Interference of fluctuating condensates

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Outline

Measuring correlation functions in intereference experiments

1. Interference of independent condensates
2. Interference of interacting 1D systems
3. Interference of 2D systems
4. Full distribution function of fringe visibility in intereference experiments. Connection to quantum impurity problem

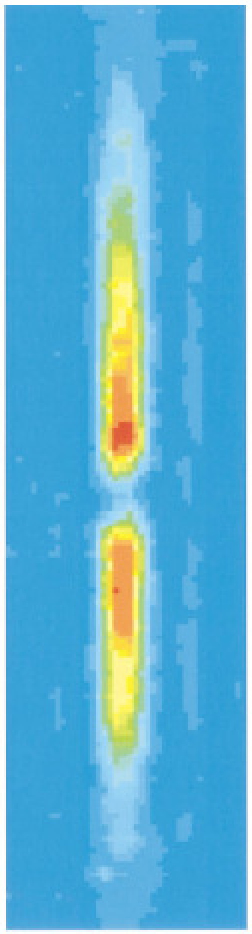
Studying decoherence in interference experiments.
Effects of finite temperature

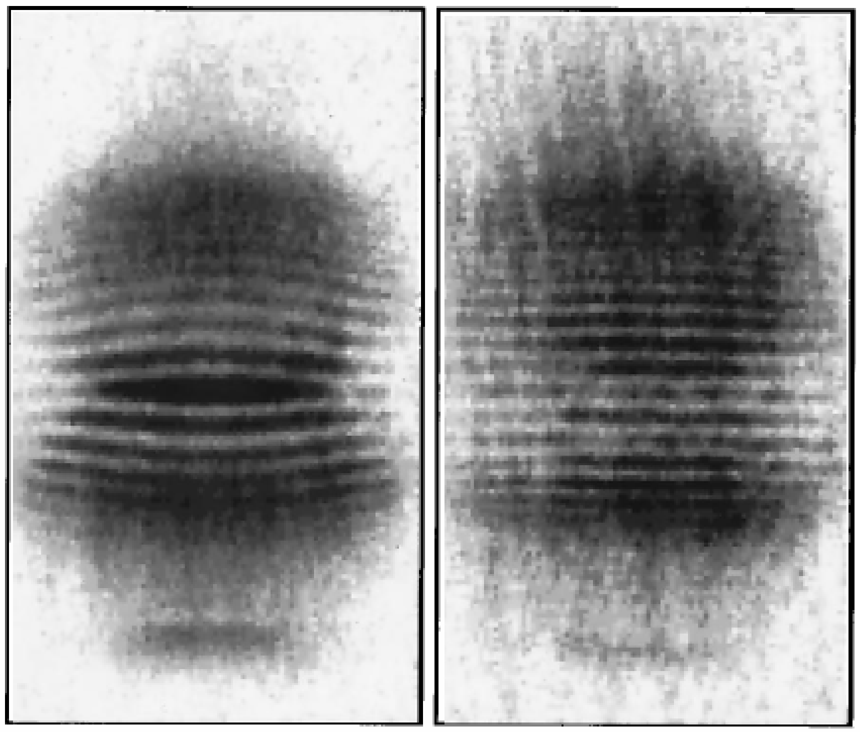
Distribution function of magnetization for lattice spin systems

Measuring correlation functions in intereference experiments

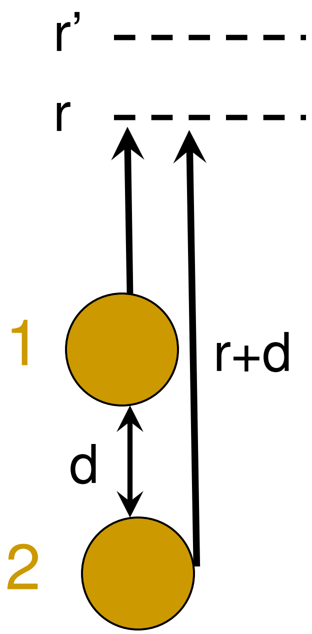
Interference of two independent condensates

Andrews et al., Science 275:637 (1997)





Interference of two independent condensates



$$\rho_{\rm int}(r) = a_1^\dagger(r)\, a_2(r) + \text{c.c.}$$

$$a_1(r) = e^{i\phi_1 + i k_1 r} \qquad k_1 = \frac{m v_1}{\hbar} = \frac{m r}{\hbar t}$$

$$a_2(r) = e^{i\phi_2 + i k_2 r} \qquad k_2 = \frac{m v_2}{\hbar} = \frac{m (r + d)}{\hbar t}$$

$$\rho_{\rm int}(r) = e^{i(k_2 - k_1) r}\, e^{i(\phi_2 - \phi_1)} + \text{c.c.}$$

$$\rho_{\rm int}(r) = e^{i \frac{m d r}{\hbar t}}\, e^{i(\phi_2 - \phi_1)} + \text{c.c.}$$

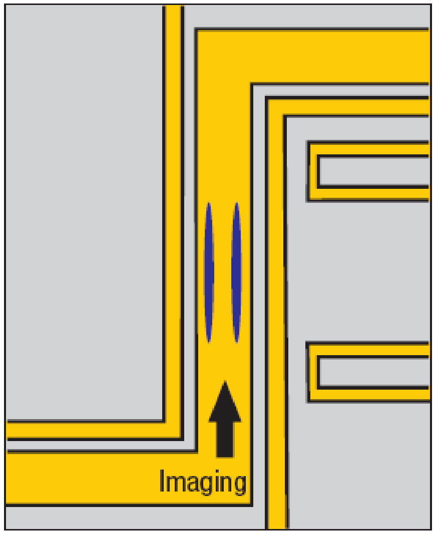
Clouds 1 and 2 do not have a well defined phase difference.
However each individual measurement shows an interference pattern

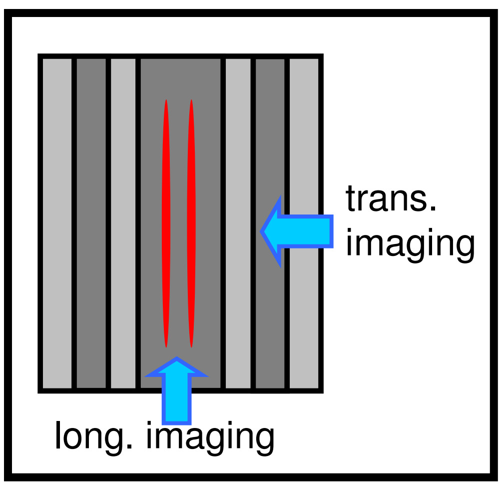
$$\langle \rho_{\rm int}(r) \rangle = 0$$

$$\langle \rho_{\rm int}(r)\, \rho_{\rm int}(r') \rangle = e^{i \frac{m d}{\hbar t}(r - r')} + \text{c.c.}$$

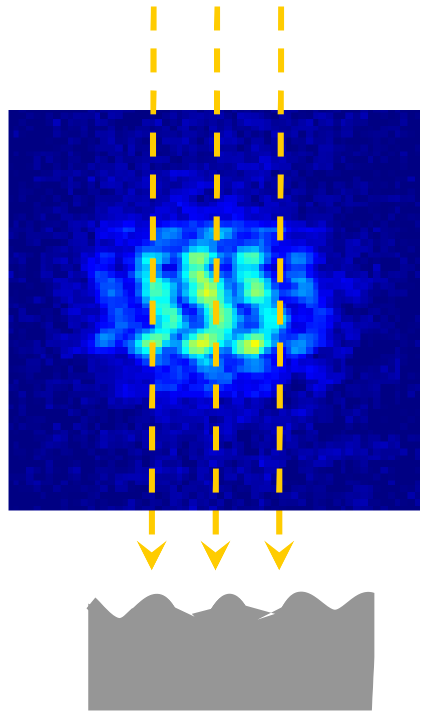
Interference of one dimensional condensates

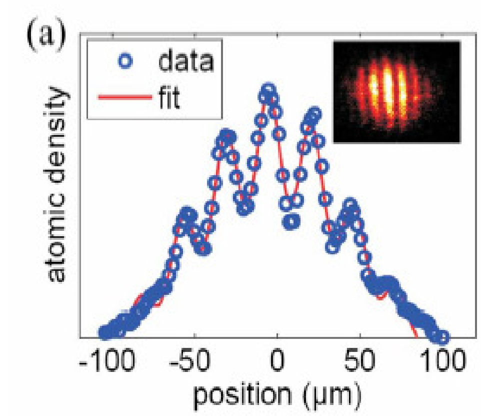
Experiments: Schmiedmayer et al., Nature Physics (2005,2006)





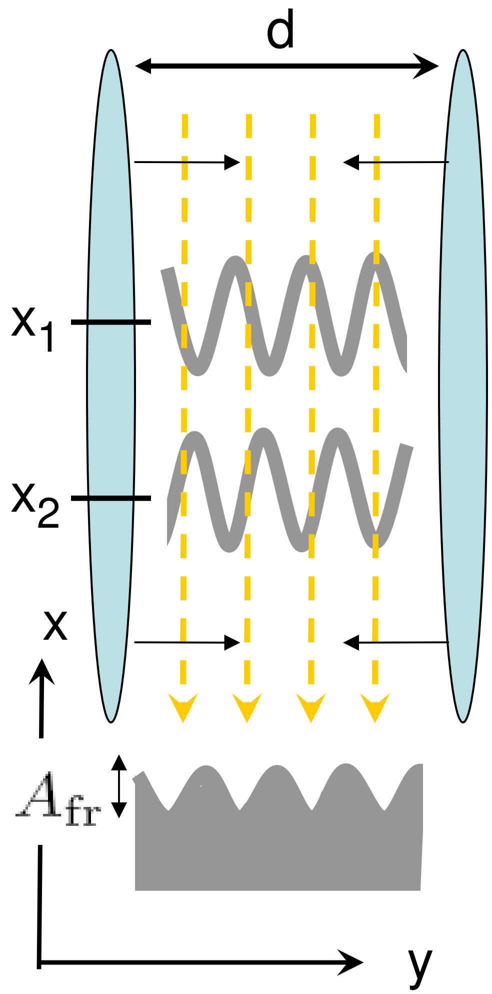
Reduction of the contrast
due to fluctuations





Figures courtesy of J. Schmiedmayer

Interference of one dimensional condensates



Amplitude of interference fringes, $A_{\rm fr}$, contains information about phase fluctuations within individual condensates

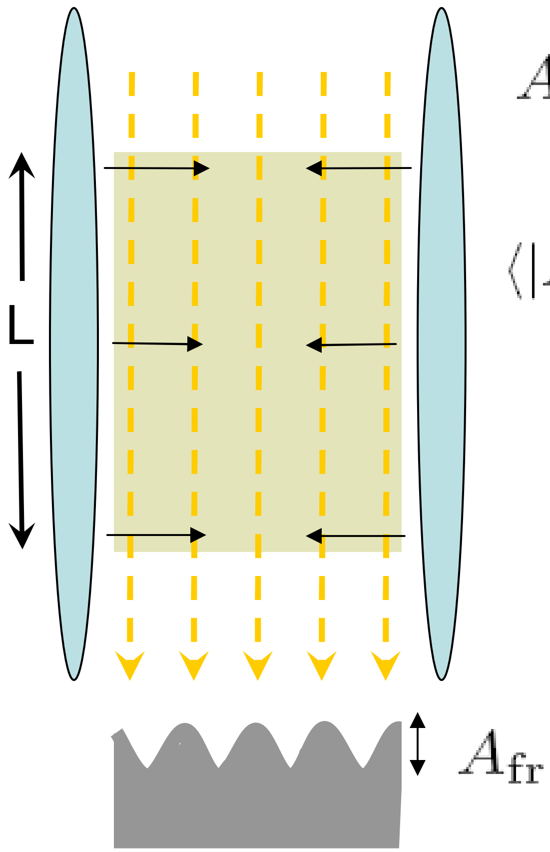
$$d\rho_{\rm int}(x,y) = (e^{i\frac{mdy}{\hbar t}} a_1^\dagger(x) a_2(x) + {\rm c.c.})\, dx$$

$$\rho_{\rm int}(y) = e^{i\frac{mdy}{\hbar t}} \int_0^L dx\, a_1^\dagger(x) a_2(x) + {\rm c.c.}$$

$$\rho_{\rm int}(y) = A_{\rm fr}\, e^{i\Delta\phi + i\frac{mdy}{\hbar t}} + {\rm c.c.}$$

Interference amplitude and correlations

Polkovnikov, Altman, Demler, PNAS 103:6125 (2006)



$$A_{\mathrm{fr}}\, e^{i\Delta\phi} = \int_0^L dx\; a_1^\dagger(x)\, a_2(x)$$

$$\langle |A_{\mathrm{fr}}|^2 \rangle = \int_0^L \int_0^L dx\, dy\, \langle a_1^\dagger(x)\, a_2(x)\, a_2^\dagger(y)\, a_1(y) \rangle$$

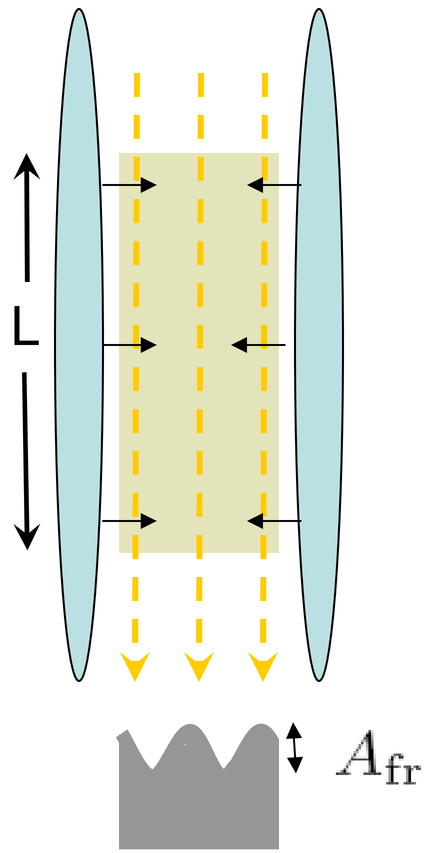
$$\simeq L \int_0^L dx\, \langle a_1(x)\, a_1^\dagger(0) \rangle \langle a_2(0) a_2^\dagger(x) \rangle$$

For identical condensates

$$\langle |A_{\mathrm{fr}}|^2 \rangle = L \int_0^L dx\, (G(x))^2$$

Instantaneous correlation function $G(x) = \langle a(x)\, a^\dagger(0) \rangle$

Interference between Luttinger liquids



Luttinger liquid at T=0

$$G(x) \sim \rho \left(\frac{\xi_h}{x} \right)^{1/2K}$$

K – Luttinger parameter

$$\langle |A_{\rm fr}|^2 \rangle \sim (\rho \xi_h)^{1/K} (L\rho)^{2-1/K}$$

For non-interacting bosons $K = \infty$ and $A_{\rm fr} \sim L$

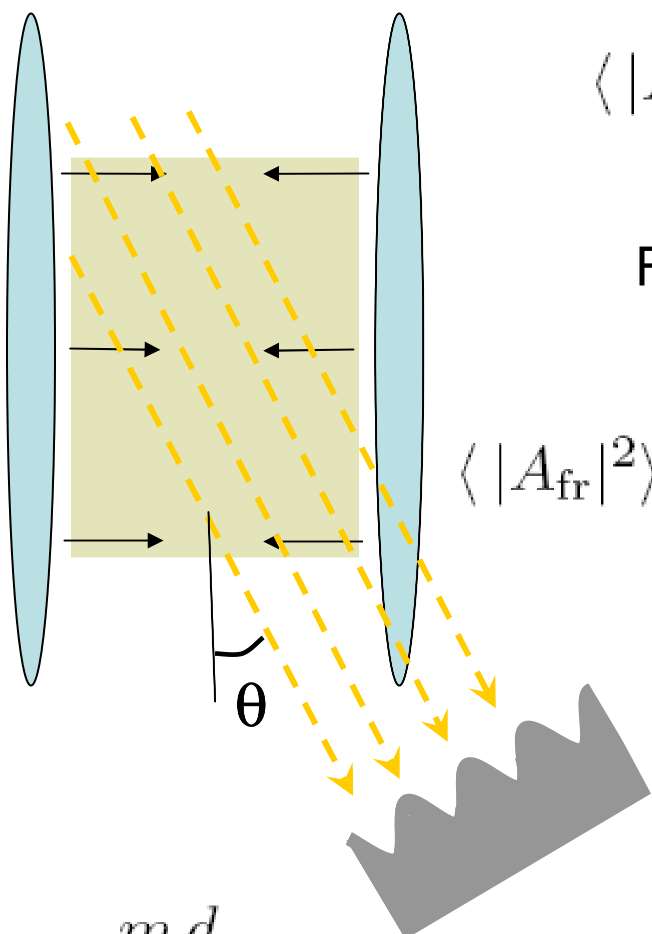
For impenetrable bosons $K = 1$ and $A_{\rm fr} \sim \sqrt{L}$

Luttinger liquid at finite temperature

$$\langle |A_{\rm fr}|^2 \rangle \sim L \rho^2 \xi_h \left(\frac{\hbar^2}{m \xi_h^2} \frac{1}{T} \right)^{1-1/K}$$

Analysis of $A_{\rm fr}$ can be used for thermometry

Rotated probe beam experiment



$$\langle |A_{\mathrm{fr}}|^2 \rangle = L \int_0^L dx \; \cos(qx)\,(G(x))^2$$

For large imaging angle, $qL >> 1$

$$\langle |A_{\mathrm{fr}}|^2 \rangle \sim L\,\rho^2\,\xi_h\,\sin\left(\frac{\pi}{K}\right)\Gamma\left(1-\frac{2}{K}\right)(\xi_h\,q)^{1/K-1}$$

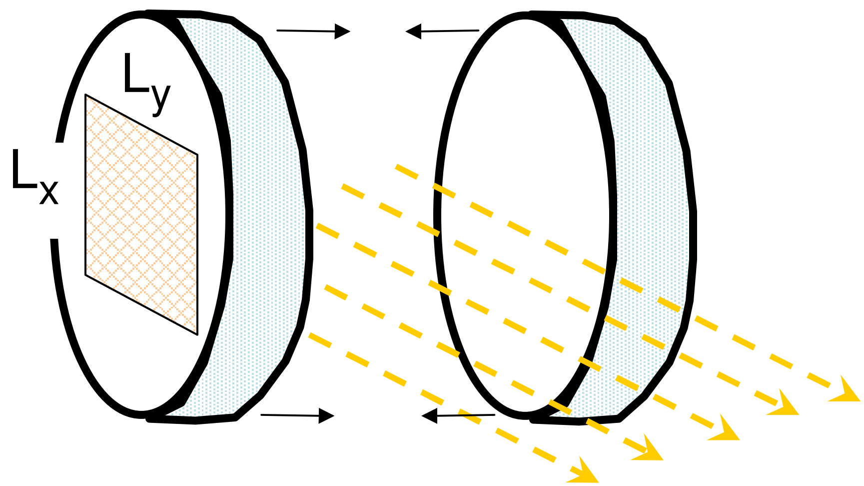
Luttinger parameter *K* may be extracted from the angular dependence of $A_{\mathrm{fr}}(\theta)$

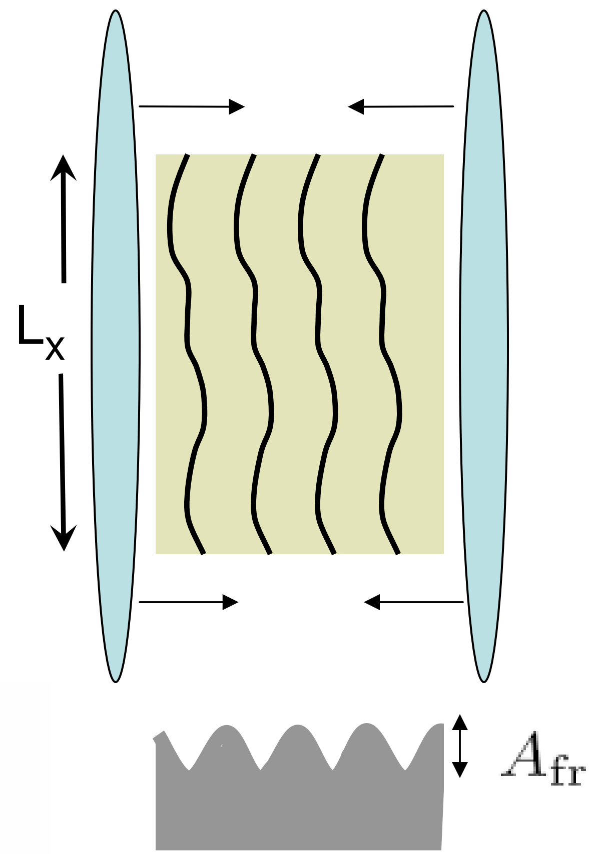
$$q = \frac{m\,d}{\hbar\,t}\tan\theta$$

Interference between two-dimensional BECs at finite temperature. Kosteritz-Thouless transition

Interference of two dimensional condensates

Experiments: Stock, Hadzibabic, Dalibard, et al., PRL 95:190403 (2005)



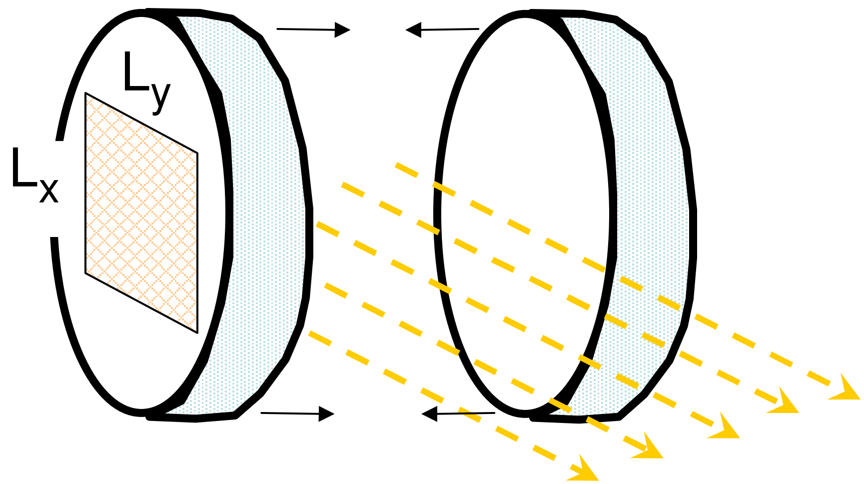


Probe beam parallel to the plane of the condensates

$$\langle\, |A_{\rm fr}|^2 \rangle \;=\; L_x\, L_y \int_0^{L_x} \int_0^{L_y} d^2\vec{r}\; (\, G(\vec{r}\,)\,)^2$$

$$G(\vec{r}\,) \;=\; \langle\, a(\vec{r}\,)\, a^\dagger(0)\, \rangle$$

Interference of two dimensional condensates. Quasi long range order and the KT transition



Theory: Polkovnikov, Altman, Demler, PNAS 103:6125 (2006)

Above KT transition

$$G(r) \sim e^{-r/\xi}$$

$$\langle |A_{\text{fr}}|^2 \rangle \sim L_x L_y$$

$$\log \xi(T) \sim 1/\sqrt{T - T_{\text{KT}}}$$

Below KT transition

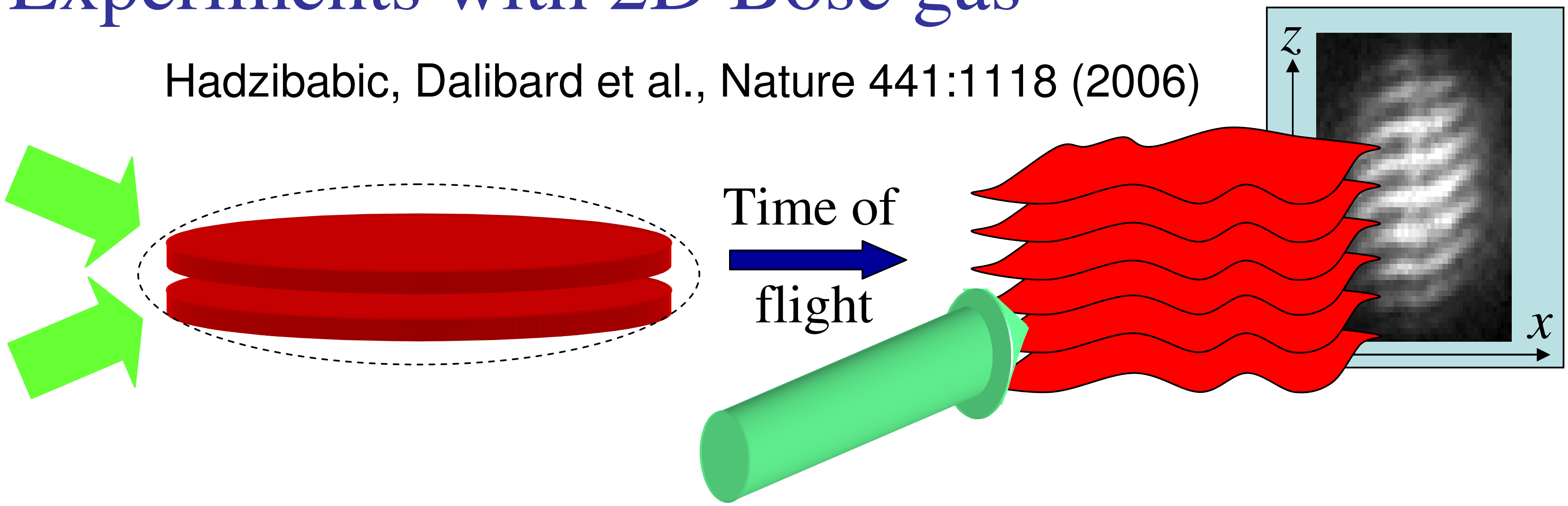
$$G(r) \sim \rho \left(\frac{\xi_h}{r} \right)^{\alpha}$$

$$\alpha(T) = \frac{m T}{2 \pi \rho_s(T) \hbar^2}$$

$$\langle |A_{\text{fr}}|^2 \rangle \sim (L_x L_y)^{2-\alpha}$$

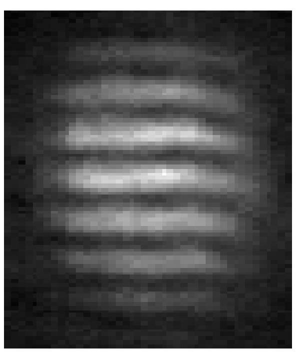
Experiments with 2D Bose gas

Hadzibabic, Dalibard et al., Nature 441:1118 (2006)

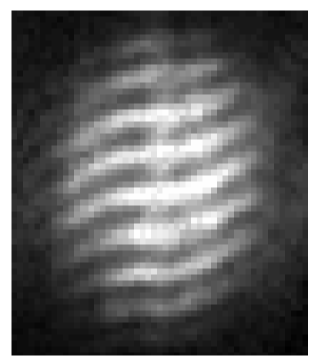


Typical interference patterns

low temperature

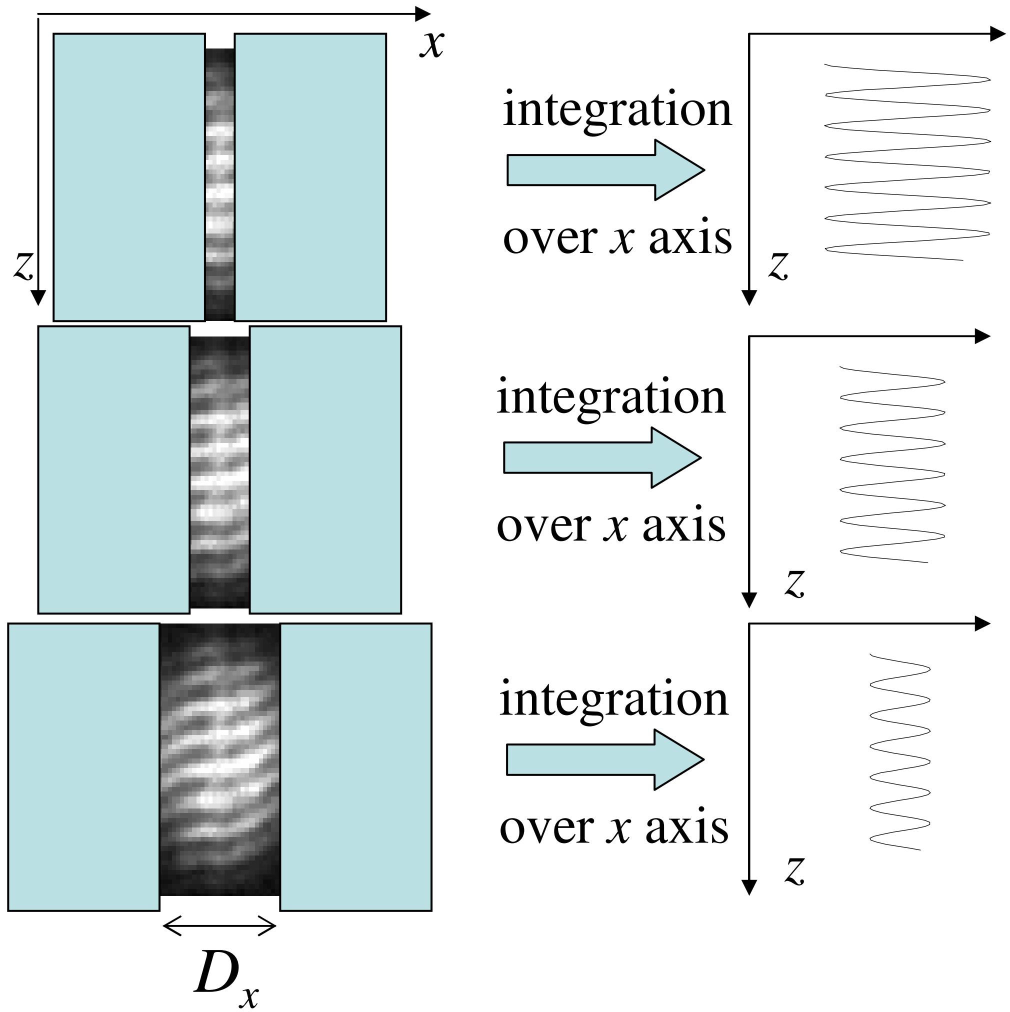


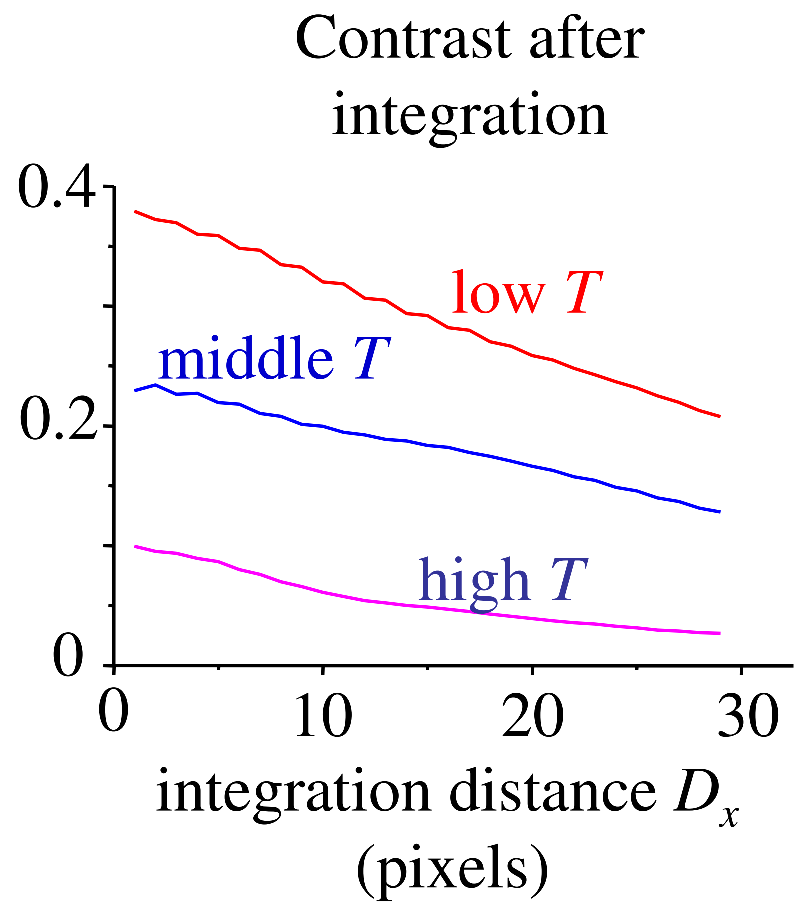
higher temperature



Experiments with 2D Bose gas

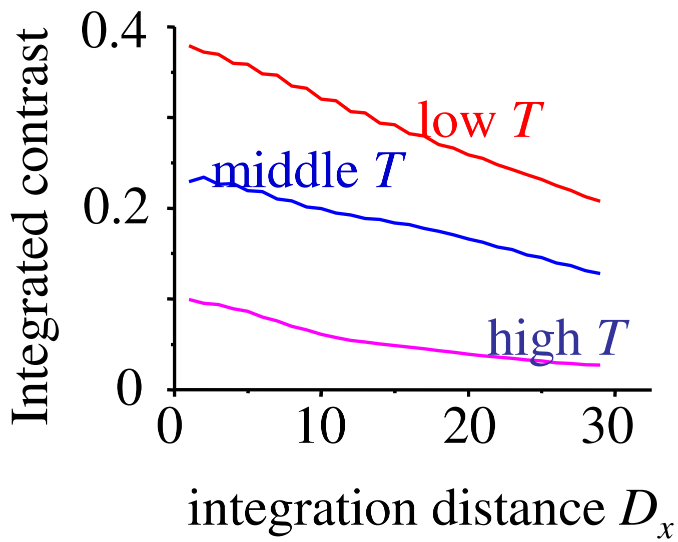
Hadzibabic et al., Nature 441:1118 (2006)



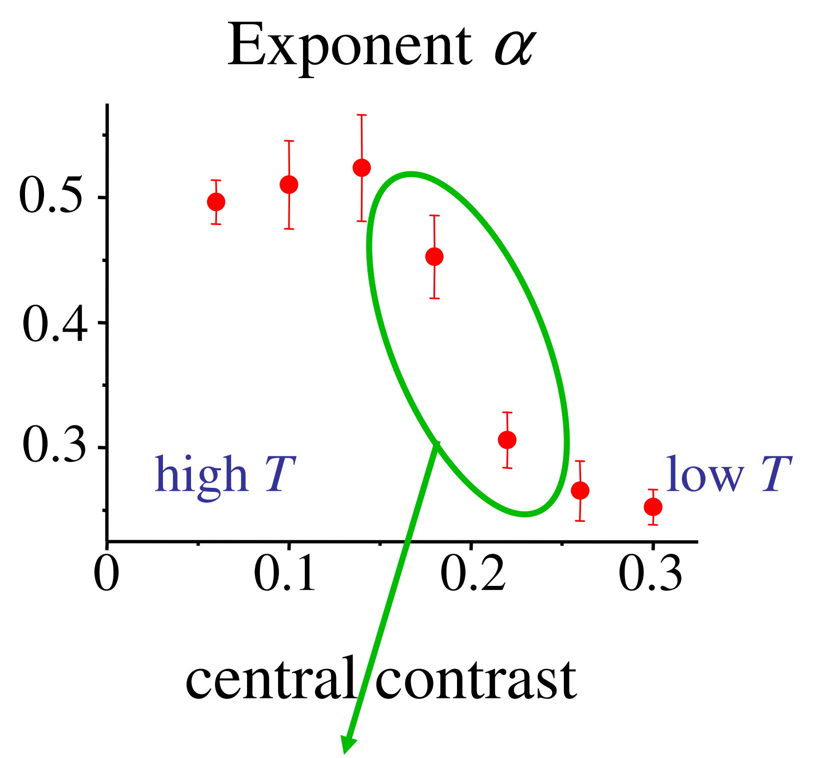


Experiments with 2D Bose gas

Hadzibabic et al., Nature 441:1118 (2006)



fit by: $$C^2 \sim \frac{1}{D_x}\int^{D_x}[g_1(0,x)]^2 dx \sim \left(\frac{1}{D_x}\right)^{2\alpha}$$



"Sudden" jump!?

⟹ if $g_1(r)$ decays exponentially with $\ell_{\rm coh} \ll D_x$: $\alpha = 1/2$

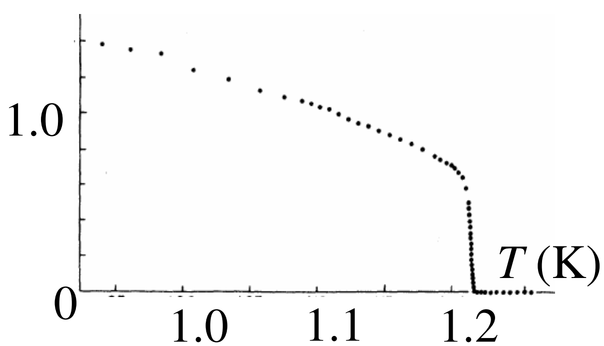
⟹ if $g_1(r)$ decays algebraically or exponentially with a large $\ell_{\rm coh}$:

$$\alpha < 1/2$$

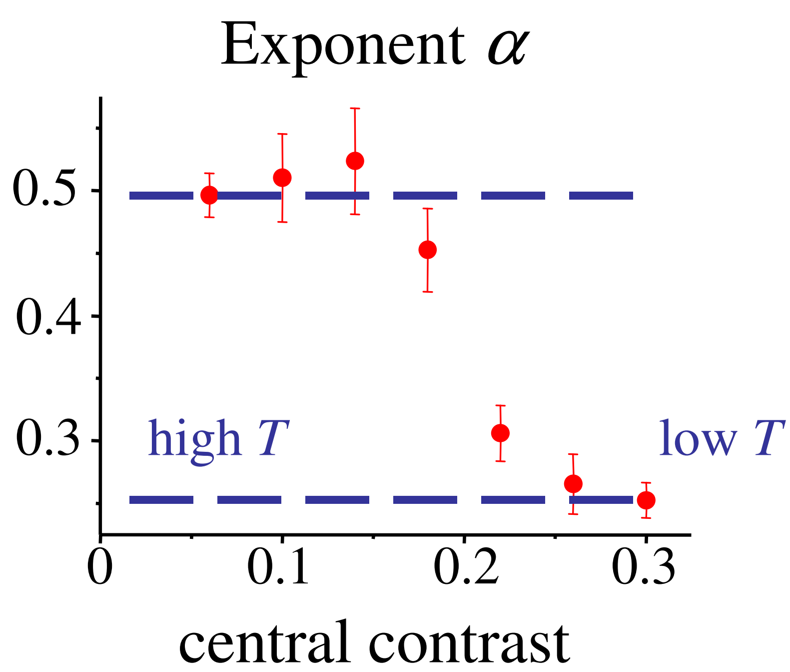
Experiments with 2D Bose gas

Hadzibabic et al., Nature 441:1118 (2006)

c.f. Bishop and Reppy



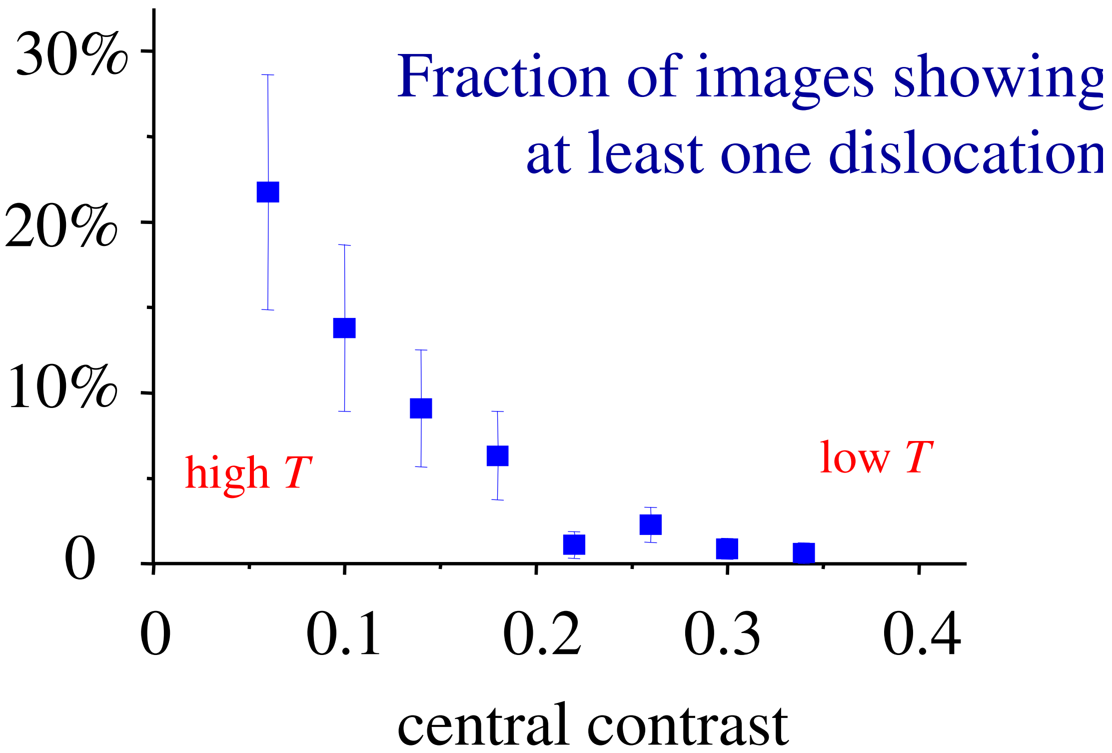
He experiments:
universal jump in
the superfluid density

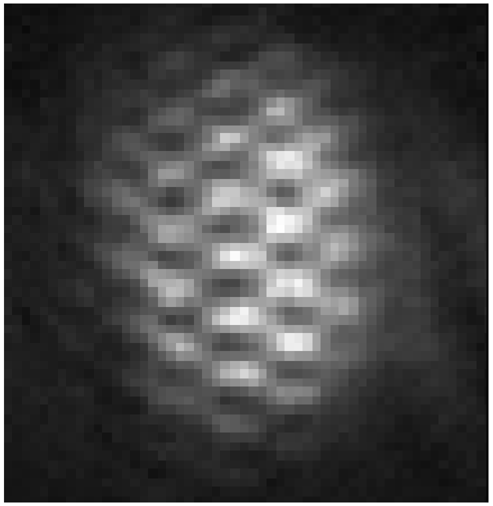


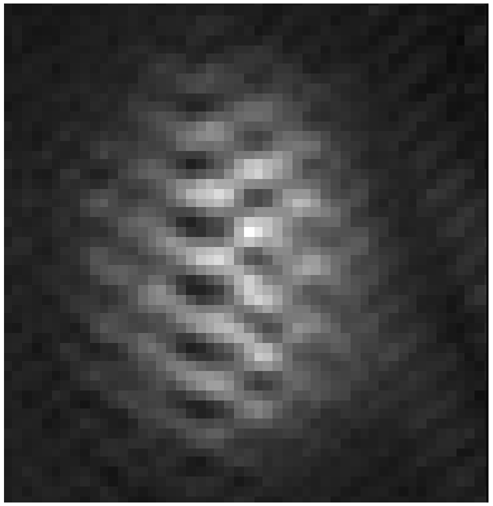
Ultracold atoms experiments:
jump in the correlation function.
KT theory predicts α=1/4
just below the transition

Experiments with 2D Bose gas. Proliferation of thermal vortices

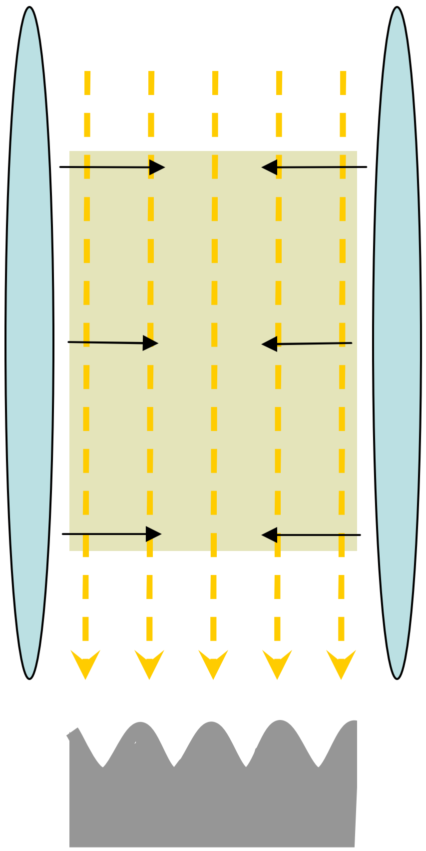
Hadzibabic et al., Nature 441:1118 (2006)







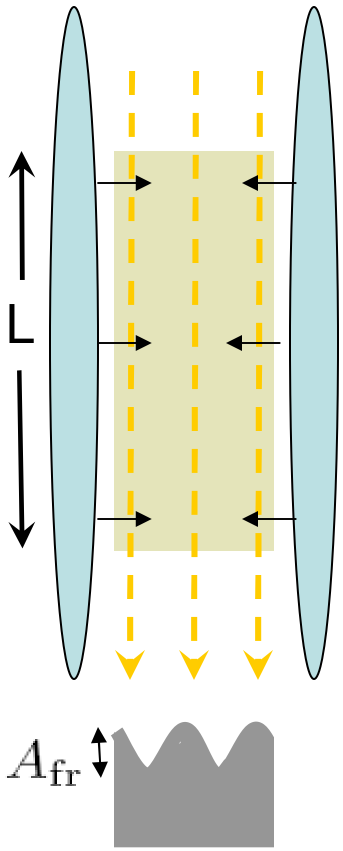
The onset of proliferation coincides with α shifting to 0.5!



Full distribution function of fringe amplitudes for interference experiments between two 1d condensates

Gritsev, Altman, Demler, Polkovnikov, Nature Physics 2:705(2006)

Higher moments of interference amplitude



$A_{\rm fr}$ is a quantum operator. The measured value of $|A_{\rm fr}|$ will fluctuate from shot to shot.
Can we predict the distribution function of $|A_{\rm fr}|$?

Higher moments

$$\langle\, |A_{\rm fr}|^{2n} \rangle = \int_0^L dz_1 \ldots dz_n' \, |\langle\, a^\dagger(z_1) \ldots a^\dagger(z_n)\, a(z_1') \ldots a(z_n')\,\rangle|^2$$

Changing to periodic boundary conditions (long condensates)

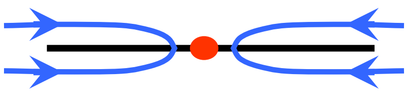
$$\langle\, |A_{\rm fr}|^{2n} \rangle = \langle\, |A_{\rm fr}|^2 \rangle^n \times Z_{2n}$$

$$Z_{2n} = \prod_{ij} \int_0^{2\pi} \cdots \int_0^{2\pi} \frac{du_i}{2\pi} \frac{dv_j}{2\pi} \left| \frac{\prod_{i<j} 2\sin(\frac{u_i-u_j}{2}) \prod_{i<j} 2\sin(\frac{v_i-v_j}{2})}{\prod_{ij} 2\sin(\frac{u_i-v_j}{2})} \right|^{1/K}$$

Explicit expressions for Z_{2n} are available but cumbersome

Fendley, Lesage, Saleur, J. Stat. Phys. 79:799 (1995)

Impurity in a Luttinger liquid



$$S = \frac{\pi K}{2} \int dx\, d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right] + 2g \int d\tau \cos\phi\,(x=0\,,\tau)$$

Expansion of the partition function in powers of *g*

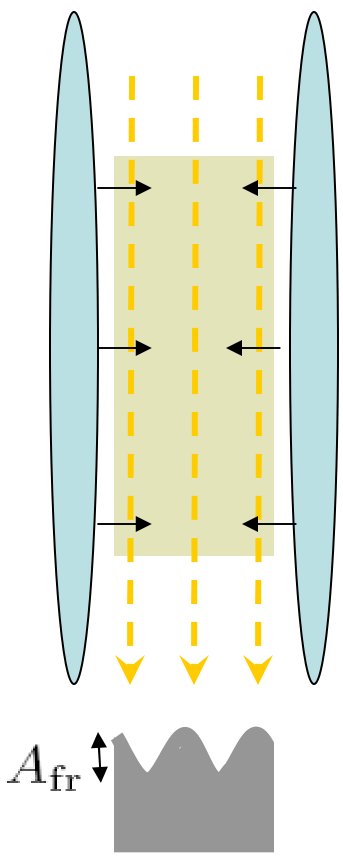
$$Z_{\rm imp} = \sum_n \frac{g^{2n}}{(2n)!} \int d\tau_1 \ldots d\tau_n \,(e^{i\phi} + e^{-i\phi})_{\tau_1} \ldots (e^{i\phi} + e^{-i\phi})_{\tau_{2n}}$$

$$Z_{\rm imp} = \sum \frac{g^{2n}}{(n!)^2} Z_{2n}$$

$$Z_{2n} = \prod_{ij} \int_0^{2\pi} \cdots \int_0^{2\pi} \frac{du_i}{2\pi} \frac{dv_j}{2\pi} \left| \frac{\prod_{i<j} 2\sin(\frac{u_i - u_j}{2}) \prod_{i<j} 2\sin(\frac{v_i - v_j}{2})}{\prod_{ij} 2\sin(\frac{u_i - v_j}{2})} \right|^{1/K}$$

Partition function of the impurity contains correlation functions taken at the same point and at different times. Moments of interference experiments come from correlations functions taken at the same time but in different points. Euclidean invariance ensures that the two are the same

Relation between quantum impurity problem and interference of fluctuating condensates



Normalized amplitude of interference fringes $\quad a^2 = |A_{\rm fr}|^2/\langle\, |A_{\rm fr}|^2 \rangle$

Distribution function of fringe amplitudes $\quad W(\,K,\, a^2\,)$

Relation to the impurity partition function

$$Z_{\rm imp}(\,K,\, g\,) \;=\; \int_0^{\infty} da^2\, W(\,K\,,\, a^2\,)\, I_0(\,2g\, a\,)$$

Distribution function can be reconstructed from $Z_{\rm imp}(\,K,\, g\,)$ using completeness relations for the Bessel functions

$$\boxed{W(K,a^2) \;=\; 2\int_0^{\infty} g\, dg\, Z_{\rm imp}(K, ig) J_0(2ga^2)}$$

Bethe ansatz solution for a quantum impurity

$Z_{\rm imp}(K,\,g\,)$ can be obtained from the Bethe ansatz following
Zamolodchikov, Phys. Lett. B 253:391 (91); Fendley, et al., J. Stat. Phys. 79:799 (95)
Making analytic continuation is possible but cumbersome

Interference amplitude and spectral determinant

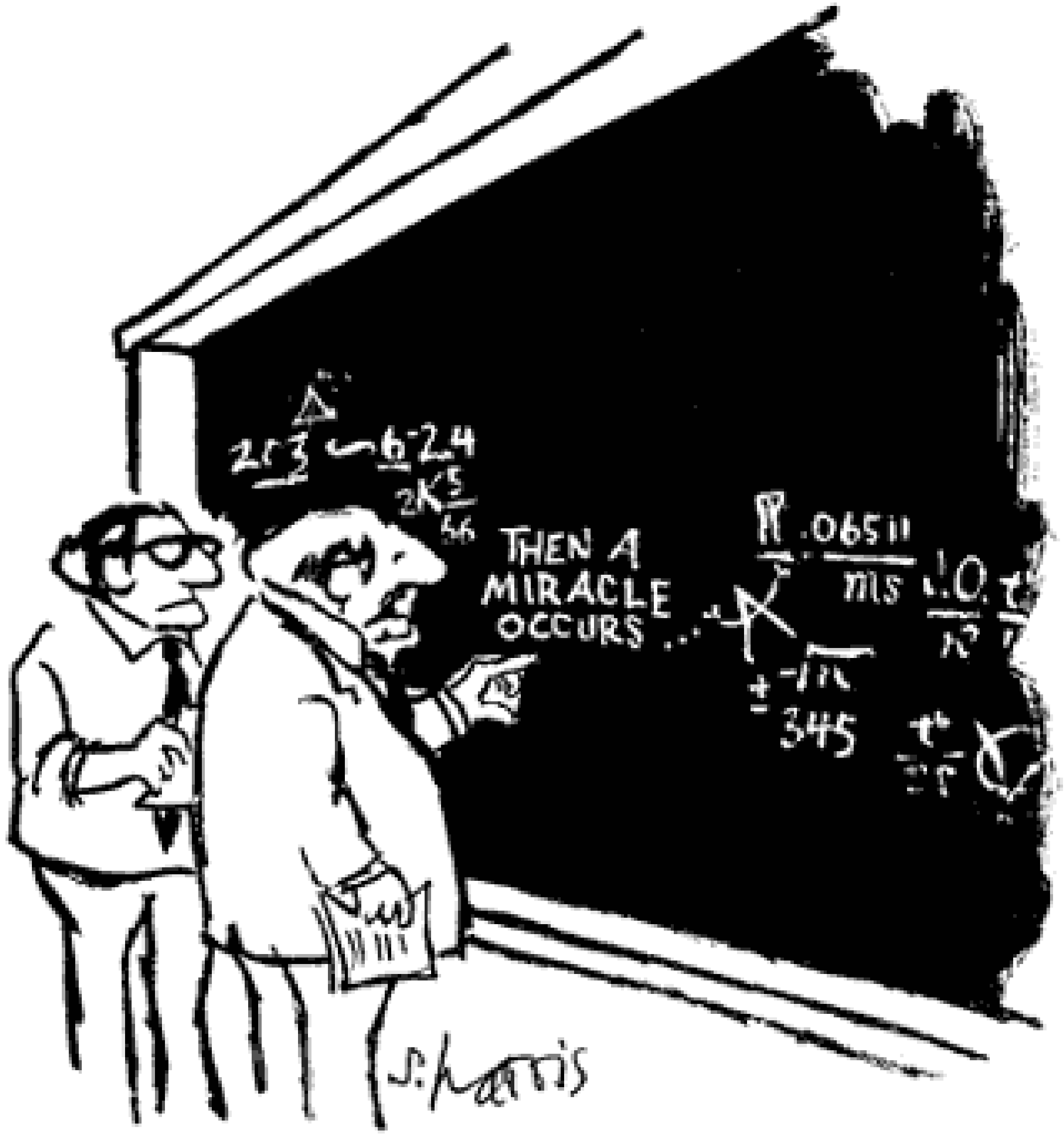
$Z_{\rm imp}(\,K,\,ig\,)$ is related to the Schroedinger equation

Dorey, Tateo, J.Phys. A. Math. Gen. 32:L419 (1999)
Bazhanov, Lukyanov, Zamolodchikov, J. Stat. Phys. 102:567 (2001)

$$-\frac{d^2\,\Psi}{d\,x^2} + (\,x^{4K-2} + \frac{3}{4x^2}\,)\,\Psi = E\,\Psi$$

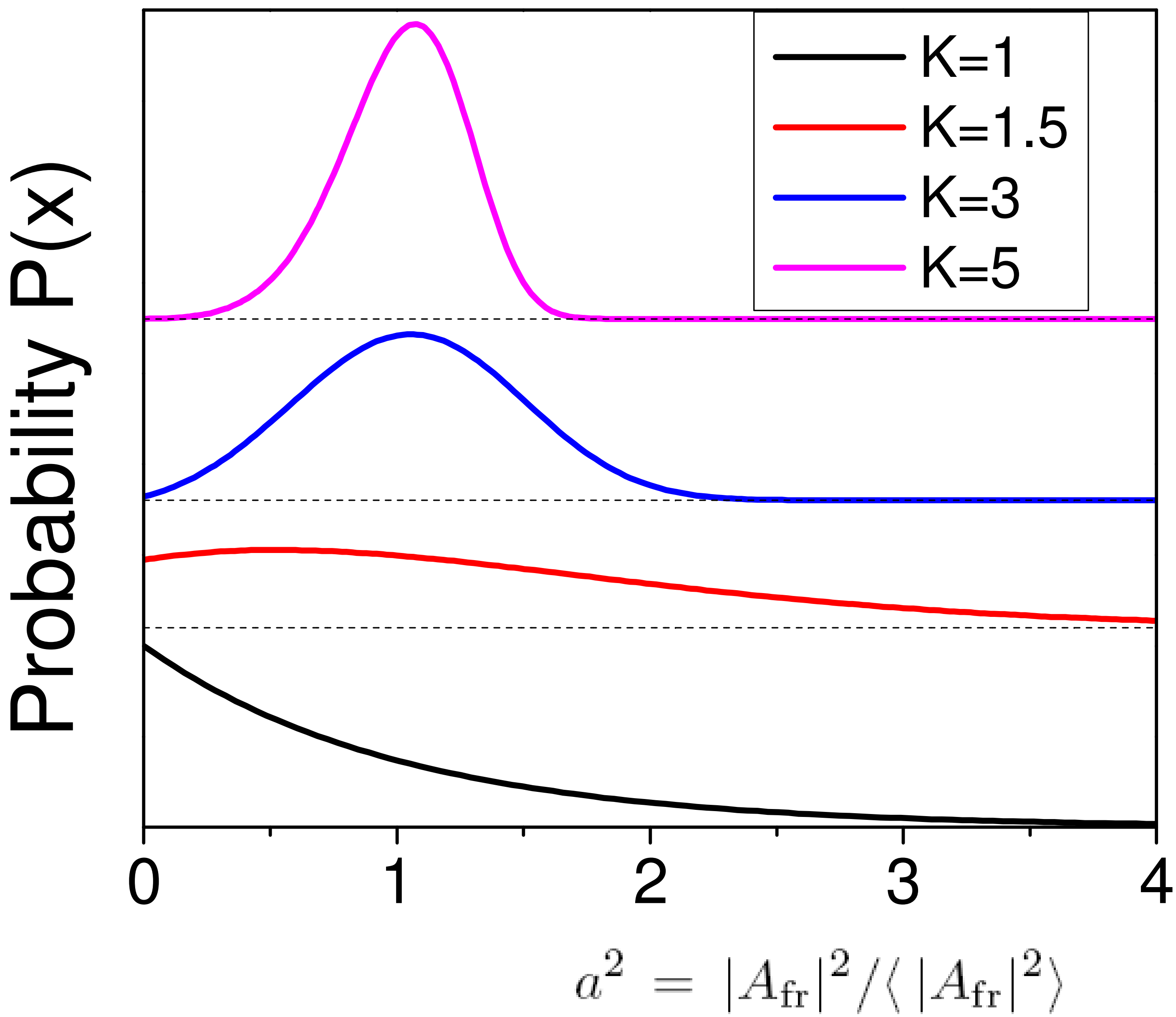
Spectral determinant $\quad D(E) = \prod_{n=1}^{\infty}(\,1 - \frac{E}{E_n})$

$$Z_{\rm imp}(\,K,\,ig\,) = D\left(\frac{g^2}{\pi^2}\,(4K)^{2-K^{-1}}\,[\Gamma(1-\frac{1}{2K})]^2\,\sin^2(\frac{\pi}{2K})\right)$$



“I think you should be more explicit here in step two.”

Evolution of the distribution function



Narrow distribution for $K \to \infty$ Approaches Gumbel distribution.

Width $\sigma = \frac{\pi}{6K}$

Wide Poissonian distribution for $K \to 1$

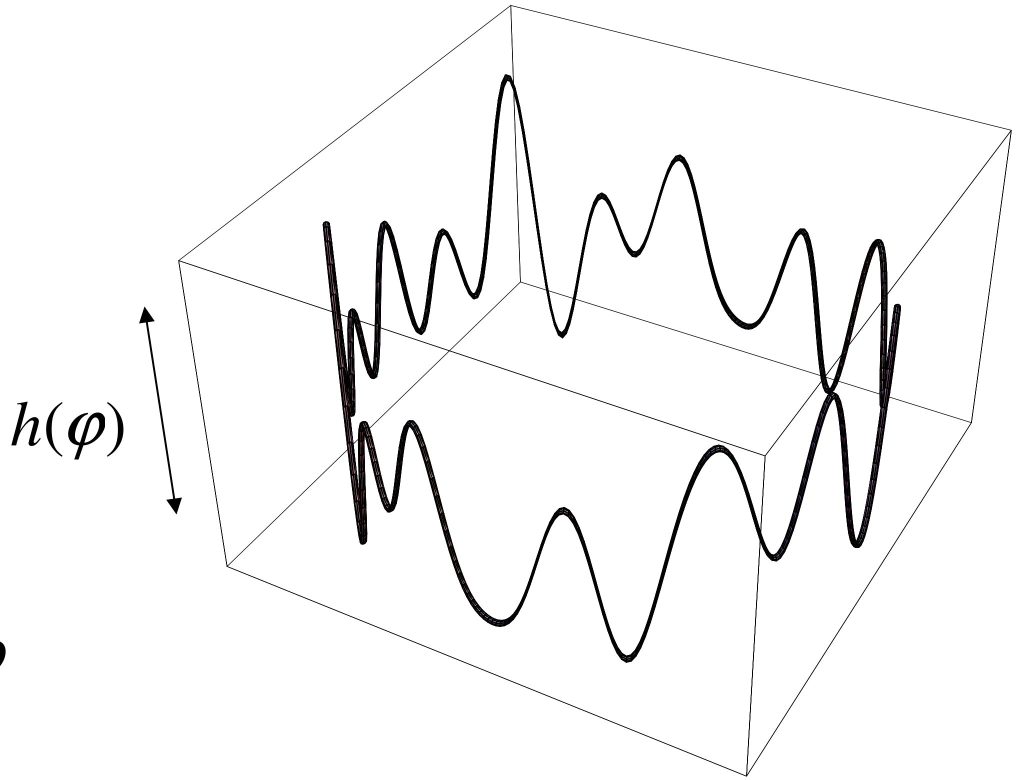
Gumbel distribution and 1/f noise

For K>>1

Fringe visibility

⇅

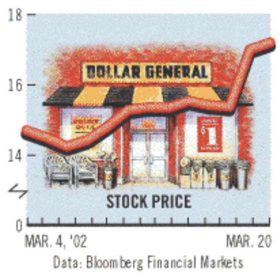
Roughness $= \int h(\varphi)^2 d\varphi$



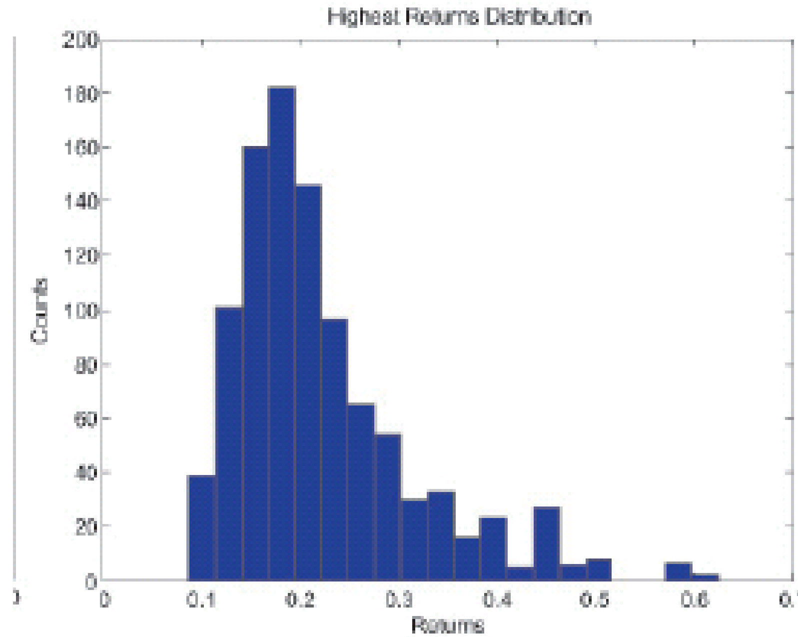
1/f Noise and Extreme Value Statistics
T.Antal et al. Phys.Rev.Lett. 87, 240601(2001)

Gumbel Distribution in Statistics

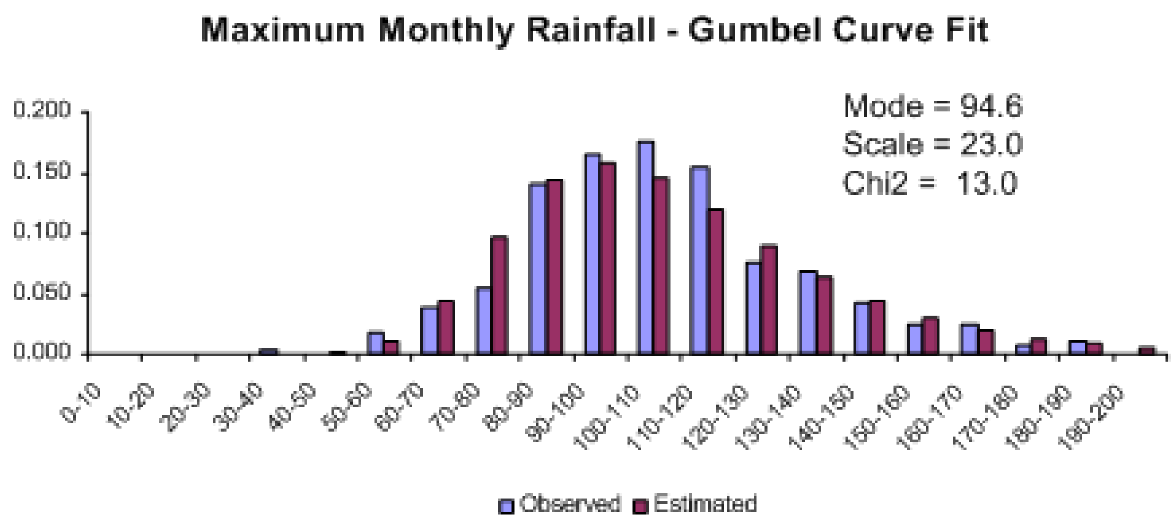
Describes Extreme Value Statistics, appears in climate studies, finance, etc.



Stock performance: distribution of “best performers” for random sets chosen from S&P500



Distribution of largest monthly rainfall over a period of 291 years at Kew Gardens



Distribution function for open boundary conditions, finite temperature, 2D systems, …

A. Imambekov et al.

$$Z_{2n} = \int_0^1 \dots \int_0^1 du_1 \dots du_n dv_1 \dots dv_n \left| \frac{\prod_{i<j} |u_i - u_j| \prod_{i<j} |v_i - v_j|}{\prod_{ij} |u_i - v_j|} \right|^{1/K}$$

$$= \int_{-1/2}^{1/2} \dots \int_{-1/2}^{1/2} du_1 \dots dv_n e^{\frac{1}{K}\left(\sum_{i<j} f(u_i,u_j) + \sum_{i<j} f(v_i,v_j) - \sum_{ij} f(u_i,v_j)\right)}$$

$$f^p(x,y) = Log \frac{1}{\pi} \sin |\pi(x-y)|$$ Periodic boundary conditions

$$f(x,y) = Log(|x-y|)$$ Open boundary conditions

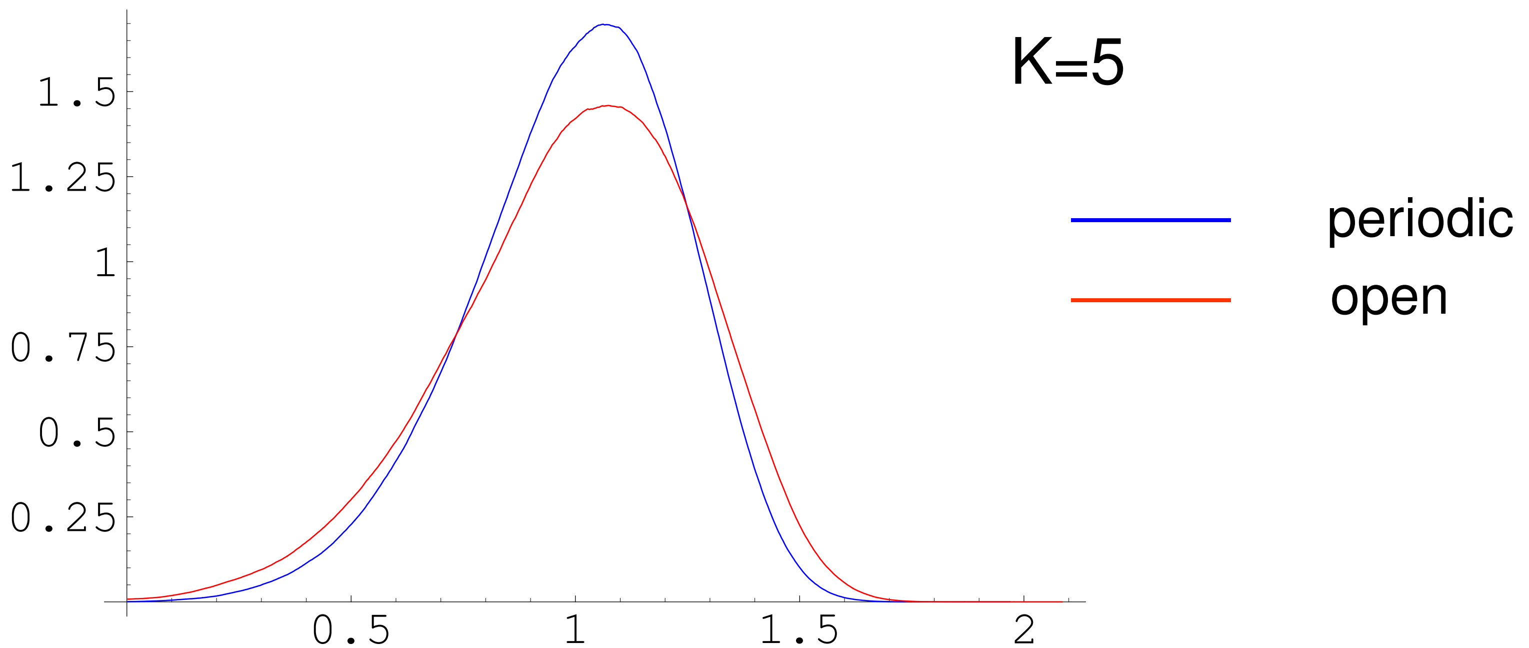
$$f(x,y,a) = Log \frac{a}{\pi} \sinh |\frac{\pi(x-y)}{a}|, a = \frac{\xi_T}{L}$$ Finite temperature

Partition function of classical plasma

1. High temperature expansion (expansion in 1/K)

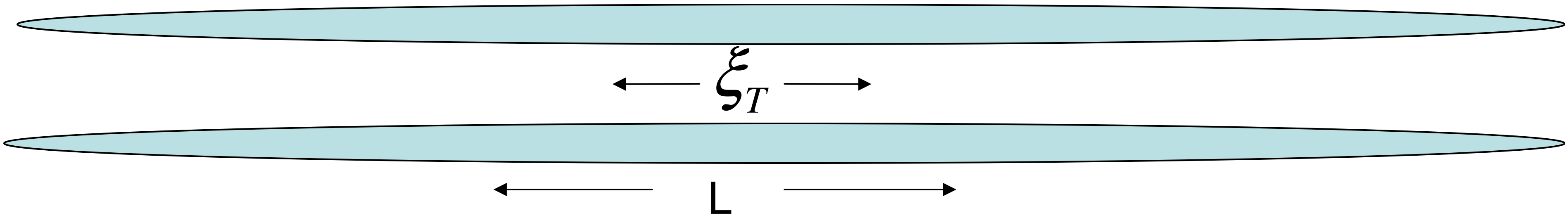
2. Non-perturbative solution

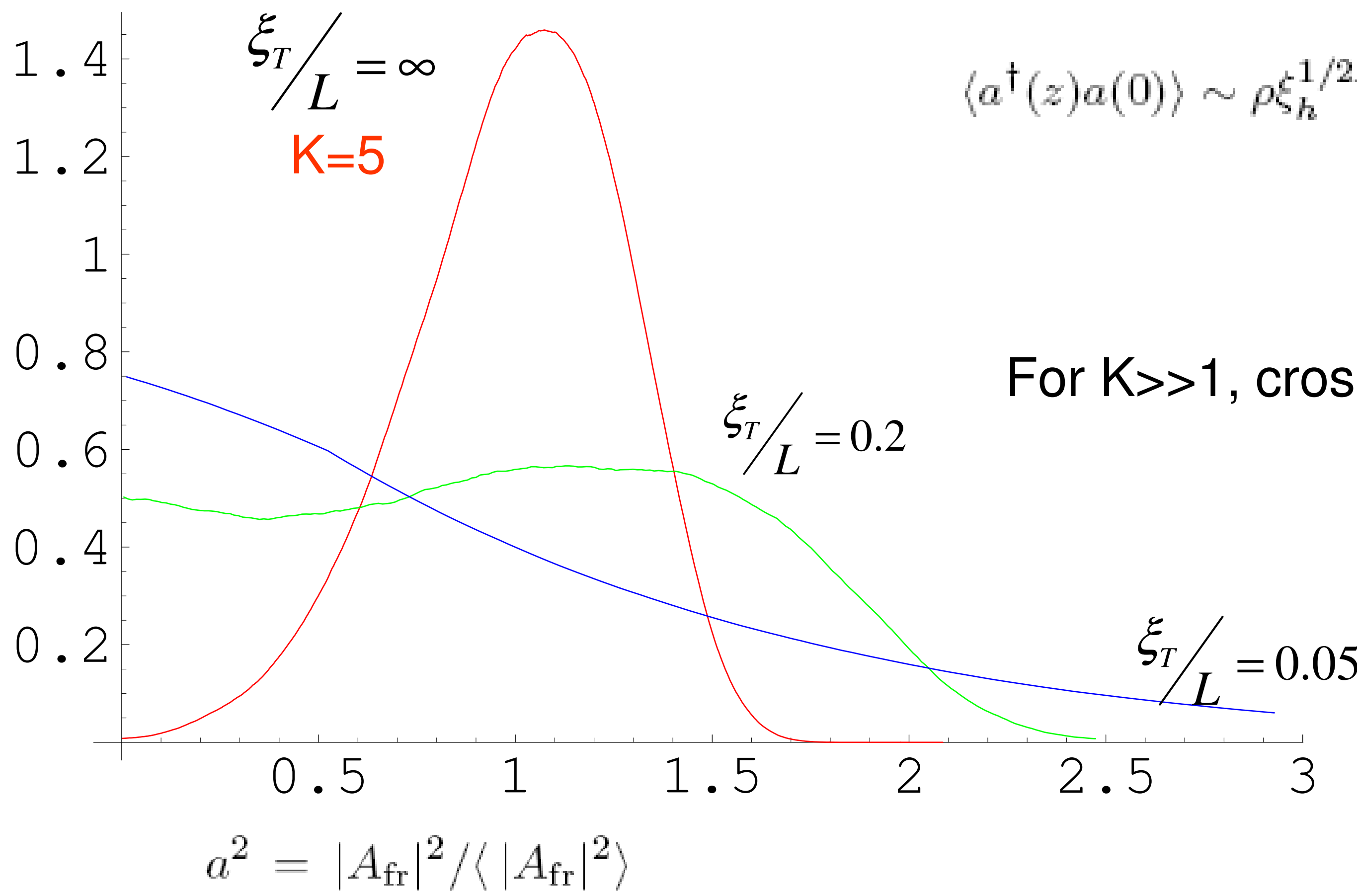
Periodic versus Open boundary conditions



$$a^2 = |A_{\rm fr}|^2 / \langle\, |A_{\rm fr}|^2 \rangle$$

Distribution functions at finite temperature

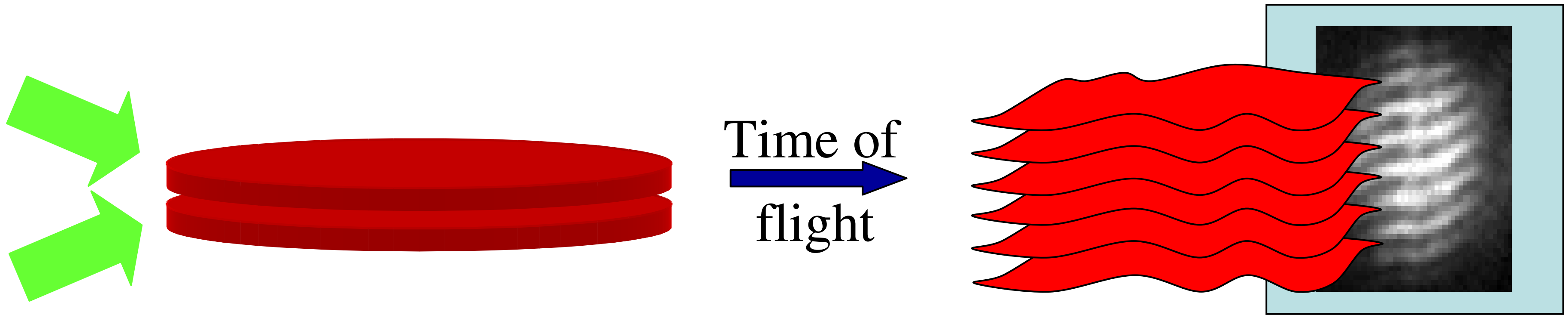


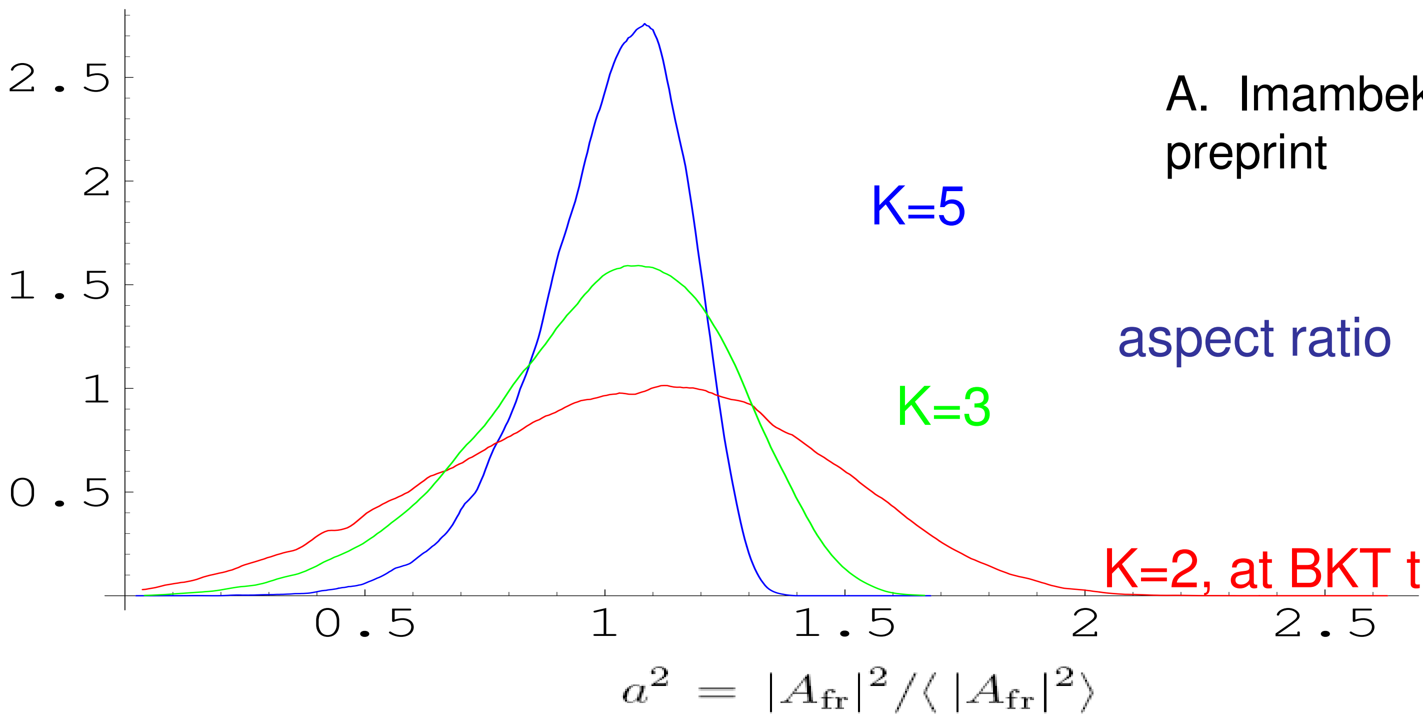


$$\langle a^\dagger(z) a(0) \rangle \sim \rho \xi_h^{1/2K} \left(\frac{\pi / \xi_T}{\sinh(\pi z / \xi_T)} \right)^{1/2K}$$

For K>>1, crossover at $\xi_T K \sim L$

Interference between fluctuating 2d condensates. Distribution function of the interference amplitude





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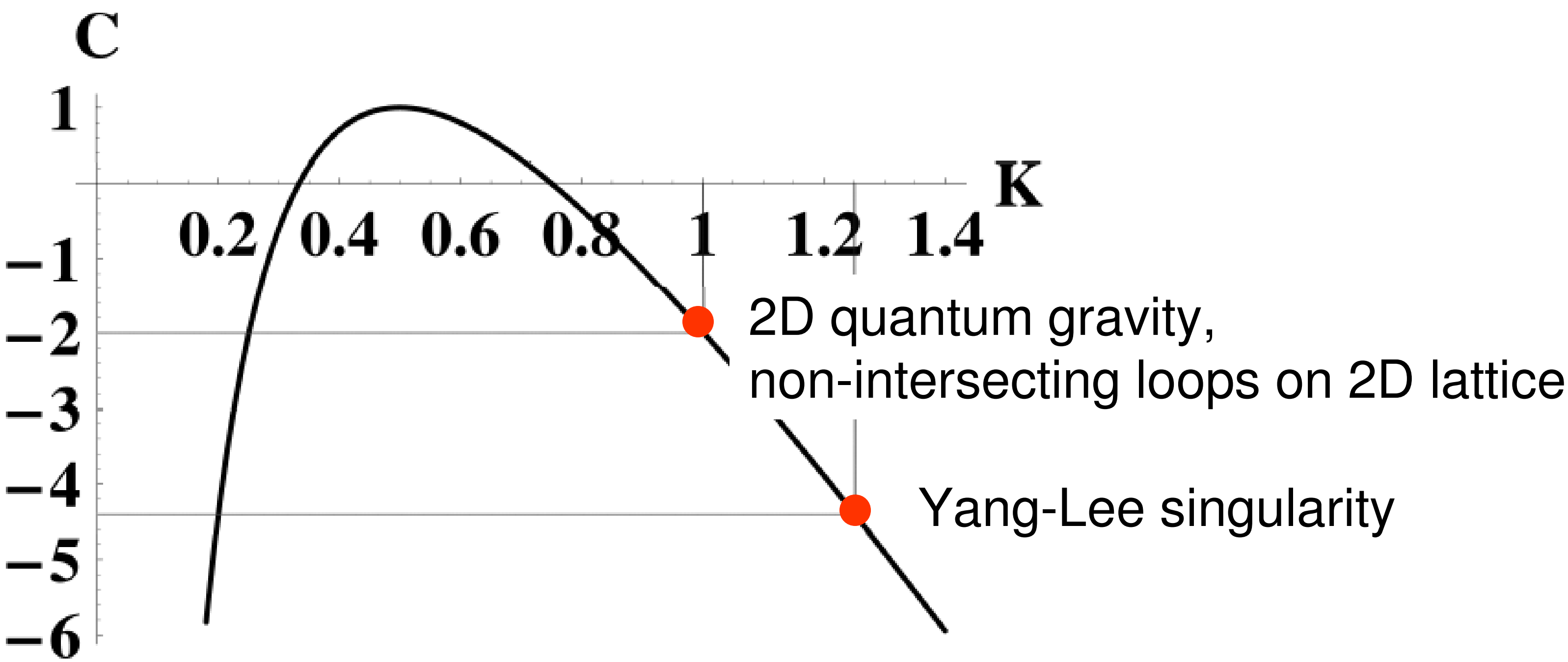
aspect ratio 1 (Lx=Ly).

From interference amplitudes to conformal field theories

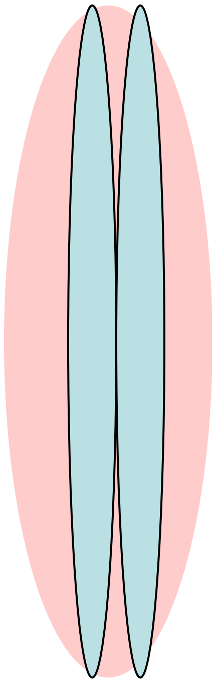
$Z_{\rm imp}(\,K,\,ig\,)$ correspond to vacuum eigenvalues of *Q* operators of CFT
Bazhanov, Lukyanov, Zamolodchikov, Comm. Math. Phys.1996, 1997, 1999

When K>1, $Z_{\rm imp}(\,K,\,ig\,)$ is related to Q operators of CFT with c<0. This includes 2D quantum gravity, non-intersecting loop model on 2D lattice, growth of random fractal stochastic interface, high energy limit of multicolor QCD, …

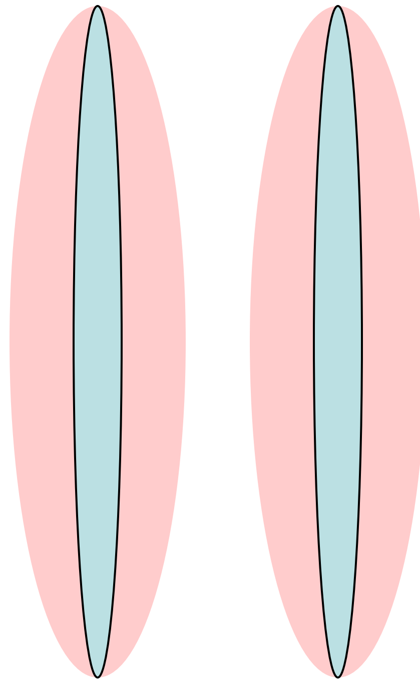


Condensate decoherence at finite temperature probed with interference experiments

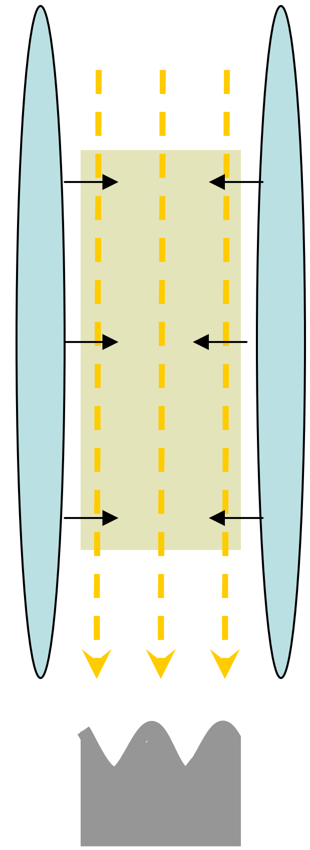
Studying dynamics using interference experiments. Thermal decoherence



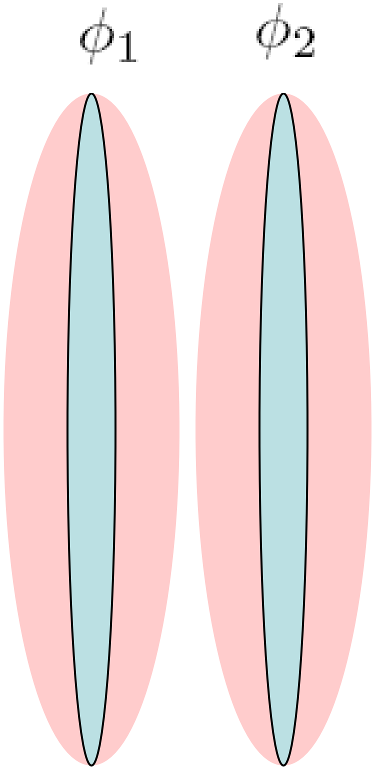
Prepare a system by splitting one condensate



Take to the regime of zero tunneling



Measure time evolution of fringe amplitudes



Finite temperature phase dynamics

$$\mathcal{H}_0 = \int dx\ \left[g\, n_1^2(x) + \rho\,(\partial_x \phi_1)^2\right] + \int dx\ \left[g\, n_2^2(x) + \rho\,(\partial_x \phi_2)^2\right]$$

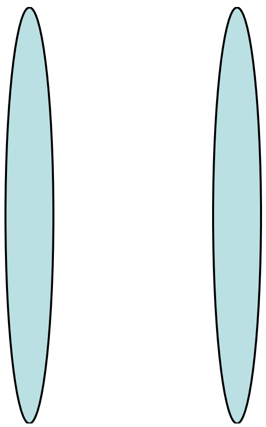
Temperature leads to phase fluctuations within individual condensates

Interference experiments measure only the relative phase

$$\phi_{av} = \frac{\phi_1 + \phi_2}{2} \qquad \phi = \phi_1 - \phi_2$$

Relative phase dynamics

ϕ_1 ϕ_2



$\phi = \phi_1 - \phi_2$

$\Delta n = (n_1 - n_2)/2$

Conjugate variables

$$\mathcal{H} = \int d^d r \left[\frac{g}{2} (\Delta n)^2 + \frac{\rho}{2} (\nabla \phi)^2 \right]$$

Hamiltonian can be diagonalized in momentum space

A collection of harmonic oscillators with $\omega_q = \sqrt{g\rho}\,|q|$

Need to solve dynamics of harmonic oscillators at finite T

Coherence $\langle \Psi(t) | e^{i\phi} | \Psi(t) \rangle = e^{-\frac{1}{2}\sum_q \langle \phi_q^2(t) \rangle}$

Relative phase dynamics

High energy modes, $\hbar\omega_{\rm osc} > k_{\rm B}T$, quantum dynamics

Low energy modes, $\hbar\omega_{\rm osc} < k_{\rm B}T$, classical dynamics

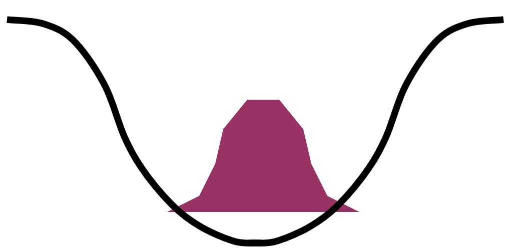
Combining all modes

$$t < \frac{h}{k_{\rm B}T}$$

Quantum dynamics

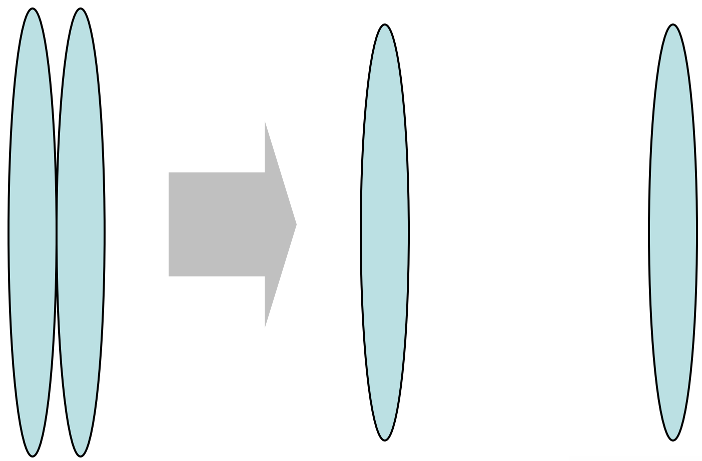
$$t > \frac{h}{k_{\rm B}T}$$

Classical dynamics



For studying dynamics it is important to know the initial width of the phase

Relative phase dynamics



$$\frac{N}{2} \pm \delta N \qquad \frac{N}{2} \mp \delta N$$

Naive estimate

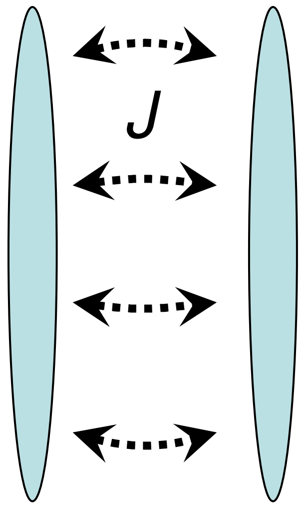
$$\delta N \sim \sqrt{N}$$

Long Phase Coherence Time and Number Squeezing of two Bose-Einstein Condensates on an Atom Chip

G.-B. Jo, Y. Shin, S. Will, T. A. Pasquini, M. Saba, W. Ketterle, and D. E. Pritchard*
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

M. Vengalattore, M. Prentiss
MIT-Harvard Center for Ultracold Atoms, Jefferson Laboratory, Physics Department, Harvard University, Cambridge, MA 02138, USA
(Dated: August 27, 2006)

Relative phase dynamics



$$\mathcal{H} = \frac{U}{2}(\Delta n)^2 - J\cos\phi$$

$$J(t) = J_0\, e^{-t/\tau_s}$$

Separating condensates at finite rate

Instantaneous Josephson frequency

$$\omega_J = \sqrt{U\,J}$$

Adiabatic regime $\dot{\omega}_J < \omega_J^2$

Instantaneous separation regime $\dot{\omega}_J > \omega_J^2$

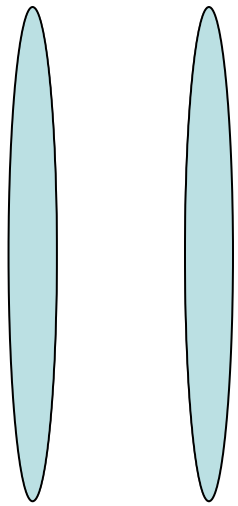
Adiabaticity breaks down when $\omega_J \sim 1/\tau_s$

Charge uncertainty at this moment

$$U\,\delta N^2 \sim \omega_J \sim 1/\tau_s$$

Squeezing factor $$\frac{\delta N}{\sqrt{N}} \sim \sqrt{\frac{1}{N\,U\,\tau_s}} \sim \sqrt{\frac{1}{\mu\,\tau_s}}$$

Relative phase dynamics



Quantum regime $\frac{h}{\mu} < t < \frac{h}{k_B T}$

1D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} \, e^{-t/2\pi K \tau_s}$

2D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} \left(\frac{t_0}{t} \right)^{1/16 T_{KT} \tau_s}$

Classical regime $t > \frac{h}{k_B T}$

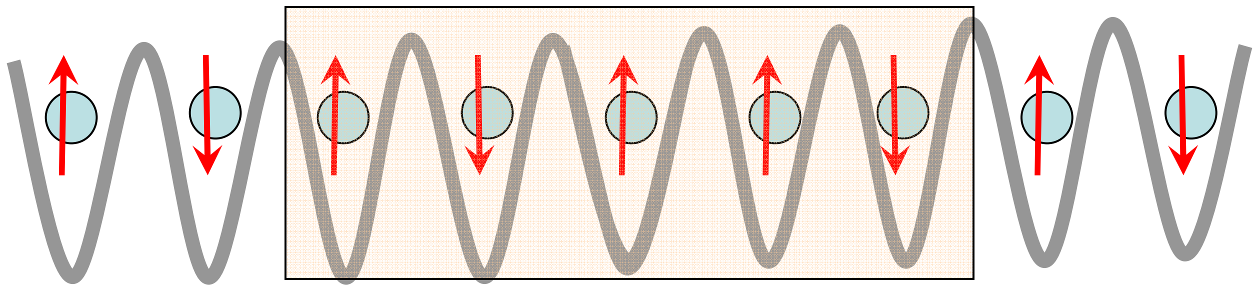
1D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\left(\frac{t}{t_T}\right)^{2/3}}$ $t_T \sim \frac{\mu K}{T^2}$

2D systems $\langle e^{i\phi(t)} \rangle \sim \left(\frac{t_0}{t} \right)^{\frac{T}{8 T_{KT}}}$

Probing spin systems using distribution function of magnetization

Probing spin systems using distribution function of magnetization

R. Cherng, E. Demler, cond-mat/0609748



Magnetization in a finite system $$M^z_{\rm tot} = \sum_{i=1}^{L} M^z(i)$$

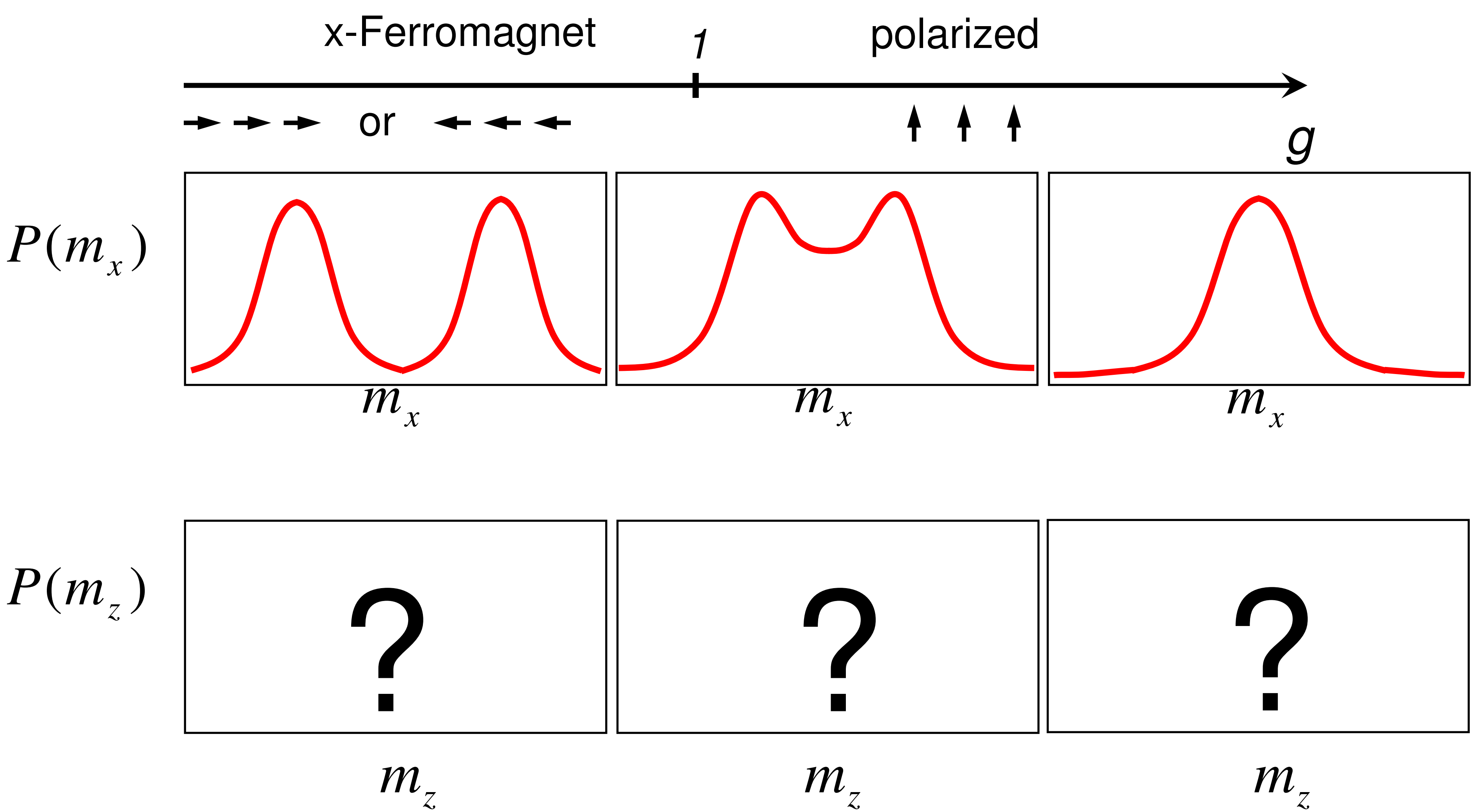
Average magnetization $$\langle M^z_{\rm tot} \rangle = L \langle M^z \rangle$$

Higher moments of $M^z_{\rm tot}$ contain information about higher order correlation functions

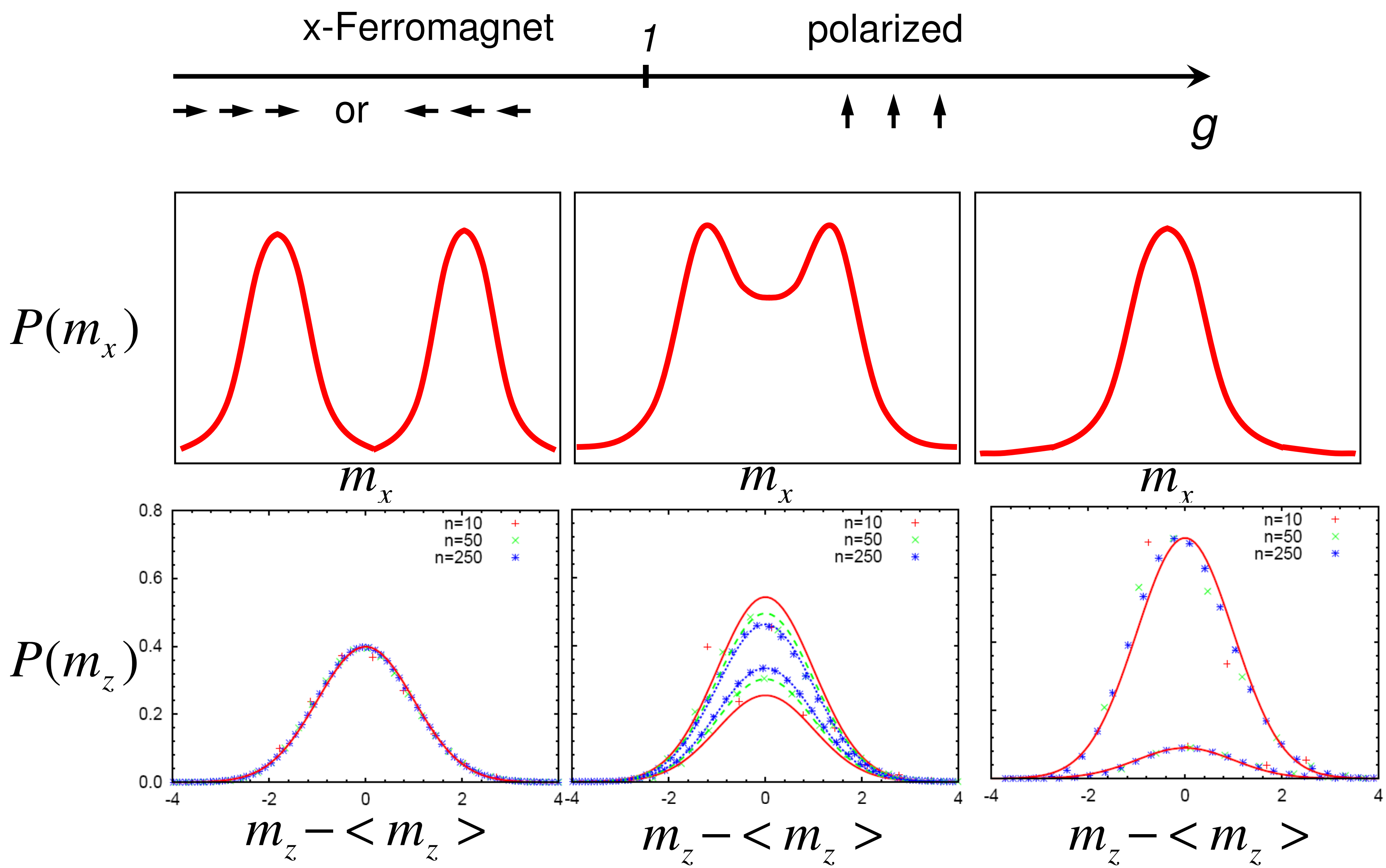
$$\langle (M^z_{\rm tot} - \langle M^z_{\rm tot} \rangle)^2 \rangle = L \sum_{ij} \langle (M_z(i) - \langle M^z \rangle) \, (M^z(j) - \langle M^z \rangle) \rangle$$

Probing spin systems using distribution function of magnetization

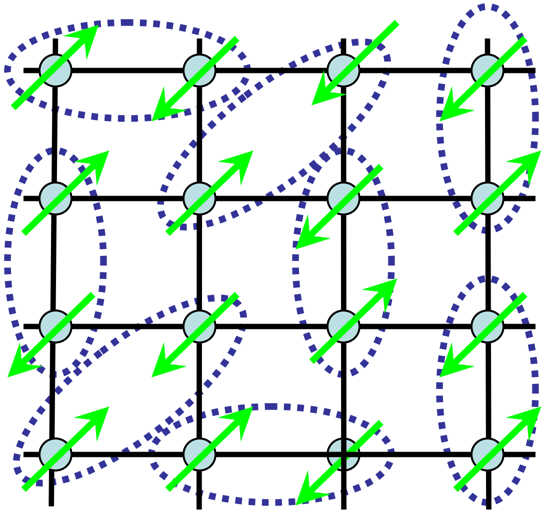
$$\mathcal{H} = -J \sum_i \left[2S^x(i) S^x(i+1) + g S^z(i) \right]$$



Distribution Functions



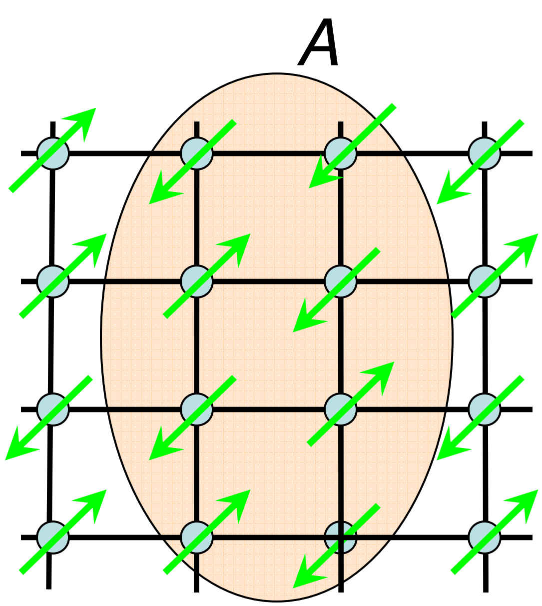
Using noise to detect spin liquids



Spin liquids have no broken symmetries
No sharp Bragg peaks

Algebraic spin liquids have long range spin correlations

$$\langle S_i \, S_j \rangle \; = \; \frac{e^{i\,Q\,r_{ij}}}{|\,r_i \; - \; r_j\,|^{1+\eta}}$$



No static magnetization $\quad \langle S_{\mathrm{A}} \rangle \; = \; 0$

Noise in magnetization exceeds shot noise

$$\langle S_{\mathrm{A}}^2 \rangle \; = \sum_{ij} \langle S_i \, S_j \rangle \; \sim \; A \int_{\mathrm{A}} \frac{r\,dr}{|\,r\,|^{1+\eta}} \sim A^{1+\frac{1-\eta}{2}}$$

Conclusions

Interference of extended condensates can be used to probe equilibrium correlation functions in one and two dimensional systems

Analysis of time dependent decoherence of a pair of condensates can be used to probe low temperature dissipation

Measurements of the distribution function of magnetization provide a new way of analyzing correlation functions in spin systems realized with cold atoms