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PUBLISHED ONLINE: 29 AUGUST 2010 | DOI: 10.1038/NPHYS1754

nature physics

# Quantum critical states and phase transitions in the presence of non-equilibrium noise

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**Quantum critical points are characterized by scale-invariant correlations and therefore by long-range entanglement. As such, they present fascinating examples of quantum states of matter and their study is an important theme in modern physics. However, little is known about the fate of quantum criticality under non-equilibrium conditions. Here we investigate the effect of external noise sources on quantum critical points. It is natural to expect that noise will have a similar effect to finite temperature, that is, destroying the subtle correlations underlying the quantum critical behaviour. Surprisingly, we find that the ubiquitous 1/*f* noise does preserve the critical correlations. The emergent states show an intriguing interplay of intrinsic quantum critical and external-noise-driven fluctuations. We illustrate this general phenomenon with specific examples describing solid-state and ultracold-atoms systems. Moreover, our approach shows that genuine quantum phase transitions can exist even under non-equilibrium conditions.**

An important motivation for investigating the behaviour of non-equilibrium quantum states comes from state-of-the-art experiments in atomic physics. Of particular interest in this regard are systems of ultracold polar molecules[1,2] and long chains of ultracold trapped ions[3]. On the one hand, these systems offer unique possibilities to realize strongly correlated many-body states, which undergo interesting quantum phase transitions[4–6]. However, on the other hand, they are controlled by large external electric fields, which are inherently noisy and easily drive the system out of equilibrium[7,8]. It is natural to ask what remains of the quantum states, and in particular, the critical behaviour under such conditions.

The effect of non-equilibrium noise on quantum critical points is also relevant to more traditional solid-state systems. Josephson junctions, for example, are known to be affected by non-equilibrium circuit noise, such as $1/f$ noise. Without this noise, a single quantum Josephson junction should undergo a textbook quantum phase transition[9,10]: depending on the value of a shunt resistor, the junction can be in either a normal or a superconducting state. A phase transition occurs at a universal value of the shunt resistance $R_s = R_Q = h/(2e)^2$, independent of the strength of the Josephson coupling. This is closely related to the problem of macroscopic quantum tunnelling of a two-level system (or q-bit) coupled to a dissipative environment[11].

There is a large body of work on $1/f$ noise as a source of decoherence for superconducting q-bits (see, for example, refs 12, 13). However, the effect of such noise on the quantum phase transitions and the non-equilibrium steady states of Josephson junctions poses fundamental open questions. Do the different phases (superconducting or normal) retain their integrity in the presence of the noise? Is the phase transition between them sharply defined?

In certain cases it was argued that a non-equilibrium drive may act as an effective temperature[14,15]. And temperature is known to be a relevant perturbation, which destroys quantum criticality[16,17]. In contrast, we find that the external $1/f$ noise is only a marginal perturbation at the critical point in many cases of interest. This is exemplified in the section entitled ‘Phase transition in a noisy Josephson junction’. In the section entitled ‘One-dimensional chains of polar molecules or trapped ions’, we investigate the potentially richer physics of one-dimensional systems. These systems form a critical state at $T = 0$, which can undergo pinning in the presence of a commensurate lattice or a single impurity. Pinning occurs as a quantum phase transition at a critical value of the correlation exponent[18] (for application to ion traps, see ref. 6). Another interesting phenomenon in ion chains is the zigzag instability[19], which is expected to evolve into a true quantum phase transition in the limit of long chains.

Again the relevant issue is the fate of these critical states and quantum phase transitions in the presence of noisy electrodes. Such noise has been characterized in recent experiments with ion traps[7,8], where it was found to have a $1/f$ power spectrum and attributed to localized charge patches on the electrodes. A crucial result of our analysis is that such noise preserves the critical states, and the exponents are continuously tuned by it. The fact that the system is out of equilibrium is betrayed by the linear response to an external probe, such as light scattering. The energy dissipation function of the scattered light can become negative for sufficiently strong external noise, exhibiting gain instead of loss.

The long-wavelength description of the noise-driven steady state allows us to study its stability to various static perturbations within a renormalization group framework. In this way we describe pinning by a static impurity and by a lattice potential. We show that pinning–depinning occurs as a phase transition driven by interplay of the intrinsic quantum fluctuations and the external noise. Before proceeding we note previous work that found modified quantum criticality in cases where the non-equilibrium conditions were due to an imposed current[20,21].

## Phase transition in a noisy Josephson junction

In our discussion of the Josephson junction, we consider the standard circuit shown in Fig. 1a. The offset charge $eN_0$ on the capacitor has random time-dependent fluctuations with a

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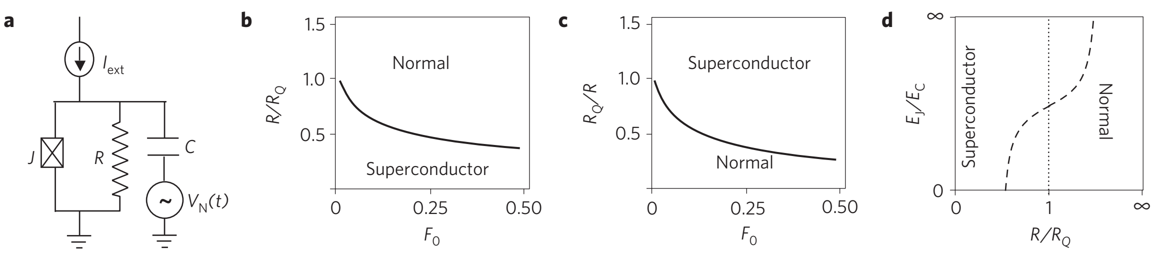


**Figure 1 | Effects of non-equilibrium noise on the localization quantum phase transition of a single shunted Josephson junction: a**, Electronic circuit relevant to a resistively shunted Josephson junction with charging noise. **b**, Critical resistance $R/R_Q$ as a function of the noise strength $F_0$, in the weak coupling limit. **c**, Critical conductance $R/R_Q$ as a function of the noise strength $F_0$, in the strong-coupling limit. **b** and **c** are related by the duality transformation $R/R_Q \to R_Q/R$ and 'superconductor' ↔ 'normal'. **d**, Schematic phase diagram at equilibrium (dotted line) and in the presence of non-equilibrium $1/f$ noise (dashed line).

$1/f$ spectrum[22] $\langle N_0^*(\omega)N_0(\omega)\rangle = F_0/|\omega|$. This is modelled by the fluctuating voltage source $V_N(t) = eN_0(t)/C$.

Consider first the system at vanishing Josephson coupling, which is then just an $RC$ circuit. Treating the resistor as an ohmic bath in thermal contact with the system[23] results in the Langevin equation for a damped quantum oscillator:

$$\frac{1}{2}c\ddot{\theta} + \eta\dot{\theta} = \zeta(t) + \frac{1}{2}\dot{N}_0(t) \tag{1}$$

Here $c = \hbar C/2e^2$ and $\eta = (1/2\pi)R_Q/R$. The random forcing term $\zeta(t)$ originates from the equilibrium bath, and therefore at $T = 0$ has the power spectrum $\langle \zeta_\omega^* \zeta_\omega\rangle = \eta|\omega|$. The other random forcing term is the time derivative of the charge noise. As the charge fluctuations have a spectrum $\sim F_0/|\omega|$, the power spectrum of $\dot{N}_0$ is $\sim F_0|\omega|$, which mimics the resistor noise. Unlike the resistor, however, external fluctuations do not have an associated dissipation term. This is because the noise source is not in thermal contact with the system. Thus, the fluctuation dissipation theorem is violated in the presence of the non-equilibrium noise source.

Using the linear equation of motion (1) we can compute the phase autocorrelation function:

$$\langle \cos[\theta(t) - \theta(0)]\rangle \sim t^{-(1+F_0/\eta)/\pi\eta} \tag{2}$$

Interestingly, the non-equilibrium noise does not destroy the power-law scaling, but modifies the exponent. We conclude that a critical (scale-invariant) non-equilibrium steady state is obtained in the presence of the external noise.

The important question to address in the context of weak coupling, is under what conditions the critical steady-state we just described is stable to introduction of the Josephson coupling as a perturbation. That is, we should find how the perturbation transforms under a scale transformation that leaves the critical steady state invariant. To investigate this we turn to formulation of the dynamics in terms of the Keldysh action described in the Methods section. The quadratic action (7) describing the $RC$ circuit is scale invariant, whereas the Josephson coupling term

$$S_J = J\int dt\left[\cos\theta_f(t) - \cos\theta_b(t)\right] \tag{3}$$

is not in general. Here $\theta_f$ ($\theta_b$) is the field on the forward (backward) part of the Keldysh contour. From the decay of the correlation function (2) we can directly read off the anomalous scaling dimension of the perturbation, which is $\alpha = 1 - (1 + F_0/\eta)/2\pi\eta$. When $\alpha > 0$ the perturbation grows under renormalization and ultimately destabilizes the critical steady state. We therefore predict a phase transition at a critical resistance $R^*/R_Q = (\sqrt{8\pi F_0 + 1} - 1)/4\pi F_0$, below which the Josephson coupling term becomes relevant. Note that we recover the equilibrium dissipative transition at $R^* = R_Q$ in a 'quiet' circuit ($F_0 = 0$). We can tune across the transition also by maintaining a constant resistance $R < R_Q$ and increasing the non-equilibrium noise 'power' $F_0$, as shown in Fig. 1b.

Within the weak-coupling theory we do not have direct access to the properties of the steady state at $R < R^*$. However, because the Josephson coupling grows under renormalization, it is reasonable to expect that the junction would be superconducting. To determine this with more confidence we shall now take the opposite, strong-coupling viewpoint.

We employ a well-known duality between weak and strong coupling[9,24], under which Cooper pair tunnelling $J\int dt\cos(\theta)$ is mapped to tunnelling of phase slips across the junction $S_g = g\int dt\cos(\phi)$. Concomitantly the resistance $R/R_Q$ is mapped to a normalized conductance $R_Q/R$. In the strong-coupling limit of the Josephson junction $J \gg e^2/C$, the dual action, with a phase-slip tunnelling $S_g$, is at weak coupling. The scaling analysis can proceed in the same way as above, giving a transition at the value of shunt resistance $R^*/R_Q = 4\pi F_0/(\sqrt{8\pi F_0 + 1} - 1)$. For $R < R^*$ the phase-slip tunnelling $S_g$ is irrelevant. Therefore, at asymptotically long times all phase-slip events are paired, making the superconducting state stable for $R < R^*$, at least in the strong-coupling limit.

The combined results of the weak and strong coupling analysis imply a phase diagram of the form shown in Fig. 1d. At weak coupling the critical resistance, in the presence of noise, occurs at $R^*$ that is smaller than $R_Q$, whereas at strong coupling $R^*$ is larger than $R_Q$. The dashed line in this figure shows a simple interpolation of the phase boundary between the two limiting regimes. However, we cannot exclude the possibility that new phases, such as a metallic phase, arise at intermediate coupling.

## One-dimensional chains of polar molecules or trapped ions

We now investigate the interplay between critical quantum fluctuations and external classical noise in one-dimensional systems. Good laboratories for studying such effects are ions in ring or linear Paul traps, as well as polar molecules confined to one dimension. As a result of the confinement to one dimension, both systems are affected by quantum fluctuations. On the other hand, they are also subject to noisy electric fields that can influence the steady-state correlations.

In ion traps, the fluctuations of the electric potential, which couples to the ionic charge density, have been carefully characterized[7,8]. The noise power spectrum was found to be very close to $1/f$ and with spatial structure indicating moderately short-range correlations. In the molecule system, electric fields are used to polarize the molecules, and fluctuations in these fields couple to the molecule density through the molecular polarizability.

Our starting point for theoretical analysis is the universal harmonic theory describing long-wavelength phonons in the one-dimensional system[25], which is written in terms of the displacement field $\phi(x,t)$ of the particles from a putative

Wigner lattice. The long-wavelength density fluctuations are represented by the gradient of the displacement field, $(-1/\pi)\partial_x\phi(x,t)$. The part of the density with Fourier components of wavelengths near the interparticle spacing are encoded by $\rho_0\cos(2\pi\rho_0 x+2\phi(x,t))$ (refs 18,25), where $\rho_0$ is the average density. The operator $O_{\rm DW}=\rho_0\cos(2\phi(x,t))$ is the density-wave (or solid) order parameter field of the Wigner lattice. As in the Josephson junction, we wish to address two questions. (1) How does the external noise affect the steady state, which in equilibrium exhibits algebraic correlations. (2) How does it influence phase transitions, such as the lattice pinning transition.

We model the external electric noise as a random time-dependent field coupled to the particle density. In general, the noise couples to both components of the density through the terms $-f(x,t)\pi^{-1}\partial_x\phi(x,t)$ and $\zeta(x,t)\rho_0\cos(2\phi(x,t))$. For now we assume that the noise source is correlated over sufficiently long distances, so that its component at spatial frequencies near the particle density ($\zeta(x,t)$) is very small and can be neglected. In this case the long-wavelength theory remains harmonic. We shall characterize the noise by its power spectrum $F(q,\omega)=\langle f(q,\omega)f(-q,-\omega)\rangle$. We take this to be $1/f$ noise with short-range spatial correlations, that is, $F(q,\omega)=F_0/|\omega|$.

When the system is irradiated with external noise we expect it to absorb energy. To stabilize a steady state we need a dissipative bath that can take this energy from the system. In the Josephson junction problem, the resistor naturally played this role. Is there a similar dissipative coupling in the one-dimensional systems under consideration here?

In the ion traps, there is a natural dissipative coupling because these systems can be continuously laser cooled. Thus, the system can reach a steady state, which reflects a balance between the laser cooling and the external noise (see Supplementary Information). The polar molecules do not couple to a natural source of dissipation; however, a thermal bath can in principle be realized by immersion in a large atomic condensate[26]. In the Supplementary Information we show that the bath generated by a two-dimensional weakly interacting condensate provides the needed dissipation.

The combined effects of interactions, external noise and dissipation are described by a quadratic Keldysh action as shown in the Methods section (8). This is the natural extension from the single junction (7) to the one-dimensional chain. However, there is an important difference. The harmonic chain is scale invariant only without the noise and dissipation terms, which are strictly speaking relevant perturbations of this fixed point. Indeed, the dissipative coupling generates a relaxation timescale $\tau\sim 1/\eta$, which breaks the scale invariance. To retain the scale invariance and still drive the system out of equilibrium we can consider the interesting limiting regime in which both $\eta\to 0$ and $F_0\to 0$, while the ratio $F_0/\eta$ tends to a constant. Then the correlation function is easily calculated and seen to be a power law

$$\langle\cos(2\phi_{\rm cl}(x))\cos(2\phi_{\rm cl}(0))\rangle\sim x^{-2K(1+\pi^{-2}F_0/\eta)} \tag{4}$$

where $K$ is the Luttinger parameter, which determines the decay of correlations at equilibrium ($F_0=0$). The same exponent holds for the temporal correlations. We see that the dimensionless ratio $F_0/\eta$, which measures the deviation from equilibrium, acts as a marginal perturbation. In practice $\eta$ and $F_0$ are non-vanishing. Then the result (4) will be valid at scales shorter than $1/\eta$. Correlations will decay exponentially at longer scales. Thus, $\eta$ serves as an infrared cutoff of the critical steady state. In practice however the system size or cutoff of the $1/f$ spectrum may set more stringent infrared cutoffs.

The density–density correlations can be measured directly by light scattering. The (energy integrated) light diffraction pattern in the far-field limit gives directly the static structure factor $S(q)=\langle\rho_{-\mathbf{q}}\rho_{\mathbf{q}}\rangle$ of the sample. In particular, the power-law singularity in $S(q)$ near wave vector $q_0\sim 2\pi\rho_0$ is just the Fourier transform of the power-law decay of the Wigner crystal correlations (4).

We can also compute the decay of phase correlations $\langle\cos[\theta(x)-\theta(0)]\rangle$, which in the system of cold molecules may be measured by interference experiments[27]. By considering the dual representation of the harmonic action (8) we find a decay exponent $(1+F_0/\eta)/2K$.

At equilibrium both crystalline and phase correlations are controlled by $K$ alone: reducing $K$ (by increasing interactions) leads to a slower decay of density-wave correlations and concomitantly faster decay of phase correlations. This duality, a consequence of minimal uncertainty between phase and density in the harmonic ground state, is violated in the presence of noise. Increasing the noise leads to a faster decay of both the density and phase correlations.

## Response

Under the non-equilibrium conditions, the fluctuation dissipation theorem does not hold in general and we should consider response functions separately from the correlations. Here we discuss the Fourier transform of the density–density response function, which gives the linear response of the system to a weak periodic potential with wave vector $q$, oscillating at a frequency $\omega$. This is the response function probed by Bragg spectroscopy[28–30].

The combined response at small wave vectors $q\ll q_0=2\pi\rho_0$ is unmodified by the noise. This is because the probe field couples linearly to $\phi$ through the smooth part of the density $\sim\partial_x\phi$. As the system is harmonic, any two perturbations that couple linearly to the oscillator field simply add up independently. Hence, we have $\chi''(q,\omega)=K|q|\Theta(\omega)\delta(\omega-q)$ as in equilibrium. On the other hand, the response at wave vectors near the inverse interparticle distance involves a nonlinear coupling through the component of the density $\rho_{q_0}(x,t)=\cos(q_0x+2\phi)$. We calculate this exactly and obtain (see Supplementary Information)

$$\begin{aligned}\chi''(q,\omega)&=C(K,K_\star)(\omega^2-\delta q^2)^{K_\star-1}\Theta(\omega^2-\delta q^2)\\ C(K,K_\star)&=\frac{1}{4\Gamma^2(K_\star)}\frac{\sin(\pi K)}{\sin(\pi K_\star)}\end{aligned} \tag{5}$$

Here $\Gamma$ is the Gamma function, $\delta q\equiv q-q_0$ and we have defined $K_\star\equiv K(1+\pi^{-2}F_0/\eta)$.

The response near $q=0$ and $q=q_0$ is shown in Fig. 2. By comparing the plot in Fig. 2a, showing the case of vanishing noise, to Fig. 2b where $F_0/\eta=4\pi^2$, we see that the noise turned the divergent response at $q=q_0$ to a power-law suppression. What betrays the fact that the spectrum (Fig. 2b) stems from non-equilibrium conditions and not just weaker interactions? Consider the relation between the response and the energy dissipation functions $\dot{E}(q,\omega)=\omega\chi''(q,\omega)$, that is the rate by which the probe is doing work on the system. Inspecting the pre-factor $C(K,K_\star)$ in equation (5) we find, that for sufficiently strong noise $F_0/\eta>\pi^2(1-K)/K$ (for $K<1$), the energy dissipated by the probe can become negative, which would be strictly prohibited if the system was at equilibrium.

The situation is analogous to a laser, where gain is achieved by pumping the medium out of equilibrium to ‘population inversion’. Here the external $1/f$ noise plays the role of the pump. In contrast to a laser the gain spectrum is continuous and reflects the critical properties of the many-body steady state. Equation (5) implies a commensurability effect between the noise and the intrinsic interactions that leads to oscillations between gain and loss as a function of the noise power.

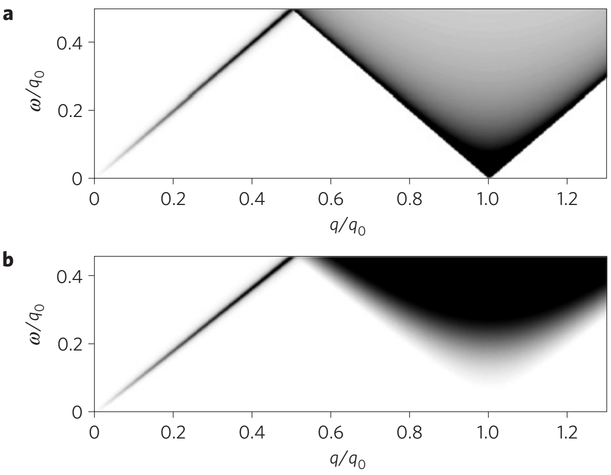


**Figure 2 | Effects of non-equilibrium noise on the response to Bragg spectroscopy. a**, Imaginary part of the response function $\chi''(q,\omega)$ in a one-dimensional system with $K = 0.5$, at equilibrium ($F_0 = 0$). **b**, The same plot as in **a**, in the presence of a strong $1/f$ noise with $F_0/\eta = 4\pi^2$.

## Non-equilibrium phase transitions

We have seen that by changing the $1/f$ noise one can continuously tune the critical exponent associated with the power-law decay of correlations in one-dimensional quantum systems. As in the case of the Josephson junction, we can ask if it is possible to use the new knob to tune across a phase transition.

A textbook[18] phase transition in one-dimensional quantum systems is that of pinning by a commensurate periodic lattice potential. In equilibrium it occurs below a universal critical value of the Luttinger parameter $K_c = 2$, regardless of the strength of the potential. In the context of the real-time dynamics, the periodic potential is added as a perturbation to the action (8)

$$S_g = g \int \mathrm{d}x\mathrm{d}t\ \cos(2\phi(x,t)) \tag{6}$$

The scaling of the perturbation (6) in the critical steady state is determined with the help of the correlation function equation (4). We find that the action of the periodic lattice has the scaling dimension $\alpha_p = 2 - K(1 + \pi^{-2}F_0/\eta)$. This implies an instability, which signals a phase transition to a pinned state for $F_0/\eta < \pi^2(2K^{-1} - 1)$. In particular for $F_0 = 0$ we recover the equilibrium pinning (or Mott) transition at the universal value of the Luttinger parameter $K_c = 2$. Note that for $K > 2$ the system is always unpinned because $F_0$ is non-negative.

A pinning transition can also occur in the presence of a single impurity[31]. The main difference from the previous case is that this perturbation is completely local and therefore its scaling dimension is reduced by 1 relative to the periodic potential: $\alpha_i = 1 - K(1 + \pi^{-2}F_0/\eta)$. Accordingly the depinning transition occurs at a lower critical noise $F_0/\eta = \pi^2(K^{-1} - 1)$ than in the case of the periodic potential.

## Discussion and conclusions

We described a new class of non-equilibrium quantum critical states and phase transitions, which emerge in the presence of external classical noise sources. Physical examples include a Josephson junction and one-dimensional chains of trapped ions or polar molecules coupled to $1/f$ noise. In contrast to thermal noise, which destroys quantum criticality, the $1/f$ noise preserves the algebraic decay of correlations and thus acts as a marginal perturbation at the quantum critical point in these systems. A noise that deviates from $1/f$ at low frequencies, for example $1/f^{1+\epsilon}$, is relevant (irrelevant) for $\epsilon > 0$ ($\epsilon < 0$). However, for $|\epsilon| \ll 1$ the critical correlations will be maintained below the crossover scale $t_* \sim t_0 \exp(1/|\epsilon|)$, where $t_0$ is the short-time cutoff.

The critical exponents associated with both phase and density correlations are varied continuously by the noise, which also destroys the well-known duality between the two. An even more pronounced effect of the non-equilibrium conditions is betrayed by the dissipative response of the critical steady state to an external probe field, which for strong noise can change sign and turn from loss to gain.

Quantum phase transitions, such as pinning of the crystal by an impurity or by a commensurate lattice potential, can take place in the presence of the external, non-equilibrium noise. In particular the system can be tuned across the depinning transition by tuning the noise power.

It would be interesting to extend these ideas to higher-dimensional systems, such as one- or two-dimensional arrays of coupled tubes of polar molecules. The natural phases in equilibrium are the broken-symmetry phases, either superfluid or charge-density wave. The intriguing sliding Luttinger liquid phase, which retains the one-dimensional power-law correlations despite the higher-dimensional coupling, is expected to be stable only in a narrow parameter regime[32]. As the $1/f$ noise acts to suppress both the phase and density correlations it will act to stabilize this phase in a much wider regime. It would also be interesting to consider the effect of the noise on more complex phase transitions, such as the zigzag instability of ion chains[19] as well as Josephson junction arrays[33].

## Methods

**Keldysh action of the quantum Josephson Junction.** The linear quantum Langevin equation (1) is equivalent to the quadratic Keldysh action[34]

$$S_0 = \sum_{\omega,q} (\theta_{\mathrm{cl}}^* \ \ \hat{\theta}^*) \begin{pmatrix} 0 & \frac{1}{2}c\omega^2 - i\eta\omega \\ \frac{1}{2}c\omega^2 + i\eta\omega & -2i\eta|\omega| - 2i\omega^2 \frac{F_0}{|\omega|} \end{pmatrix} \begin{pmatrix} \theta_{\mathrm{cl}} \\ \hat{\theta} \end{pmatrix} \tag{7}$$

Here $\theta_{\mathrm{cl}}$ and $\hat{\theta}$ are the 'classical' and 'quantum' fields. As usual they are defined as the symmetric and antisymmetric combinations, respectively, of the fields associated with forward and backward time propagation of operators: $\theta_{\mathrm{cl}} = (\theta_f + \theta_b)/2, \hat{\theta} = \theta_f - \theta_b$. The Josephson coupling (3) is added to this action.

We note that the contribution of the non-equilibrium noise has the same scaling dimension as the terms coming from the resistor $\propto |\omega|$. In contrast the capacitive term $\propto \omega^2$ is irrelevant, at low frequencies. As a result, the fixed-point action, governing the exponent of equation (2), does not depend on $c$.

**Keldysh action for one-dimensional systems.** The Keldysh action that describes the long-wavelength density fluctuations, coupled to the external noise and the dissipative bath is given by

$$S_0 = \sum_{\omega,q} (\phi_{\mathrm{cl}}^* \ \ \hat{\phi}^*) \begin{pmatrix} 0 & \frac{1}{\pi K}(\omega^2 - q^2) - i\eta\omega \\ \frac{1}{\pi K}(\omega^2 - q^2) + i\eta\omega & -2i\eta|\omega| - 2i\frac{q^2}{\pi^2}\frac{F_0}{|\omega|} \end{pmatrix} \begin{pmatrix} \phi_{\mathrm{cl}} \\ \hat{\phi} \end{pmatrix} \tag{8}$$

Here $F_0/|\omega|$ is the power spectrum of the external noise. The factor of $q^2/\pi^2$ in front of this term appears because the noise couples to $(1/\pi)\partial_x\phi$, the smooth part of the density. $\eta$ denotes the dissipative coupling, which is derived in the Supplementary Information.

Received 3 July 2009; accepted 14 July 2010; published online 29 August 2010

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## Acknowledgements

We thank E. Berg, S. Huber, S. Kivelson, A. Lamacraft, K. Moler and E. Zeldov for stimulating discussions. This work was partially supported by the US–Israel BSF (E.A. and E.D.), ISF (E.A.) and Swiss SNF under MaNEP and division II (T.G.). E.D. acknowledges support from NSF DMR-0705472, CUA, DARPA-OLE and AFOSR-MURI. E.G.D.T. is supported by the Adams Fellowship Program of the Israel Academy of Sciences and Humanities.

## Author contributions

All authors contributed equally to this work.

## Additional information

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