Spin Bose-Glass Phase in Bilayer Quantum Hall Systems at $\nu = 2$

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We develop an effective spin theory to describe magnetic properties of the $\nu = 2$ quantum Hall bilayer systems. In the absence of disorder this theory agrees with the microscopic Hartree-Fock calculations, and for finite disorder it predicts the existence of a novel spin Bose-glass phase. The Bose glass is characterized by the presence of domains of canted antiferromagnetic phase with zero average antiferromagnetic order and short range correlations. It has infinite antiferromagnetic transverse susceptibility and finite longitudinal spin susceptibility. Transition from the canted antiferromagnetic phase to the spin Bose-glass phase is characterized by a universal value of the spin conductance. [S0031-9007(99)09076-6]

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Recently a canted antiferromagnetic phase has been predicted in bilayer quantum Hall (QH) systems at a total filling factor $\nu = 2$ on the basis of microscopic Hartree-Fock calculations and a long wavelength quantum O(3) nonlinear sigma model [1]. In this Letter we construct an alternative effective spin theory that can describe the richness of the phase diagram of a bilayer $\nu = 2$ quantum Hall system. Our effective spin theory treats the interlayer tunneling nonperturbatively, in contrast to the O(3) nonlinear sigma model which includes tunneling perturbatively through an antiferromagnetic exchange. It is in qualitative agreement with the results of microscopic Hartree-Fock calculations in [1] and extends the earlier effective field theory by allowing us to study quantitatively the effect of a finite gate voltage between the layers and calculate intersubband excitation energies. Our theory can easily incorporate the effects of disorder and we predict that for any nonzero disorder there is a new $\nu = 2$ spin Bose-glass quantum Hall phase which may be visualized as domains of canted antiferromagnetic phase surrounded by domains of fully polarized ferromagnetic or spin singlet (SS) phases. In this system the Bose-glass phase we predict is quite novel, and we elaborate in this Letter on the origin and the properties of this new QH glass phase. Related disorder induced spin phase has been discussed in a different setting in Ref. [2].

In the absence of interlayer interaction each layer of the $\nu = 2$ bilayer system would be in a fully spin polarized ferromagnetic (FPF) $\nu = 1$ incompressible QH state with spins in both layers pointing in the direction of the applied magnetic field. Tunneling between the layers favors the formation of spin singlet states from the pairs of electrons in the opposite layers and energetically stabilizes the spin singlet state. In [1] it was observed that the competition between the two tendencies may lead to a third intermediate phase: a canted antiferromagnetic (CAF) state, where spins in the two layers have the same component along the applied field but opposite components in the perpendicular 2D plane.

We now introduce a simple lattice model which we use to describe the physics of the bilayer $\nu = 2$ QH system. We consider a bilayer square lattice model shown in Fig. 1.

Sites in each layer may be thought of as labeling different intra-Landau level states. Electrons may tunnel from one layer to another conserving the in-plane site index (i.e., between the states with the same intra-Landau level index). There is a ferromagnetic interaction between nearest-neighbor sites within individual layers and a Zeeman interaction with the applied magnetic field. We also account for the charging energy, i.e., the energy cost of creating charge imbalance between the layers through the term $\mathcal{H}_c$ below. The Hamiltonian of the system may be written as

$$\begin{aligned}
\mathcal{H} &= \mathcal{H}_T + \mathcal{H}_c + \mathcal{H}_Z + \mathcal{H}_F\,, \\
\mathcal{H}_T &= -\frac{1}{2}\Delta_{\mathrm{SAS}}\sum_i (c^\dagger_{Ti\sigma}c_{Bi\sigma} + c^\dagger_{Bi\sigma}c_{Ti\sigma})\,, \\
\mathcal{H}_c &= \frac{1}{2}\epsilon_c\sum_i [(n_{Ti}-1)^2 + (n_{Bi}-1)^2]\,, \qquad (1) \\
\mathcal{H}_Z &= -H_z\sum_i (S^z_{Ti} + S^z_{Bi})\,, \\
\mathcal{H}_F &= -J\sum_{\langle ij\rangle}(\mathbf{S}_{Ti}\mathbf{S}_{Tj} + \mathbf{S}_{Bi}\mathbf{S}_{Bj})\,,
\end{aligned}$$

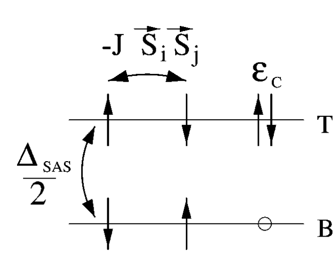

FIG. 1. Effective bilayer lattice model for the $\nu = 2$ double layer QH system.

where T and B label electrons in the top and bottom layers, respectively, i is the in-plane site (intra-Landau level) index, and σ is the spin index. $S^a_{Ti} = c^\dagger_{Ti\alpha}\boldsymbol{\sigma}^{\mathbf{a}}_{\boldsymbol{\alpha\beta}} c_{Ti\beta}$ and $n_{Ti} = \sum_\sigma c^\dagger_{Ti\sigma} c_{Ti\sigma}$ are spin and charge operators, respectively, for layer T, with analogous definitions for layer B. Parameters J and ϵ_c of this model may be easily estimated as $J = e^2/(16\sqrt{2\pi}\,\epsilon l)$, where $l = \sqrt{\hbar c/eB}$ is magnetic length, and $\epsilon_c = \frac{e^2}{\epsilon l}\sqrt{\frac{\pi}{2}}\{1 - e^{d^2/l^2}(1 - Erf[d/(l\sqrt{2})])\}$, where d is the distance between the layers and Erf is the error function [3].

Each individual rung, i.e., two sites with the same in-plane site index on the opposite layers, must be populated by two electrons. Therefore, we have six possible states for each rung, which are conveniently classified into three states that are spin triplets

$$
\begin{aligned}
|t_+\rangle &= t^\dagger_+|0\rangle = c^\dagger_{T\uparrow}c^\dagger_{B\uparrow}|0\rangle, \\
|t_0\rangle &= t^\dagger_0|0\rangle = \tfrac{1}{\sqrt{2}}(c^\dagger_{T\uparrow}c^\dagger_{B\downarrow} + c^\dagger_{T\downarrow}c^\dagger_{B\uparrow})|0\rangle, \qquad (2) \\
|t_-\rangle &= t^\dagger_-|0\rangle = c^\dagger_{T\downarrow}c^\dagger_{B\downarrow}|0\rangle,
\end{aligned}
$$

and three states that are spin singlets

$$
\begin{aligned}
|\tau_+\rangle &= \tau^\dagger_+|0\rangle = c^\dagger_{T\uparrow}c^\dagger_{T\downarrow}|0\rangle, \\
|\tau_0\rangle &= \tau^\dagger_0|0\rangle = \tfrac{1}{\sqrt{2}}(c^\dagger_{T\uparrow}c^\dagger_{B\downarrow} - c^\dagger_{T\downarrow}c^\dagger_{B\uparrow})|0\rangle, \qquad (3) \\
|\tau_-\rangle &= \tau^\dagger_-|0\rangle = c^\dagger_{B\uparrow}c^\dagger_{B\downarrow}|0\rangle.
\end{aligned}
$$

Operators t and τ satisfy bosonic commutation relations [4], and the constraint $\tau^\dagger_\alpha\tau_\alpha + t^\dagger_\alpha t_\alpha = 1$ projects into the physical Hilbert space.

In Hamiltonian (1) all the terms except $\mathcal{H}_F$ act within a single rung. It is therefore natural as a first step to diagonalize $\mathcal{H}' = \mathcal{H}_T + \mathcal{H}_c + \mathcal{H}_Z$ on one rung. A simple calculation gives for the lowest energy eigenstates of $\mathcal{H}'$:

State $|t_+\rangle$ with energy $E_t = -H_z$.

State

$$|v_+\rangle = \frac{(\sin\theta - \cos\theta)}{2}(|\tau_+\rangle + |\tau_-\rangle) - \frac{(\sin\theta + \cos\theta)}{\sqrt{2}}|\tau_0\rangle,$$

with energy $E_v = \frac{\epsilon_c}{2} - \sqrt{\Delta^2_{\mathrm{SAS}} + \epsilon_c^2/4}$. Here $\tan\theta = \epsilon_c/(2\Delta_{\mathrm{SAS}} + 2\sqrt{\Delta^2_{\mathrm{SAS}} + \epsilon_c^2/4})$.

State $|v_+\rangle$ is a spin singlet state whose energy is lowered by interlayer tunneling, and $|t_+\rangle$ is a spin triplet state favored by Zeeman interaction. We rewrite Hamiltonian (1) keeping only the lowest energy states $|v_{i+}\rangle$ and $|t_{i+}\rangle$:

$$
\begin{aligned}
\tilde{\mathcal{H}} = {} & E_t\sum_i t^\dagger_{i+}t_{i+} + E_v\sum_i v^\dagger_{i+}v_{i+} - \frac{J}{4}(\cos\theta + \sin\theta)^2 \\
& \times \sum_{\langle ij\rangle}(t^\dagger_{i+}v_{i+}v^\dagger_{j+}t_{j+} + t^\dagger_{j+}v_{j+}v^\dagger_{i+}t_{i+}) \\
& - \frac{J}{2}\sum_{\langle ij\rangle} t^\dagger_{i+}t_{i+}t^\dagger_{j+}t_{j+}\,, \qquad (4)
\end{aligned}
$$

and the hard core constraint is implied

$$v^\dagger_{i+}v_{i+} + t^\dagger_{i+}t_{i+} = 1\,. \qquad (5)$$

Another interpretation of Hamiltonian (4) is that its continuum limit gives the energy functional for a general family of states with (possibly nonuniform) expectation values $|t(x)|^2 = \langle\Omega|\Psi_{\mathrm{antisym}\uparrow}(x)^\dagger\Psi_{\mathrm{antisym}\uparrow}(x)|\Omega\rangle$, $|v(x)|^2 = \langle\Omega|\Psi_{\mathrm{sym}\downarrow}(x)^\dagger\Psi_{\mathrm{sym}\downarrow}(x)|\Omega\rangle$, and $t(x)v^*(x) = \langle\Omega|\Psi^\dagger_{\mathrm{sym}\downarrow}\Psi_{\mathrm{antisym}\uparrow}|\Omega\rangle$. Here $\Psi^\dagger_{\mathrm{sym/antisym}\sigma}$ creates an electron of given symmetry with respect to layer interchanges and spin σ, and $|\Omega\rangle$ is a ground state of the system.

The mean-field analysis of (4) may be done by considering states with simultaneously condensed v and t bosons. They correspond to the variational wave functions of the form $|\Phi\rangle = \exp\{\alpha\sum_i v^\dagger_{i+} + \beta\sum_i t^\dagger_{i+}\}|0\rangle$, where $|0\rangle$ is a vacuum state [4]. The energy of state $|\Phi\rangle$ is given by

$$
\begin{aligned}
E_0 = {} & E_v|\alpha|^2 + E_t|\beta|^2 \\
& - J(\cos\theta + \sin\theta)^2|\alpha|^2|\beta|^2 - J|\beta|^4, \qquad (6)
\end{aligned}
$$

and state $|\Phi\rangle$ obeys constraint (5) on the average provided that

$$|\alpha|^2 + |\beta|^2 = 1\,. \qquad (7)$$

Values of α and β that minimize (6) under the condition (7) are given by

$$
\begin{aligned}
&|\alpha| = 1 \quad |\beta| = 0 \quad \text{if } t_{\min} < 0\,, \\
&|\alpha| = t_{\min} \quad |\beta| = \sqrt{1 - t^2_{\min}} \quad \text{if } 0 < t_{\min} < 1\,, \qquad (8) \\
&|\alpha| = 0 \quad |\beta| = 1 \quad \text{if } t_{\min} > 1\,,
\end{aligned}
$$

where $t_{\min} = 0.5[J(\cos\theta + \sin\theta)^2 - E_t + E_v]/[J \times (\cos\theta + \sin\theta)^2 - J]$.

The first and the last cases correspond to the FPF and SS phases, respectively, and an intermediate phase with both $|\alpha|$ and $|\beta|$ finite describes the canted antiferromagnetic phase of [1] with direction of the Néel ordering given by the phase between the t_+ and v_+ condensates $\tan^{-1}N_y/N_x = \arg(\alpha^*\beta)$ (the Néel order parameter is defined as $\mathbf{N} = \sum_i \mathbf{S}_{Ti} - \sum_i \mathbf{S}_{Bi}$). In Fig. 2 we show the phase diagram obtained from Eqs. (8).

This phase diagram is in general agreement with the one obtained from the Hartree-Fock calculations in Ref. [1]; however, the bosonic theory predicts a smaller width of the CAF region. This is hardly surprising since Hartree-Fock mean-field calculations commonly overestimate the stability of the ordered phase, whereas the bosonic model goes beyond mean-field approach by taking into account the effect of charge fluctuations between the layers (see, for example, the expression for the energy of a singlet state E_v). We can also use our bosonic model to calculate the phase diagram in the presence of an interlayer charge imbalance, and these results will be reported elsewhere [3].

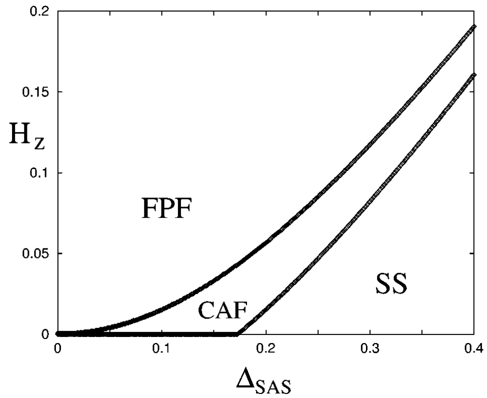

FIG. 2. Phase diagram for disorder-free system for $d = l$. All energies are in units of $e^2/\epsilon l$.

The lowest energy interband transition in the SS phase will correspond to destroying a v_+ and creating a t_+ boson. The energy for such transition is $\omega_- = -J(\cos\theta + \sin\theta)^2 + E_t - E_v$ and vanishes at the SS/CAF transition as may be seen from Eqs. (8). Analogously, in the FPF state the lowest energy interband transition will correspond to destroying t_+ and creating a v_+ boson [3].

Let us now give a simple physical picture that will illustrate the formal calculations presented above. We consider an SS state which has singlet v_+ bosons on all rungs and imagine creating a t_+ triplet on one of the rungs. Creating a localized triplet requires energy $E_t - E_v$, and this energy is unaffected by the ferromagnetic interaction since parallel and antiparallel contributions cancel for triplet interacting with neighboring singlets. However, $\mathcal{H}_F$ also gives rise to a process in which one of the spins of the triplet pair and one spin from the neighboring singlet pair are flipped simultaneously. This process is shown in Fig. 3 and may be interpreted as hopping of the triplet boson to the nearest-neighbor site. Therefore, creating a propagating triplet boson at wave vector k will give it an additional kinetic energy $\tilde{J}(\cos k_x + \cos k_y)$ due to $\mathcal{H}_F$. This allows us to have a situation when $E_t - E_v > 0$ but $E_t - E_v - 2\tilde{J} < 0$, i.e., when it is energetically unfavorable to create localized triplets but it is already favorable to create them at $k = 0$, i.e., to have a condensate of t_+ bosons. This effect is the origin of the CAF state and allows us to understand this phase as a coherent superposition of condensed t_+ and v_+ bosons.

In a real system there is always disorder. It may be due to fluctuations in the distance between the wells or the presence of impurities. For our effective spin model the major effects of disorder will be randomness in the value of tunneling Δ_{SAS} and the appearance of a random local gate voltage, in both cases leading to random local fluctuations in the energy of the v_+ boson. Then if we are close to the CAF/SS transition we may have a situation induced by disorder where $E_t - E_v^{\max} - 2\tilde{J} < 0$ and $E_t - E_v^{\min} - 2\tilde{J} > 0$. So for some regions creating nonlocal t_+ triplets will lower the energy of the system and for some regions it will lead to an energy increase. In this case the system breaks into domains, with each domain being locally a CAF phase or a SS phase (region III in Fig. 4). Each CAF domain may be thought of as being in a quantum disordered state with an undefined direction of the Néel order but finite z-magnetization [2]. Close to the CAF/FPF transition line in the disorder-free system we may have a disorder induced situation where we have CAF domains in the background of domains of the FPF phase (region I in Fig. 4). Finally, we can also have the phase where we have domains of all three kinds (region II in Fig. 4). In Fig. 4 we show the resulting phase diagram for the same values of parameters as in Fig. 2 but assuming that Δ_{SAS} may randomly vary by 10% around its average value. Such a variation in Δ_{SAS} is physically quite reasonable even in high quality 2D systems since Δ_{SAS} depends exponentially on layer thickness. There are no phase transitions between regions I, II, and III on the phase diagram in Fig. 4 but only smooth crossovers. The true quantum phase transitions occur between FPF and I, SS and III, and between CAF and one of the I, II, or III regions.

The nature of these phase transitions is also easy to understand. The SS phase is an insulating phase of zero density of t_+ bosons, the FPF state is an insulating phase with density $n = 1$, and CAF is a superfluid phase. The randomness that we consider acts as a randomness in the

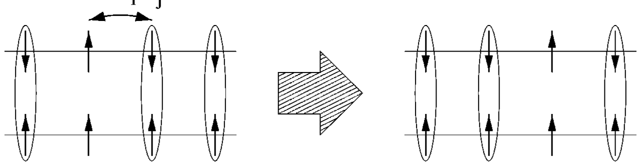

FIG. 3. Triplet "hopping" process due to ferromagnetic inplane interaction.

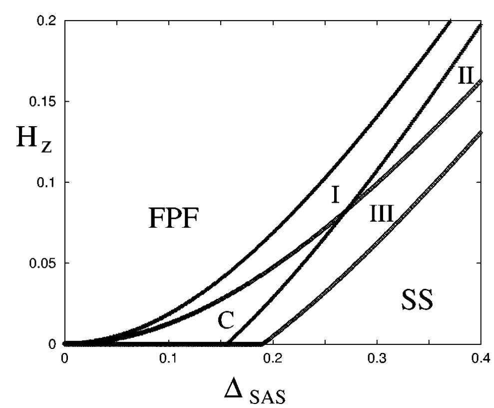

FIG. 4. Phase diagram for a disordered system. All energies are in units of $e^2/\epsilon l$ and $d = l$. Variation in Δ_{SAS} was assumed to be 10%. Region I corresponds to domains of CAF and FPF phases, region III to domains of SS and CAF, and region II to domains of all three kinds. There are no phase transitions among regions I, II, III; they all correspond to the spin Bose-glass phase.

chemical potential of these t_+ bosons, so our problem is equivalent to the problem of bosons in a random potential, the so-called dirty boson problem, considered in Refs. [5,6]. We immediately recognize I, II, and III as a single Bose-glass (SBG) phase of the singlet and triplet bosons. This observation allows us to draw several important conclusions about the properties of this SBG phase. In the SS state the $\langle S^z(-\omega)S^z(\omega)\rangle$ correlation function is zero and in the CAF phase it has a δ-function peak at zero frequency due to the Goldstone mode of the spontaneous breaking of the U(1) symmetry of spin rotations around the z axis. In the SBG phase this correlation function will be finite at small frequencies, which implies finite longitudinal spin susceptibility and is the analog of finite compressibility of the usual charge Bose glass. Our new SBG phase does not have antiferromagnetic long range order, i.e., $\langle N_{x(y)}\rangle = 0$. All the t_+ and v_+ bosons are localized in this phase; therefore, it will have only short range mean antiferromagnetic correlations. But analogous to the infinite superfluid susceptibility of charge Bose glass our SBG phase will have an infinite transverse antiferromagnetic susceptibility. Another important feature of the Bose-glass phase is a finite density of low energy excitations [5,6]. This implies that our SBG phase will have a specific heat linear in temperature, which provides another way to experimentally distinguish it from the CAF phase whose specific heat is T^2 or FPF and SS phases that have exponentially small specific heat at low temperatures. The existence of the SBG phase separating the SS, FPF, and CAF phases also has important consequences in that it changes the critical exponents for the corresponding phase transitions from the one obtained in [1] for the disorder-free system. The new critical exponents will be those of the superconductor-insulator transition in the dirty boson system studied in [5,6]. We would also like to point out that the SBG system that we suggested may be a better experimental realization of a 2D superconductor-insulator transition in a boson system than conventionally used 2D superconducting films [7] in that it is free of long range forces and allows one to vary the density of bosons by varying H_z. In addition, our predicted QH Bose-glass phase transition does not have the complication arising from parallel fermionic excitations which may play a role in the superconducting films [8]. We therefore expect, based on the arguments given in [5,6], that transition from the CAF phase to the SBG phase will be characterized by a truly universal longitudinal spin conductance, which, in principle, can be measured by measuring the spin susceptibility and the spin diffusion coefficient.

Before concluding, we remark on the feasible experimental observability of our proposed QH Bose-glass phase. First, we remark that the basic $\nu = 2$ QH phase transition and the associated softening of the relevant spin density excitations have been verified experimentally [9] via inelastic light scattering spectroscopy. Since interlayer tunneling fluctuations are invariably present in real systems, it is, in fact, quite possible that the experiments in [9] have already observed a transition to the Bose-glass phase as in our Fig. 4. Some evidence supporting this possibility comes from the fact that the softening of the spin density excitations observed in [9] did not lead to the appearance of a sharp dispersing Goldstone mode expected in the CAF phase but only to some broad zero energy spectral weight consistent with the Bose-glass phase. Future experiments in samples with deliberately controlled disorder should be carried out to conclusively verify our prediction of a QH disordered Bose-glass phase.

In conclusion, we predict a new 2D Bose-glass phase in a $\nu = 2$ QH bilayer system by introducing an effective spin theory. This phase has the usual properties of a Bose-glass phase [5,6] including a universal spin conductance at the transition. While we have specifically considered the $\nu = 2$ integer QH situation, our arguments should also go through for all $\nu = 2/(\text{odd integer})$ fractional QH states, following the reasoning of [1], and for the fractional filling there should be an exotic fractional quantum 2D Bose glass in bilayer systems [10].

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[10] Another plausible effect of disorder, which we do not discuss in this paper, is the appearance of unmatched spins 1/2 leading to the possibility of a random-singlet phase. This effect is not important at $\nu = 2$ but may become relevant for the $\nu = 2/(\text{odd integer})$ fractional QH states.