Strongly correlated systems: from electronic materials to cold atoms

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Collaborators:

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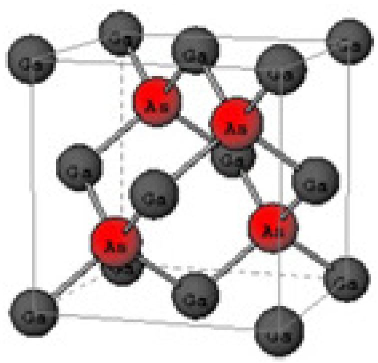
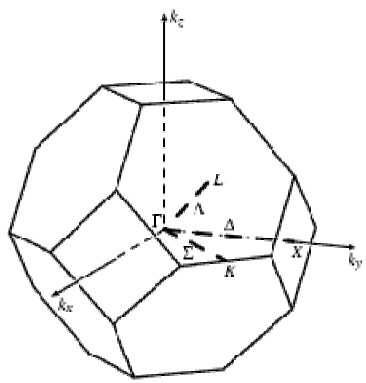
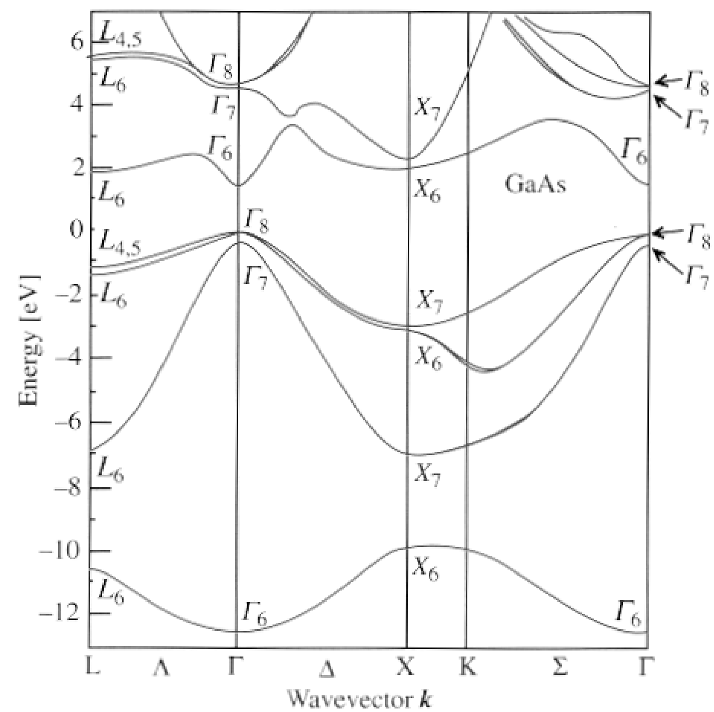
“Conventional” solid state materials

Bloch theorem for non-interacting electrons in a periodic potential

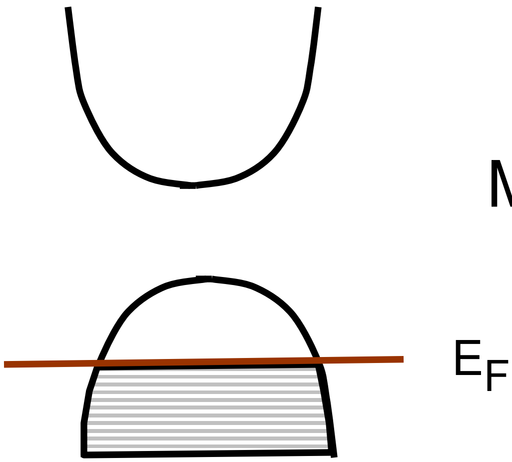




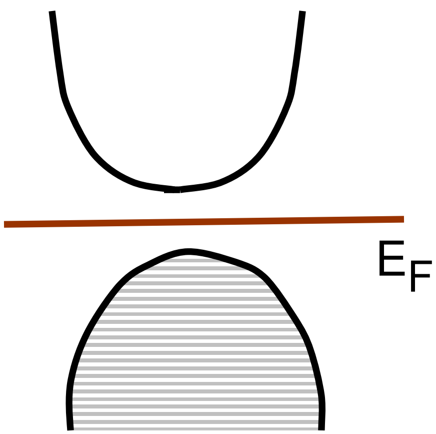


Electronic band structure of GaAs

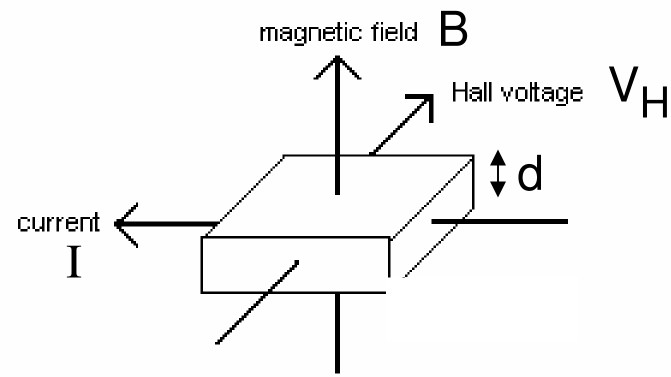
Consequences of the Bloch theorem



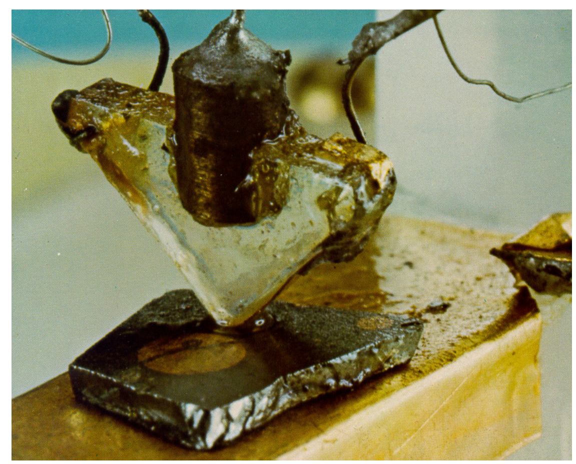
Metals



Insulators
and
Semiconductors



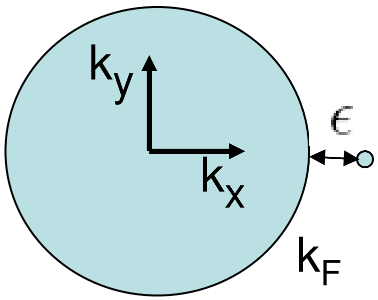
$$V_H = \frac{I\,B}{n\,e^*\,d}$$



First semiconductor transistor

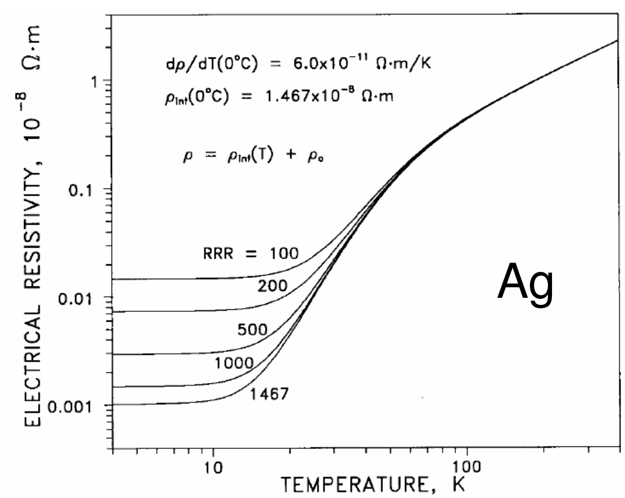
“Conventional” solid state materials

Electron-phonon and electron-electron interactions are irrelevant at low temperatures

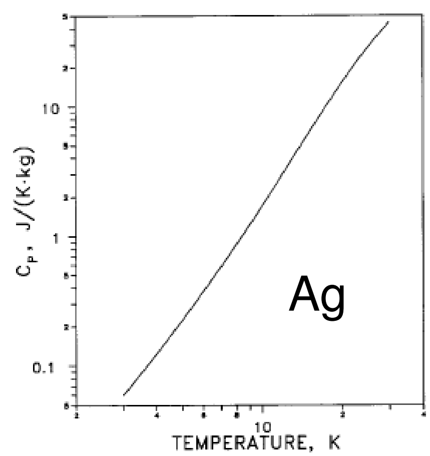


$$\frac{1}{\tau_{\mathrm{e-e}}} \sim \epsilon^2 \qquad \frac{1}{\tau_{\mathrm{e-ph}}} \sim \epsilon^3$$

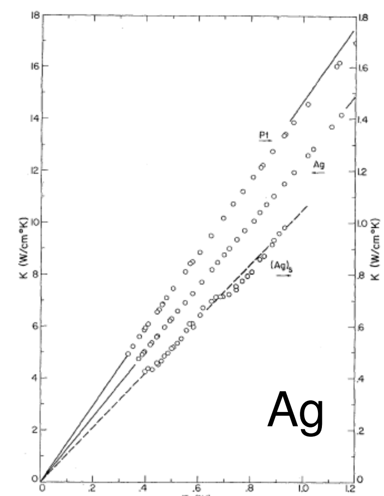
Landau Fermi liquid theory: when frequency and temperature are smaller than E_F electron systems are equivalent to systems of non-interacting fermions



$$\rho = \rho_0 + aT^2$$

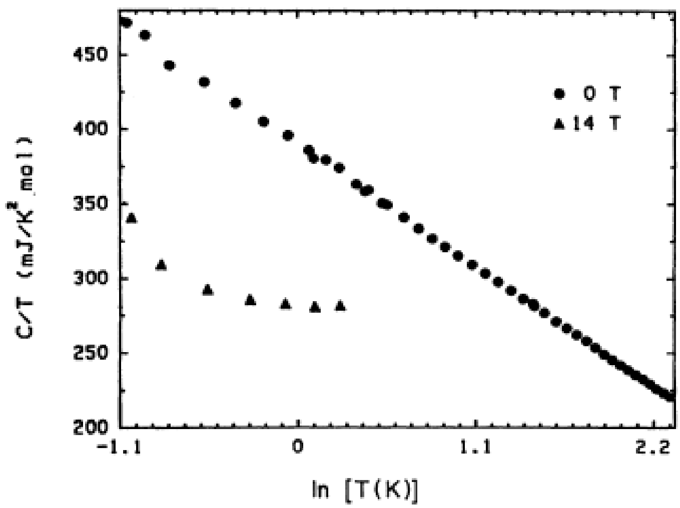


$$c/T = \mathrm{const}$$



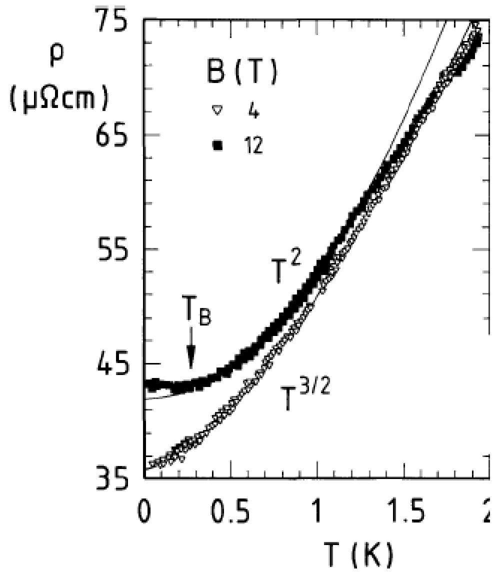
$$\kappa/T = \mathrm{const}$$

Non Fermi liquid behavior in novel quantum materials

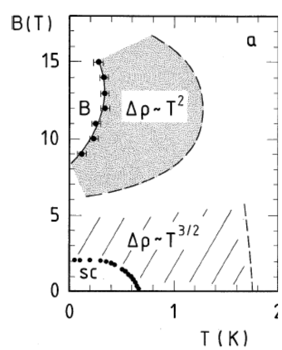


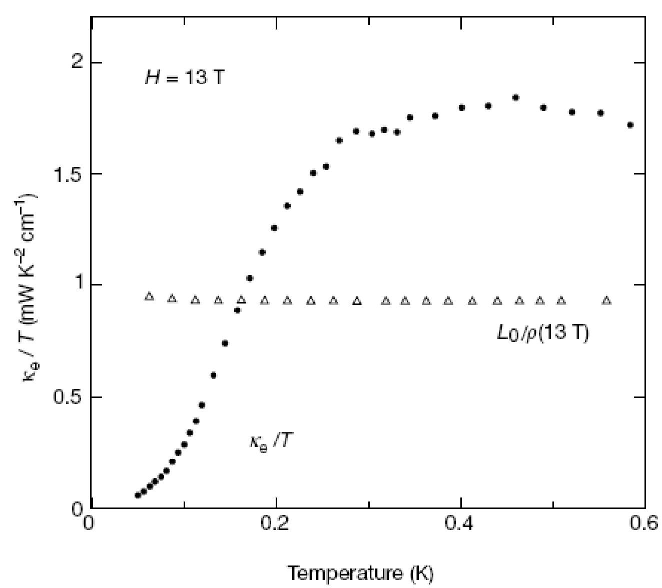
$UCu_{3.5}Pd_{1.5}$
Andraka, Stewart,
PRB 47:3208 (93)

Violation of the
Wiedemann-Franz law
in high Tc superconductors
Hill et al., Nature 414:711 (2001)



$CeCu_2Si_2$. Steglich et al.,
Z. Phys. B 103:235 (1997)





Puzzles of high temperature superconductors

Unusual "normal" state

Maple, JMMM 177:18 (1998)

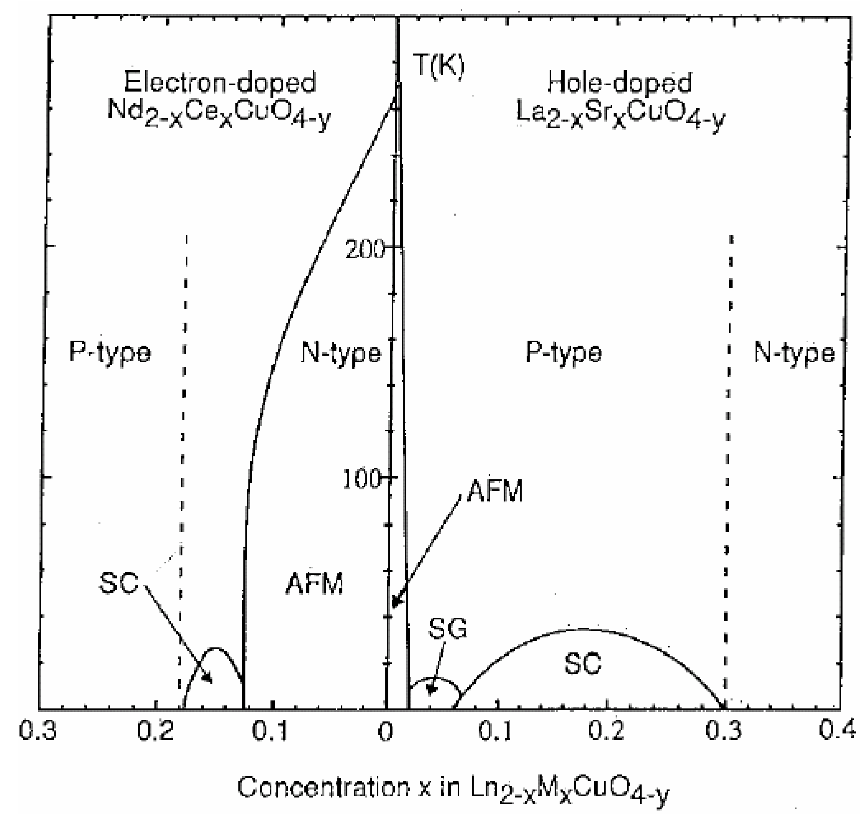
Resistivity, opical conductivity,
Lack of sharply defined quasiparticles,
Nernst effect

Mechanism of Superconductivity

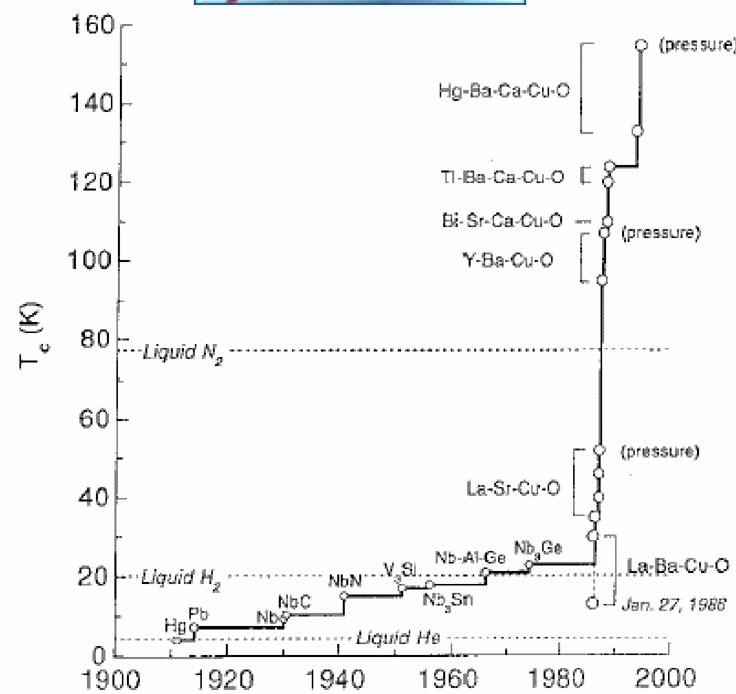
High transition temperature,
retardation effect, isotope effect,
role of elecron-electron
and electron-phonon interactions

Competing orders

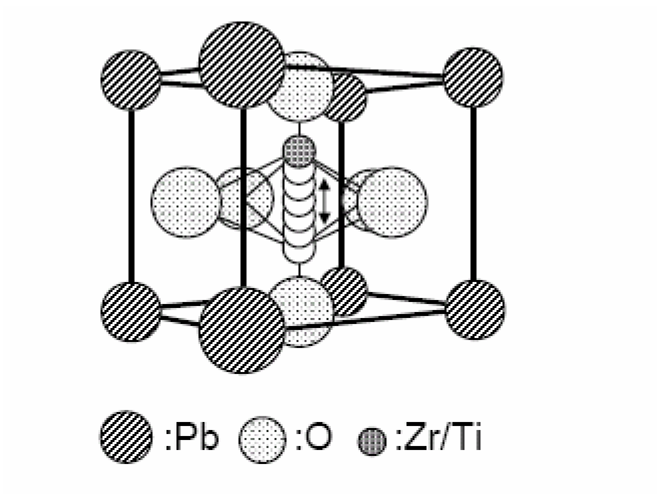
Role of magnetsim, stripes,
possible fractionalization

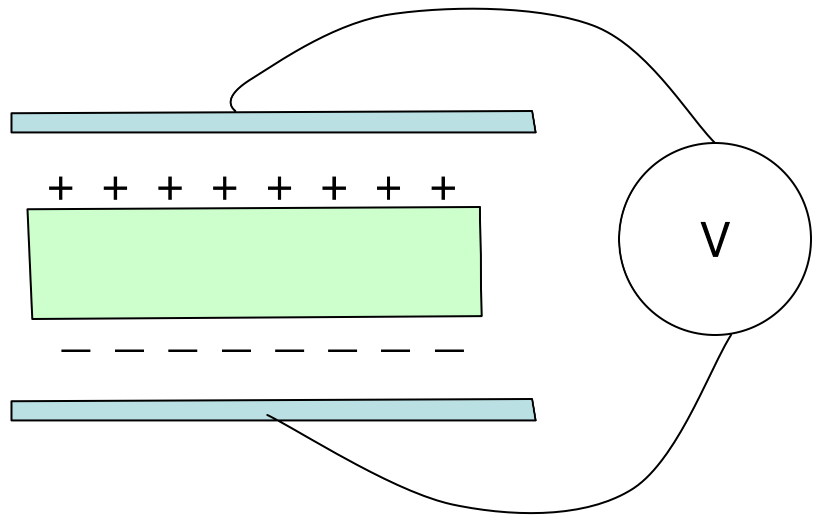


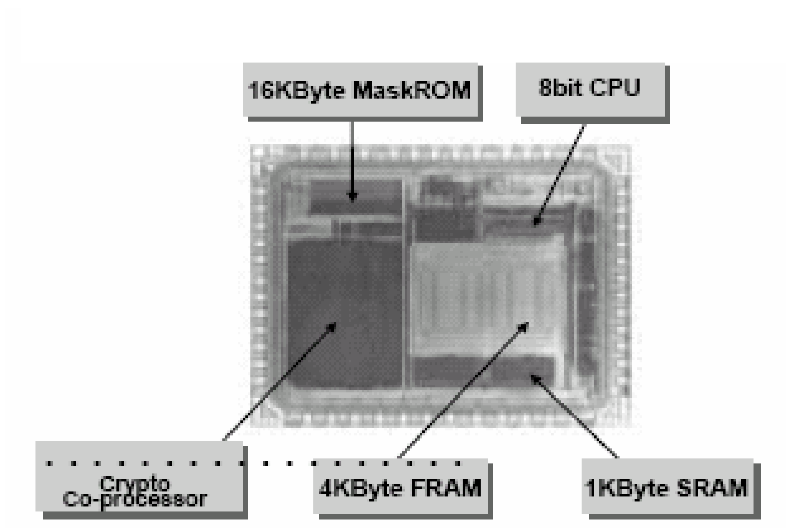
Applications of quantum materials: High Tc superconductors



Applications of quantum materials: Ferroelectric RAM







FeRAM in Smart Cards

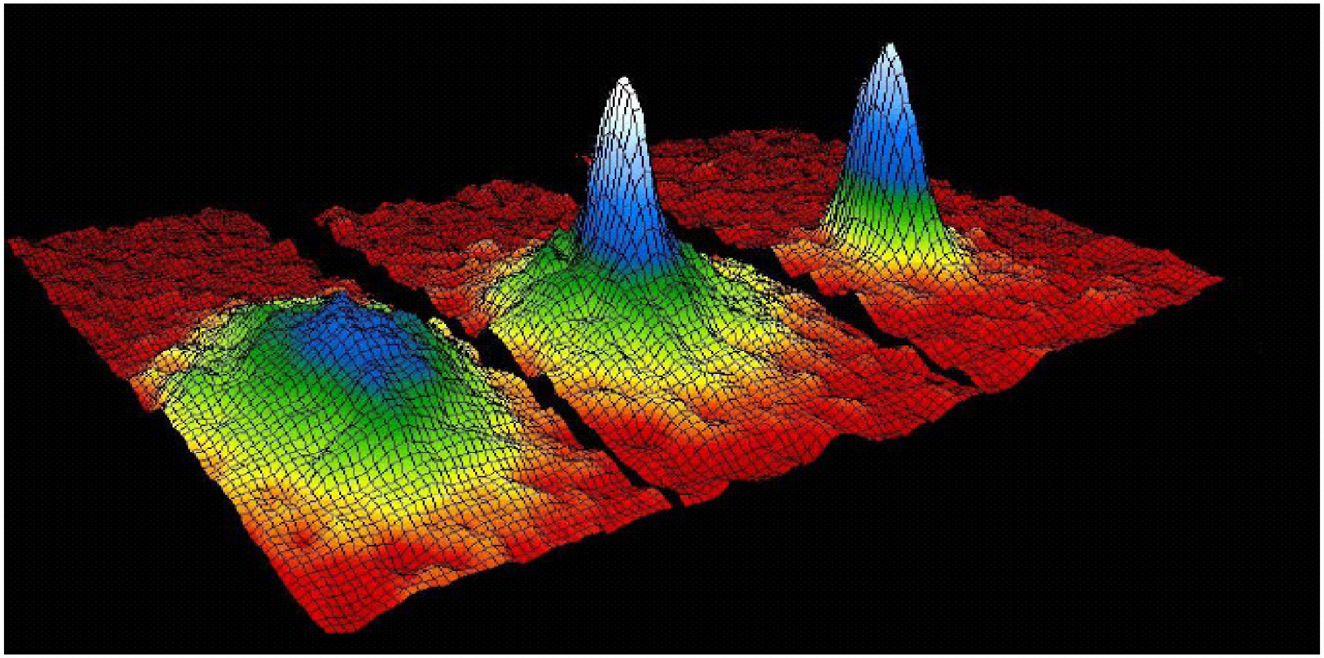
Non-Volatile Memory

High Speed Processing

Breakdown of the “standard model” of electron systems in novel quantum materials

Modeling strongly correlated systems using cold atoms

Bose-Einstein condensation



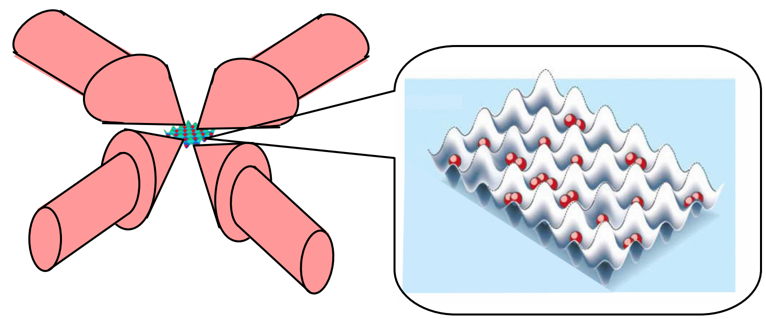
$n \sim 10^{14}\,\mathrm{cm}^{-3}$ $\qquad$ $T_{\mathrm{BEC}} \sim 1\mu\mathrm{K}$

Scattering length is much smaller than characteristic interparticle distances.
Interactions are weak

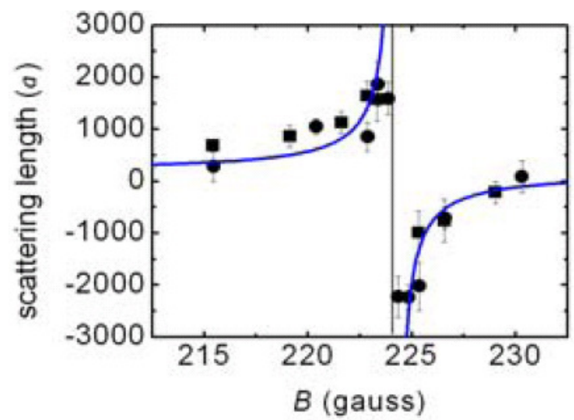
New Era in Cold Atoms Research

Focus on Systems with Strong Interactions

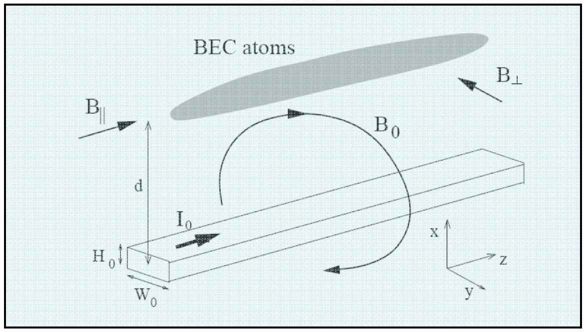
• Optical lattices



• Feshbach resonances

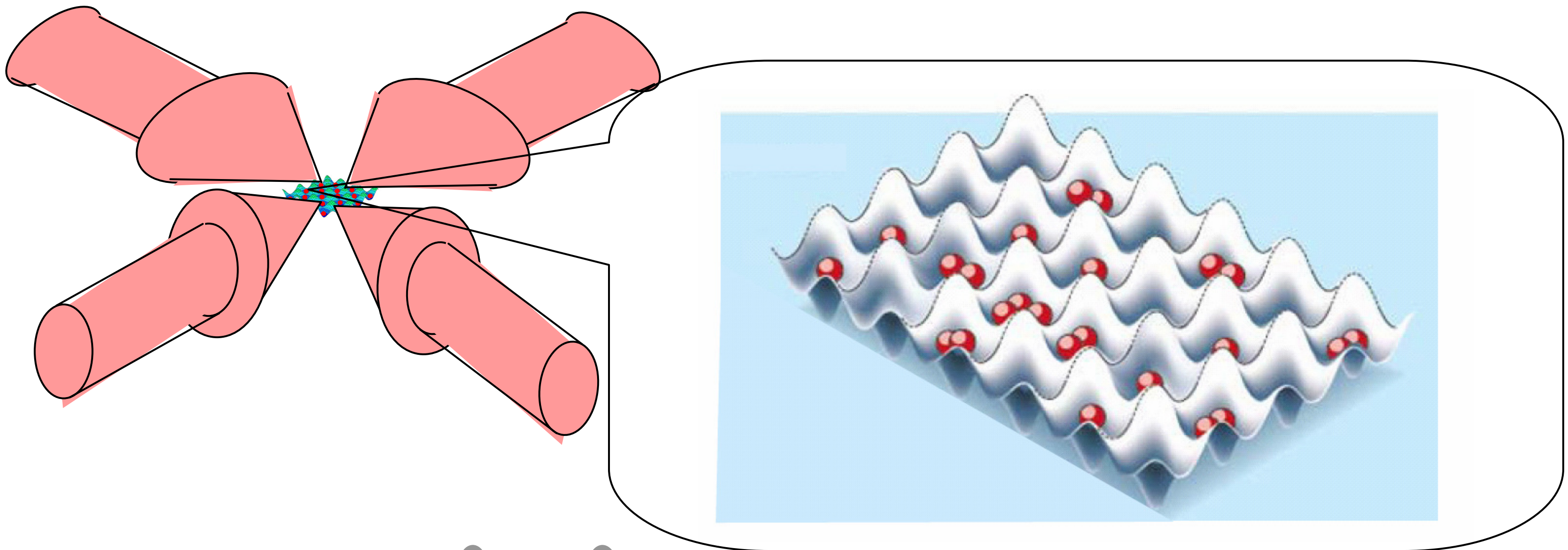


• Low dimensional systems

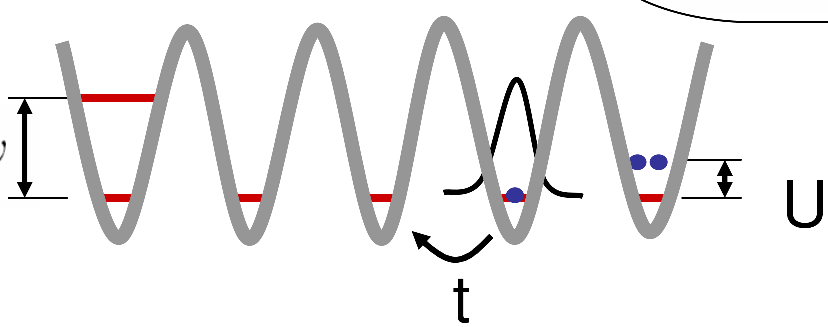


Atoms in optical lattices

Theory: Jaksch et al. PRL (1998) Experiment: Greiner et al., Nature (2001)



$t, U << \hbar\omega$

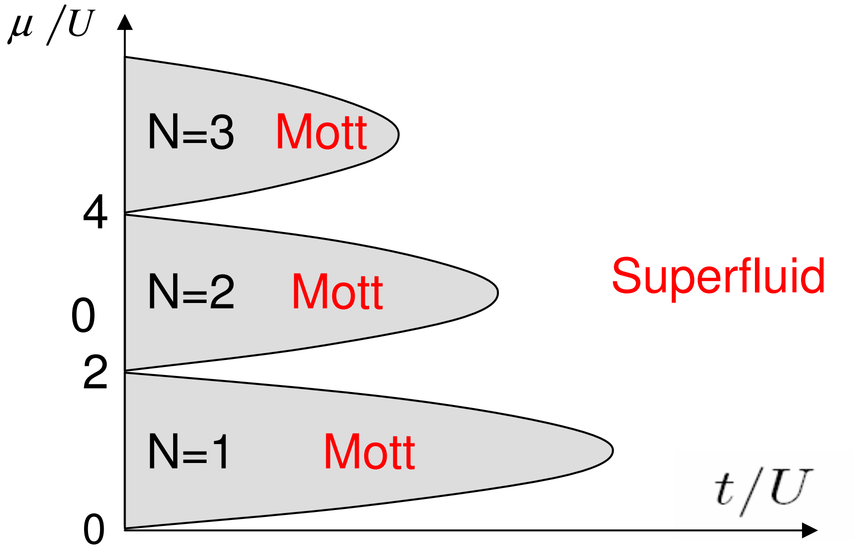


t – tunneling of atoms between neighboring wells

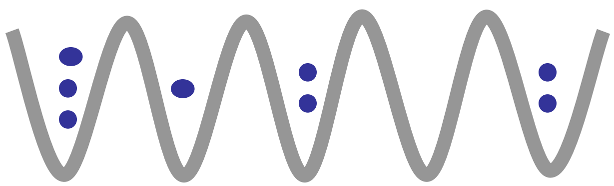
$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i \left(n_i - 1\right) - \mu \sum_i n_i$$

U – repulsion of atoms sitting in the same well

Bose Hubbard model

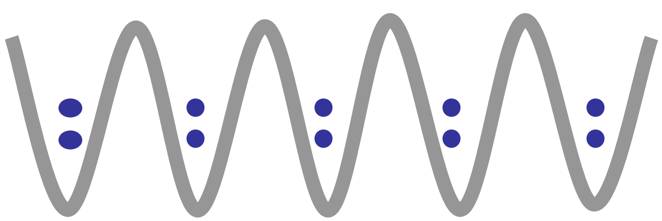


M.P.A. Fisher et al.,
PRB40:546 (1989)



$$U << Nt$$

Superfluid phase
Weak interactions

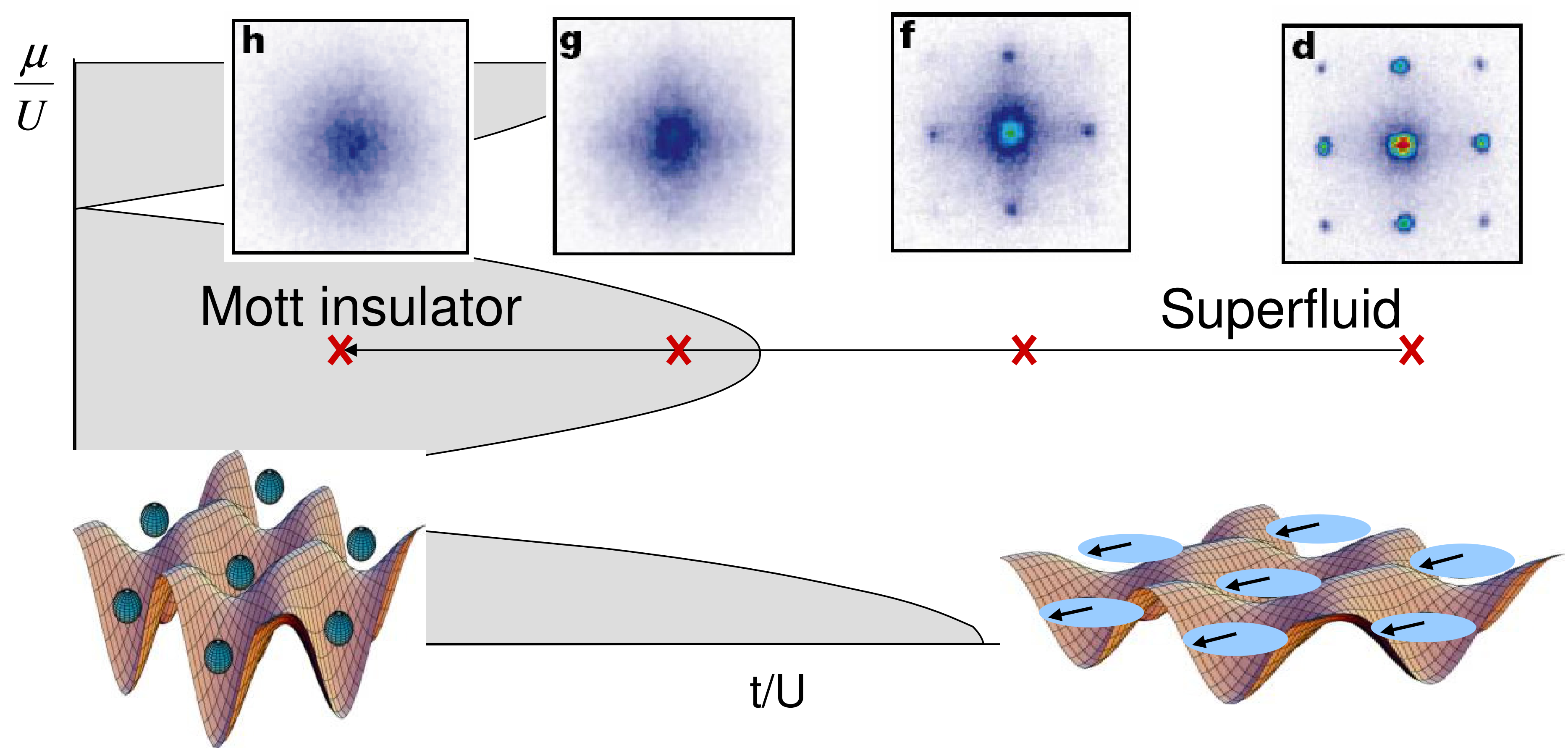


$$U >> Nt$$

Mott insulator phase
Strong interactions

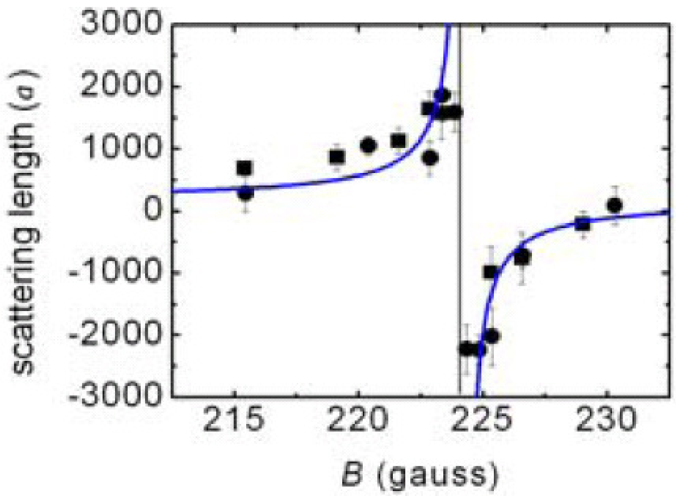
Superfluid to insulator transition in an optical lattice

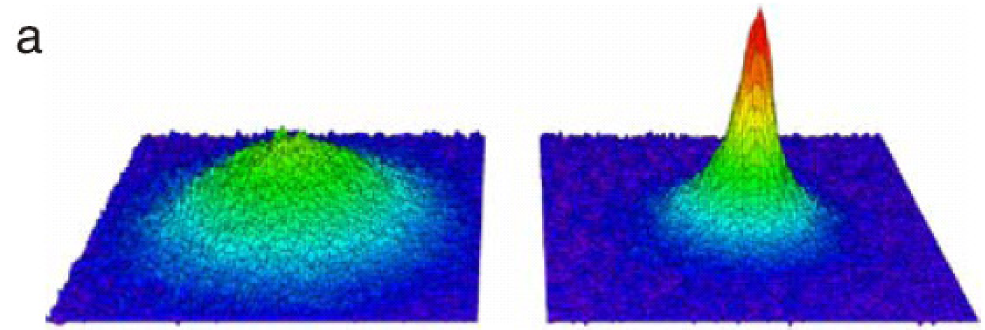
M. Greiner et al., Nature 415 (2002)

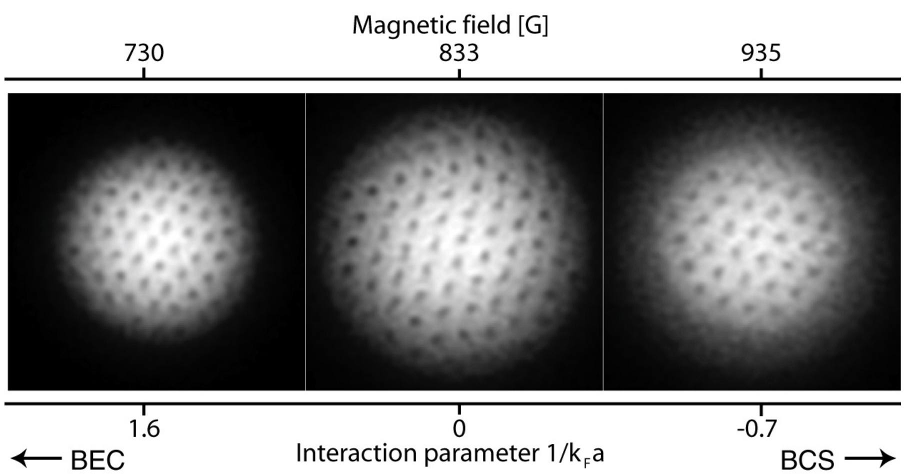


Feshbach resonance and fermionic condensates

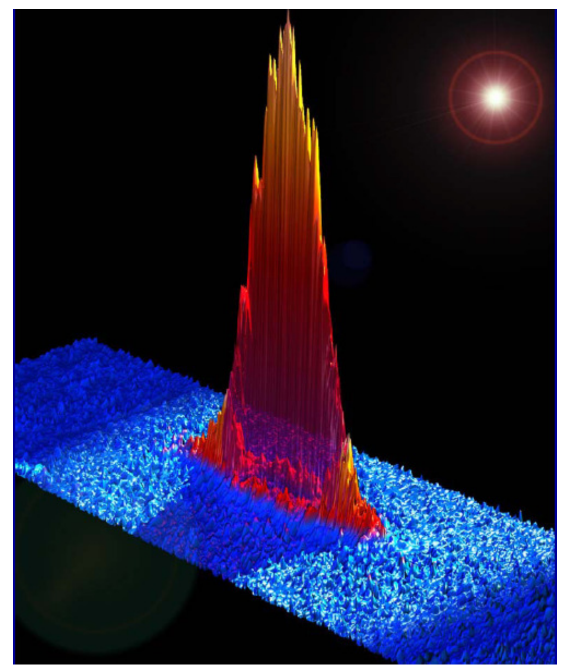
Greiner et al., Nature 426 (2003); Ketterle et al., PRL 91 (2003)





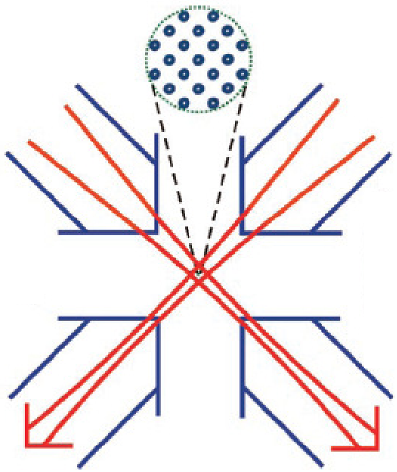


Zwirlein et al., Nature (2005)

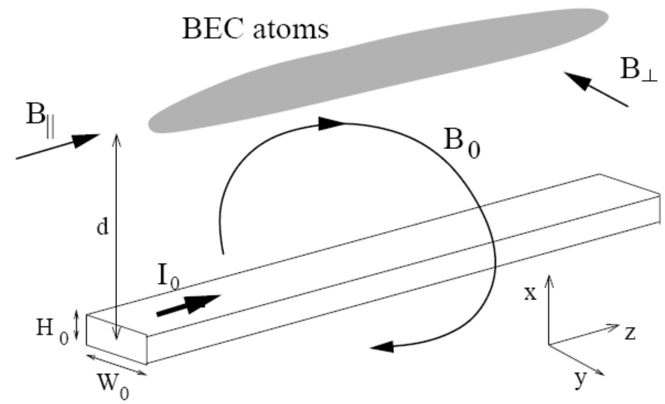


Hulet et al., Science (2005)

One dimensional systems



1D confinement in optical potential
Weiss et al., Bloch et al.,
Esslinger et al., …



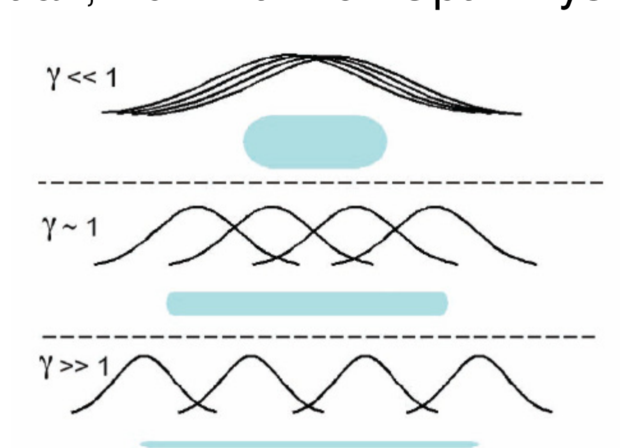
One dimensional systems in microtraps.
Thywissen et al., Eur. J. Phys. D. (99);
Hansel et al., Nature (01);
Folman et al., Adv. At. Mol. Opt. Phys. (02)

$$\mathrm{E_{kin}} \sim \frac{\hbar^2}{m\,d^2} \sim \frac{\hbar^2\,n^2}{m}$$

$$\mathrm{E_{int}} \sim g\,n$$

$$\gamma = \frac{\mathrm{E_{int}}}{\mathrm{E_{kin}}} \sim \frac{g\,m}{\hbar^2\,n}$$

Strongly interacting
regime can be reached
for low densities



New Era in Cold Atoms Research

Focus on Systems with Strong Interactions

Goals

- Resolve long standing questions in condensed matter physics
  (e.g. origin of high temperature superconductivity)

- Resolve matter of principle questions
  (e.g. existence of spin liquids in two and three dimensions)

- Study new phenomena in strongly correlated systems
  (e.g. coherent far from equilibrium dynamics)

Outline

Two component Bose mixtures in optical lattices: realizing quantum magnetic systems using cold atoms

Fermions in optical lattices: modeling high Tc cuprates

Beyond "plain vanilla" Hubbard model: boson-fermion mixtures as analogues of electron-phonon systems; using polar molecules to study long range interactions

Emphasis of this talk: detection of many-body quantum states

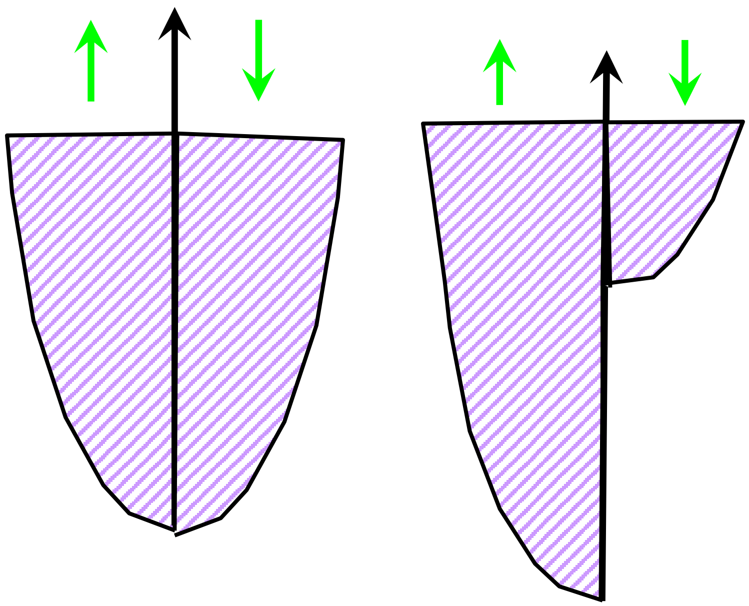
Quantum magnetism

Ferromagnetism

Magnetic needle in a compass

Magnetic memory in hard drives.
Storage density of hundreds of
billions bits per square inch.

Stoner model of ferromagnetism



Spontaneous spin polarization decreases interaction energy but increases kinetic energy of electrons

Mean-field criterion

$$I\,N(0) = 1$$

I – interaction strength

N(0) – density of states at the Fermi level

Antiferromagnetism

Antiferromagnetic Heisenberg model $\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \, \vec{S}_j$

$$| AF \rangle = | \uparrow \downarrow \rangle$$

$$| S \rangle = \frac{1}{\sqrt{2}} \left(| \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle \right)$$

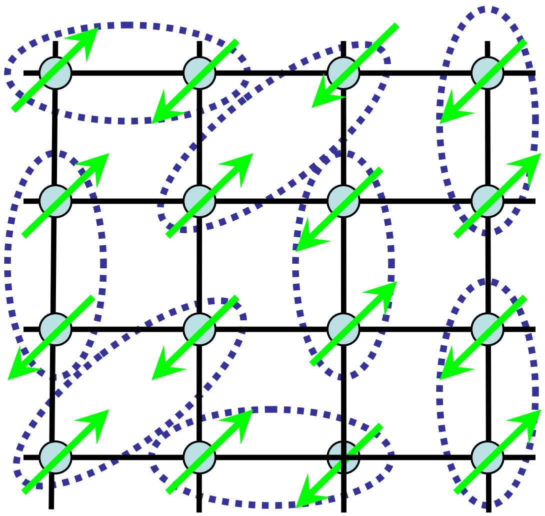
$$| t \rangle = \frac{1}{\sqrt{2}} \left(| \uparrow \downarrow \rangle + | \downarrow \uparrow \rangle \right)$$

$$\boxed{| AF \rangle = \frac{1}{\sqrt{2}} \left(| S \rangle + | t \rangle \right)}$$

Antiferromagnetic state breaks spin symmetry.
It does not have a well defined spin

Spin liquid states

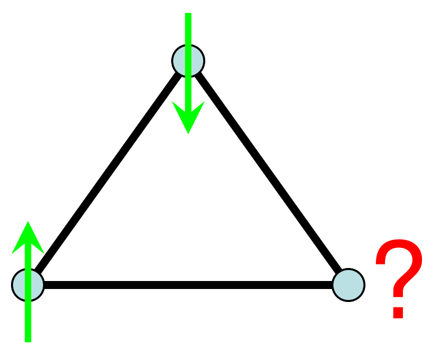
Alternative to classical antiferromagnetic state: spin liquid states



Properties of spin liquid states:

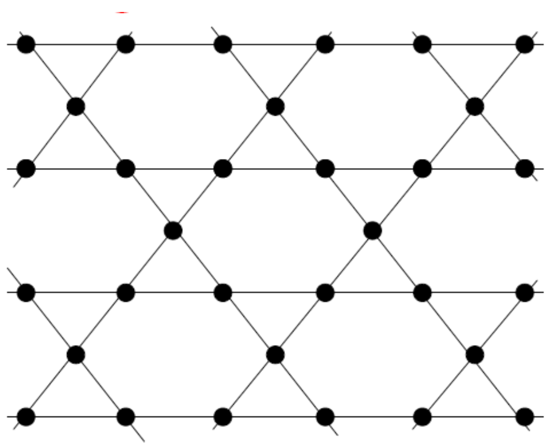
- fractionalized excitations
- topological order
- gauge theory description

Systems with geometric frustration



Spin liquid behavior in systems with geometric frustration

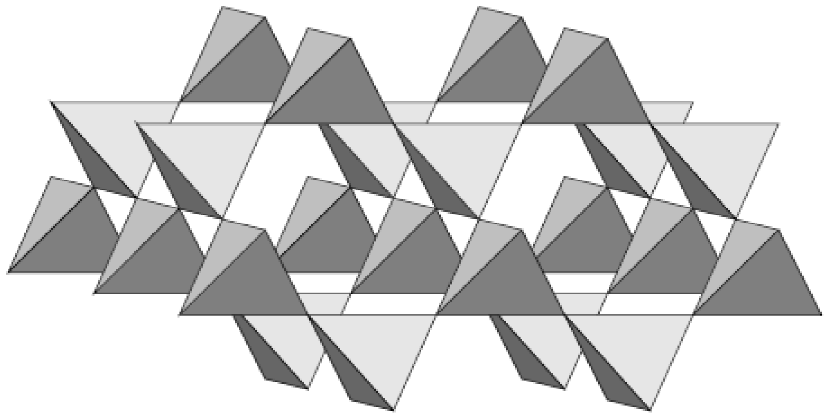
Kagome lattice



$SrCr_{9-x}Ga_{3+x}O_{19}$

Ramirez et al. PRL (90)
Broholm et al. PRL (90)
Uemura et al. PRL (94)

Pyrochlore lattice



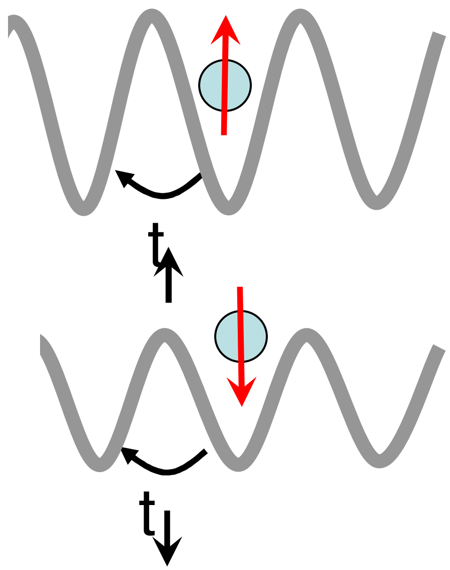
$ZnCr_2O_4$
$A_2Ti_2O_7$

Ramirez et al. PRL (02)

Engineering magnetic systems using cold atoms in optical lattices

Two component Bose mixture in optical lattice

Example: $^{87}\mathrm{Rb}$. Mandel et al., Nature 425:937 (2003)



$$|\uparrow\rangle = |F=1, m_F=-1\rangle$$

$$|\downarrow\rangle = |F=2, m_F=-2\rangle$$

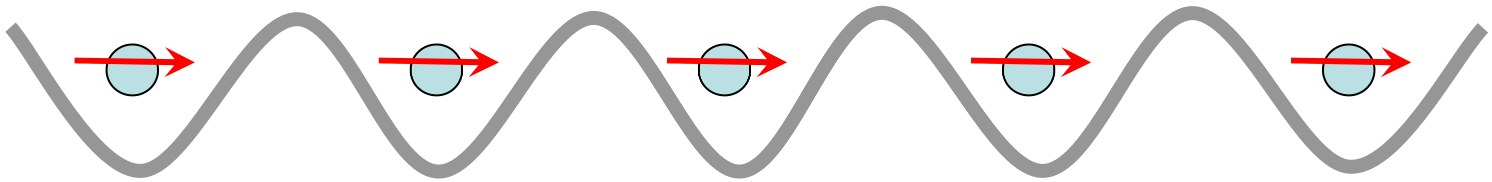
Two component Bose Hubbard model

$$\begin{aligned}\mathcal{H} = & -t_\uparrow \sum_{\langle ij\rangle} b^\dagger_{i\uparrow} b_{j\uparrow} - t_\downarrow \sum_{\langle ij\rangle} b^\dagger_{i\downarrow} b_{j\downarrow} + U_{\uparrow\uparrow} \sum_i n_{i\uparrow}(n_\uparrow - 1) \\ & + U_{\downarrow\downarrow} \sum_i n_{i\downarrow}(n_\downarrow - 1) + U_{\uparrow\downarrow} \sum_i n_{i\uparrow} n_\downarrow\end{aligned}$$

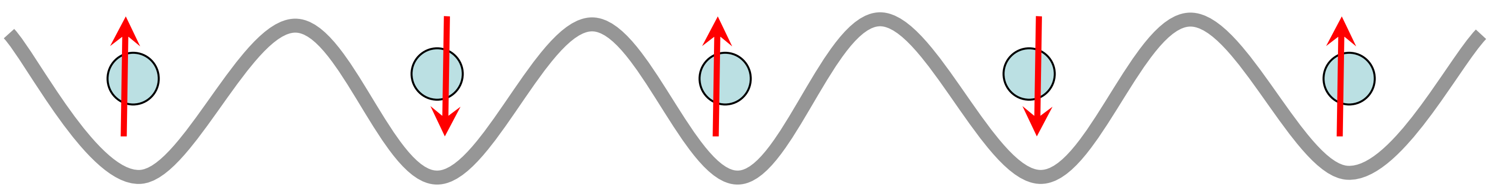
Two component Bose mixture in optical lattice. Magnetic order in an insulating phase

Insulating phases with N=1 atom per site. Average densities $n_\uparrow = n_\downarrow = \frac{1}{2}$

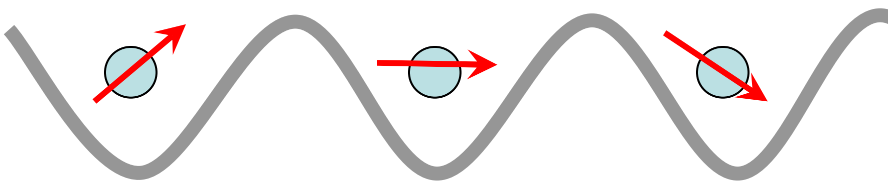
Easy plane ferromagnet $|\,\Psi\,\rangle = \prod_i \left(b^\dagger_{i\uparrow} + e^{i\phi}\, b^\dagger_{i\downarrow} \right) |\,0\,\rangle$



Easy axis antiferromagnet $|\,\Psi\,\rangle = \prod_{i \in A} b^\dagger_{i\uparrow} \prod_{i \in B} b^\dagger_{i\downarrow}$



Quantum magnetism of bosons in optical lattices



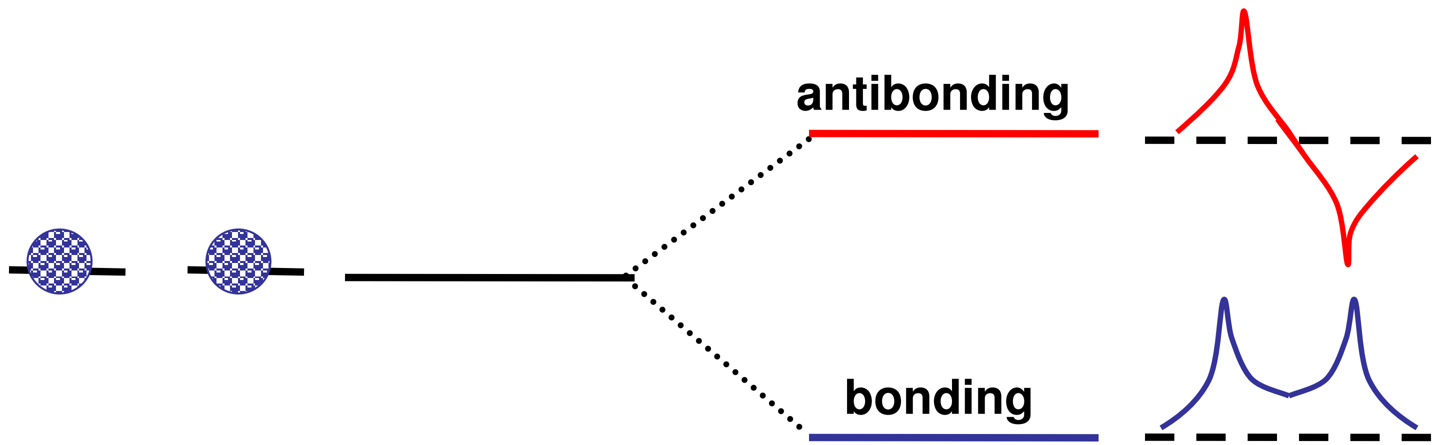
Duan, Lukin, Demler, PRL (2003)

$$\mathcal{H} = J_z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + J_\perp \sum_{\langle ij \rangle} \left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right)$$

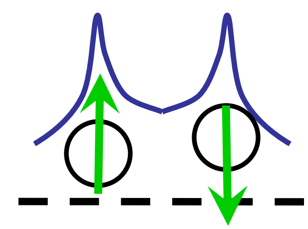
$$J_z = \frac{t_\uparrow^2 + t_\downarrow^2}{2U_{\uparrow\downarrow}} - \frac{t_\uparrow^2}{U_{\uparrow\uparrow}} - \frac{t_\downarrow^2}{U_{\downarrow\downarrow}} \qquad J_\perp = - \frac{t_\uparrow t_\downarrow}{U_{\uparrow\downarrow}}$$

- Ferromagnetic $\quad U_{\uparrow\downarrow} >> U_{\uparrow\uparrow},\ U_{\downarrow\downarrow}$
- Antiferromagnetic $\quad U_{\uparrow\downarrow} << U_{\uparrow\uparrow},\ U_{\downarrow\downarrow}$

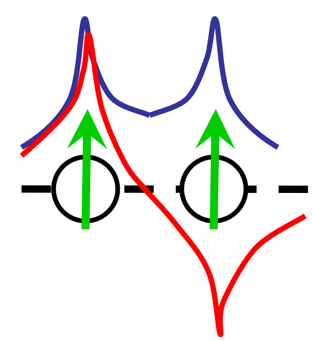
Exchange Interactions in Solids



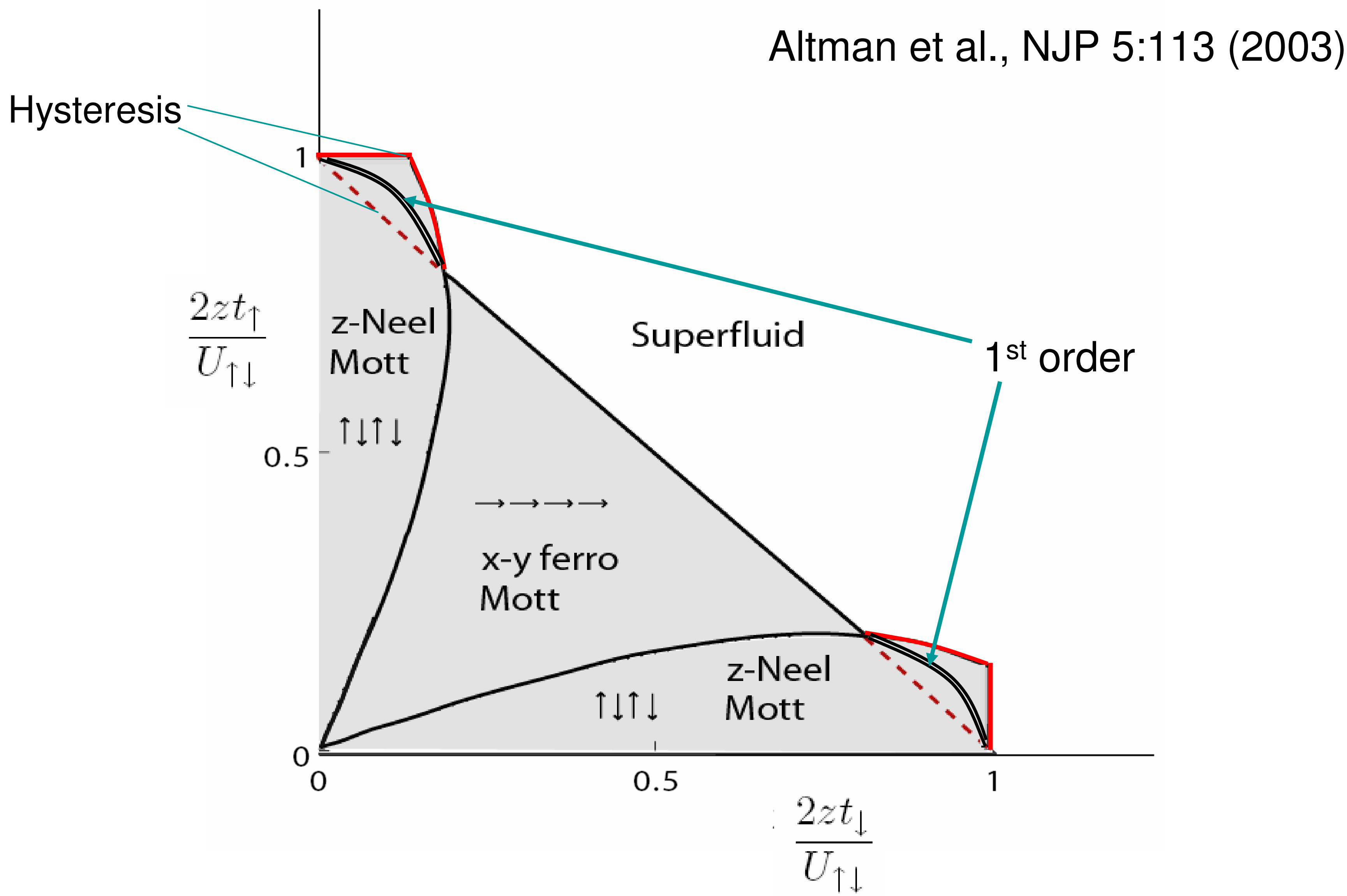
Kinetic energy dominates: antiferromagnetic state



Coulomb energy dominates: ferromagnetic state



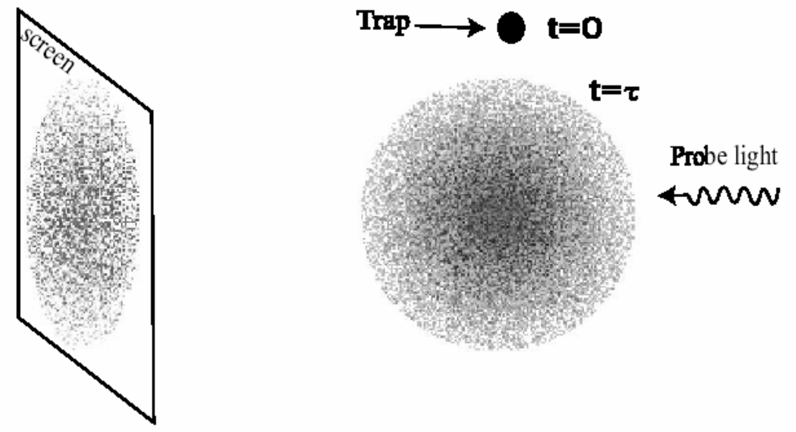
Two component Bose mixture in optical lattice.
Mean field theory + Quantum fluctuations



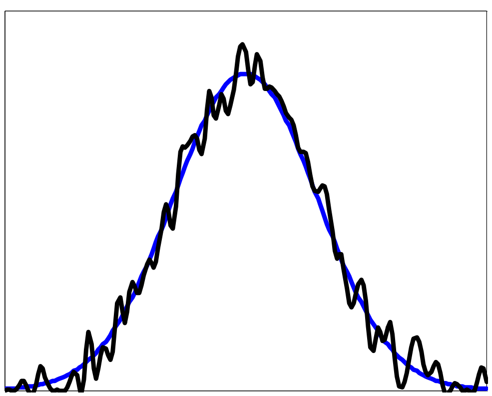
How to detect antiferromagnetic order?

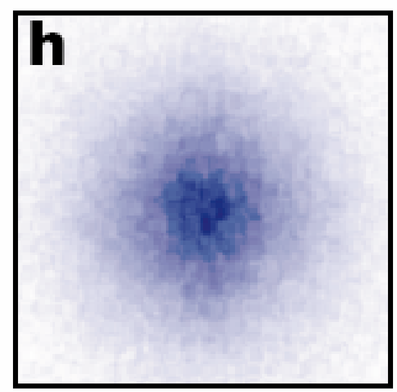
Quantum noise measurements in time of flight experiments

Time of flight experiments



Quantum noise interferometry of atoms in an optical lattice



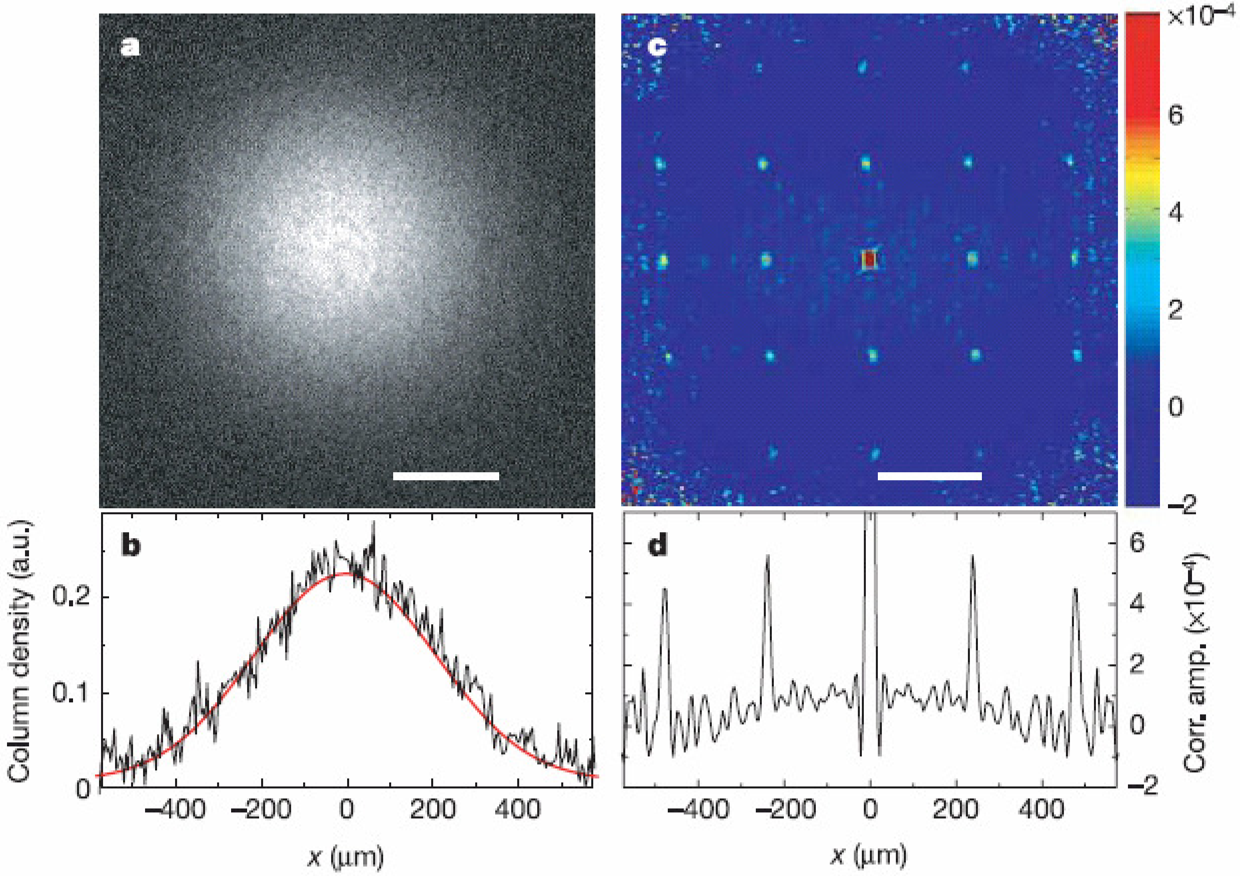


Second order coherence $G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$

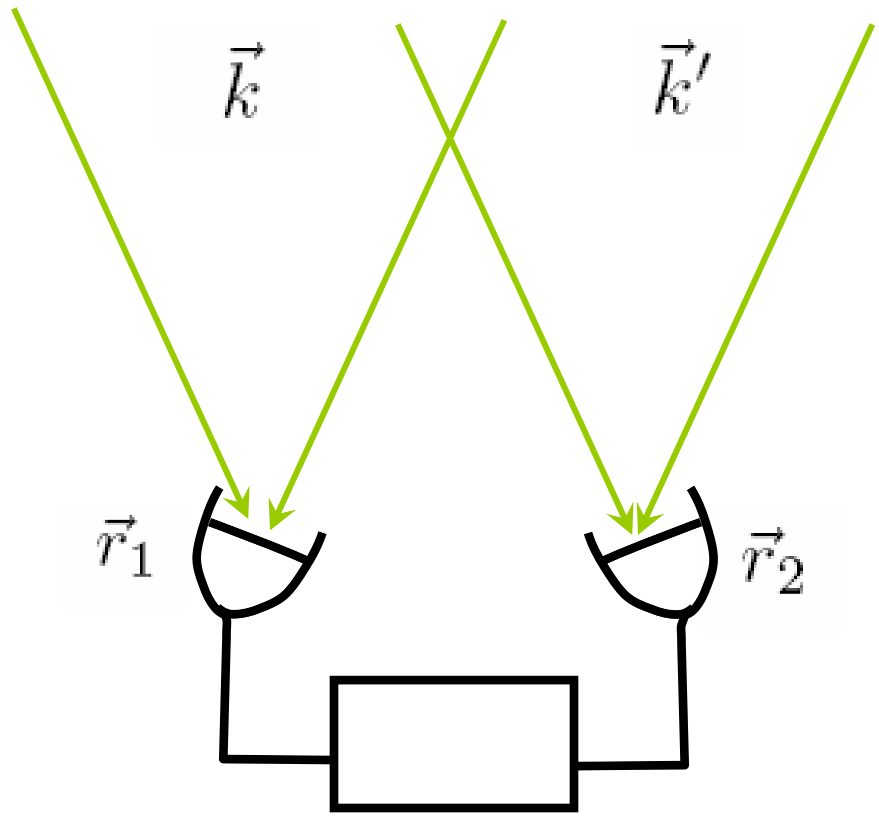
Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

Theory: Altman et al., PRA 70:13603 (2004)

Experiment: Folling et al., Nature 434:481 (2005)

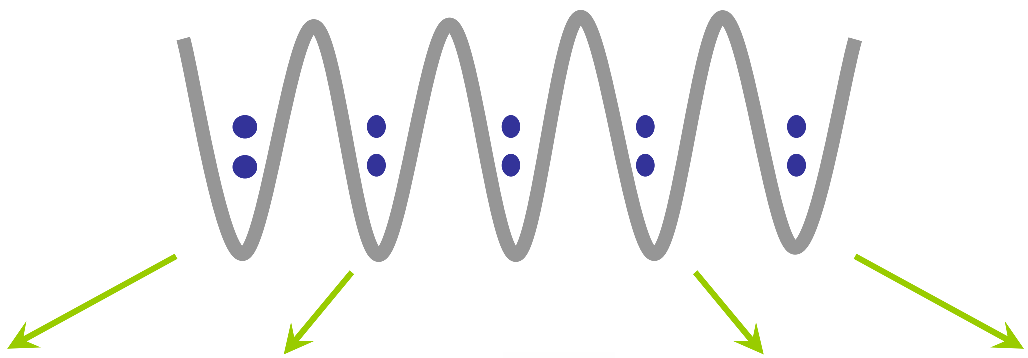


Hanburry-Brown-Twiss stellar interferometer



$$\langle I(\vec{r}_1)\ I(\vec{r}_2)\rangle = A + B\ \cos\Big((\vec{k}-\vec{k}')\ (\vec{r}_1-\vec{r}_2)\Big)$$

Second order coherence in the insulating state of bosons



Bosons at quasimomentum $\vec{k}$ expand as plane waves

with wavevectors $\vec{k}, \; \vec{k} + \vec{G}_1, \; \vec{k} + \vec{G}_2$

First order coherence: $\langle \, \rho(\vec{r}) \, \rangle$

Oscillations in density disappear after summing over $\vec{k}$

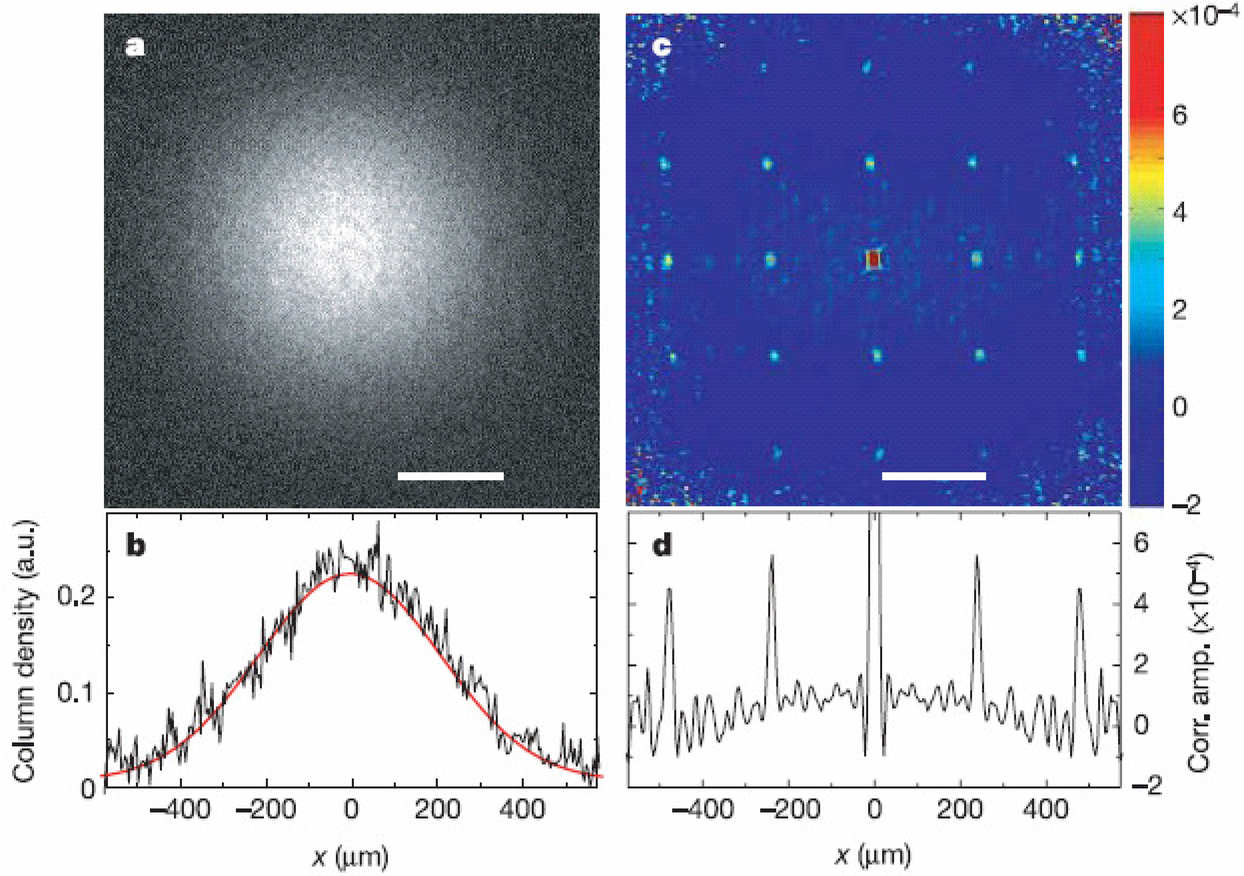
Second order coherence: $\langle \, \rho(\vec{r}_1) \; \rho(\vec{r}_2) \, \rangle$

Correlation function acquires oscillations at reciprocal lattice vectors

$$\langle \, \rho(\vec{r}_1) \; \rho(\vec{r}_2) \, \rangle = A_0 + A_1 \; \cos\left(\; \vec{G}_1(\vec{r}_1 - \vec{r}_2) \; \right) + A_2 \; \cos\left(\; \vec{G}_2(\vec{r}_1 - \vec{r}_2) \; \right) + \ldots$$

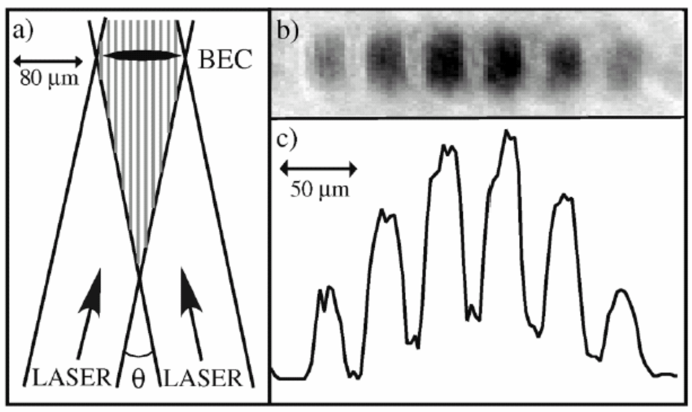
Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

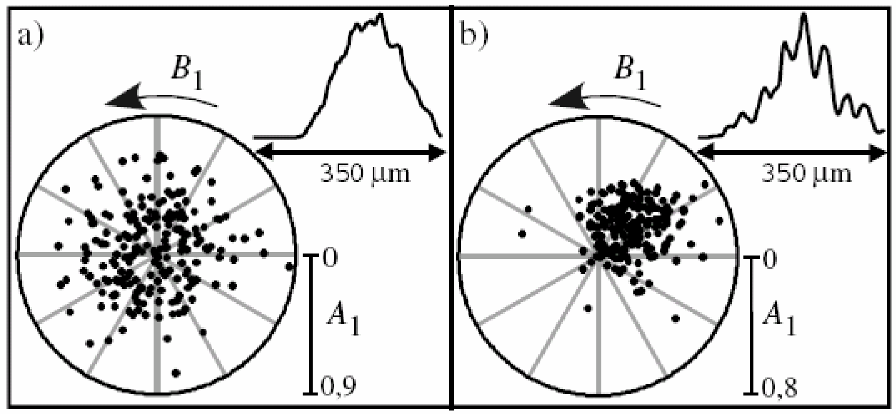
Experiment: Folling et al., Nature 434:481 (2005)



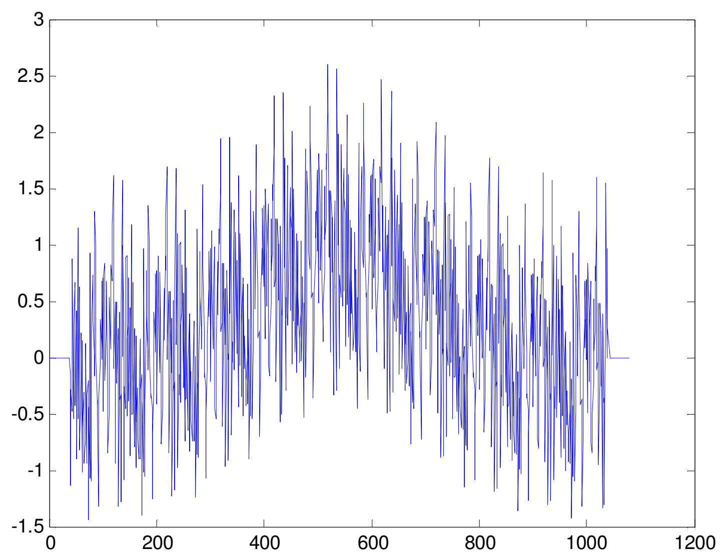
Interference of an array of independent condensates

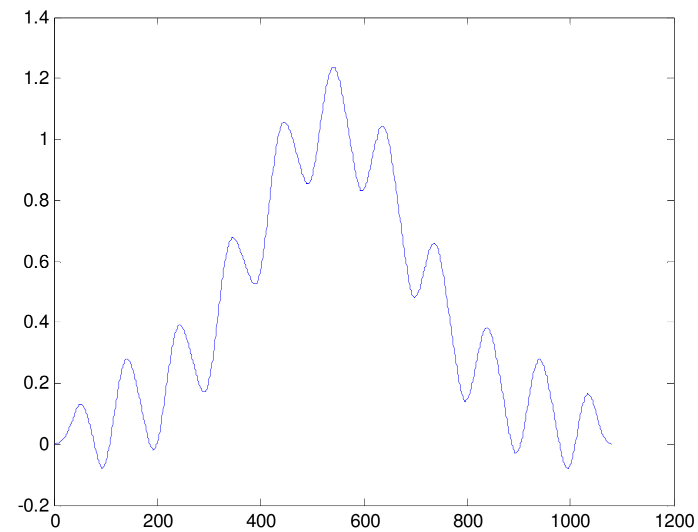
Hadzibabic et al., PRL 93:180403 (2004)





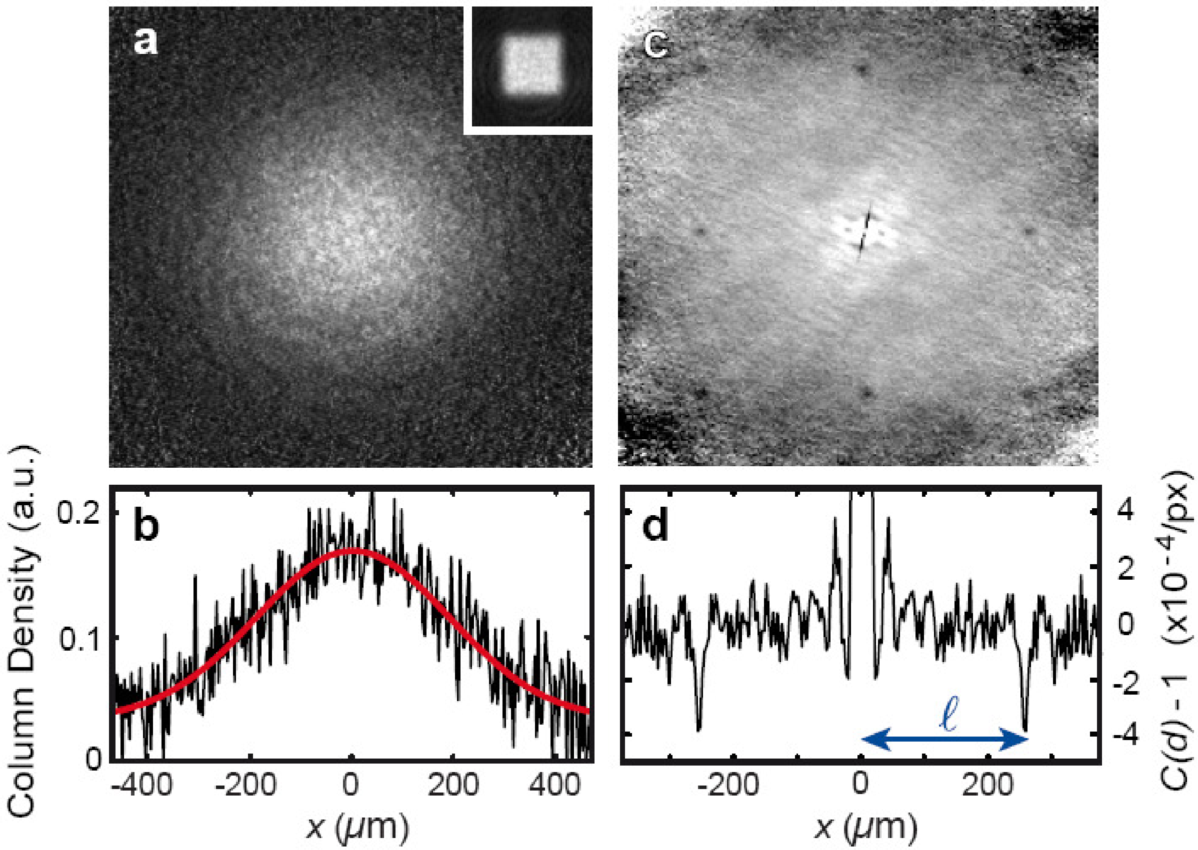
Smooth structure is a result of finite experimental resolution (filtering)



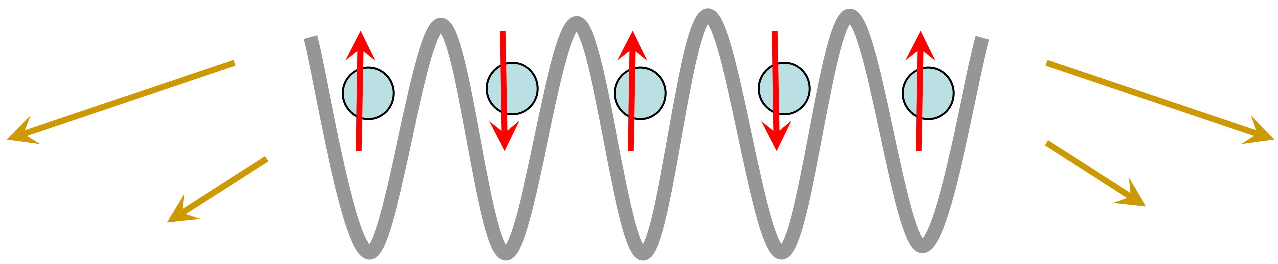


Second order coherence in the insulating state of fermions. Hanburry-Brown-Twiss experiment

Experiment: T. Rom et al. Nature in press



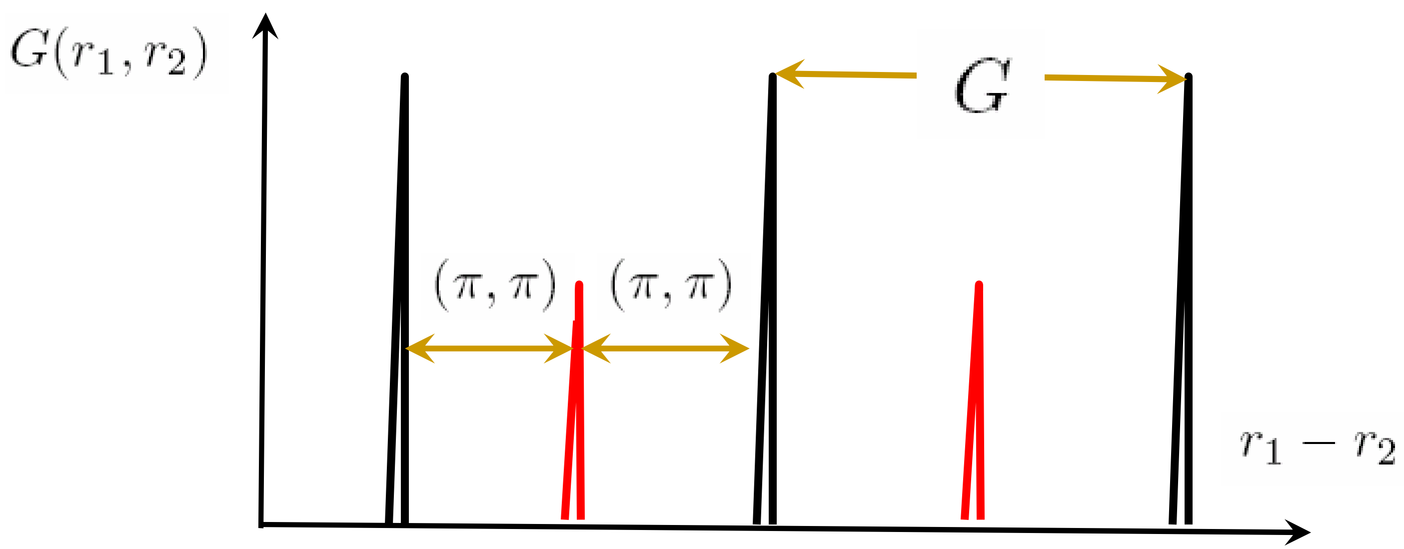
Probing spin order of bosons



$$r = \frac{\hbar\; k\; t}{m}$$

Correlation Function Measurements

$$\begin{aligned} G(r_1, r_2) &= \langle\, n(r_1)\; n(r_2)\,\rangle_{\mathrm{TOF}} - \langle\, n(r_1)\,\rangle_{\mathrm{TOF}}\; \langle\, n(r_2)\,\rangle_{\mathrm{TOF}} \\ &\sim \langle\, n(k_1)\; n(k_2)\,\rangle_{\mathrm{LAT}} - \langle\, n(k_1)\,\rangle_{\mathrm{LAT}}\; \langle\, n(k_2)\,\rangle_{\mathrm{LAT}} \end{aligned}$$



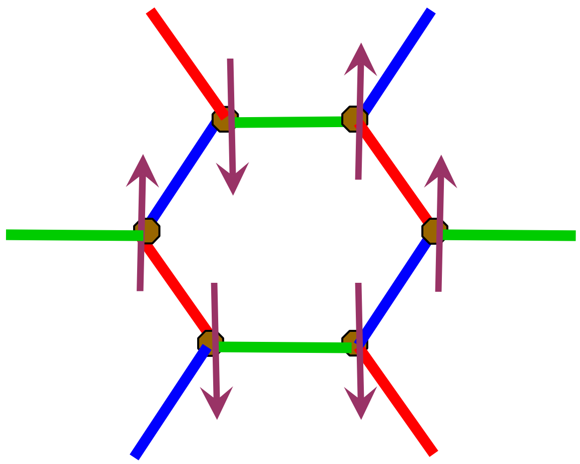
Extra Bragg peaks appear in the second order correlation function in the AF phase

Realization of spin liquid using cold atoms in an optical lattice

Theory: Duan, Demler, Lukin PRL 91:94514 (03)

Kitaev model

$$H = -J_x \sum \sigma_i^x \sigma_j^x - J_y \sum \sigma_i^y \sigma_j^y - J_z \sum \sigma_i^z \sigma_j^z$$



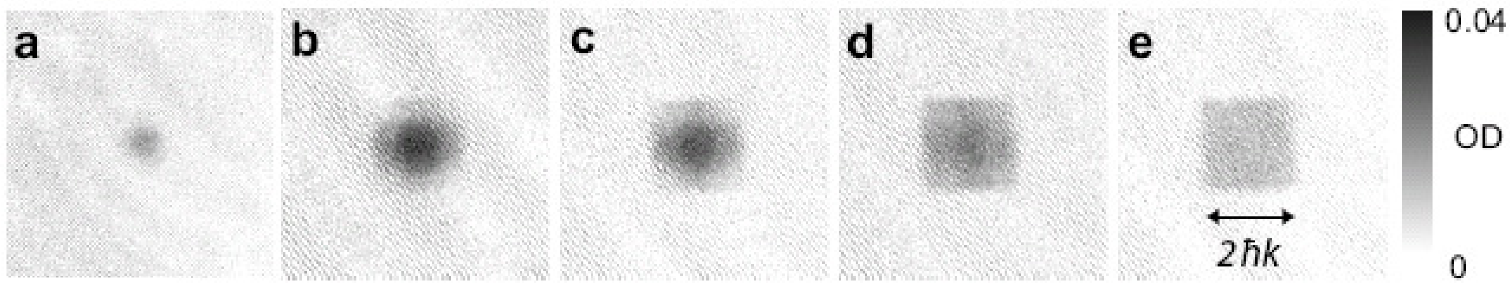
Questions:
Detection of topological order
Creation and manipulation of spin liquid states
Detection of fractionalization, Abelian and non-Abelian anyons
Melting spin liquids. Nature of the superfluid state

Simulation of condensed matter systems: fermionic Hubbard model and high Tc superconductivity

Hofstetter et al., PRL 89:220407 (2002)

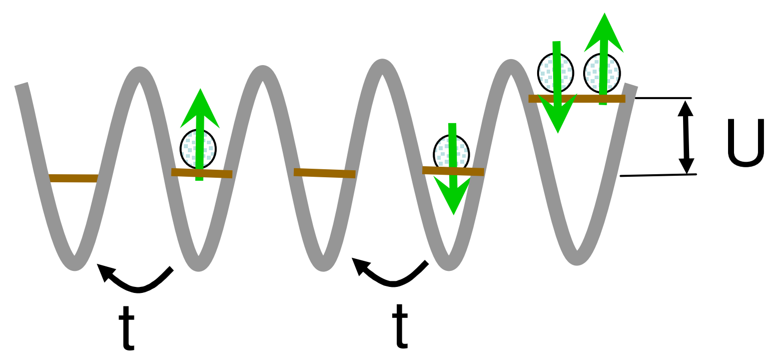
Fermionic atoms in an optical lattice

Experiment: Esslinger et al., PRL 94:80403 (2005)

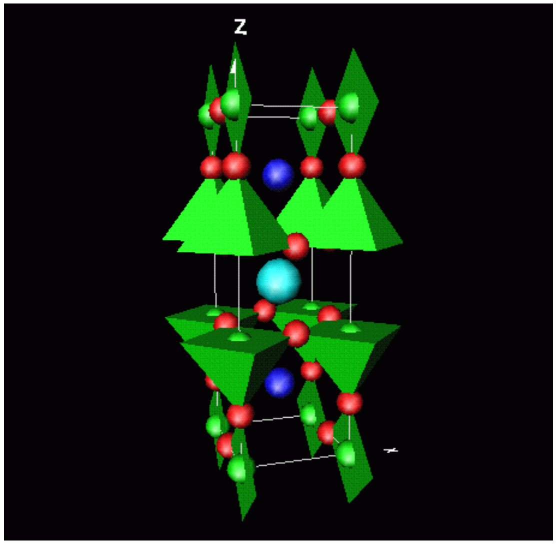


Fermionic Hubbard model

$$\mathcal{H} = -t \sum_{\langle ij \rangle\,\sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i$$

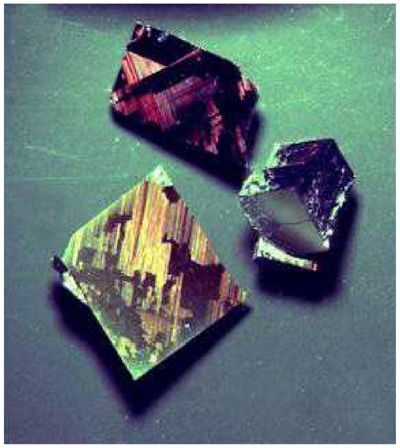


High temperature superconductors



$YBa_2Cu_3O_7$

Superconducting
Tc 93 K



Picture courtesy of UBC Superconductivity group

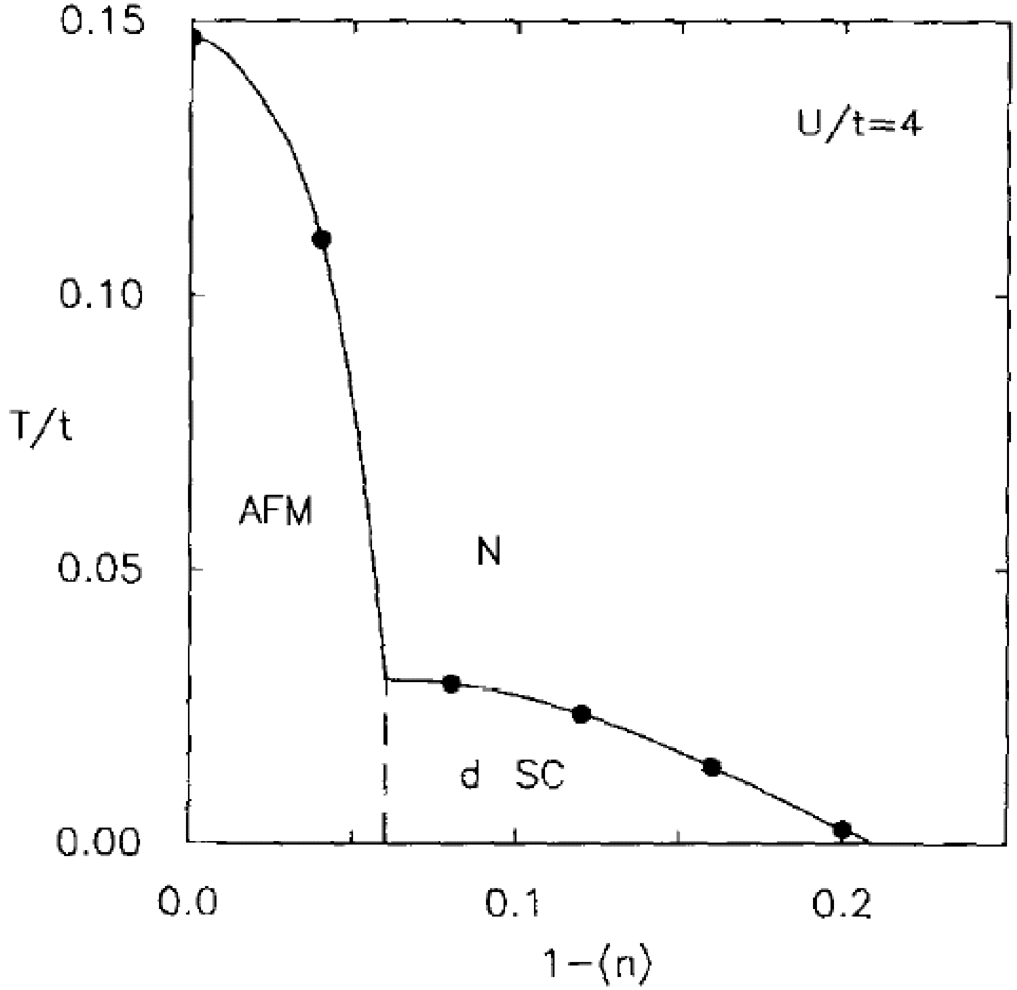
Hubbard model – minimal model for cuprate superconductors

P.W. Anderson, cond-mat/0201429

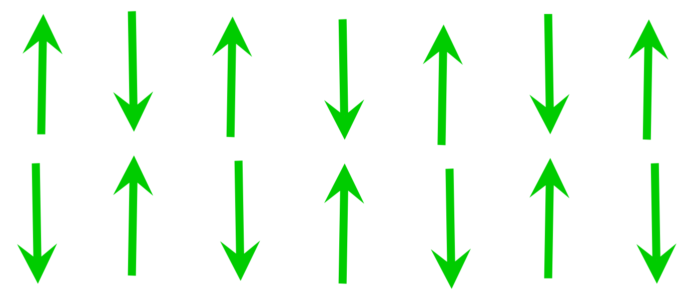
After many years of work we still do not understand the fermionic Hubbard model

Positive U Hubbard model

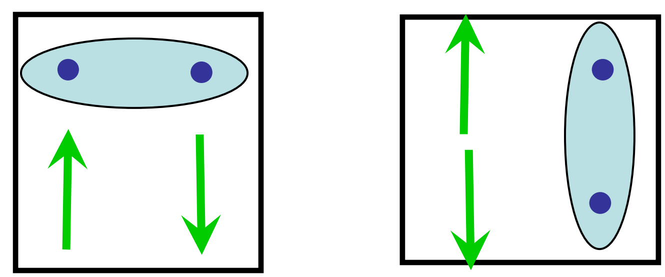
Possible phase diagram. Scalapino, Phys. Rep. 250:329 (1995)



Antiferromagnetic insulator

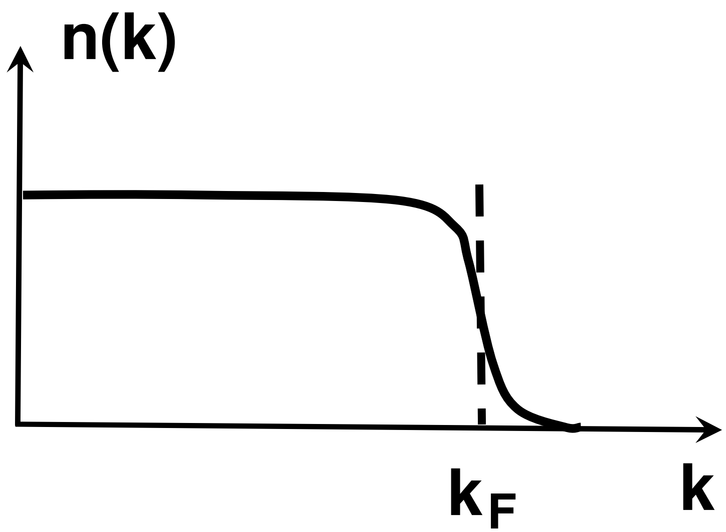


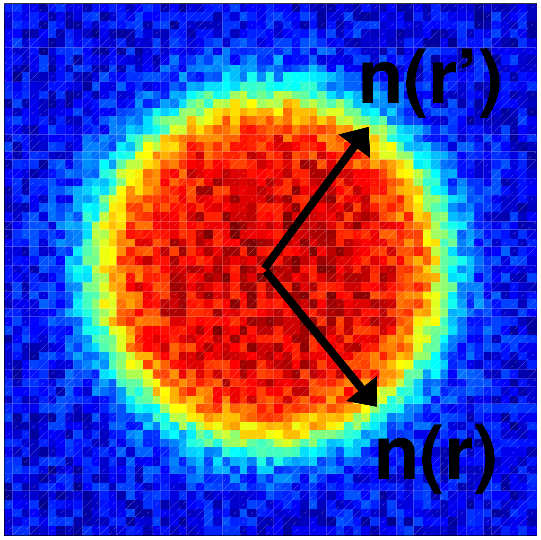
D-wave superconductor

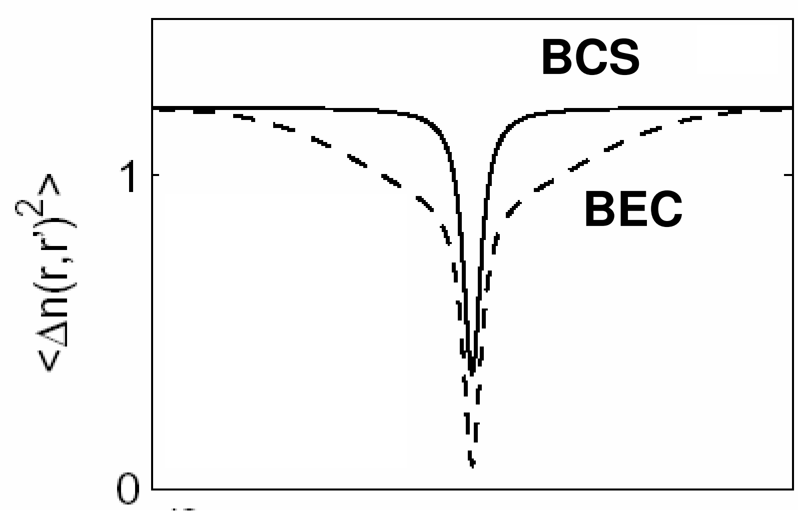


Second order interference from the BCS superfluid

Theory: Altman et al., PRA 70:13603 (2004)





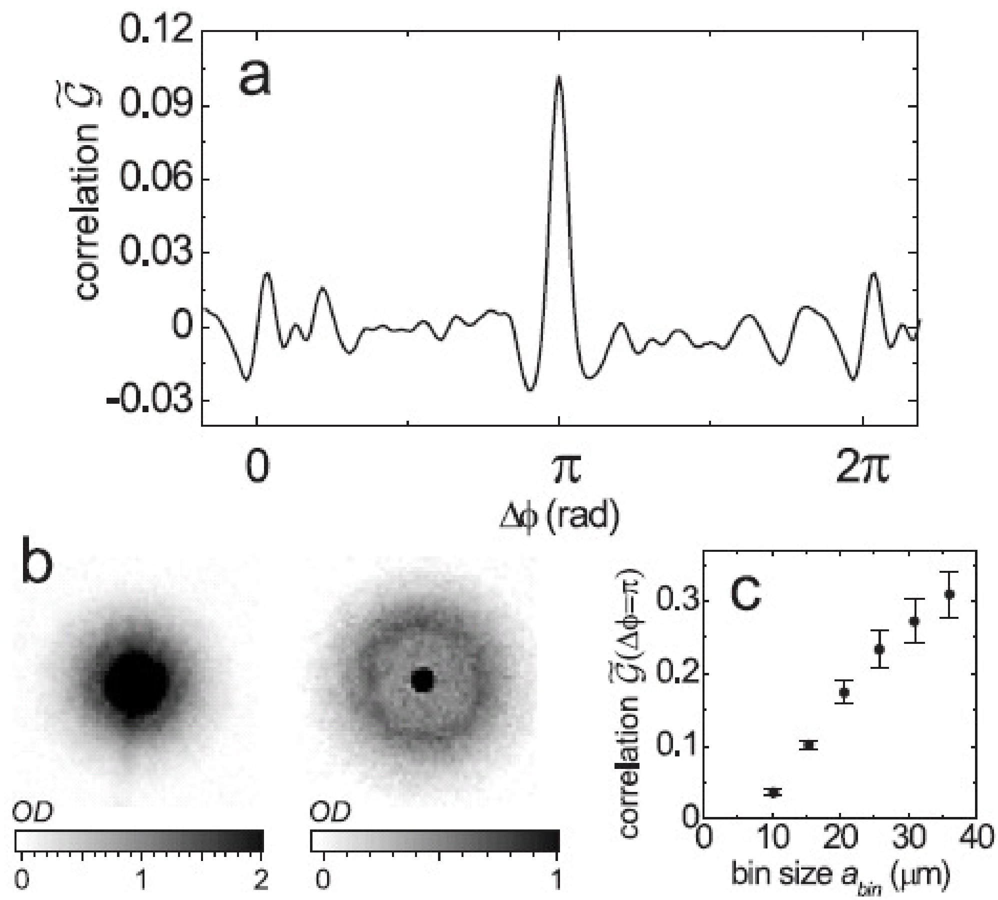


$$\Delta n(\mathbf{r},\mathbf{r}') \equiv n(\mathbf{r}) - n(\mathbf{r}')$$

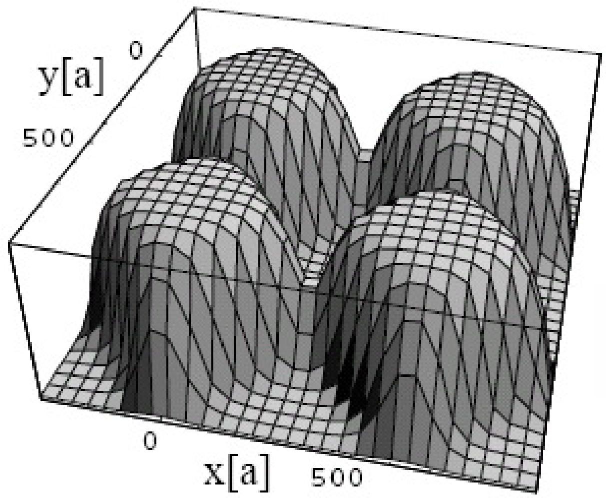
$$\Delta n(\mathbf{r},-\mathbf{r}) \left| \Psi_{BCS} \right\rangle = 0$$

Momentum correlations in paired fermions

Greiner et al., PRL 94:110401 (2005)



Fermion pairing in an optical lattice



Second Order Interference
In the TOF images

$$G(r_1, r_2) = \langle n(r_1) n(r_2) \rangle - \langle n(r_1) \rangle \langle n(r_2) \rangle$$

Normal State

$$G_{\mathrm{N}}(r_1, r_2) = \delta(r_1 - r_2)\rho(r_1) - \rho^2(r_1) \sum_G \delta(r_1 - r_2 - \frac{G\hbar t}{m})$$

Superfluid State

$$G_{\mathrm{S}}(r_1, r_2) = G_{\mathrm{N}}(r_1, r_2) + \Psi(r_1) \sum_G \delta(r_1 + r_2 + \frac{G\hbar t}{m})$$

$\Psi(r) = |u(Q(r))v(Q(r))|^2$ **measures the Cooper pair wavefunction**

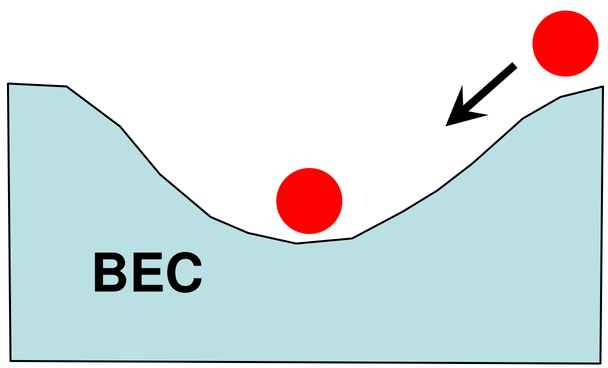
$$Q(r) = \frac{mr}{\hbar t}$$

One can identify unconventional pairing

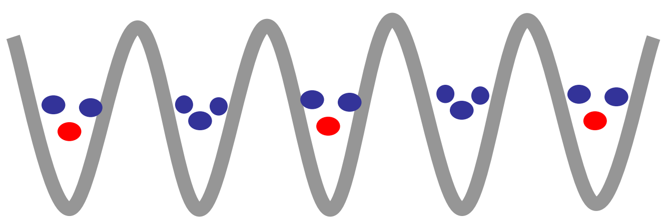
Beyond ”plain vanilla” Hubbard model

Boson Fermion mixtures

Experiments: ENS, Florence, JILA, MIT, Rice, ETH, Hamburg, …

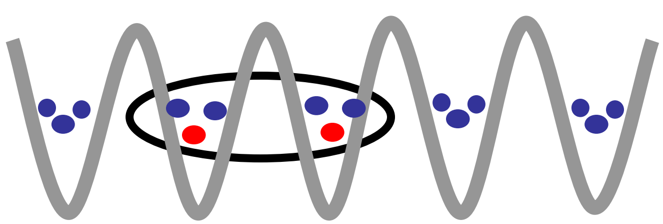


Bosons provide cooling for fermions and mediate interactions. They create non-local attraction between fermions



Charge Density Wave Phase

Periodic arrangement of atoms



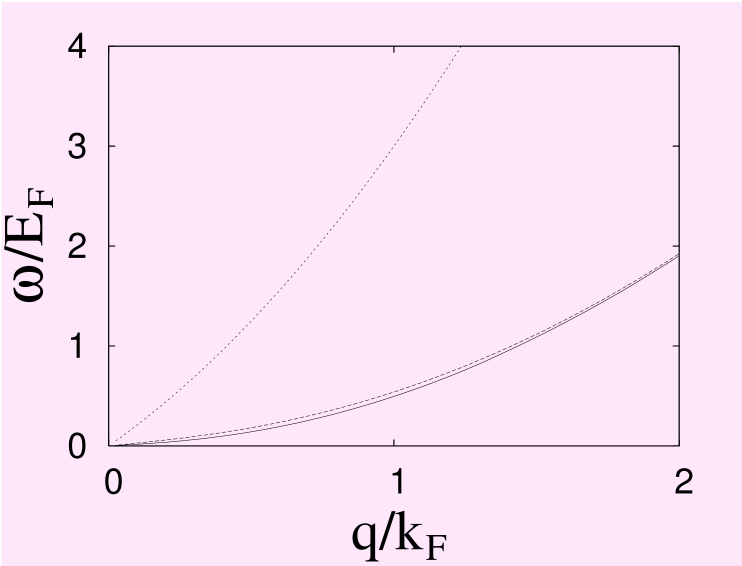
Non-local Fermion Pairing

P-wave, D-wave, …

Boson Fermion mixtures

$$
\begin{aligned}
\mathcal{H} &= \mathcal{H}_{bb} + \mathcal{H}_{\mathrm{ff}} + \mathcal{H}_{bf} \\
\mathcal{H}_{bb} &= -t_b \sum_{\langle ij \rangle} b_i^\dagger b_j + U_{bb} \sum_i n_{bi}(n_{bi} - 1) \\
\mathcal{H}_{ff} &= -t_f \sum_{\langle ij \rangle} f_i^\dagger f_j \\
\mathcal{H}_{bf} &= U_{bf} \sum_i n_{bi} n_{fi}
\end{aligned}
$$

"Phonons" :
Bogoliubov (phase) mode



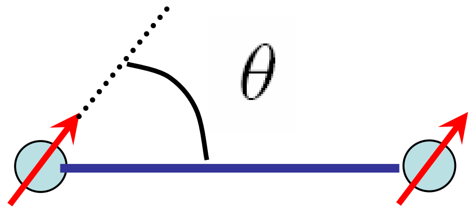
Effective fermion-"phonon" interaction

$$
\begin{aligned}
\tilde{\mathcal{H}}_{bb} &= \sum_q \omega_q \beta_q^\dagger \beta_q \\
\tilde{\mathcal{H}}_{bf} &= \sum_{kq} g_q \left(\beta_q + \beta_{-q}^\dagger\right) f_{k+q}^\dagger f_k
\end{aligned}
$$

Fermion-"phonon" vertex $g_q \sim |q|$
Similar to electron-phonon systems

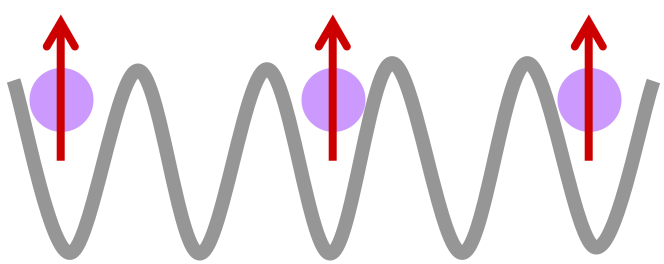
Polar molecules in optical lattices

Adding long range repulsion between atoms

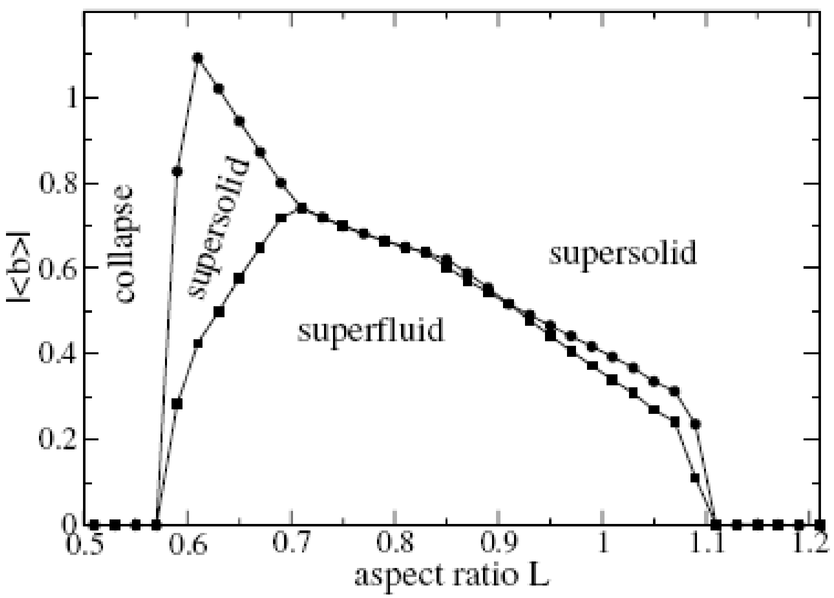


$$V_{\rm int} = d^2 \frac{1 - 3\cos^2\theta}{|\mathbf{r} - \mathbf{r}'|^3} + \frac{4\pi\hbar^2 a}{m}\delta(\mathbf{r} - \mathbf{r}')$$

Experiments:
Florence, Yale, Harvard, …



Bosonic molecules in optical lattice



Goral et al., PRL88:170406 (2002)

Conclusions

We understand well: electron systems in semiconductors and simple metals. Interaction energy is smaller than the kinetic energy. Perturbation theory works

We do not understand: strongly correlated electron systems in novel materials. Interaction energy is comparable or larger than the kinetic energy. Many surprising new phenomena occur, including high temperature superconductivity, magnetism, fractionalization of excitations

Ultracold atoms have energy scales of 10^{-6}K, compared to 10^{4} K for electron systems. However, by engineering and studying strongly interacting systems of cold atoms we should get insights into the mysterious properties of novel quantum materials

Our big goal is to develop a general framework for understanding strongly correlated systems. This will be important far beyond AMO and condensed matter