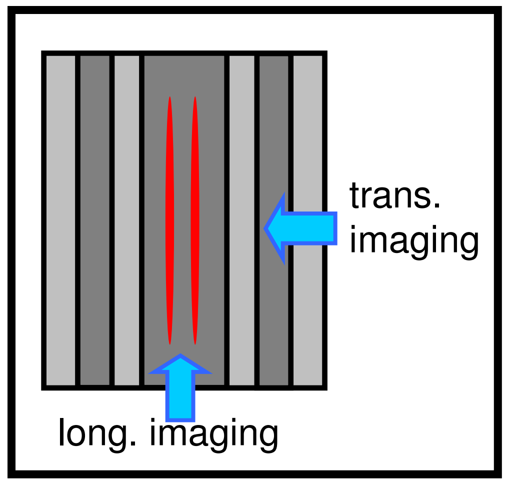
Interference of one dimensional condensates

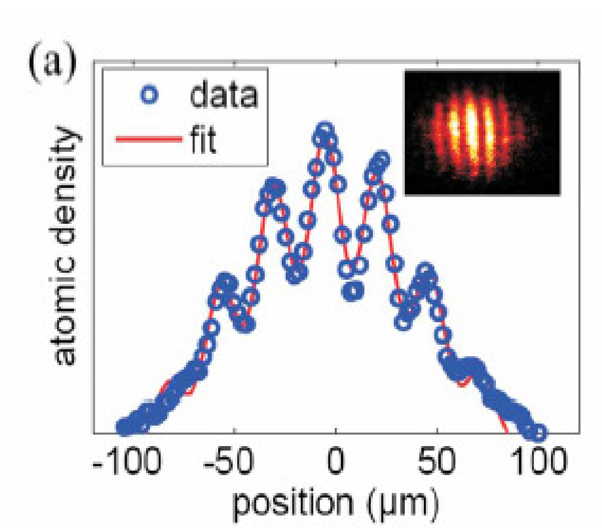
Experiments with 1d condensates: Sengstock , Phillips, Weiss, Bloch, Esslinger, …

Interference of 1d condensates: Schmiedmayer et al., Nature Physics (2005,2006)

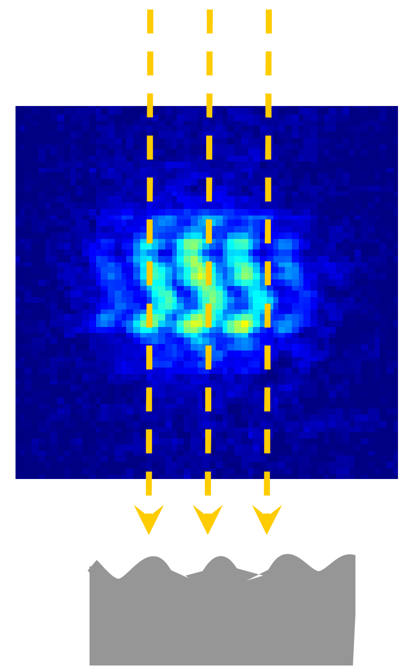


Longitudial imaging





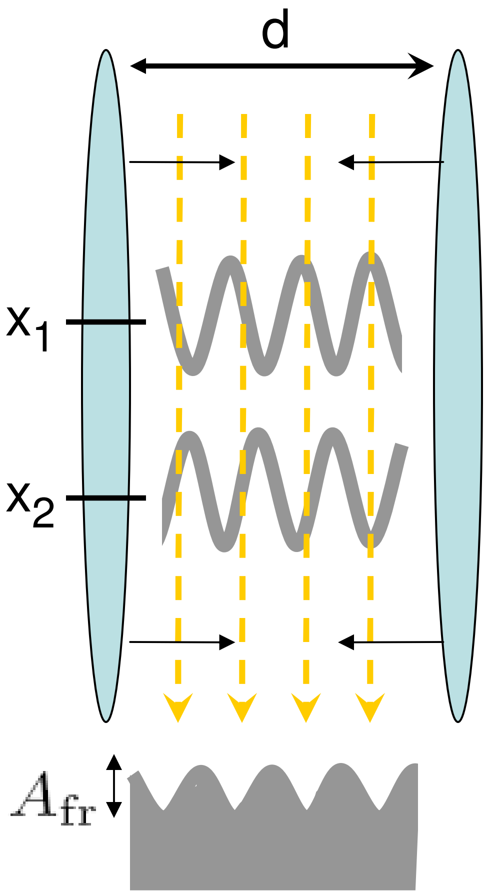
Transverse imaging



Figures courtesy of J. Schmiedmayer

Interference of one dimensional condensates

Polkovnikov, Altman, Demler, PNAS 103:6125 (2006)



Amplitude of interference fringes, A_{fr}

$$|A_{\mathrm{fr}}|\, e^{i\Delta\phi} = \int_0^L dx\ a_1^\dagger(x)\, a_2(x)$$

For independent condensates A_{fr} is finite but $\Delta\phi$ is random

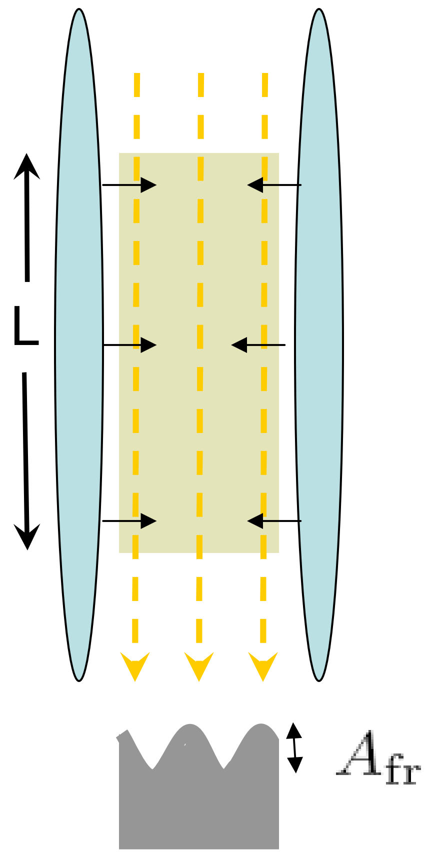
$$\langle |A_{\mathrm{fr}}|^2 \rangle = \int_0^L \int_0^L dx\, dy\, \langle a_1^\dagger(x)\, a_2(x)\, a_2^\dagger(y)\, a_1(y) \rangle$$

$$\simeq L \int_0^L dx\, \langle a_1(x)\, a_1^\dagger(0) \rangle \langle a_2(0) a_2^\dagger(x) \rangle$$

For identical condensates $\langle |A_{\mathrm{fr}}|^2 \rangle = L \int_0^L dx\, (G(x))^2$

Instantaneous correlation function $G(x) = \langle a(x)\, a^\dagger(0) \rangle$

Interference between 1d condensates at T=0



Luttinger liquid at T=0

$$G(x) \sim \rho \left(\frac{\xi_h}{x} \right)^{1/2K}$$

K – Luttinger parameter

$$\langle |A_{\mathrm{fr}}|^2 \rangle \sim (\rho \xi_h)^{1/K} (L\rho)^{2-1/K}$$

For non-interacting bosons $K = \infty$ and $A_{\mathrm{fr}} \sim L$

For impenetrable bosons $K = 1$ and $A_{\mathrm{fr}} \sim \sqrt{L}$

Luttinger liquid at finite temperature

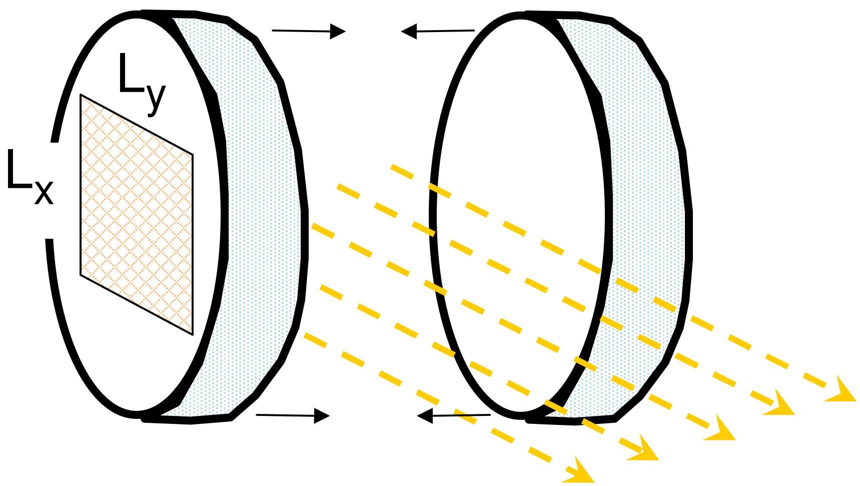
$$\langle |A_{\mathrm{fr}}|^2 \rangle \sim L \rho^2 \xi_h \left(\frac{\hbar^2}{m \xi_h^2} \frac{1}{T} \right)^{1-1/K}$$

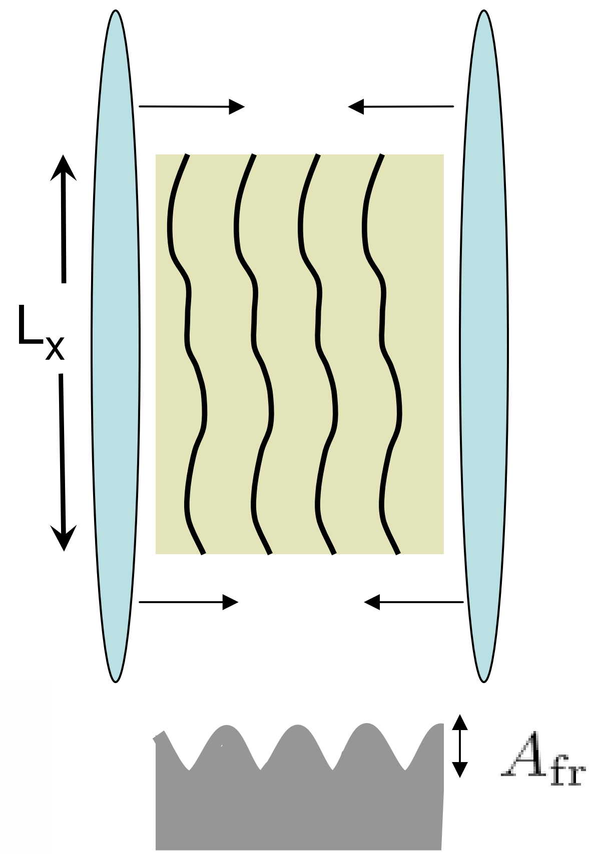
Analysis of A_{fr} can be used for thermometry

Interference of two dimensional condensates

Experiments: Hadzibabic et al. Nature (2006)

Gati et al., PRL (2006)



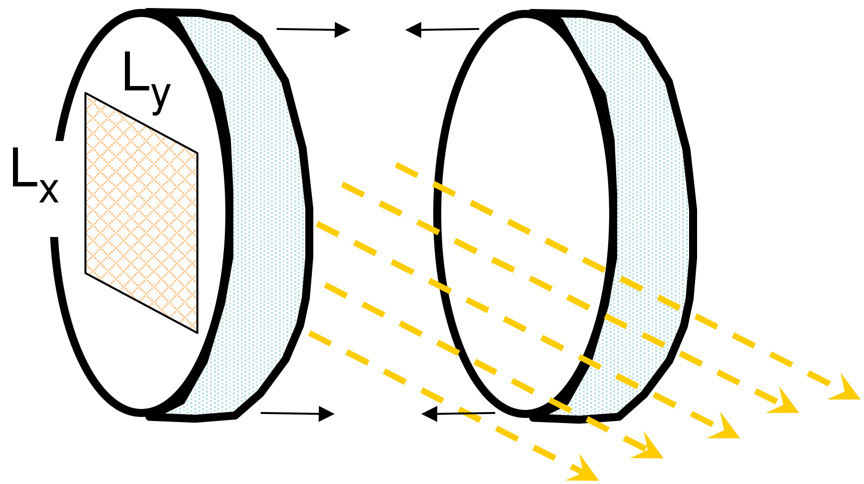


Probe beam parallel to the plane of the condensates

$$\langle\, |A_{\rm fr}|^2 \rangle \,=\, L_x\, L_y \int_0^{L_x} \int_0^{L_y} d^2\vec{r}\, (\, G(\vec{r}\,)\,)^2$$

$$G(\vec{r}\,) \,=\, \langle\, a(\vec{r}\,)\, a^\dagger(0)\, \rangle$$

Interference of two dimensional condensates. Quasi long range order and the BKT transition



Above BKT transition

$$G(r) \sim e^{-r/\xi}$$

$$\langle |A_{\rm fr}|^2 \rangle \sim L_x L_y$$

$$\log \xi(T) \sim 1/\sqrt{T - T_{\rm KT}}$$

Below BKT transition

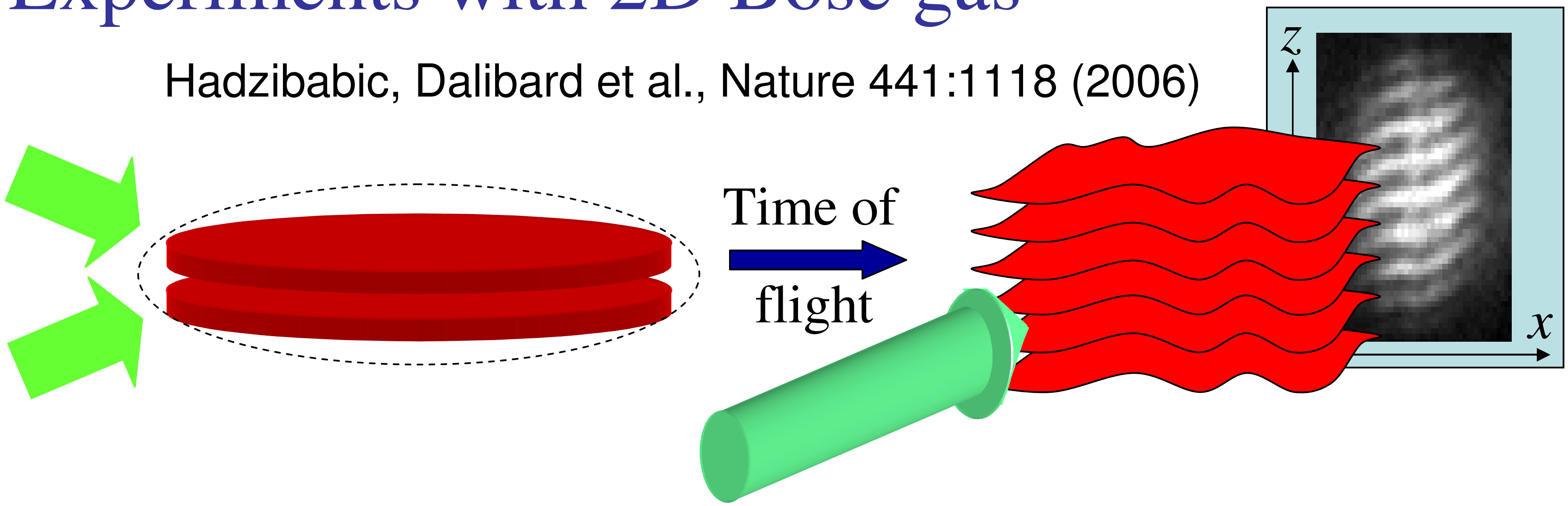
$$G(r) \sim \rho \left(\frac{\xi_h}{r} \right)^{\alpha}$$

$$\alpha(T) = \frac{m T}{2 \pi \rho_s(T) \hbar^2}$$

$$\langle |A_{\rm fr}|^2 \rangle \sim (L_x L_y)^{2-\alpha}$$

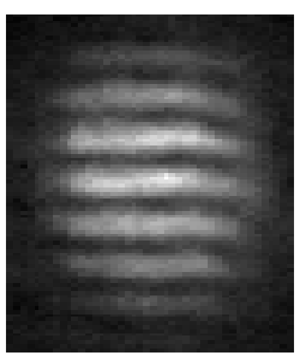
Experiments with 2D Bose gas

Hadzibabic, Dalibard et al., Nature 441:1118 (2006)

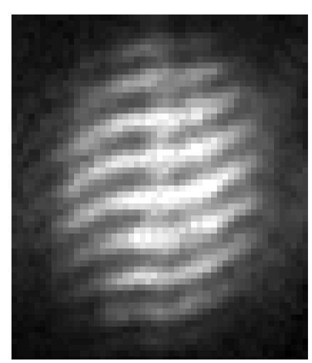


Typical interference patterns

low temperature



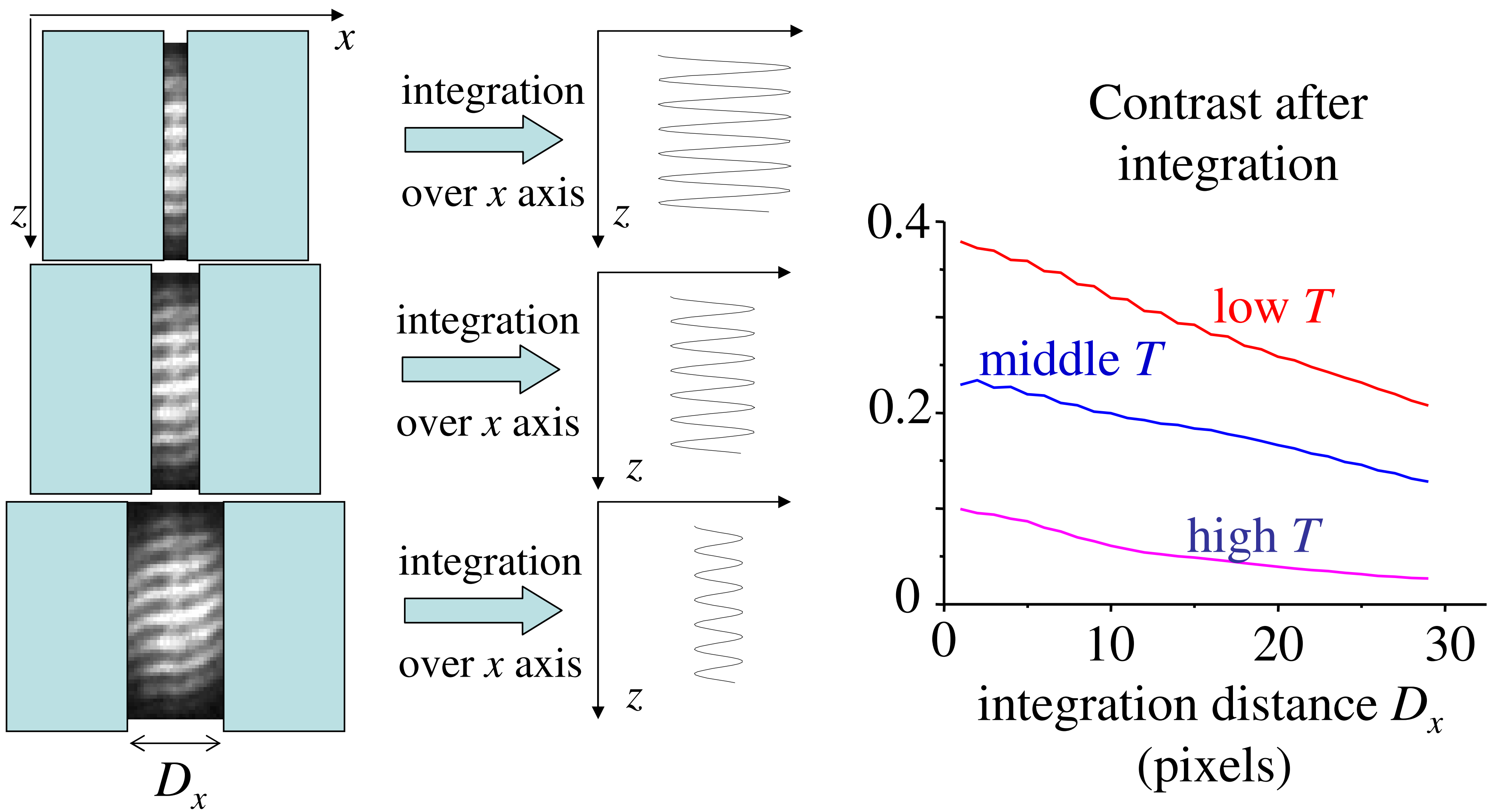
higher temperature



Figures courtesy of
Z. Hadzibabic and J. Dalibard

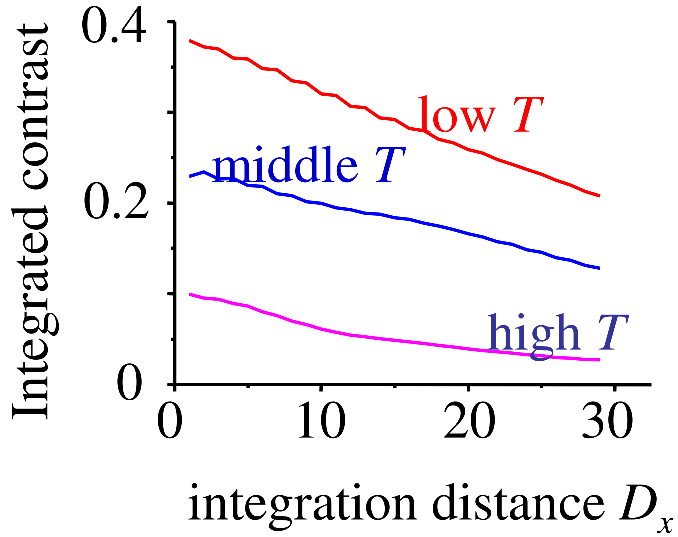
Experiments with 2D Bose gas

Hadzibabic et al., Nature 441:1118 (2006)



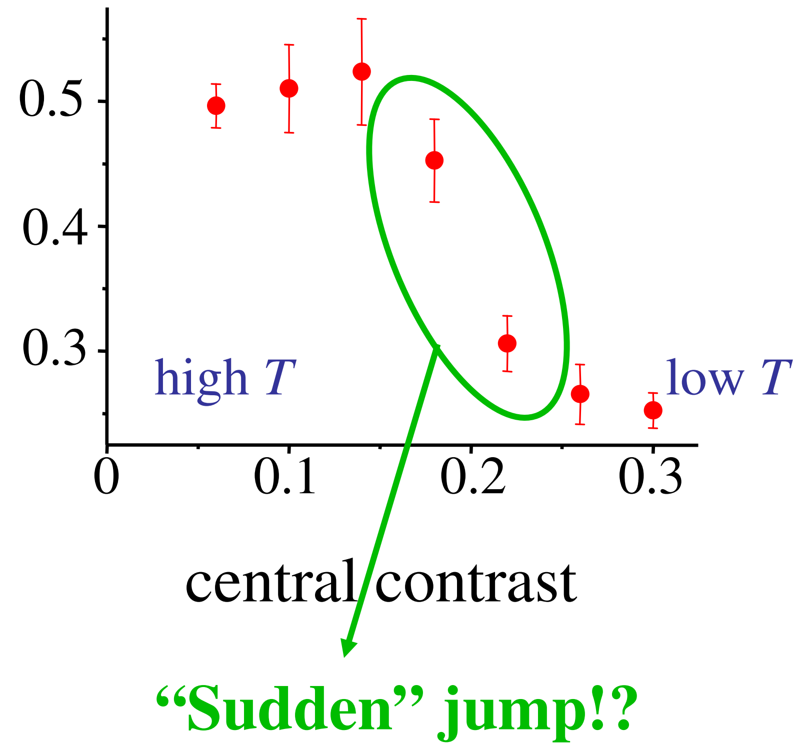
Experiments with 2D Bose gas

Hadzibabic et al., Nature 441:1118 (2006)



fit by: $$C^2 \sim \frac{1}{D_x}\int^{D_x}[g_1(0,x)]^2 dx \sim \left(\frac{1}{D_x}\right)^{2\alpha}$$

Exponent α



⇨ if $g_1(r)$ decays exponentially with $\ell_{\text{coh}} \ll D_x$: $\alpha = 1/2$

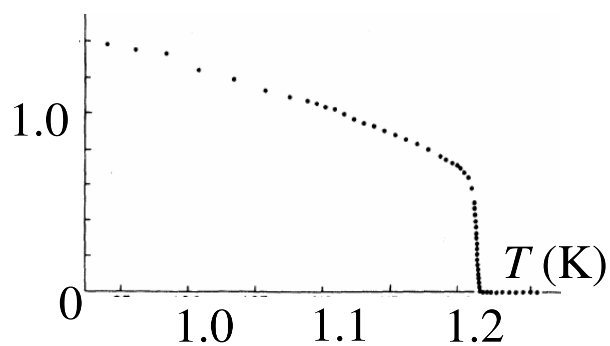
⇨ if $g_1(r)$ decays algebraically or exponentially with a large ℓ_{coh}:

$$\alpha < 1/2$$

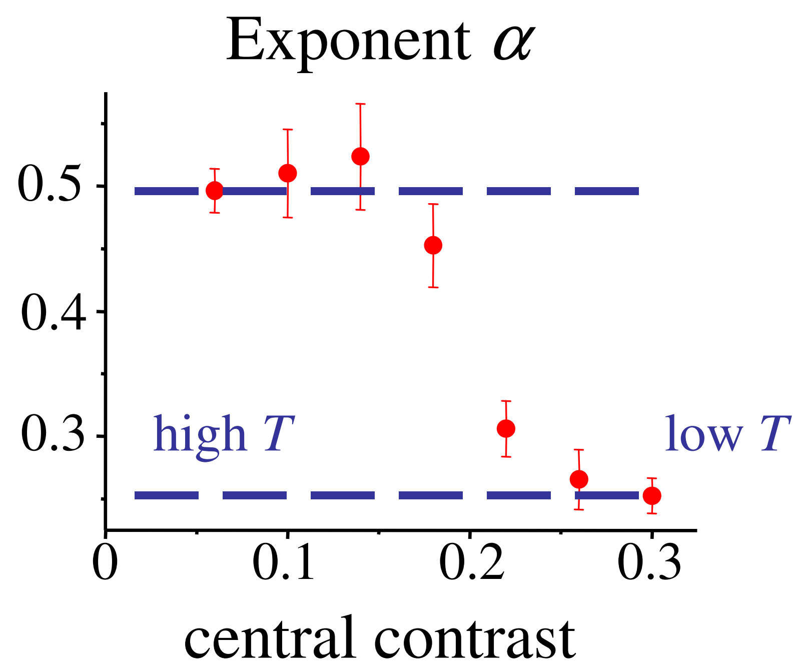
Experiments with 2D Bose gas

Hadzibabic et al., Nature 441:1118 (2006)

c.f. Bishop and Reppy



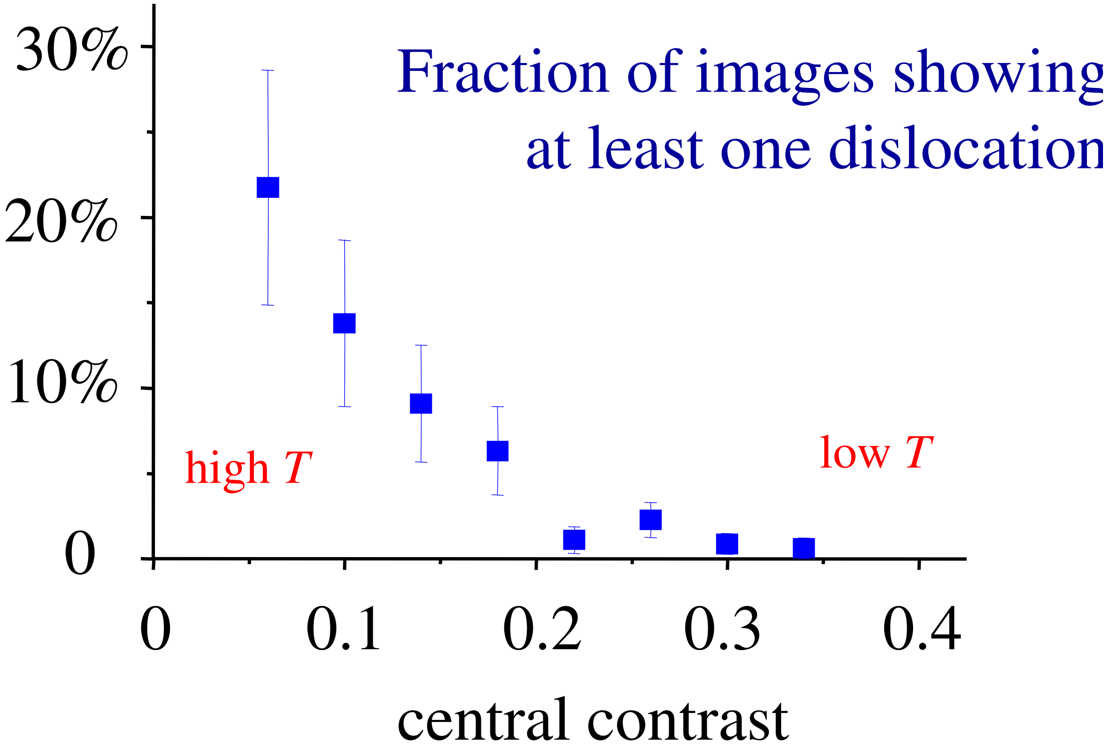
He experiments:
universal jump in
the superfluid density

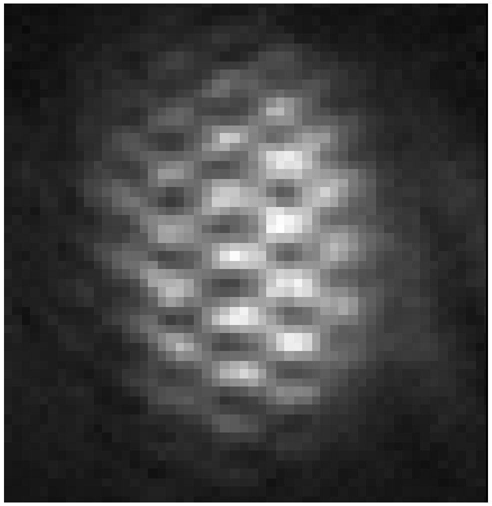


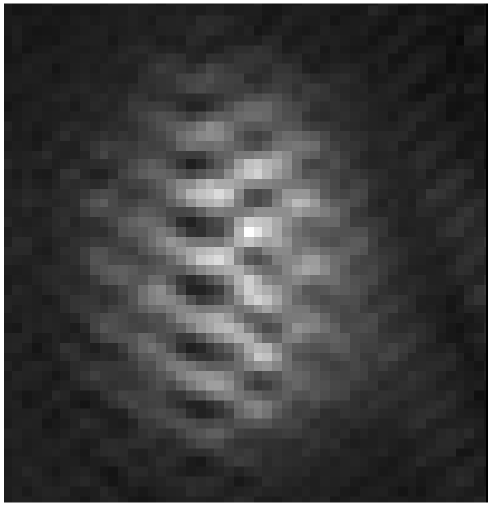
Ultracold atoms experiments:
jump in the correlation function.
BKT theory predicts α=1/4
just below the transition

Experiments with 2D Bose gas. Proliferation of thermal vortices

Hadzibabic et al., Nature 441:1118 (2006)





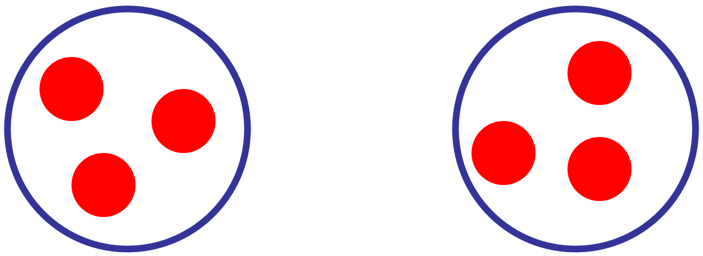


The onset of proliferation coincides with α shifting to 0.5!

Fundamental noise in interference experiments

Amplitude of interference fringes is a quantum operator. The measured value of the amplitude will fluctuate from shot to shot. We want to characterize not only the average but the fluctuations as well.

Shot noise in interference experiments



Interference with a finite number of atoms.
How well can one measure the amplitude of interference fringes in a single shot?

One atom:	No
Very many atoms:	Exactly
Finite number of atoms:	?

Consider higher moments of the interference fringe amplitude

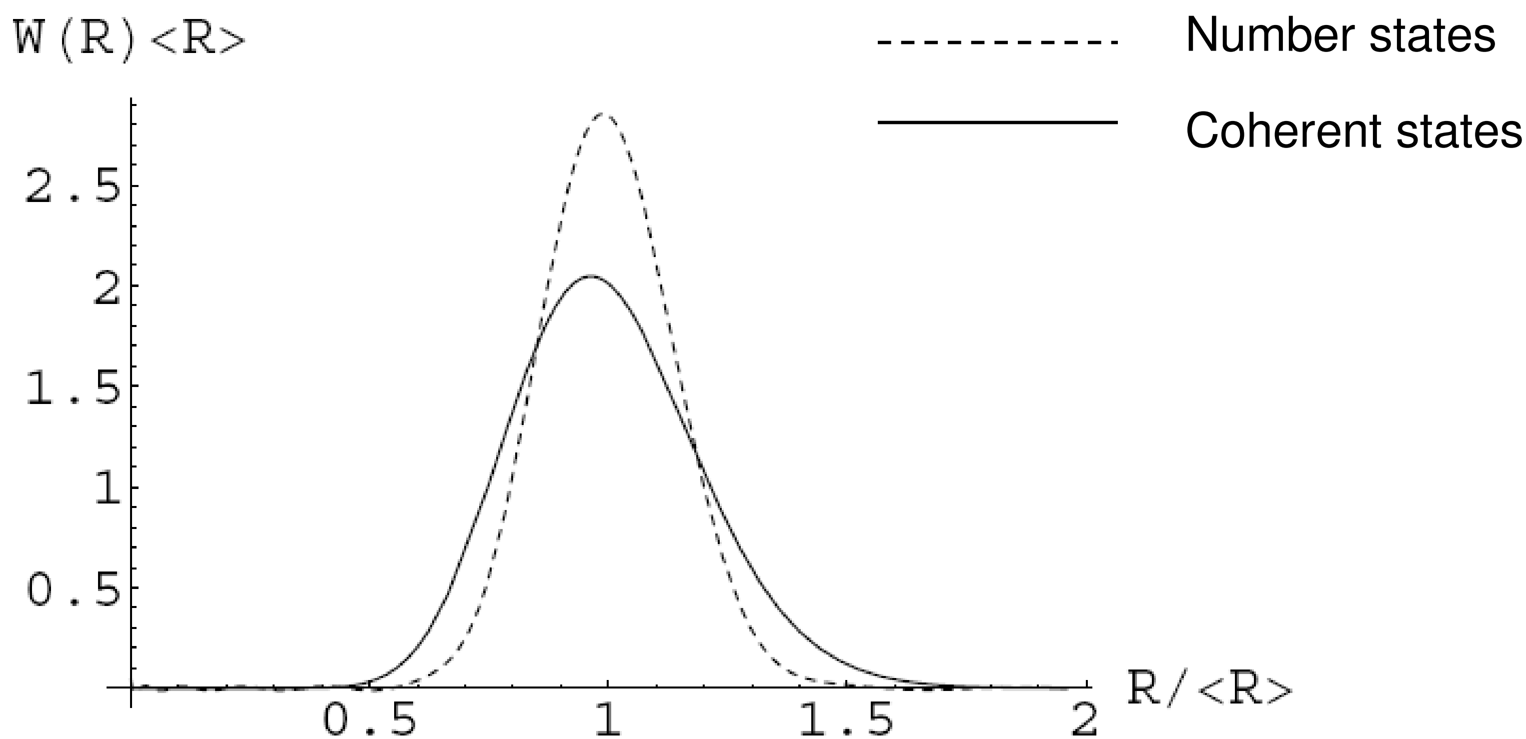
$\langle |A|^2 \rangle$, $\langle |A|^4 \rangle$, and so on

Obtain the entire distribution function of $|A|^2$

Shot noise in interference experiments

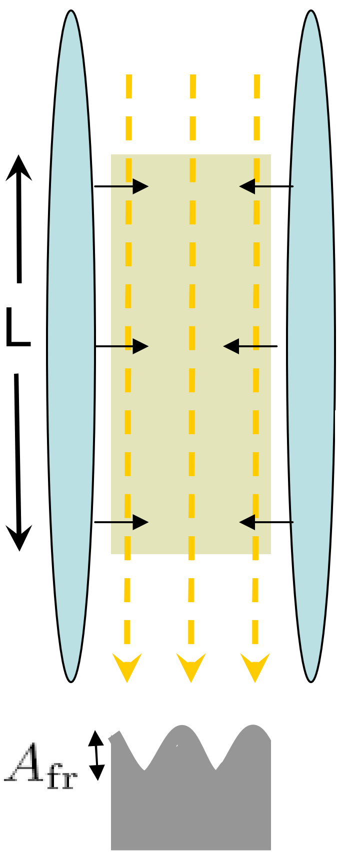
Polkovnikov, Europhys. Lett. 78:10006 (1997)
Imambekov, Gritsev, Demler, 2006 Varenna lecture notes

Interference of two condensates with 100 atoms in each cloud



Distribution function of fringe amplitudes for interference of fluctuating condensates

Gritsev, Altman, Demler, Polkovnikov, Nature Physics (2006)
Imambekov, Gritsev, Demler, cond-mat/0612011



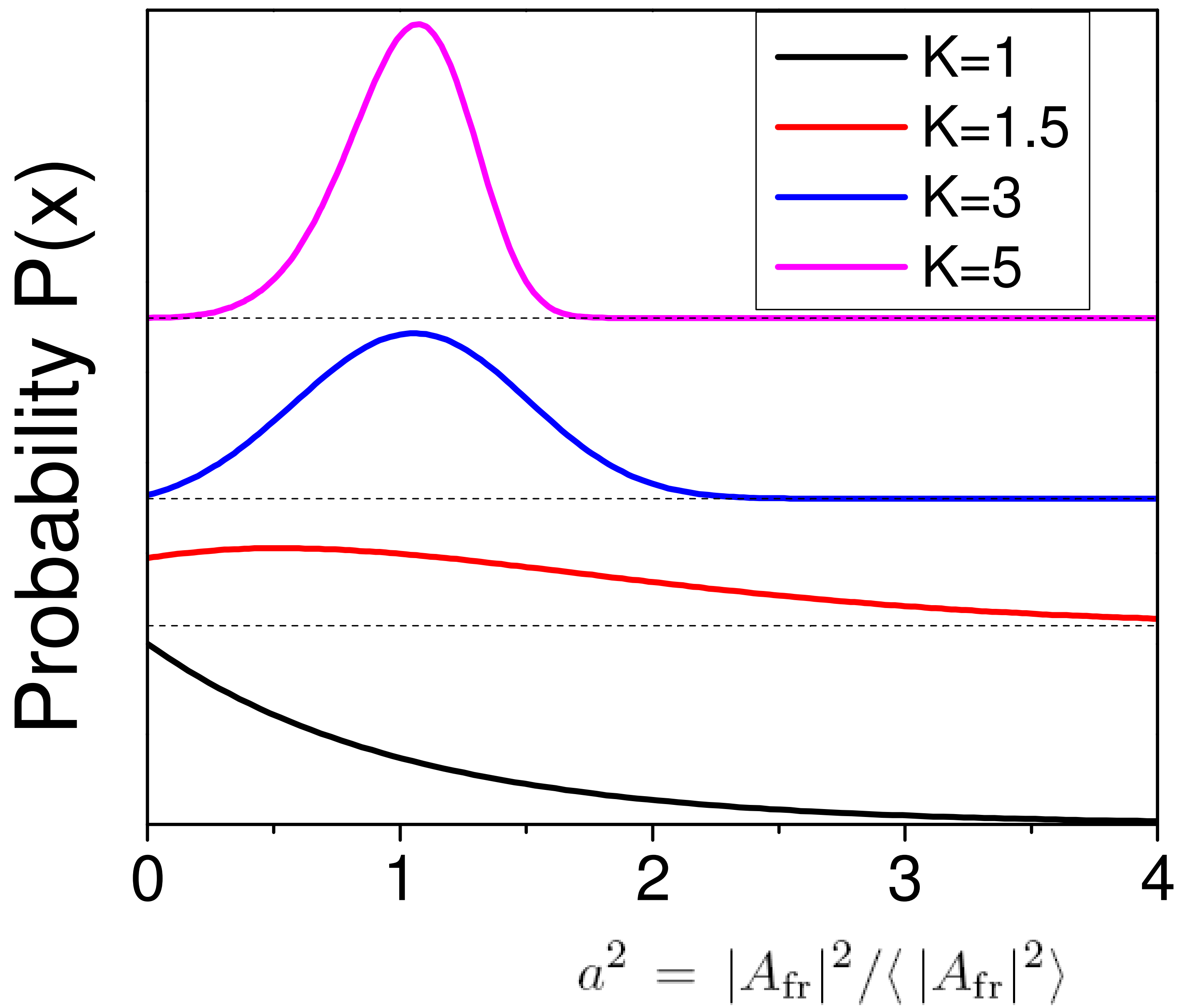
$A_{\rm fr}$ is a quantum operator. The measured value of $|A_{\rm fr}|$ will fluctuate from shot to shot.

$$\langle\, |A_{\rm fr}|^{2n} \rangle \;=\; \int_0^L dz_1 \,\ldots\, dz'_n \,|\langle\, a^\dagger(z_1) \,\ldots\, a^\dagger(z_n)\, a(z'_1) \,\ldots\, a(z'_n)\,\rangle|^2$$

Higher moments reflect higher order correlation functions

We need the full distribution function of $|A_{\rm fr}|$

Interference of 1d condensates at T=0. Distribution function of the fringe contrast

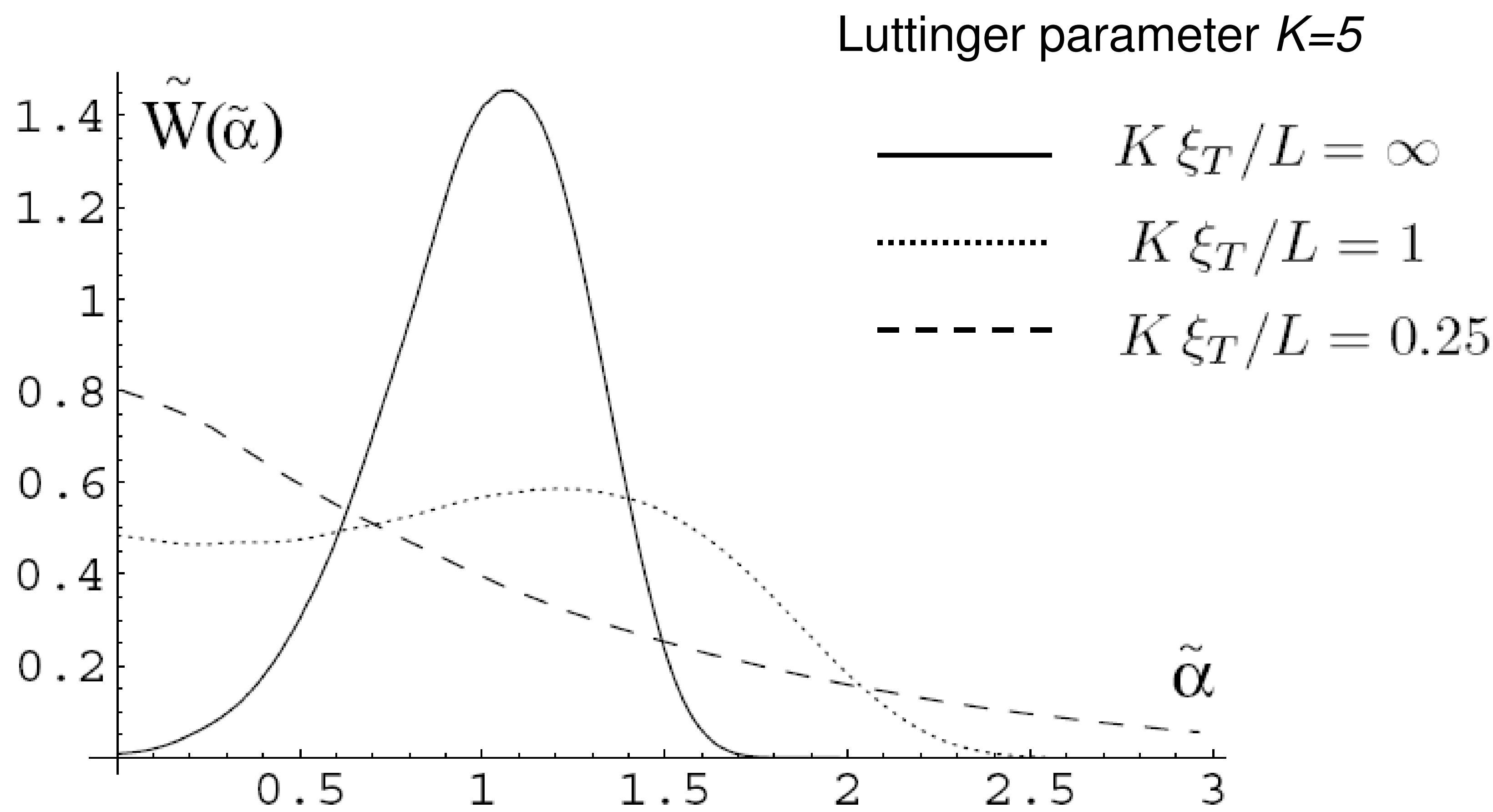


Narrow distribution for $K \to \infty$ Approaches Gumbel distribution.

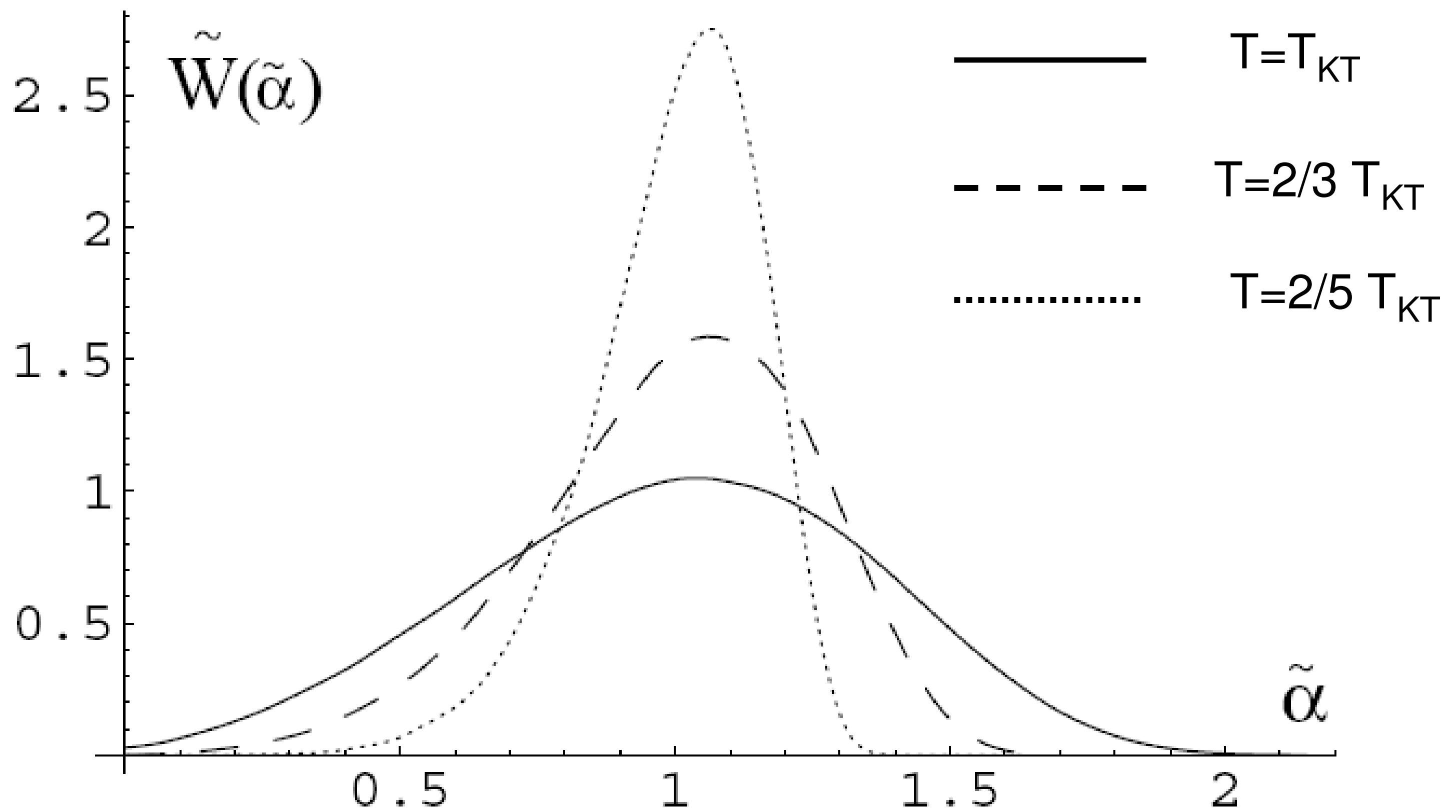
Width $\sigma = \dfrac{\pi}{6K}$

Wide Poissonian distribution for $K \to 1$

Interference of 1d condensates at finite temperature. Distribution function of the fringe contrast

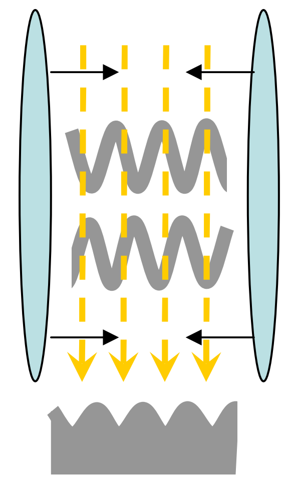


Interference of 2d condensates at finite temperature. Distribution function of the fringe contrast



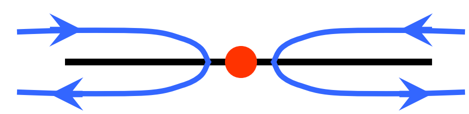
From visibility of interference fringes to other problems in physics

Interference between interacting 1d Bose liquids. Distribution function of the interference amplitude

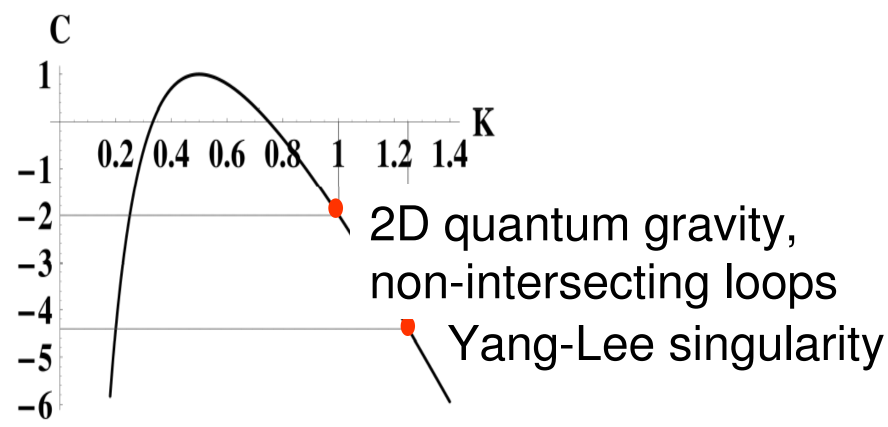


$A_{\rm fr}$ is a quantum operator. The measured value of $|A_{\rm fr}|$ will fluctuate from shot to shot.
How to predict the distribution function of $|A_{\rm fr}|$

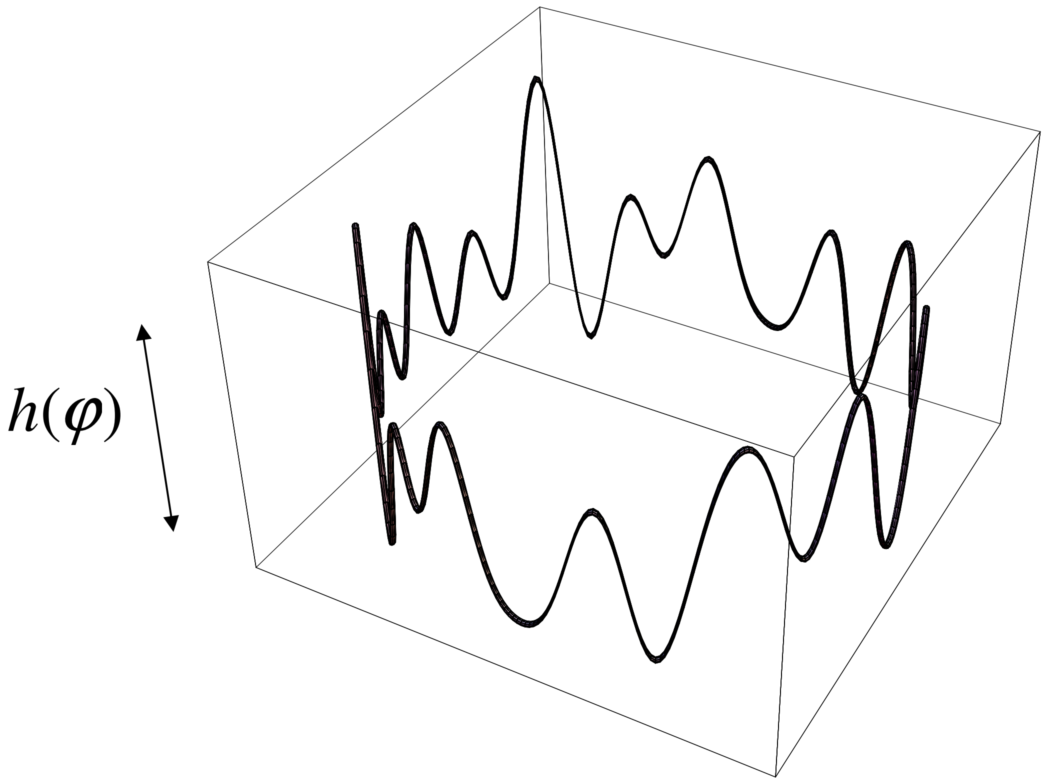
Quantum impurity problem: interacting one dimensional electrons scattered on an impurity



Conformal field theories with negative central charges: 2D quantum gravity, non-intersecting loop model, growth of random fractal stochastic interface, high energy limit of multicolor QCD, …



Fringe visibility and statistics of random surfaces



Fringe visibility

$\Updownarrow$

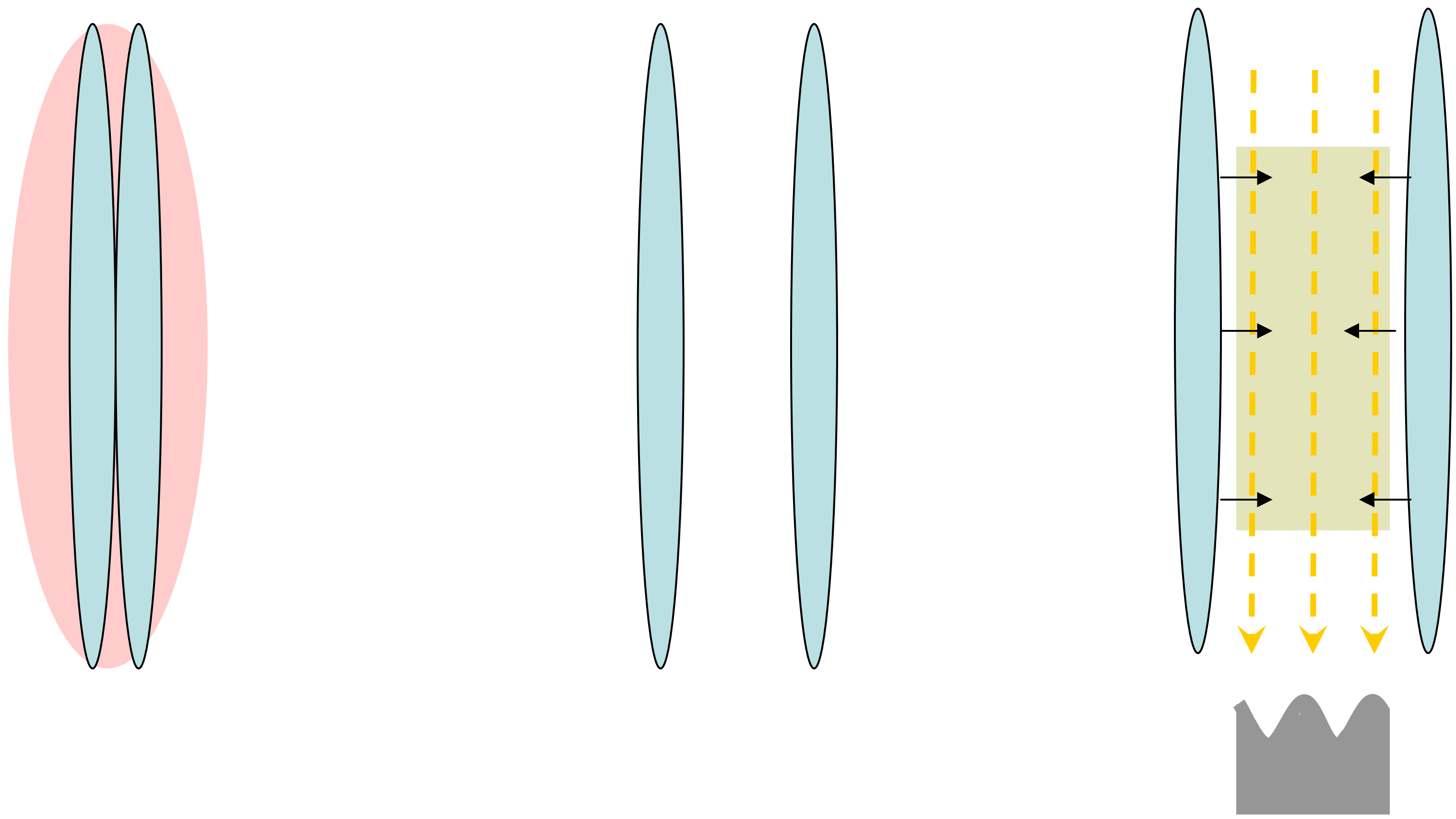
Roughness $= \int h(\varphi)^2 \, d\varphi$

Proof of the Gumbel distribution of interfernece fringe amplitude for 1d weakly interacting bosons relied on the known relation between 1/f Noise and Extreme Value Statistics

T.Antal et al. Phys.Rev.Lett. 87, 240601(2001)

Non-equilibrium coherent dynamics of low dimensional Bose gases probed in interference experiments

Studying dynamics using interference experiments. Quantum and thermal decoherence

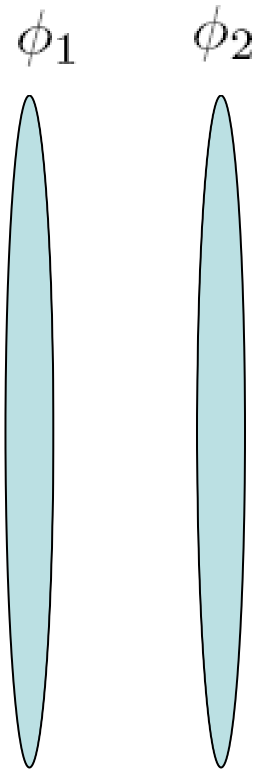


Prepare a system by splitting one condensate

Take to the regime of zero tunneling

Measure time evolution of fringe amplitudes

Relative phase dynamics



$$\mathcal{H}_0 = \int dx\ \left[g\, n_1^2(x) + \rho\,(\partial_x \phi_1)^2\right] + \int dx\ \left[g\, n_2^2(x) + \rho\,(\partial_x \phi_2)^2\right]$$

Interference experiments measure only the relative phase

$$\phi = \phi_1 - \phi_2 \qquad\qquad \Delta n = (\, n_1 - n_2 \,)/2$$

Relative phase

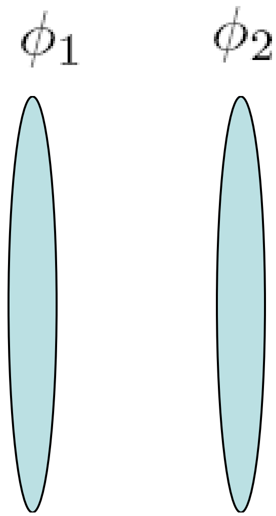
Particle number imbalance

$$[\Delta n(x_1)\,, \phi(x_2)\,] = -i\,\delta(x_1 - x_2)$$

Conjugate variables

Earlier work was based on
a single mode approximation,
e.g. Gardner, Zoller; Leggett

Relative phase dynamics



$$\boxed{\mathcal{H} = \int d^d r \, [\, \frac{g}{2} \, (\Delta n)^2 \, + \, \frac{\rho}{2} \, (\nabla \phi)^2 \,]}$$

$$\phi = \phi_1 - \phi_2$$

$$\Delta n = (\, n_1 - n_2 \,)/2$$

Hamiltonian can be diagonalized
in momentum space

A collection of harmonic oscillators
with $\omega_q = \sqrt{g\rho} \, | \, q \, |$

Need to solve dynamics of harmonic
oscillators at finite T

Coherence $\langle \, \Psi(t) \, | \, e^{i\phi} \, | \, \Psi(t) \, \rangle \, = \, e^{-\frac{1}{2} \sum_q \langle \phi_q^2(t) \rangle}$

Relative phase dynamics

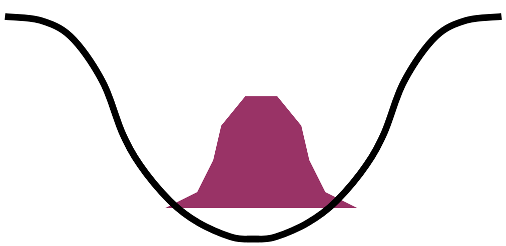
High energy modes, $\hbar\omega_{\mathrm{osc}} > k_{\mathrm{B}}T$, quantum dynamics

Low energy modes, $\hbar\omega_{\mathrm{osc}} < k_{\mathrm{B}}T$, classical dynamics

Combining all modes

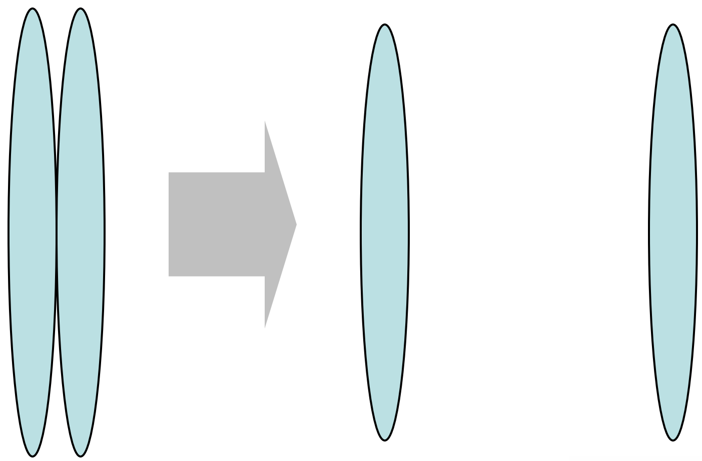
$t < \frac{h}{k_{\mathrm{B}}T}$ Quantum dynamics

$t > \frac{h}{k_{\mathrm{B}}T}$ Classical dynamics



For studying dynamics it is important to know the initial width of the phase

Relative phase dynamics



$$\frac{N}{2} \pm \delta N \qquad \frac{N}{2} \mp \delta N$$

Naive estimate

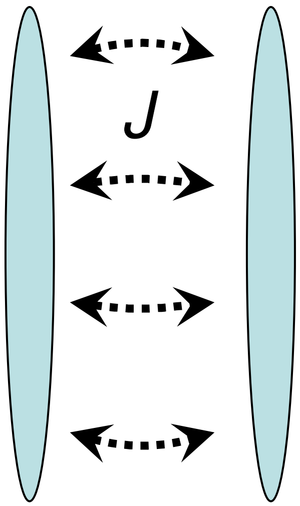
$$\delta N \sim \sqrt{N}$$

Long Phase Coherence Time and Number Squeezing of two Bose-Einstein Condensates on an Atom Chip

G.-B. Jo, Y. Shin, S. Will, T. A. Pasquini, M. Saba, W. Ketterle, and D. E. Pritchard*
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics,
Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

M. Vengalattore, M. Prentiss
MIT-Harvard Center for Ultracold Atoms, Jefferson Laboratory,
Physics Department, Harvard University, Cambridge, MA 02138, USA
(Dated: August 27, 2006)

Relative phase dynamics



Separating condensates at finite rate

Instantaneous Josephson frequency

$$\omega_{\mathrm{J}} = \sqrt{U\,J}$$

Adiabatic regime $\dot{\omega}_{\mathrm{J}} < \omega_{\mathrm{J}}^2$

Instantaneous separation regime $\dot{\omega}_{\mathrm{J}} > \omega_{\mathrm{J}}^2$

$$\mathcal{H} = \frac{U}{2}(\Delta n)^2 - J\cos\phi$$

$$J(t) = J_0\, e^{-t/\tau_{\mathrm{s}}}$$

Adiabaticity breaks down when $\omega_{\mathrm{J}} \sim 1/\tau_{\mathrm{s}}$

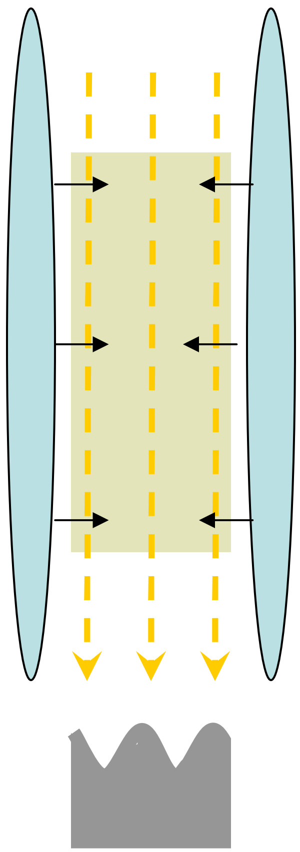
Charge uncertainty at this moment

$$U\,\delta N^2 \sim \omega_{\mathrm{J}} \sim 1/\tau_{\mathrm{s}}$$

Squeezing factor $$\frac{\delta N}{\sqrt{N}} \sim \sqrt{\frac{1}{N\,U\,\tau_{\mathrm{s}}}} \sim \sqrt{\frac{1}{\mu\,\tau_{\mathrm{s}}}}$$

Relative phase dynamics

Burkov, Lukin, Demler, cond-mat/0701058



Quantum regime $\frac{h}{\mu} < t < \frac{h}{k_B T}$

1D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} \, e^{-t/2\pi K \tau_s}$

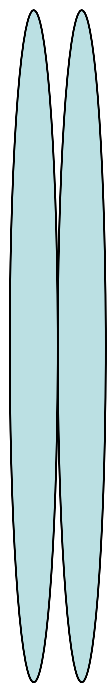
2D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} \left(\frac{t_0}{t} \right)^{1/16 T_{KT} \tau_s}$

Classical regime $t > \frac{h}{k_B T}$

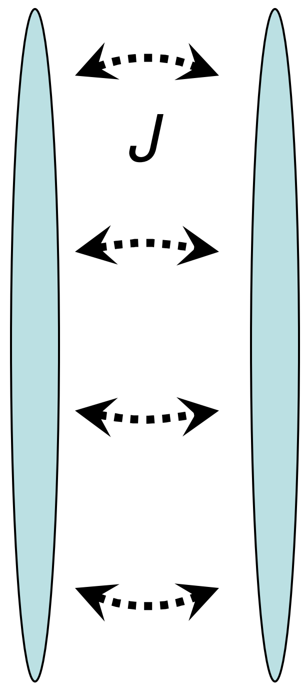
1D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\left(\frac{t}{t_T}\right)^{2/3}}$ $\quad t_T \sim \frac{\mu K}{T^2}$

2D systems $\langle e^{i\phi(t)} \rangle \sim \left(\frac{t_0}{t} \right)^{\frac{T}{8 T_{KT}}}$

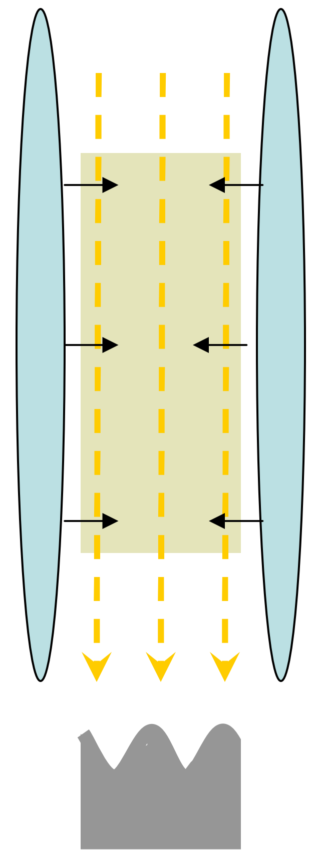
Quantum dynamics of coupled condensates. Studying Sine-Gordon model in interference experiments



Prepare a system by splitting one condensate

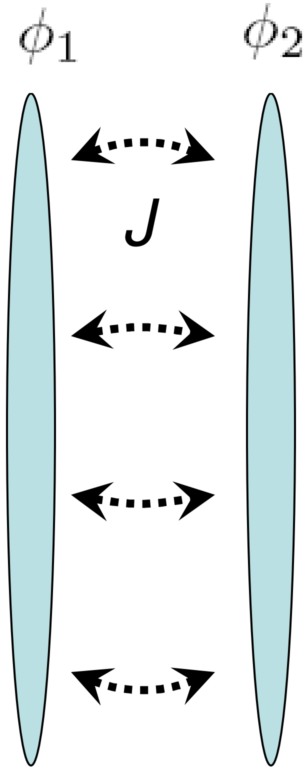


Take to the regime of finite tunneling. System described by the quantum Sine-Gordon model



Measure time evolution of fringe amplitudes

Coupled 1d systems



$$\mathcal{H}_0 = \int dx\, \left[g\, n_1^2(x) + \rho\, (\partial_x \phi_1)^2\right] + \int dx\, \left[g\, n_2^2(x) + \rho\, (\partial_x \phi_2)^2\right]$$

Interactions lead to phase fluctuations within individual condensates

$$\mathcal{H}_{tun} = -J \int dx\, \cos(\phi_1 - \phi_2)$$

Tunneling favors aligning of the two phases

$$\phi = \phi_1 - \phi_2$$

Interference experiments measure the relative phase

$$\Delta n = (\, n_1 - n_2\,)/2$$

$$\mathcal{H}[\phi] = \int dx\, d\tau \left[\frac{1}{2K}(\Delta n)^2 + \frac{K}{2}\,(\partial_x \phi)^2\right] - J \int dx\, d\tau\, \cos\phi$$

Quantum Sine-Gordon model

Hamiltonian

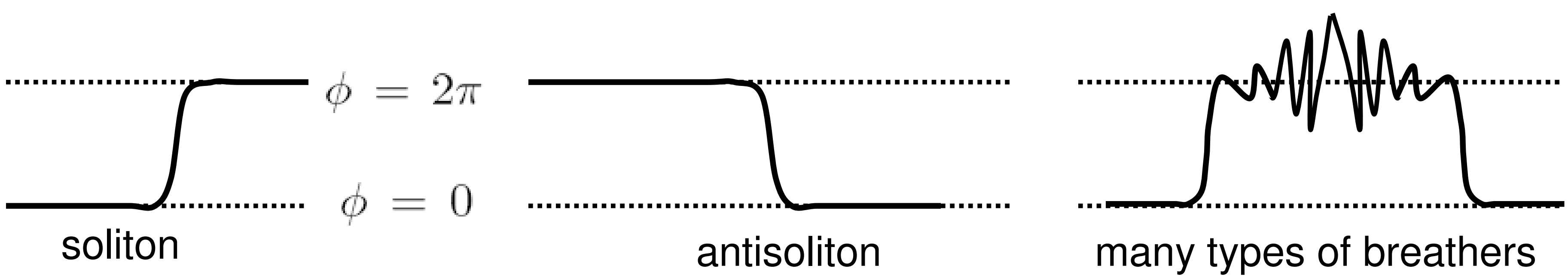
$$\mathcal{H}[\phi] = \int dx\, d\tau \left[\frac{1}{2K} (\Delta n)^2 + \frac{K}{2} (\partial_x \phi)^2 \right] - J \int dx\, d\tau\ \cos\phi$$

Imaginary time action

$$\mathcal{S}[\phi] = \frac{K}{2} \int dx\, d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right] + J \int dx\, d\tau\ \cos\phi$$

Quantum Sine-Gordon model is exactly integrable

Excitations of the quantum Sine-Gordon model



Dynamics of quantum sine-Gordon model

Hamiltonian formalism

$$\mathcal{H}[\phi] = \int dx\, d\tau \left[\frac{1}{2K} (\Delta n)^2 + \frac{K}{2} (\partial_x \phi)^2 \right] - J \int dx\, d\tau\, \cos\phi$$

Initial state $\phi(t=0) = 0$

Quantum action in space-time

$$\mathcal{S}[\phi] = \frac{K}{2} \int dx\, d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right] + J \int dx\, d\tau\, \cos\phi$$

Initial state provides a boundary condition at *t=0*

Solve as a boundary sine-Gordon model

Boundary sine-Gordon model

Exact solution due to Ghoshal and Zamolodchikov (93)
Applications to quantum impurity problem: Fendley, Saleur, Zamolodchikov, Lukyanov,…

$$S = \int_{x\tau} \left[\frac{K}{2} \left(\frac{\partial \phi}{\partial \tau}\right)^2 + \frac{K}{2} \left(\frac{\partial \phi}{\partial x}\right)^2 - m \cos \phi \right] - M \int_{\tau} cos \frac{\phi(x=0)}{2}$$

Limit $M \to \infty$ enforces boundary condition $\phi(x=0) = 0$

Sine-Gordon
+ boundary condition in space

quantum impurity problem

Boundary Sine-Gordon Model

space and time enter equivalently

Sine-Gordon
+ boundary condition in time

two coupled 1d BEC

Boundary sine-Gordon model

Initial state is a generalized squeezed state

$$|\,\psi(t=0)\,\rangle = e^{\{\sum_\gamma g_\gamma A^\dagger_\gamma(\theta=0)+\sum_{\alpha\beta}\int_\theta K_{\alpha\beta}(\theta)A^\dagger_\alpha(-\theta)A^\dagger_\beta(\theta)\,\}}|\,\mathrm{vac}\,\rangle$$

$A^\dagger_\alpha(\theta)$ creates solitons, breathers with rapidity θ

$A^\dagger_\gamma(\theta=0)$ creates even breathers only

Matrix $K_{\alpha\beta}(\theta)$ and g_γ are known from the exact solution of the boundary sine-Gordon model

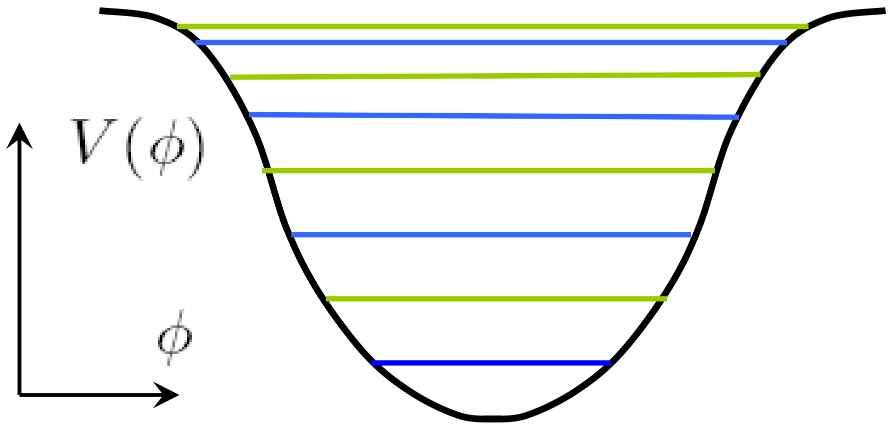
Time evolution $A^\dagger_\alpha(\theta,t)=A^\dagger_\alpha(\theta)\,e^{-iE_\alpha(\theta)t}$

Coherence $\langle\,\psi(t)\,|\,e^{i\phi}\,|\,\psi(t)\,\rangle$

Matrix elements can be computed using form factor approach
Smirnov (1992), Lukyanov (1997)

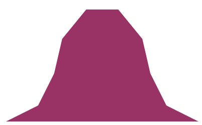
Quantum Josephson Junction

$$\boxed{\mathcal{H} = \frac{g}{2}\, n^2 \;-\; J\, \cos\phi}$$



Limit of quantum sine-Gordon model when spatial gradients are forbidden

Initial state



$$|\,\psi(t=0)\,\rangle = \sum_n C_{2n}\,|\,2n\,\rangle$$

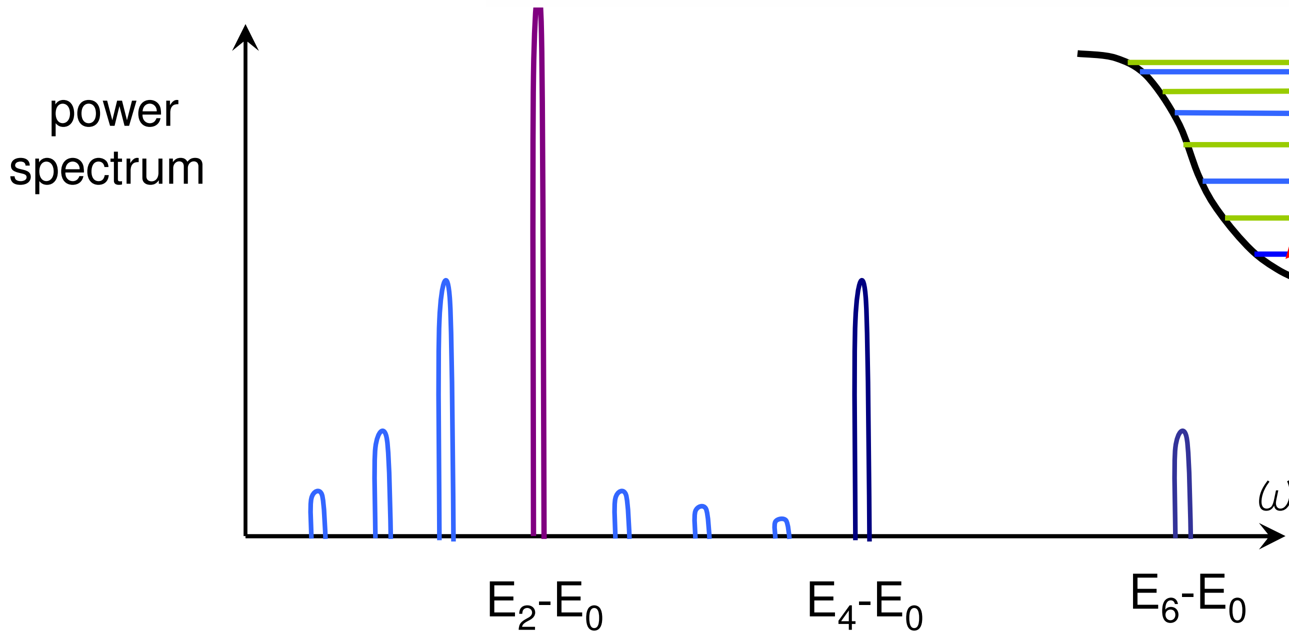
Eigenstates of the quantum Jos. junction Hamiltonian are given by Mathieu's functions

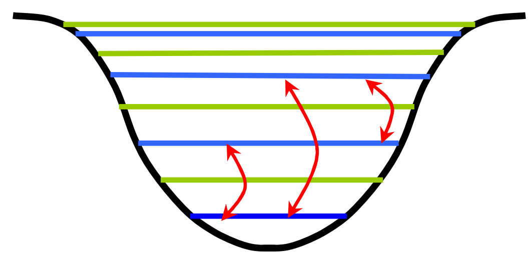
Time evolution $$|\,\psi(t)\,\rangle = \sum_n C_{2n}\, e^{-iE_{2n}t}\,|\,2n\,\rangle$$

Coherence $$\langle\,\psi(t)\,|\,e^{i\phi}\,|\,\psi(t)\,\rangle$$

Dynamics of quantum Josephson Junction

Power spectrum $$P(\omega) = \left| \int_t e^{i\omega t} \langle \psi(t) \,|\, e^{i\phi} \,|\, \psi(t) \rangle \right|^2$$





Main peak $\omega = E_2 - E_0$

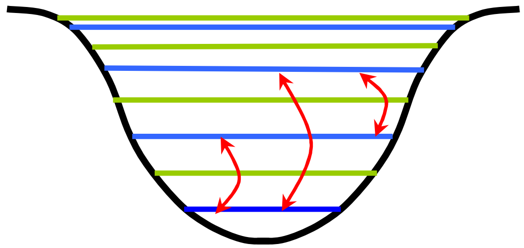
"Higher harmonics" $\omega = E_4 - E_0 \,,\, E_6 - E_0 \,,\, \ldots$

Smaller peaks $\omega = E_{2n+2} - E_{2n} \,,\, E_{2n+4} - E_{2n} \,,\, \ldots$

Dynamics of quantum sine-Gordon model

$$|\,\psi(t)\,\rangle = e^{\{\sum_\gamma g_\gamma A^\dagger_\gamma(\theta=0)+\sum_{\alpha\beta}\int_\theta K_{\alpha\beta}(\theta)A^\dagger_\alpha(-\theta)A^\dagger_\beta(\theta)\,\}}|\,\mathrm{vac}\,\rangle$$

Coherence $\langle\,\psi(t)\,|\,e^{i\phi}\,|\,\psi(t)\,\rangle$



Main peak $\displaystyle\int_\theta \langle\,\mathrm{vac}\,|\,e^{i\phi}\,|\,\mathrm{B}_1(\theta)\,\mathrm{B}_1(-\theta)\,\rangle$

"Higher harmonics" $\displaystyle\int_\theta \langle\,\mathrm{vac}\,|\,e^{i\phi}\,|\,\mathrm{B}_n(\theta)\,\mathrm{B}_n(-\theta)\,\rangle$

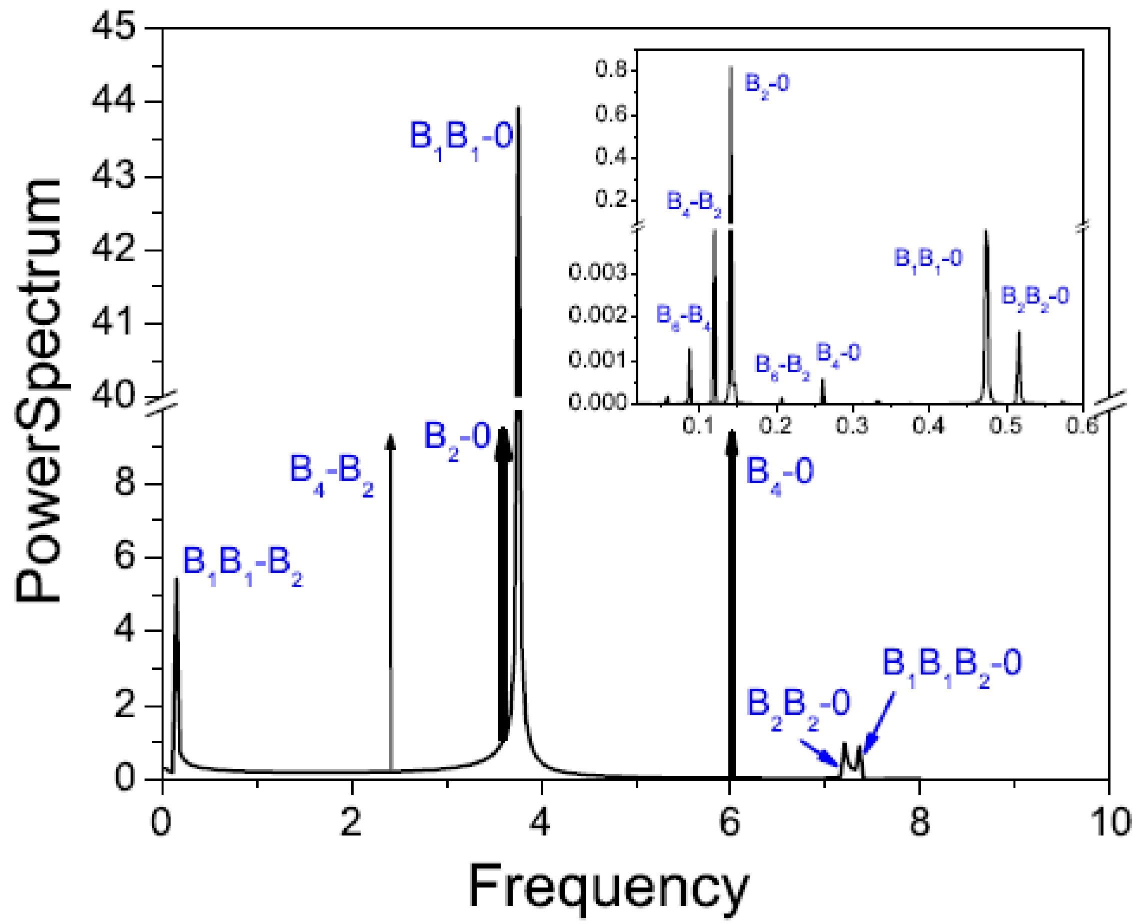
Smaller peaks $\displaystyle\int_{\theta\theta'} \langle\,\mathrm{B}_m(\theta')\,\mathrm{B}_m(-\theta')\,|\,e^{i\phi}\,|\,\mathrm{B}_n(\theta)\,\mathrm{B}_n(-\theta)\,\rangle$

Sharp peaks $\langle\mathrm{vac}\,|\,e^{i\phi}\,|\,B_{2n}(\theta=0)\,\rangle$

Dynamics of quantum sine-Gordon model

Gritsev, Demler, Lukin, Polkovnikov, cond-mat/0702343

Power spectrum $P(\omega) = \left| \int_t e^{i\omega t} \langle\, \psi(t) \,|\, e^{i\phi} \,|\, \psi(t) \,\rangle \,\right|^2$



A combination of broad features and sharp peaks. Sharp peaks due to collective many-body excitations: breathers

Conclusions

Interference of extended condensates can be used to probe equilibrium correlation functions in one and two dimensional systems

Interference experiments can be used to study non-equilibrium dynamics of low dimensional superfluids and perform spectroscopy of the quantum sine-Gordon model