Fractional Quantum Hall states in optical lattices

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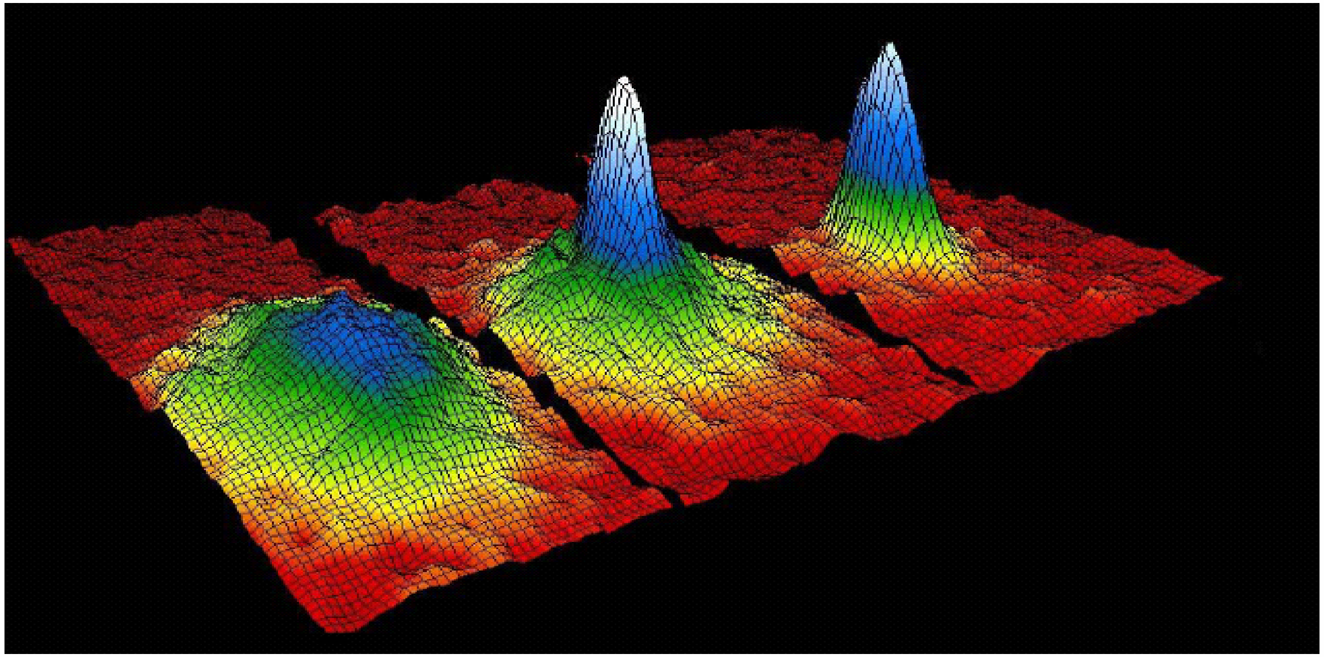
Bose-Einstein Condensation



Cornell et al., Science 269, 198 (1995)

$$n \sim 10^{14}\mathrm{cm}^3 \quad T_{\mathrm{BEC}} \sim 1\mu\mathrm{K}$$

Ultralow density condensed matter system

Interactions are weak and can be described theoretically from first principles

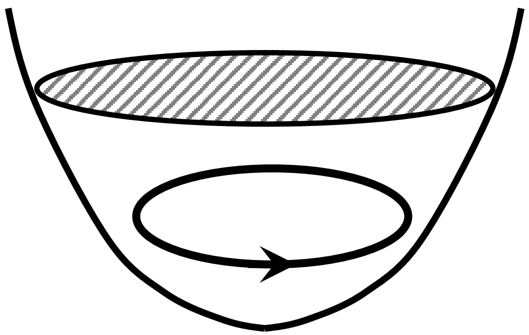
New Era in Cold Atoms Research

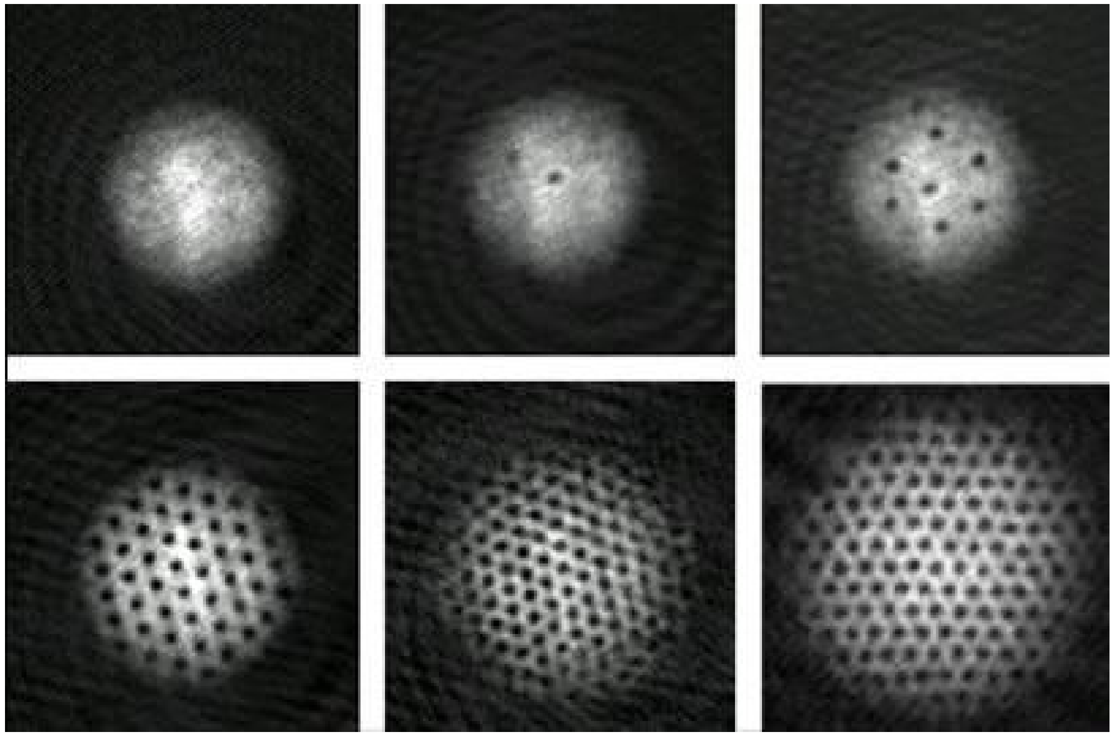
Focus on systems with strong interactions

- Optical lattices
- Feshbach resonances
- Rotating condensates
- One dimensional systems
- Systems with long range dipolar interactions

Vortex lattice in rotating BEC

Pictures courtesy of JILA
http://jilawww.colorado.edu/bec



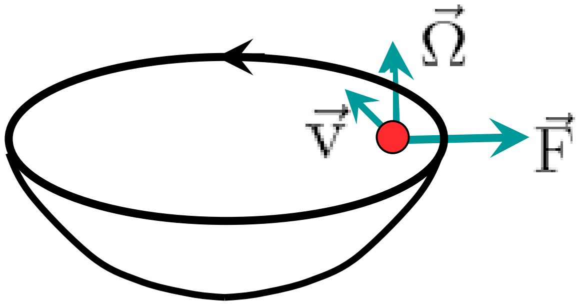


Lindeman criterion suggests that the vortex lattice melts when $N/N_{\mathrm{v}} \sim 10$. Cooper et al., Sinova et al.

QH states in rotating BEC

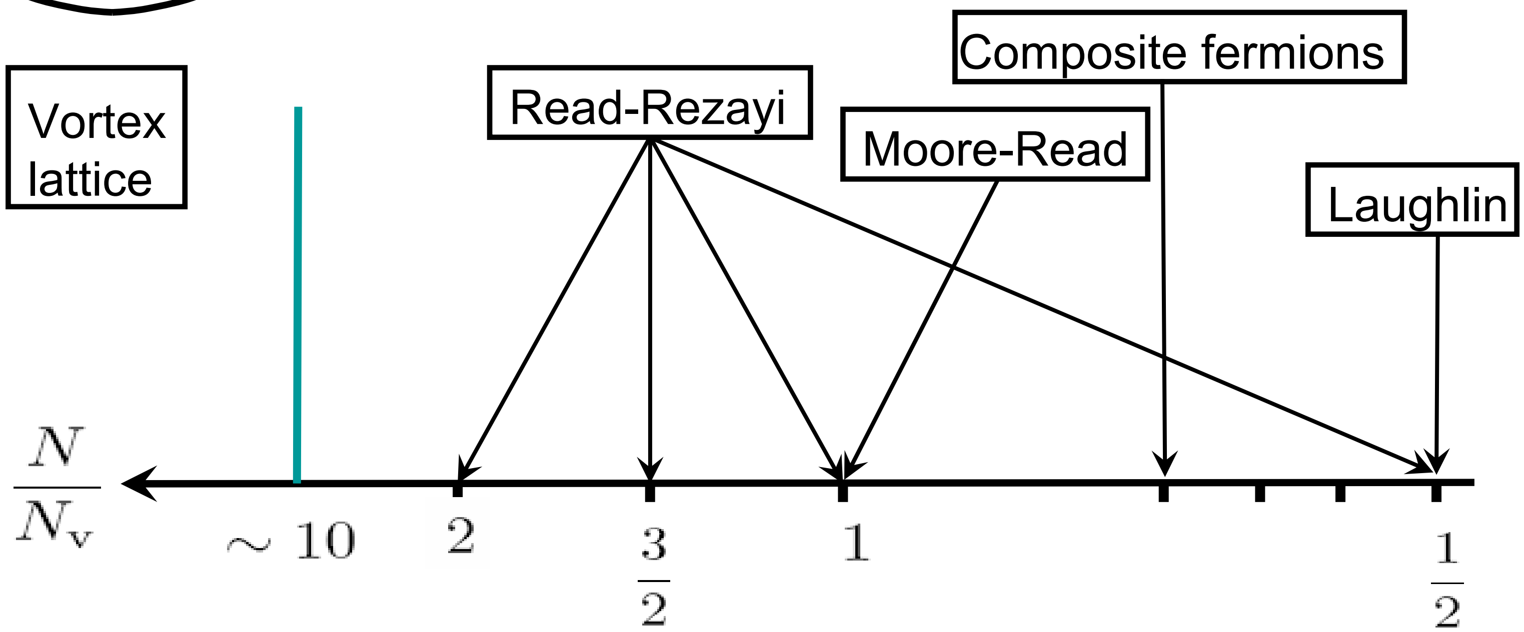
Fractional quantum Hall states have been predicted at fast rotation frequencies:

Wilkin and Gunn, Ho, Paredes et al., Cooper et al,…

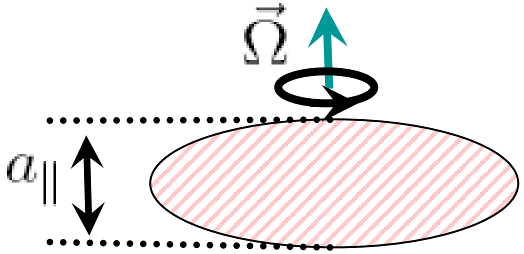


Corriolis force: $\vec{F} = 2m\,\vec{v} \times \vec{\Omega}$

Lorentz force: $\vec{F} = q\,\vec{v} \times \vec{B}$



QHE in rotating BEC



a - scattering length

It is difficult to reach small filling factors

$$\frac{N}{N_{\mathrm{v}}} \sim \left[N \, \frac{a_{\parallel}}{a} \, \frac{(\omega_{\perp} - \Omega)}{\omega_{\perp}} \right]^{\frac{1}{2}}$$

Current experiments: Schweikhard et al., PRL 92:40404 (2004)

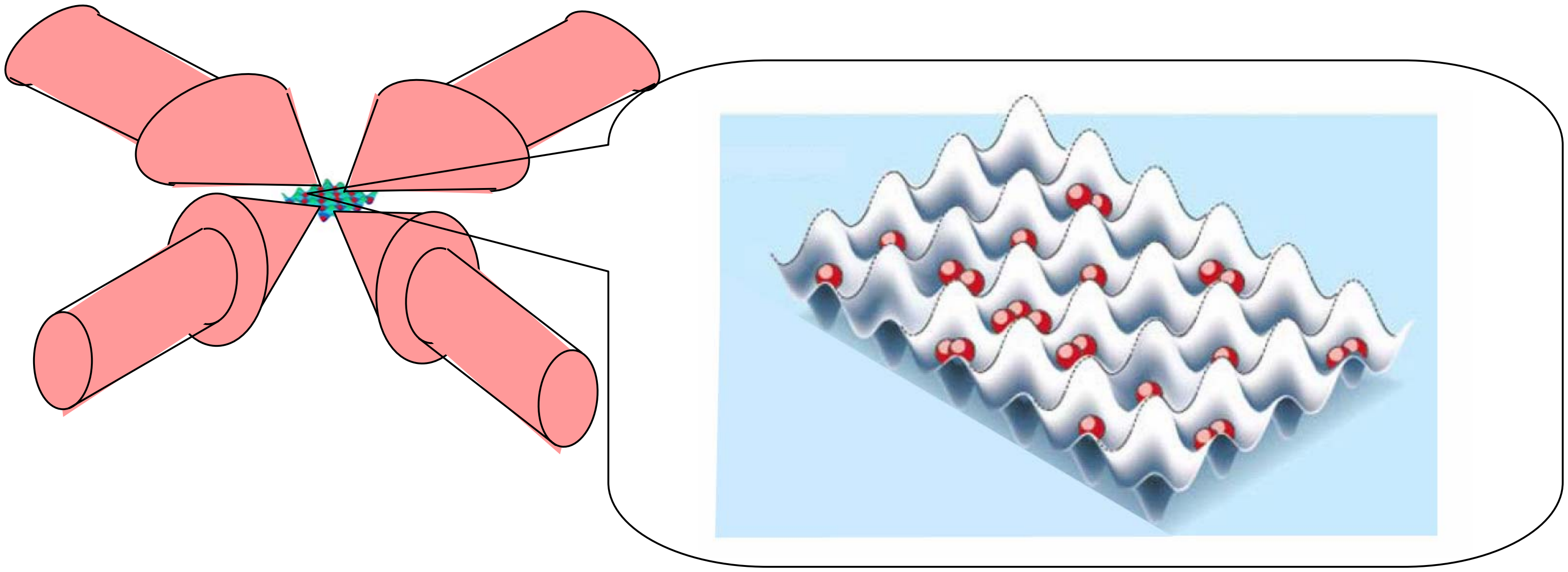
$$\frac{\omega_{\perp} - \Omega}{\omega_{\perp}} \simeq 0.01 \qquad\qquad \frac{N}{N_{\mathrm{v}}} \simeq 500$$

Small energies in the QH regime require very low temperatures

$$E \sim \frac{a}{a_{\parallel}} \, \frac{N}{N_V} \, \hbar\omega_{\perp}$$

This work: Use optical lattices

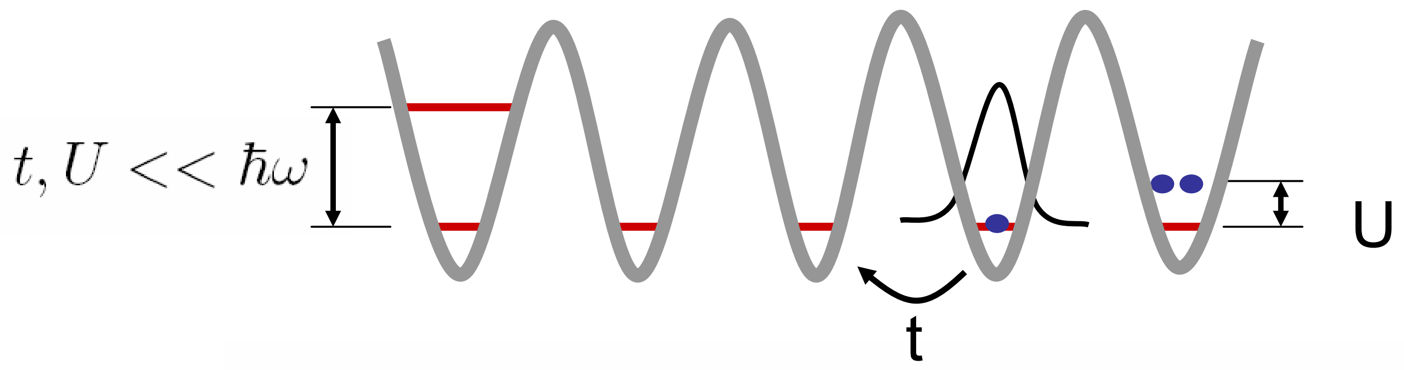
Atoms in optical lattices



Theory: Jaksch et al. PRL (1998)

Experiment: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Phillips et al., J. Physics B (2002)
Esslinger et al., PRL (2004);

Bose Hubbard Model

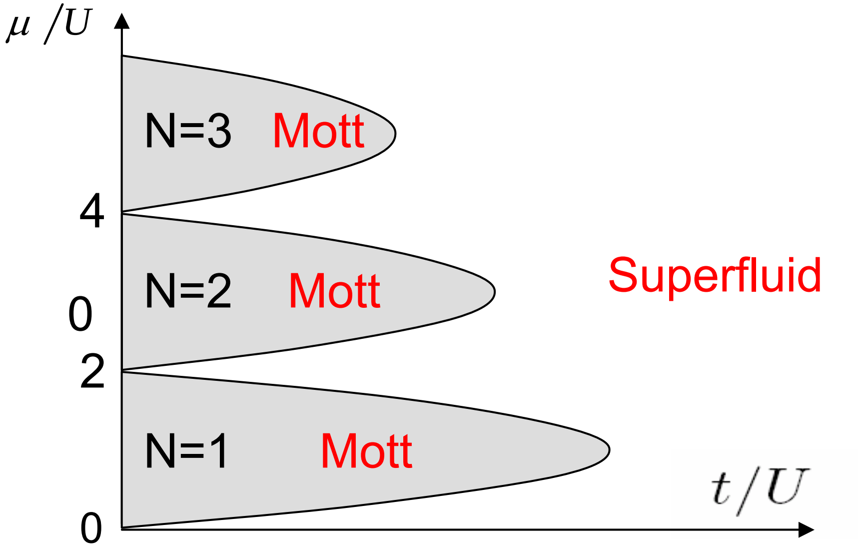


$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i^2 - \mu \sum_i n_i$$

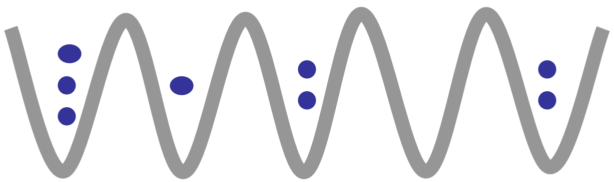
t – tunneling of atoms between neighboring wells

U – repulsion of atoms sitting in the same well

Bose Hubbard model. Mean-field phase diagram

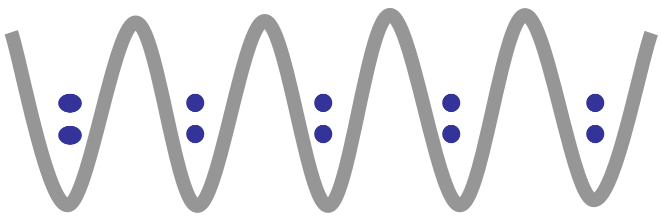


M.P.A. Fisher et al.,
PRB40:546 (1989)



$$U << Nt$$

Superfluid phase
Weak interactions

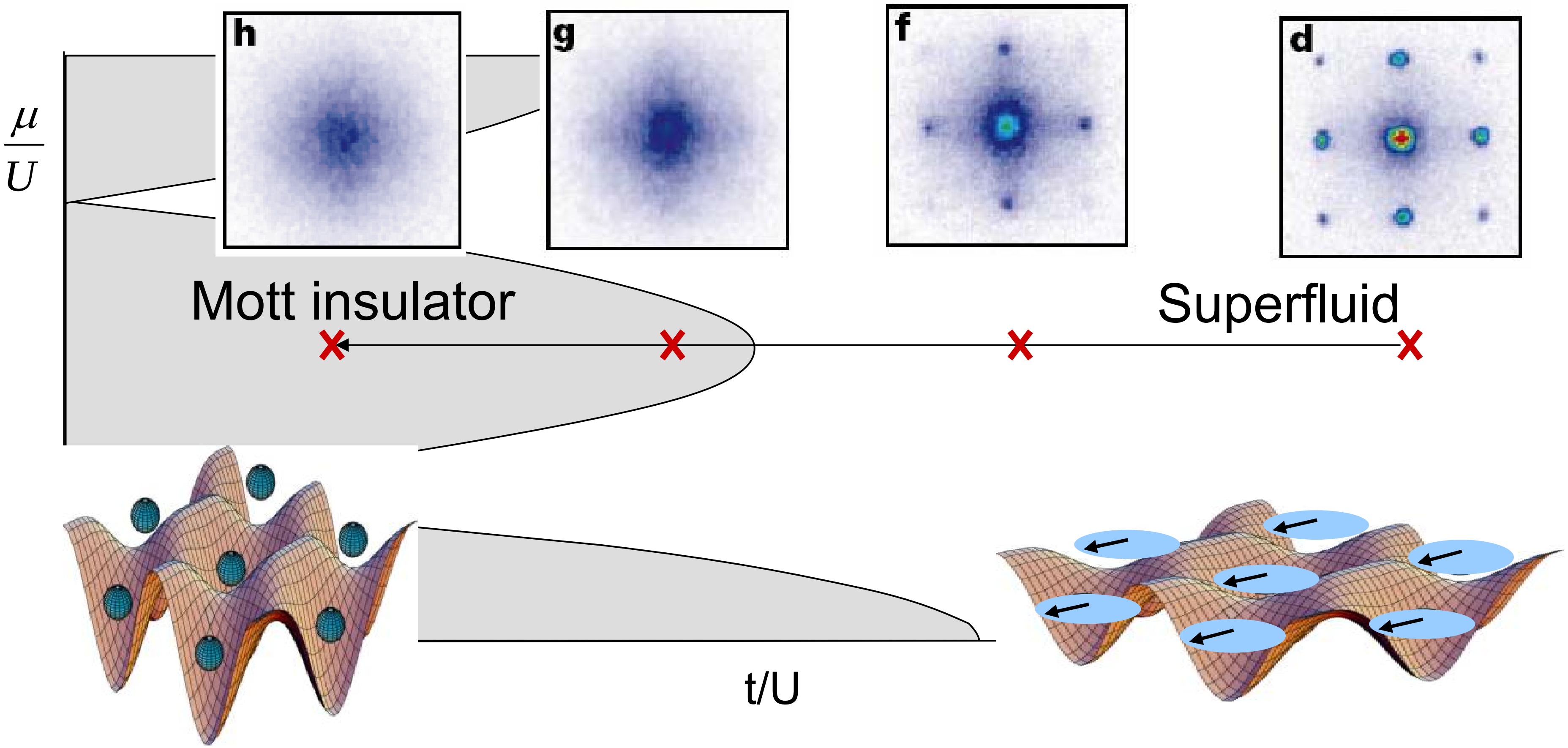


$$U >> Nt$$

Mott insulator phase
Strong interactions

Superfluid to insulator transition

Greiner et al., Nature 415 (2002)



Outline

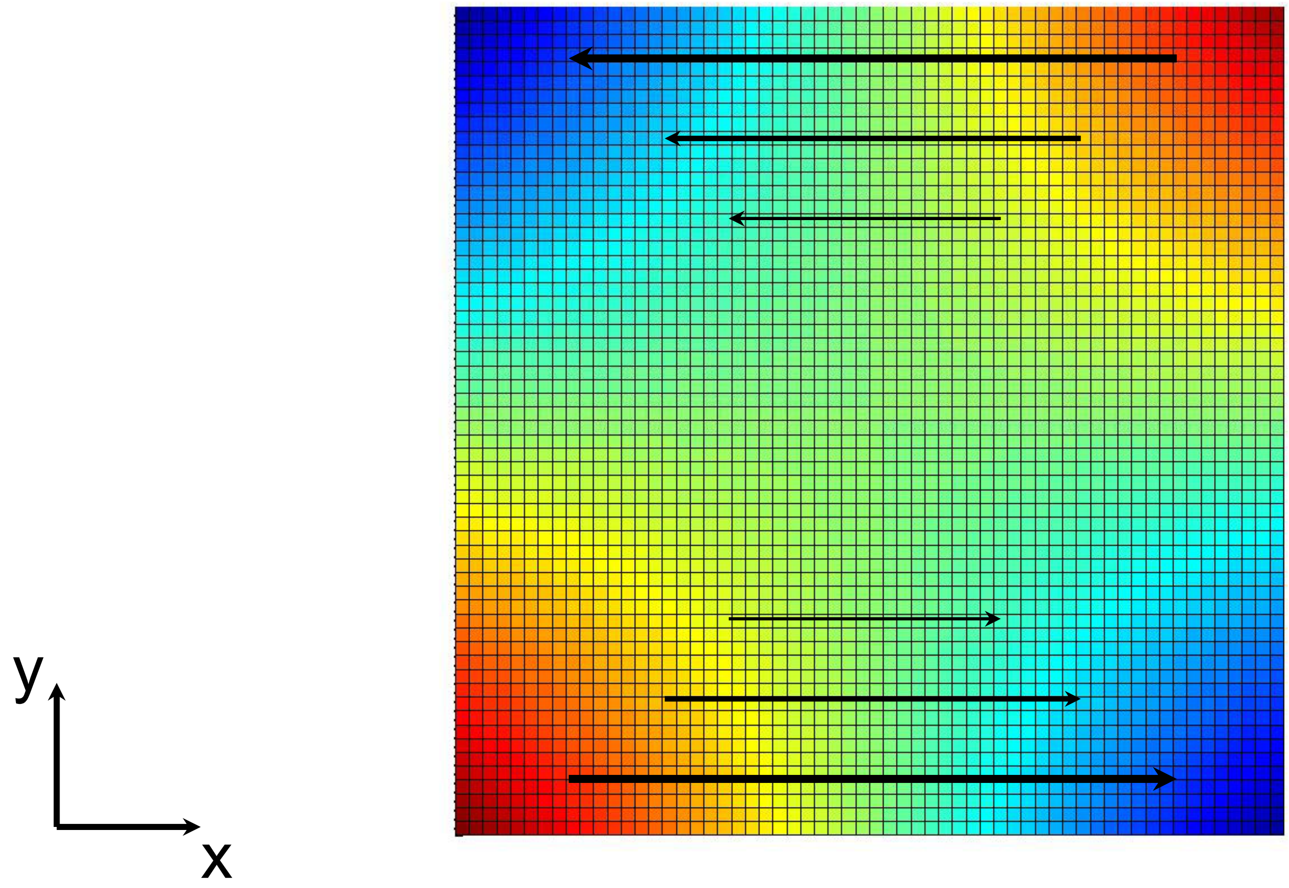
1. How to get an effective magnetic field for neutral atoms

2. Fractional Quantum Hall states of bosons on a lattice

3. How to detect the FQH states of cold atoms

Magnetic field

1. Oscillating quadropole potential: $V = A \cdot x \cdot y \cdot \sin(\omega t)$
2. Modulate tunneling



See also Jaksch and Zoller, New J. Phys. **5**, 56 (2003)

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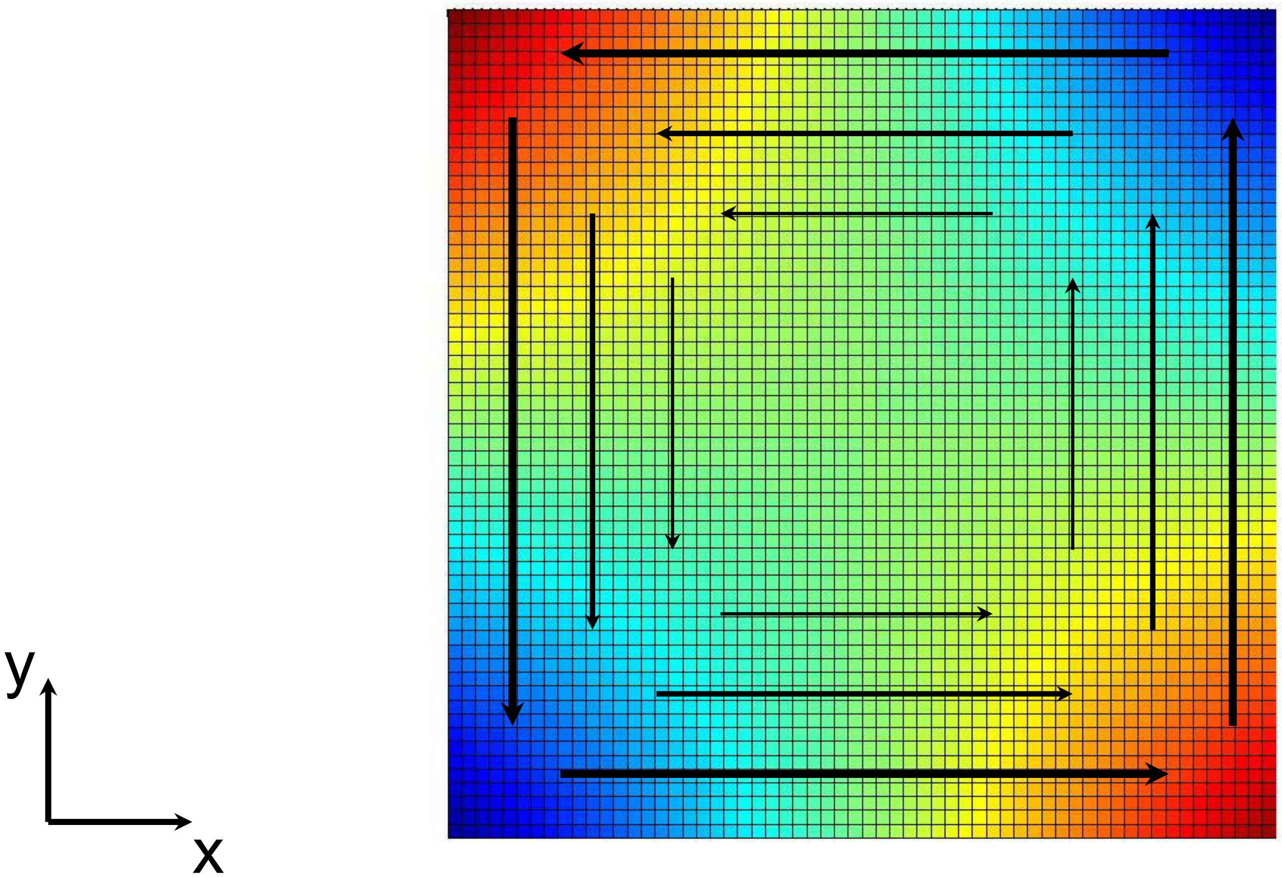


See also Jaksch and Zoller, New J. Phys. **5**, 56 (2003)

Magnetic field

1. Oscillating quadropole potential: V= A ·x·y ·sin(ωt)
2. Modulate tunneling

Proof:

$$U\left(t=\frac{n2\pi}{\omega}\right)=U\left(t=\frac{2\pi}{\omega}\right)^{n}=\left(e^{-i\beta T_x/2\hbar}e^{-2iAxy/\omega\hbar}e^{-i\beta T_y/\hbar}e^{2iAxy/\omega\hbar}e^{-i\beta T_x/2\hbar}\right)^{n}$$

$$=e^{-i\,H_{\mathrm{eff}}t/\hbar}$$

$$H_{\mathrm{eff}}\approx J\sum_x |x\rangle\langle x+1|+|x+1\rangle\langle x|+J\sum_y |y\rangle\langle y+1|e^{-2i\pi\alpha x}+e^{2i\pi\alpha x}|y+1\rangle\langle y|$$

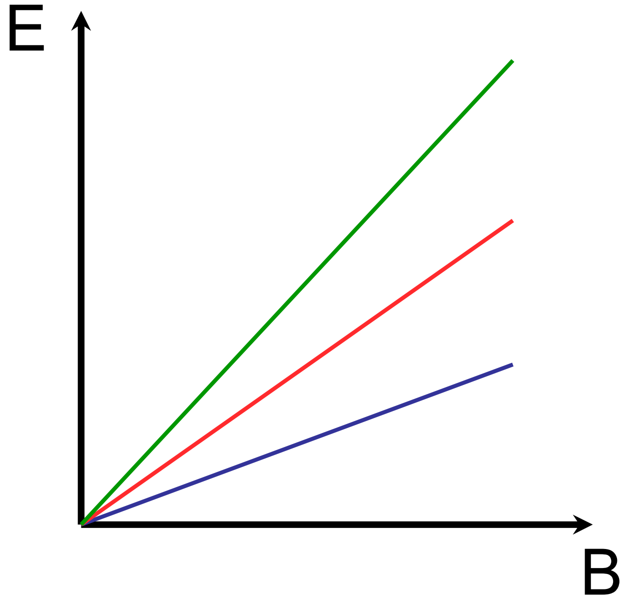
α: Flux per unit cell $0\leq \alpha \leq 1$

See also Jaksch and Zoller, New J. Phys. **5**, 56 (2003)

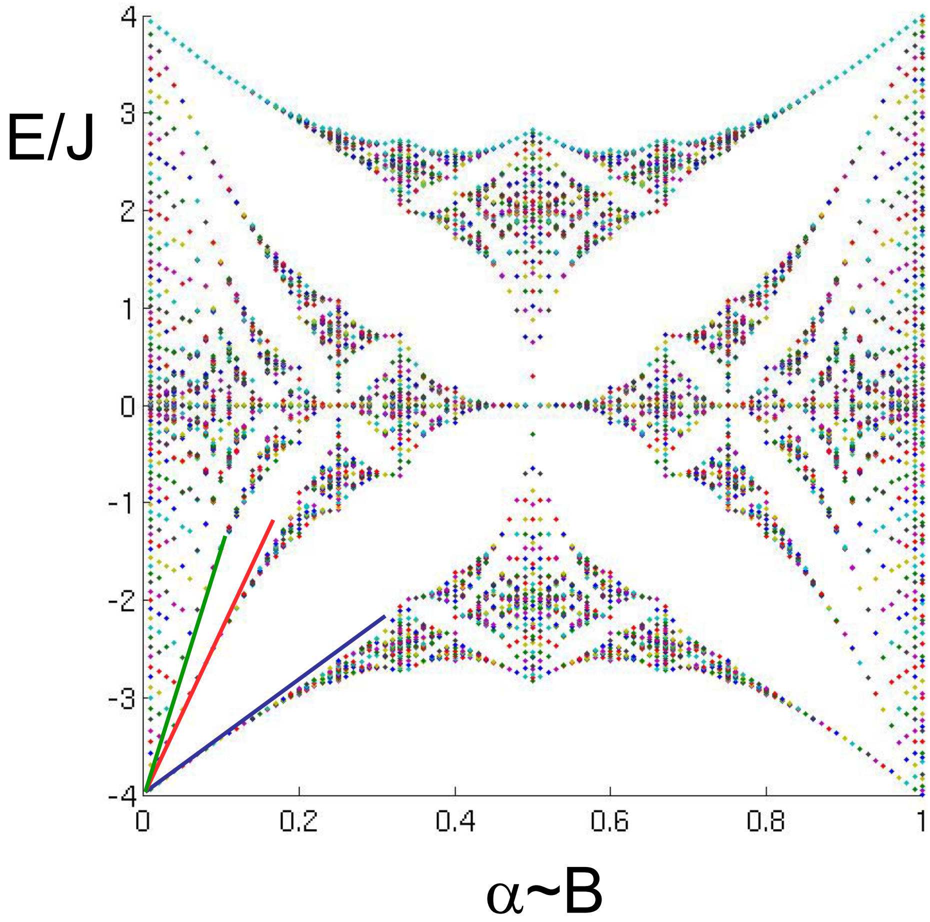
Particles in magnetic field

Continuum: Landau levels

$$E_n = \hbar \frac{eB}{mc}(n + 1/2)$$



Lattice: Hofstadter Butterfly



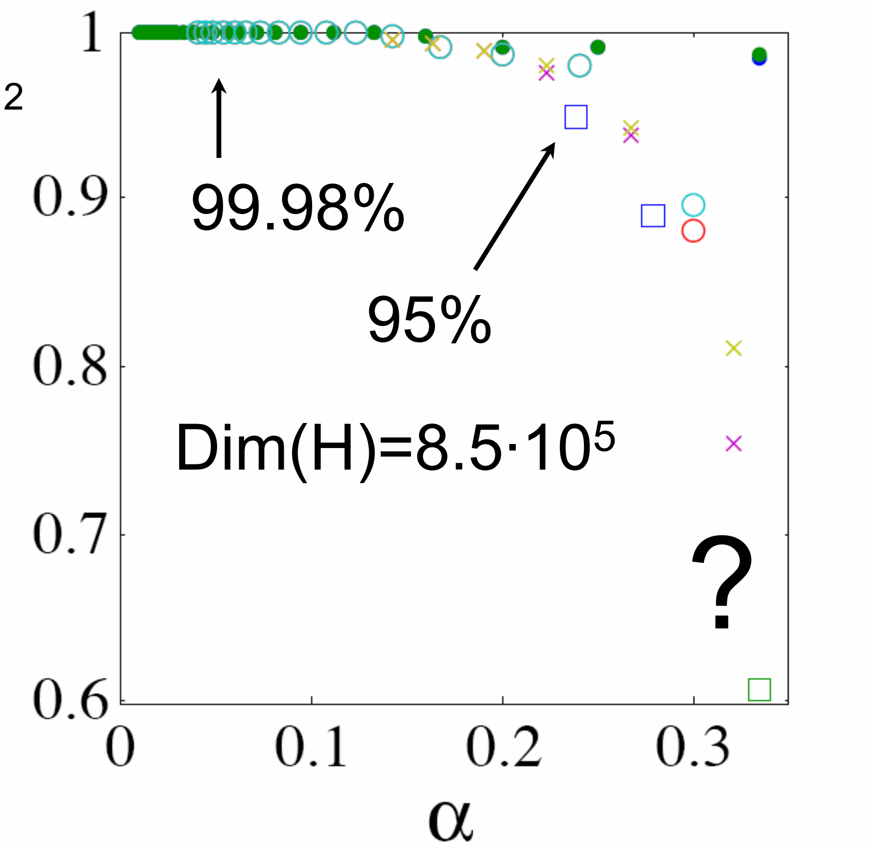
Similar $\alpha \ll 1$

Quantum Hall states in a lattice

Is the state there? $\Rightarrow$ Diagonalize H (assume J « U = ∞,
periodic boundary conditions)

Laughlin: $\Psi(r_1,\ldots\ldots,r_N) = \exp\left(-\sum |z|^2/4\right)\prod_{k<l}(z_k - z_l)^m$ $\quad z = x + i\,y$

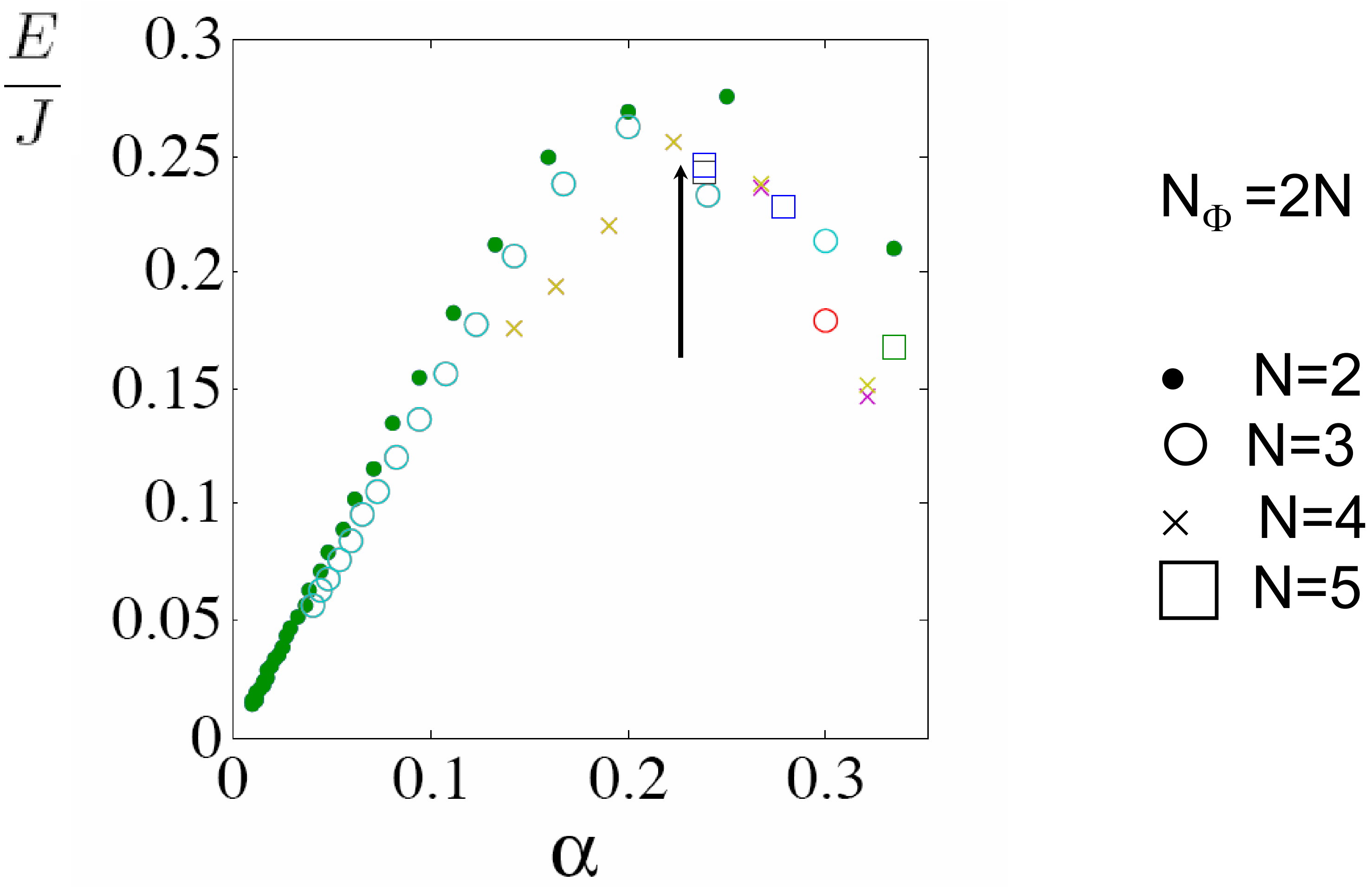
$\left|\left\langle \Psi_{\text{Ground}} \middle| \Psi_{\text{Laughlin}} \right\rangle\right|^2$



$N_\Phi = 2N$

- ● N=2
- ○ N=3
- × N=4
- □ N=5

Energy gap

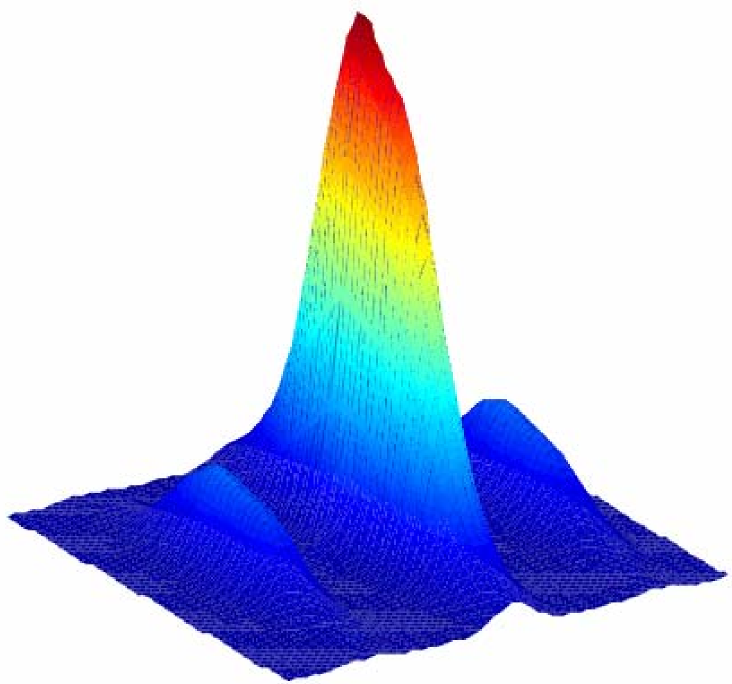


$$E \sim 0.25J > \frac{a}{a_{\|}} \frac{N}{N_V} \hbar\omega_{\perp}$$

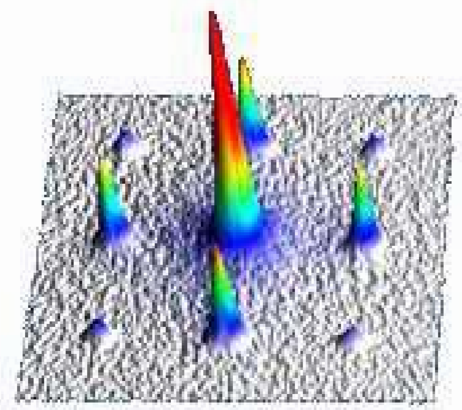
Detection

Ideally: Hall conductance, excitations

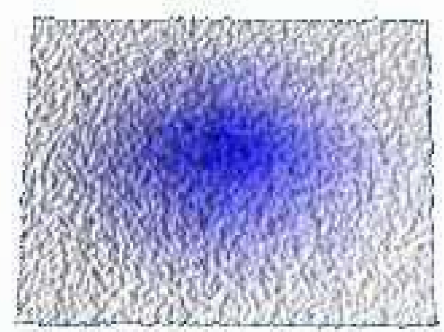
Realistically: expansion image



Quantum Hall

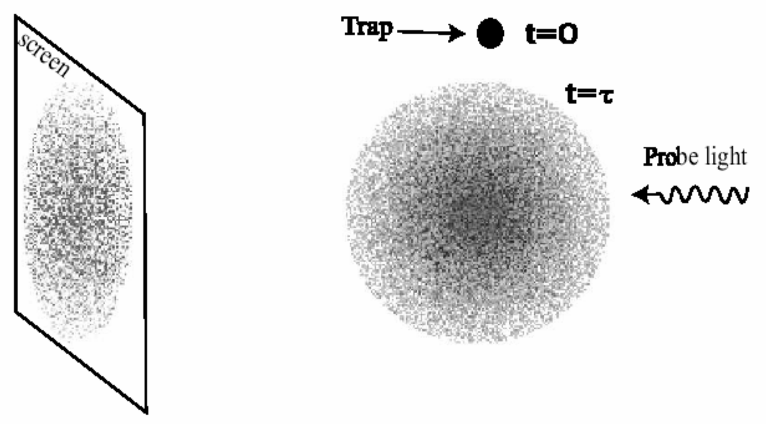


Superfluid



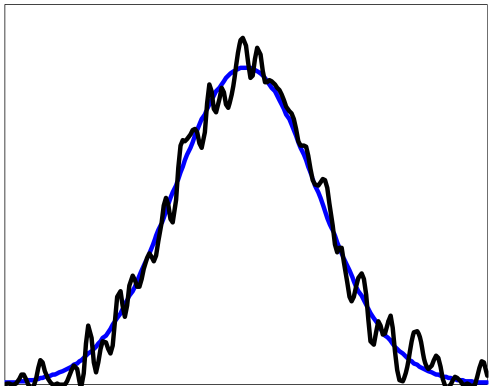
Mott

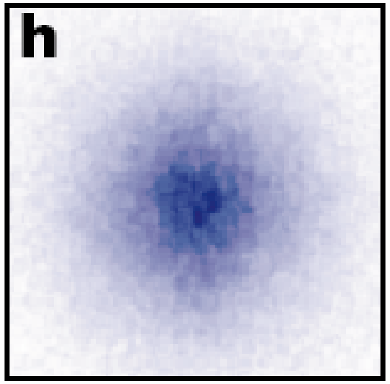
Time of flight experiments



Quantum noise interferometry of atoms in optical lattices

Altman et al., PRA(2004); Read and Cooper, PRA (2004)



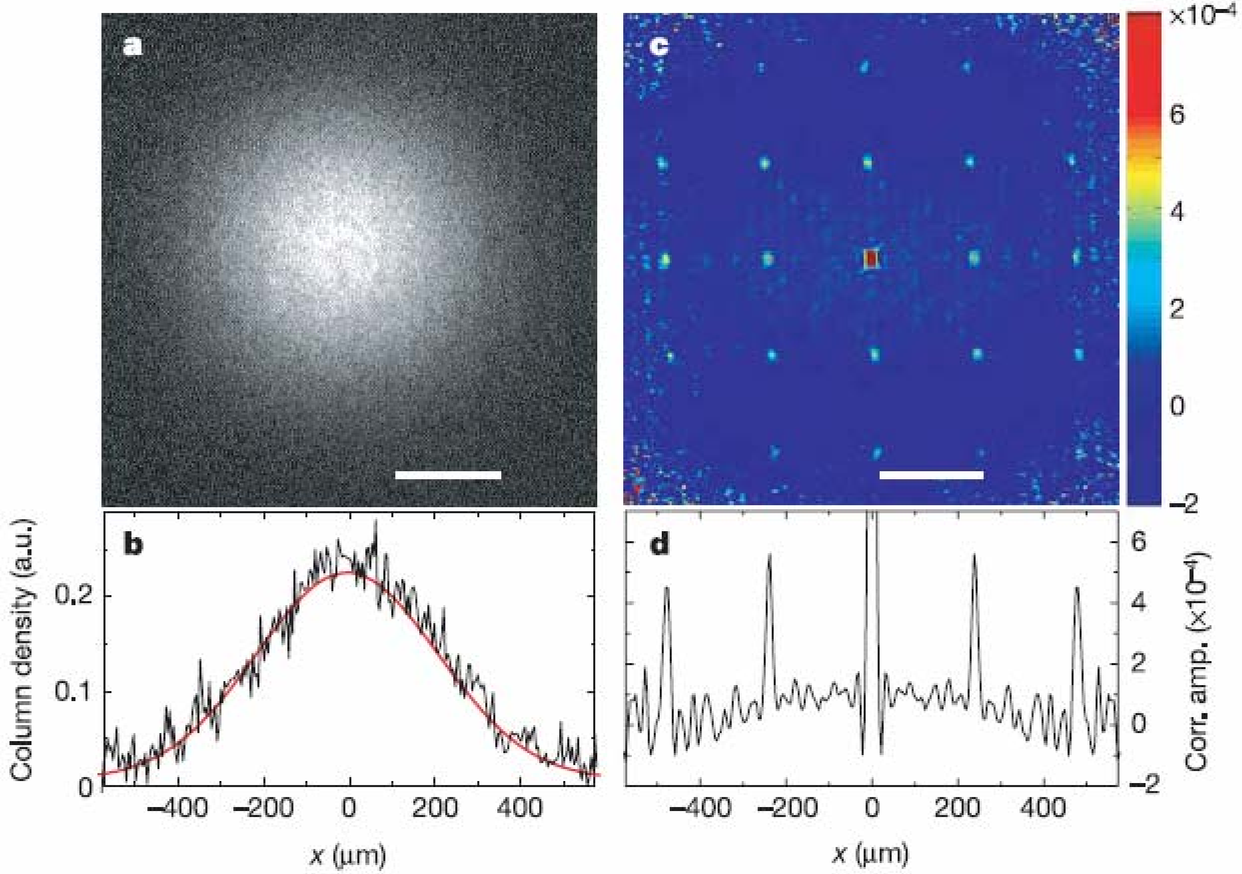


Second order coherence $G(r_1, r_2) = \langle n(r_1)n(r_2)\rangle - \langle n(r_1)\rangle\langle n(r_2)\rangle$

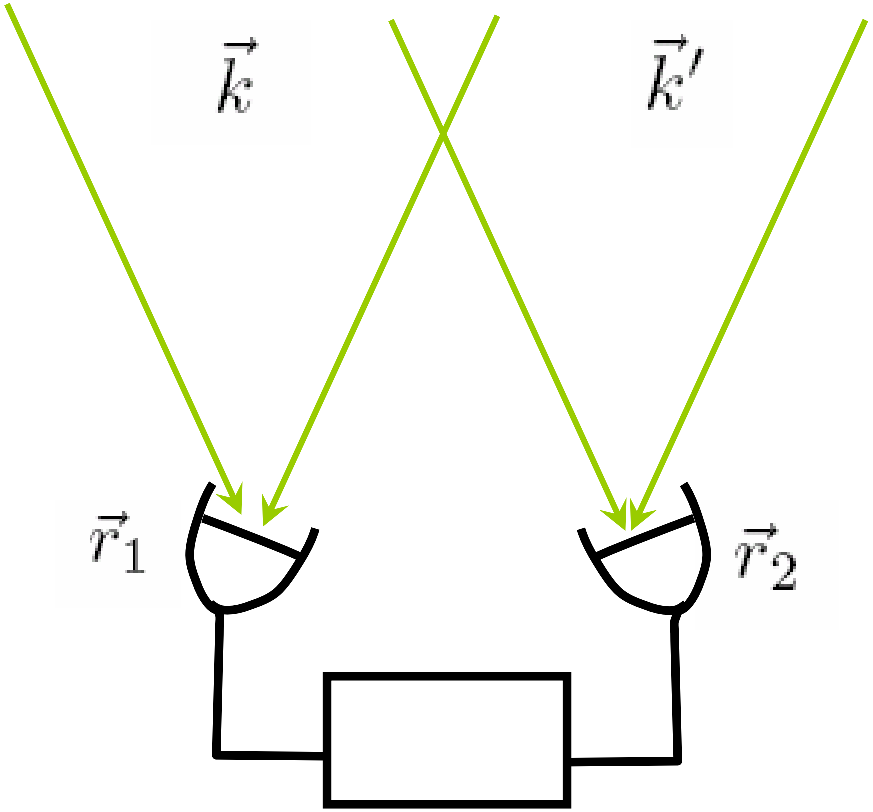
Second order coherence in the insulating state of bosons. Hanburry-Brown-Twiss experiment

Theory: Altman et al., PRA 70:13603 (2004)

Experiment: Folling et al., Nature 434:481 (2005)

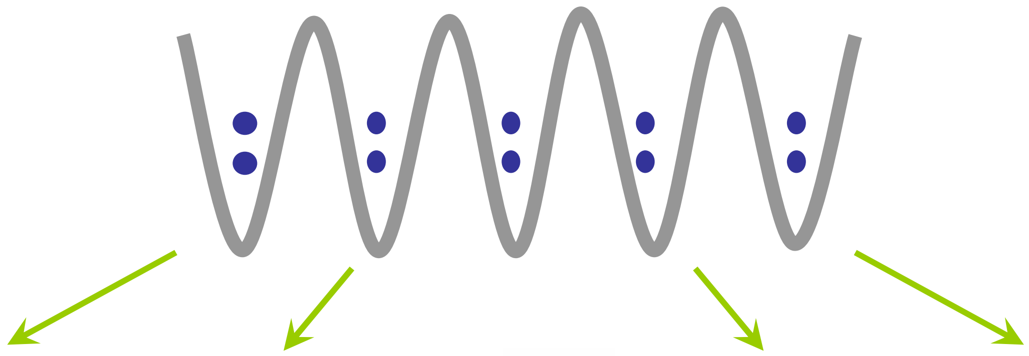


Hanburry-Brown-Twiss stellar interferometer



$$\langle I(\vec{r}_1)\ I(\vec{r}_2)\rangle = A + B\ \cos\Big((\vec{k}-\vec{k}')\ (\vec{r}_1-\vec{r}_2)\Big)$$

Second order coherence in the insulating state of bosons



Bosons at quasimomentum $\vec{k}$ expand as plane waves

with wavevectors $\vec{k},\ \vec{k}+\vec{G}_1,\ \vec{k}+\vec{G}_2$

First order coherence: $\langle\ \rho(\vec{r})\ \rangle$

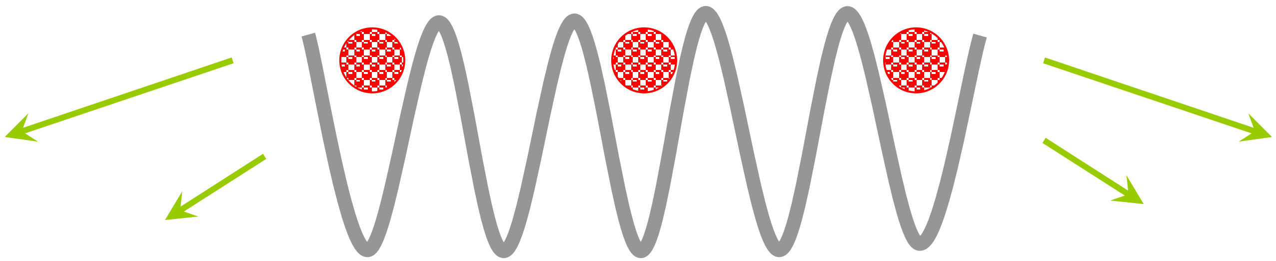
Oscillations in density disappear after summing over $\vec{k}$

Second order coherence: $\langle\ \rho(\vec{r}_1)\ \rho(\vec{r}_2)\ \rangle$

Correlation function acquires oscillations at reciprocal lattice vectors

$$\langle\ \rho(\vec{r}_1)\ \rho(\vec{r}_2)\ \rangle = A_0 + A_1\ \cos\left(\ \vec{G}_1(\vec{r}_1-\vec{r}_2)\ \right) + A_2\ \cos\left(\ \vec{G}_2(\vec{r}_1-\vec{r}_2)\ \right) + \ldots$$

Second order coherence in the FQH state



$$r = \frac{\hbar\ k\ t}{m}$$

$$\begin{aligned} G(r_1, r_2) &= \langle\ n(r_1)\ n(r_2)\ \rangle_{\mathrm{TOF}} - \langle\ n(r_1)\ \rangle_{\mathrm{TOF}}\ \langle\ n(r_2)\ \rangle_{\mathrm{TOF}} \\ &\sim \langle\ n(k_1)\ n(k_2)\ \rangle_{\mathrm{LAT}} - \langle\ n(k_1)\ \rangle_{\mathrm{LAT}}\ \langle\ n(k_2)\ \rangle_{\mathrm{LAT}} \end{aligned}$$

In the Landau gauge for states in the LLL, momentum corresponds to the guiding center coordinate. From $G(r_1, r_2)$ one can calculate $g(r_1 - r_2) = \langle n(r_1) n(r_2) \rangle_{\mathrm{LAT}}$

Read and Cooper, PRA (2004)

Conclusions

- Effective magnetic field can be created for cold neutral atoms in an optical lattice
- Fractional Quantum Hall states can be realized with atoms in optical lattices
- Detection remains an interesting open problem

Future

- Quasi particles
- Exotic states
- Magnetic field generation