Fractional Quantum Hall states in optical lattices

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Bose-Einstein Condensation


\[ n \sim 10^{14} \text{ cm}^3 \quad T_{\text{BEC}} \sim 1 \mu K \]

Ultralow density condensed matter system

Interactions are weak and can be described theoretically from first principles
New Era in Cold Atoms Research

Focus on systems with strong interactions

- Optical lattices
- Feshbach resonances
- Rotating condensates
- One dimensional systems
- Systems with long range dipolar interactions
Vortex lattice in rotating condensates

Pictures courtesy of JILA
http://jilawww.colorado.edu/bec

Lindeman criterion suggests that the vortex lattice melts when \( N/N_v \sim 10 \). Cooper et al., Sinova et al.
QH states in rotating condensates

Fractional quantum Hall states have been predicted at fast rotation frequencies:
Wilkin and Gunn, Ho, Paredes et.al., Cooper et al,…

Coriolis force: $\mathbf{F} = 2m \mathbf{v} \times \mathbf{\Omega}$
Lorentz force: $\mathbf{F} = q \mathbf{v} \times \mathbf{B}$

Vortex lattice

Read-Rezayi
Moore-Read
Composite fermions
Laughlin

$\frac{N}{N_v}$
$\sim 10$
$2$
$\frac{3}{2}$
$1$
$\frac{1}{2}$
QHE in rotating BEC

It is difficult to reach small filling factors

\[ \frac{N}{N_v} \approx \left[ N \frac{a_{||}}{a} \frac{(\omega_\perp - \Omega)}{\omega_\perp} \right]^{\frac{1}{2}} \]


\[ \frac{\omega_\perp - \Omega}{\omega_\perp} \approx 0.01 \quad \frac{N}{N_v} \approx 500 \]

Small energies in the QH regime require very low temperatures

\[ E \sim \frac{a}{a_{||}} \frac{N}{N_v} \hbar \omega_\perp \]

This work: Use optical lattices

\( a_{||} \) - scattering length
Atoms in optical lattices

Theory: Jaksch et al. PRL 81:3108(1998)

Experiment: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Esslinger et al., PRL (2004);
Bose Hubbard Model. Mean-field Phase Diagram.


Superfluid phase

\[ U < < Nt \]
Weak interactions

Mott insulator phase

\[ U > > Nt \]
Strong interactions
Superfluid to Insulator Transition

Greiner et al., Nature 415 (02)
1. How to get an effective magnetic field for neutral atoms

2. Fractional Quantum Hall states of bosons on a lattice

3. How to detect the FQH state of cold atoms
Magnetic field

1. Oscillating quadropole potential: \( V = A \cdot x \cdot y \cdot \sin(\omega t) \)
2. Modulate tunneling

See also Jaksch and Zoller, New J. Phys. 5, 56 (2003)
Magnetic field

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Magnetic field

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Proof:

\[
U \left( t = \frac{n2\pi}{\omega} \right) = U \left( t = \frac{2\pi}{\omega} \right)^n = \left( e^{-i\beta T_x/\hbar} e^{-2i Axy/\omega \hbar} e^{-i\beta T_y/\hbar} e^{2i Axy/\omega \hbar} e^{-i\beta T_x/2 \hbar} \right)^n \\
= e^{-i \frac{H_{\text{eff}}}{\hbar}}
\]

\[ H_{\text{eff}} \approx J \sum_x |x\rangle \langle x+1| + |x+1\rangle \langle x| + J \sum_y |y\rangle \langle y+1| + e^{-2i\pi \alpha} + e^{2i\pi \alpha} \]

\( \alpha: \) Flux per unit cell \( 0 \leq \alpha \leq 1 \)

See also Jaksch and Zoller, New J. Phys. 5, 56 (2003)
Particles in magnetic field

Continuum: Landau levels

\[ E_n = \hbar \frac{eB}{mc} (n + 1/2) \]

Lattice: Hofstadter Butterfly

Similar \( \alpha \ll 1 \)
Hall states in a lattice

Is the state there? $\Rightarrow$ Diagonalize $H$ (assume $J \ll U = \infty$, periodic boundary conditions)

$\langle \Psi_{\text{Ground}} | \Psi_{\text{Laughlin}} \rangle^2$

99.98%

95%

Dim($H$) = $8.5 \cdot 10^5$

$N = 2 N_\Phi$

- $N = 2$
- $N = 3$
- $N = 4$
- $N = 5$
\[ \Delta E \gtrsim 0.25 \, J \gtrsim 0.1 \frac{a_{\text{scattering}}}{l_{\perp}} \hbar \omega \]
Detection

Ideally: Hall conductance, excitations

Realistically: expansion image
Conclusions

• Effective magnetic field can be created for cold neutral atoms in an optical lattice
• Fractional Quantum Hall states can be realized with atoms in optical lattices
• Detection remains an interesting open problem

Future

- Quasi particles
- Exotic states
- Magnetic field generation