Phase-Sensitive Measurements of Order Parameters for Ultracold Atoms through Two-Particle Interferometry

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Nontrivial symmetry of order parameters is crucial in some of the most interesting quantum many-body states of ultracold atoms as well as condensed matter systems. Examples in cold atoms include p-wave Feshbach molecules and d-wave paired states of fermions that could be realized in optical lattices in the Hubbard regime. Identifying these states in experiments requires measurements of the relative phase of different components of the entangled pair wave function. We propose and discuss two schemes for such phase-sensitive measurements, based on two-particle interference revealed in atom-atom or atomic density correlations. Our schemes can also be used for relative phase measurements for nontrivial particle-hole order parameters, such as d-density wave order.

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The concept of an order parameter, which characterizes states with spontaneously broken symmetries, has been successfully applied to a wide range of physical phenomena such as the Higgs mechanism in high energy physics [1], superfluidity in neutron stars [2], superconductivity [3], gaseous Bose-Einstein condensates [4], and charge and spin ordering in electron systems [5]. In condensed matter systems, order parameters can often be characterized by nontrivial orbital symmetries. For example, high T_c cuprates exhibit d-wave pairings [6], while superfluidity of ^{3}He or superconductivity in Sr_2RuO_4 exhibit triplet p-wave pairings [7]. Other examples of order parameters with nontrivial orbital symmetries are high angular momentum Pomeranchuk instabilities of electron systems [8] and unconventional charge and spin density wave states [5]. Experimental verifications of these states are yet a challenging problem. Only phase-sensitive experiments, such as the observations of Josephson effects in corner SQUID junctions [9] and π-ring tricrystal experiments [10], have been considered as the definitive proof of the unconventional pairing for both cuprates and ruthenates [11].

During the past few years, considerable progress has been achieved in creating analogues of strongly correlated electron systems, by using ultracold atoms (see Ref. [12] for reviews). One of the most challenging problems is the search for d-wave pairing in the repulsive Hubbard model [13]. Realizations of other exotic states in cold-atom systems, such as d-density wave states [14], have been theoretically proposed. These states are characterized by order parameters with nontrivial angular dependence of the relative phase between the components of the entangled wave function. Hence, it is important to understand how tools of atomic physics can be used to perform tests of such quantum many-body states of ultracold atoms [15].

In this Letter, we discuss a scheme for performing such phase-sensitive measurements. It is based on the analysis of atom-atom correlations resulting from two-particle interference [16]. Our proposal builds on the theoretical ideas [17] of using noise correlations in atomic density to characterize many-body states and on the experimental demonstration of measurements of atom-atom correlations or of atomic density noise spectroscopy with ultracold atoms [18–21]. This method should provide unambiguous evidence for nontrivial pairings, including p- and d-wave [13,22,23], as well as for nontrivial particle-hole correlations such as in a d-density wave state [5,14]. It should also allow direct observation of two-particle coherence and nontrivial angular momentum of ultracold diatomic molecules [23,24].

We first consider a Feshbach molecule that consists of a pair of atoms, with zero center of mass momentum:

$$|\Psi_{\mathrm{mol}}\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\,\psi(\mathbf{k}) c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow}|0\rangle. \tag{1}$$

The two atoms making up the molecule can be either bosons or fermions. For concreteness, in this Letter we focus on the case of two fermions in different hyperfine states labeled by $\sigma = \uparrow\downarrow$, in analogy with states of a spin 1/2 particle. Here $\psi(\mathbf{k})$ is the wave function of a molecule, and $c^\dagger_{\mathbf{k}\sigma}$ is a creation operator of a fermion atom in the state with momentum $\mathbf{k}$ and hyperfine state σ. The symmetry of $\psi(\mathbf{k})$ determines the nature of the paired state. We assume that the potential binding the two atoms is removed instantaneously and the released atoms subsequently evolve as free particles. Experimentally, this can be achieved

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either by changing the magnetic field abruptly near a Feshbach resonance or by applying an rf pulse [20,24]. The released pair of atoms is in a superposition of opposite momenta states $|\mathbf{k}, -\mathbf{k}\rangle$ with amplitudes $\psi(\mathbf{k})$. Our goal is to measure the relative phases between $\psi(\mathbf{k})$ for different $\mathbf{k}$.

Scheme I.—We first explain the main idea through the scheme of Fig. 1(a), analogous to the quantum optics scheme of Ref. [25]. Atomic mirrors and beam splitters are based on atomic Bragg diffraction on laser standing waves and are used to reflect and mix states with momenta $\mathbf{p}$ and $\mathbf{q}$ on one side and $-\mathbf{p}$ and $-\mathbf{q}$ on the other side. Time- and space-resolved detectors on opposite sides (e.g., $D1$ and $D3$) allow measurements of correlations resulting from the interference between $\psi(\mathbf{p})$ and $\psi(\mathbf{q})$ and, thus, can reveal the relative phase between these components. We can express the original fermion operators in terms of the operators after the mixing as follows:

$$\begin{aligned} e^{-i\theta_{\mathbf{q}\uparrow}} c^\dagger_{\mathbf{q}\uparrow} &= \cos\beta d^\dagger_1 - i\sin\beta e^{i\chi_\uparrow} d^\dagger_2, \\ e^{-i\theta_{\mathbf{p}\uparrow}} c^\dagger_{\mathbf{p}\uparrow} &= -i\sin\beta e^{-i\chi_\uparrow} d^\dagger_1 + \cos\beta d^\dagger_2, \\ e^{-i\theta_{-\mathbf{p}\downarrow}} c^\dagger_{-\mathbf{p}\downarrow} &= \cos\beta d^\dagger_3 - i\sin\beta e^{i\chi_\downarrow} d^\dagger_4, \\ e^{-i\theta_{-\mathbf{q}\downarrow}} c^\dagger_{-\mathbf{q}\downarrow} &= -i\sin\beta e^{-i\chi_\downarrow} d^\dagger_3 + \cos\beta d^\dagger_4. \end{aligned} \tag{2}$$

Here $d^\dagger_i$ are creation operators for particles observed in detectors D_i ($i = 1, \ldots, 4$). The mixing angle β (of the order of $\pi/4$) and spin-dependent phases χ_σ can be controlled through the amplitudes, durations, and relative phases of the Bragg laser pulses. We denote by $\theta_{\mathbf{k}\sigma}$ the phase accumulated by an atomic component with momentum $\mathbf{k}$ and spin σ during the propagation between the source and the beam splitters.

Assuming that molecular wave functions for wave vectors $\mathbf{q}$ and $\mathbf{p}$ differ only in phase, i.e., $\psi(\mathbf{k}) = |\psi|e^{i\phi_\mathbf{k}}$, the coincidence counts of $n_i = d^\dagger_i d_i$ are

$$\begin{aligned} \langle n_1 n_3\rangle_c &= |\psi|^2 \sin^2(2\beta)\cos^2\left(\frac{\phi_\mathbf{q} - \phi_\mathbf{p} + \Phi_\mathrm{I}}{2}\right), \\ \langle n_1 n_4\rangle_c &= |\psi|^2\left[1 - \sin^2(2\beta)\cos^2\left(\frac{\phi_\mathbf{q} - \phi_\mathbf{p} + \Phi_\mathrm{I}}{2}\right)\right], \\ \Phi_\mathrm{I} &= \theta_{\mathbf{q}\uparrow} + \theta_{-\mathbf{q}\downarrow} - \theta_{\mathbf{p}\uparrow} - \theta_{-\mathbf{p}\downarrow} + \chi_\uparrow - \chi_\downarrow, \end{aligned} \tag{3}$$

and similarly for $\langle n_2 n_3\rangle_c$ and $\langle n_2 n_4\rangle_c$. The oscillatory behavior of the correlation as a function of Φ_I probes the coherence of pairing in the molecule. To vary Φ_I, one can, for instance, change the phases χ_σ. Moreover, if we know the precise value of Φ_I, such coincidence signals yield the relative phase $\phi_\mathbf{q} - \phi_\mathbf{p}$ between different molecular components. In the absence of precise knowledge of Φ_I, the phase difference $\phi_\mathbf{q} - \phi_\mathbf{p}$ could be extracted through a scheme analogous to white light fringes in classical optics, whose pattern and shape can reveal the existence of fundamental phase factors [26]. Note, however, that $\mathbf{k}$ dependence of the phase factors acquired during the propagation and the reflection may render these methods unreliable. Thus, we consider a second scheme which avoids such a problem.

Scheme II.—In this alternative scheme, we apply a $\pi/2$ Bragg pulse at the very beginning of the expansion to mix atomic components with momenta $\mathbf{q}\uparrow$ and $\mathbf{p}\uparrow$, as well as $-\mathbf{q}\downarrow$ and $-\mathbf{p}\downarrow$. This realizes, in a single operation, reflections on the mirrors and mixing on the beam splitters. In scheme II, there is a common mode propagation after the Bragg pulse, and phases acquired during the expansion do not affect interference. Two-atom interference is revealed by coincidence counts with point detectors just as in the previous scheme. The scheme can be generalized to the many-body case by replacing coincident counts between point detectors with density imaging and studying noise correlations between patterns registered on opposite sides (see below).

To discuss scheme II, we start again with the example of a dissociated Feshbach molecule, described by the wave function in Eq. (1). We consider the case in which the Bragg pulses for spin-up and -down atoms differ only in the phase, and such pulses are created by the potentials $V_\sigma(\mathbf{r}) = 2V_0\cos(\mathbf{G}.\mathbf{r} - \chi_\sigma)$. Detectors D_i ($i = 1, 2, 3, 4$) detect atoms with momenta and spins $\mathbf{p}\uparrow$, $\mathbf{q}\uparrow$, $-\mathbf{q}\downarrow$, and $-\mathbf{p}\downarrow$, respectively. The only difference between this scheme and scheme I is the absence of phase factors $e^{i\theta_{\mathbf{q}\sigma}}$. As a result, coincidence counts have forms similar to Eq. (3), with Φ_I replaced by $\Phi_\mathrm{II} = \chi_\uparrow - \chi_\downarrow$. Therefore, provided that we know the phase difference $\chi_\uparrow - \chi_\downarrow$ associated with the two Bragg pulses, atom-atom coincidence counts directly reveal $\phi_\mathbf{q} - \phi_\mathbf{p}$, i.e., the pairing symmetry in Eq. (1).

Many-body state analysis.—We now apply scheme II to a BCS state of fermions $|\Psi\rangle = \prod_\mathbf{k}(u_\mathbf{k} + v_\mathbf{k} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow})|0\rangle$. This BCS wave function is general and can describe weakly coupled BCS paired states as well as a condensate of tightly bound molecules. Here, we consider the generic

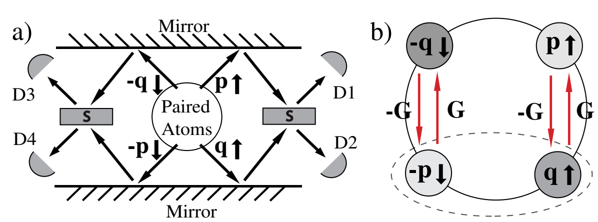

FIG. 1 (color online). Using two-atom interference to measure the relative phase between different components of molecules after dissociation. (a) Scheme I: Free propagating atoms are reflected in mirrors and mixed in beam splitters denoted by S. Coincidences are counted between detectors on opposite sides, e.g., D_1 and D_3. (b) Bragg pulses (π or $\pi/2$) with wave vectors $\mathbf{G} = \mathbf{p} - \mathbf{q}$ and $-\mathbf{G}$ are used to exchange (mirrors) or mix (beam splitters) components $\mathbf{q}\uparrow$ and $\mathbf{p}\uparrow$, as well as $-\mathbf{q}\downarrow$ and $-\mathbf{p}\downarrow$ ($|\mathbf{q}| = |\mathbf{p}|$). For long enough Bragg pulses, no other diffraction order is involved. Scheme II is realized by applying a single $\pi/2$ pulse at the beginning of the expansion.

diffraction pulse that can mix states whose momenta are separated by any integer multiple of $\mathbf{G}$. The effect of the mixing pulse is described by the transformation of particle creation operators: $c^\dagger_{\mathbf{k}\uparrow} \to \tilde{c}^\dagger_{\mathbf{k}\uparrow} = \sum_m \alpha^{\mathbf{k}\uparrow}_{0,m} e^{-im\chi_\uparrow} c^\dagger_{\mathbf{k}+m\mathbf{G}\uparrow}$ and $c^\dagger_{\mathbf{k}-\mathbf{G}\uparrow} \to \tilde{c}^\dagger_{\mathbf{k}-\mathbf{G}\uparrow} = \sum_m \alpha^{\mathbf{k}\uparrow}_{-1,m} e^{-i(m+1)\chi_\uparrow} c^\dagger_{\mathbf{k}+m\mathbf{G}\uparrow}$, and analogously for $c^\dagger_{-\mathbf{k}\downarrow}$ and $c^\dagger_{-\mathbf{k}+\mathbf{G}\downarrow}$. The scattering amplitudes $\alpha^{\mathbf{k}\sigma}_{j,m}$ are controlled by the diffraction pulse amplitude V_0 and its duration τ. We assume that, before the mixing pulse, only states with momenta $\pm\mathbf{k}$, $\pm(\mathbf{k}-\mathbf{G})$, which are close to the Fermi surface, have finite probabilities to be occupied, while states with momenta $\pm(\mathbf{k}-m\mathbf{G})$ for $m \neq 0, 1$, far from the Fermi surface, are empty. The mixing pulse then induces the interference between particles with momenta $\pm\mathbf{k}$ and $\pm(\mathbf{k}-\mathbf{G})$.

The signature of nontrivial pairing of the BCS wave function shows up in the angular dependence of the phase $\phi_\mathbf{k}$ in $v_\mathbf{k} = |v_\mathbf{k}|e^{i\phi_\mathbf{k}}$. In order to probe the relative phase $\Delta\phi = \phi_\mathbf{k} - \phi_{\mathbf{k}-\mathbf{G}}$ between pairs with momenta $\mathbf{k}$ and $\mathbf{k}-\mathbf{G}$, we consider the following density noise correlation after the interference:

$$\langle \delta n_{\mathbf{k}\uparrow} \delta n_{-\mathbf{k}+\mathbf{G}\downarrow}\rangle = \langle n_{\mathbf{k}\uparrow} n_{-\mathbf{k}+\mathbf{G}\downarrow}\rangle - \langle n_{\mathbf{k}\uparrow}\rangle\langle n_{-\mathbf{k}+\mathbf{G}\downarrow}\rangle = |v_\mathbf{k} u_{\mathbf{k}-\mathbf{G}} \alpha^{\mathbf{k}\uparrow}_{00}\alpha^{-\mathbf{k}\downarrow}_{01} e^{-i\chi_\downarrow} + u_\mathbf{k} v_{\mathbf{k}-\mathbf{G}} \alpha^{\mathbf{k}\uparrow}_{-10}\alpha^{-\mathbf{k}\downarrow}_{11} e^{-i\chi_\uparrow}|^2 - (|v_\mathbf{k}|^2 - |v_{\mathbf{k}-\mathbf{G}}|^2)(|v_\mathbf{k}|^2|\alpha^{\mathbf{k}\uparrow}_{00}|^2|\alpha^{\mathbf{k}\uparrow}_{-10}\alpha^{-\mathbf{k}\downarrow}_{11}|^2 - |v_{\mathbf{k}-\mathbf{G}}|^2|\alpha^{-\mathbf{k}\downarrow}_{01}|^2|\alpha^{-\mathbf{k}\downarrow}_{11}|^2). \quad (4)$$

In analogy with the case of a Feshbach molecule, the right-hand side of Eq. (4) contains an interference term which depends on the relative phase $\Delta\phi$ as well as on $\Phi_{\rm II} = \chi_\uparrow - \chi_\downarrow$.

Space- and time-resolved single atom detection [18,21,27] permits direct measurements of atom-atom correlations for specific momenta, corresponding to Eq. (4). Alternatively, one may look for noise correlation in absorption images after the time of flight [17]. In this case, absorption imaging, as well as finite resolution of detectors, results in the integration of the atomic density. In order to take into account these effects, we integrate Eq. (4) over ranges of momenta as shown in Fig. 2(a). We present in Fig. 2(b) the result of this integration, which displays noise correlation in integrated density vs the phase difference $\chi_\uparrow - \chi_\downarrow$ of the diffraction pulses. Here the integration range is $|\Delta k_y| = |\mathbf{G}|/10$, $|\Delta k_x| = |\mathbf{G}|/10$, and $|\Delta k_z| = 5|\mathbf{G}|$, and the pairing gap is $\Delta \approx 0.1E_F$. The diffraction pulse amplitude is set to $V_0/E_R = 2$, where $E_R = |\mathbf{G}|^2/8m$ is the recoil energy, and its duration is chosen to have the maximum oscillation of the signal. We assume that the integration range is sufficiently small that the phases of the Cooper pairs $\phi_\mathbf{k}$ and $\phi_{\mathbf{k}-\mathbf{G}}$ are constant in the integration range.

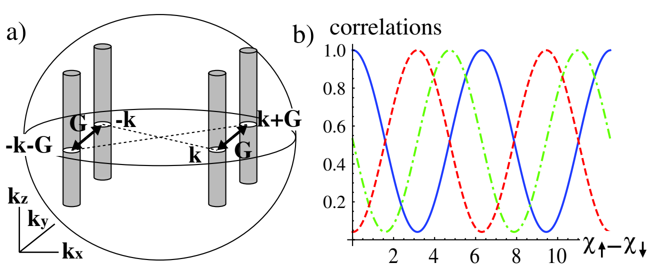

FIG. 2 (color online). (a) In order to take into account the finite resolution of detectors and the integration in absorption imaging, the density noise correlation is integrated over the cylinders shown in the figure. (b) Integrated density noise correlations $\langle \delta N_{V_\uparrow} \delta N_{V_\downarrow}\rangle$ as a function of phase difference $\chi_\uparrow - \chi_\downarrow$ for a diffraction pulse of amplitude $V_0/E_R = 2$ and a duration τ which yields the maximum oscillation of the signal. Blue, green (dash-dotted line), and red (dashed line) curves correspond to $\Delta\phi = \phi_\mathbf{k} - \phi_{\mathbf{k}-\mathbf{G}} = 0$, $\pi/2$, and π, respectively.

The oscillatory behavior of the integrated noise correlations $\langle \delta N_{V_\uparrow} \delta N_{V_\downarrow}\rangle$ as a function of $\chi_\uparrow - \chi_\downarrow$[see Fig. 2(b)] should provide an unambiguous proof of the Cooper pair coherence. Moreover, the value of the correlation at $\chi_\uparrow - \chi_\downarrow = 0$ yields information about the phase difference $\Delta\phi = \phi_\mathbf{k} - \phi_{\mathbf{k}-\mathbf{G}}$, which is the quantity we are interested in. The value of the correlation at $\chi_\uparrow - \chi_\downarrow = 0$ also depends on the scattering amplitudes $\alpha^{k\sigma}_{j,m}$ and thus on $V_0\tau$. In Fig. 3, we present the integrated noise correlation signal $\langle \delta N_{V_\uparrow} \delta N_{V_\downarrow}\rangle$ at $\chi_\uparrow - \chi_\downarrow = 0$ as a function of $V_0\tau$ for three different values of $\Delta\phi$ and find striking differences. We conclude that it should be possible to discriminate between $\Delta\phi = 0$ and $\Delta\phi = \pi$ even when full 3D resolution is not available.

In discussions so far, we assumed that the BCS pairs or molecules are at rest before dissociation. When molecules are cold but not condensed, there is a spread in the center of mass momenta determined by the temperature. Even in this case, there is still a perfect coherence between different parts of the wave function of each molecule, yielding a two-body interference. However, the average of these

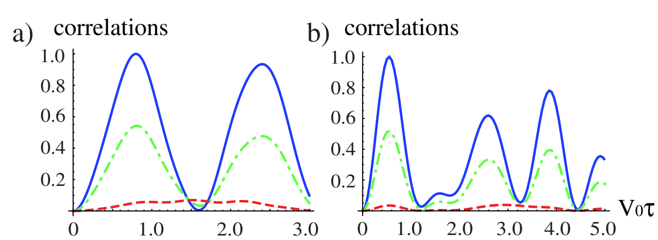

FIG. 3 (color online). Integrated noise correlations $\langle \delta N_{V_\uparrow} \delta N_{V_\downarrow}\rangle$ as a function of $V_0\tau$ (a) for a Bragg pulse amplitude $V_0/E_R = 2$ and (b) for a Bragg pulse amplitude $V_0/E_R = 20$. Blue, green (dash-dotted line), and red (dashed line) curves correspond to $\Delta\phi = \phi_\mathbf{k} - \phi_{\mathbf{k}-\mathbf{G}} = 0$, $\pi/2$, and π, respectively.

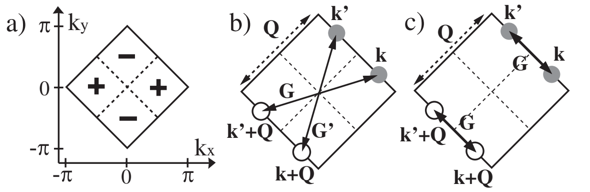

FIG. 4. (a) Illustration of the sign of the CDW amplitude $\psi_{\rm ph}(\mathbf{k}) = \langle c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}+\mathbf{Q}\sigma}\rangle$ for the d-wave CDW state. (b), (c) Phase-sensitive detection of the symmetry of the d-wave CDW state in time-of-flight experiments. For (b), two pulses which transfer momenta $\mathbf{G}$ and $\mathbf{G}'$ are applied at the beginning of expansion. In (c), a single pulse with momentum transfer $\mathbf{G}$ is applied. All the couplings through the Bragg pulses are indicated by solid arrows. Here $\mathbf{Q}$ is the wave vector of CDW.

interference terms over the center of mass momenta of individual molecules could result in the washing out of noise correlations. We expect this suppression to be moderate as long as the thermal spread of the molecule center of mass momenta is small enough.

Systems with particle-hole correlations.—There are several types of many-body states characterized by correlations in the particle-hole channel, such as charge (CDW) and spin density wave states. The most exotic of them have a finite angular momentum. This means that we have $\langle c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}+\mathbf{Q}\sigma}\rangle = \psi_{\rm ph}(\mathbf{k})$, where $\psi_{\rm ph}(\mathbf{k})$ has a nontrivial angular dependence.

Our scheme above can be generalized to provide an unambiguous phase-sensitive detection of such states as well. To be concrete, let us consider a 2D system near half filling. In this case, one can combine two different measurements of correlation functions to obtain the information on the order parameter $\psi_{\rm ph}(\mathbf{k})$, as shown in Figs. 4(b) and 4(c). In Fig. 4(b), two Bragg pulses couple $\mathbf{k}$ and $\mathbf{k}' + \mathbf{Q}$, as well as $\mathbf{k}'$ and $\mathbf{k} + \mathbf{Q}$. Here, the correlation function $\langle \delta n_{\mathbf{k}} \delta n_{\mathbf{k}'}\rangle$ contains an interference term proportional to $\psi_{\rm ph}(\mathbf{k})\psi_{\rm ph}(\mathbf{k}')$. In Fig. 4(c), a Bragg pulse couples $\mathbf{k}$ and $\mathbf{k}'$, and the correlation function $\langle \delta n_{\mathbf{k}} \delta n_{\mathbf{k}'+\mathbf{Q}}\rangle$ contains the term $\psi^*_{\rm ph}(\mathbf{k})\psi_{\rm ph}(\mathbf{k}')$. When combined, this information should not only provide evidence of the angular dependence of CDW but also allow one to distinguish site and band centered density wave states.

In conclusion, we have proposed a new method, inspired from quantum optics, for performing phase-sensitive measurements of nontrivial order parameters in entangled systems of ultracold atoms. This is a new example of ultracold atom quantum simulators, with a view toward studying open problems in strongly correlated systems.

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