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Universal dynamical phase diagram of lattice spin models and strongly correlated ultracold atoms in optical lattices

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Abstract

We study semiclassical dynamics of anisotropic Heisenberg models in two and three dimensions. Such models describe lattice spin systems and hard core bosons in optical lattices. We solve numerically Landau–Lifshitz type equations on a lattice and show that in the phase diagram of magnetization and interaction anisotropy, one can identify several distinct regimes of dynamics. These regions can be distinguished based on the character of one dimensional solitonic excitations, and stability of such solitons to transverse modulation. Small amplitude and long wavelength perturbations can be analyzed analytically using mapping of non-linear hydrodynamic equations to KdV type equations. Numerically we find that properties of solitons and dynamics in general remain similar to our analytical results even for large amplitude and short distance inhomogeneities, which allows us to obtain a universal dynamical phase diagram. As a concrete example we study dynamical evolution of the system starting from a state with magnetization step and show that formation of oscillatory regions and their stability to transverse modulation can be understood from the properties of solitons. In regimes unstable to transverse modulation we observe formation of lump type solutions with modulation in all directions. We discuss implications of our results for experiments with ultracold atoms.

Supplementary material for this article is available [online](#)

Keywords: solitons, nonlinear dynamics, ultracold atoms

(Some figures may appear in colour only in the online journal)

Motivation

Understanding nonequilibrium quantum dynamics of many-body systems is an important problem in many areas of physics. The most fundamental challenge facing this field is identifying emergent collective phenomena, which should lead to universality and common properties even in systems which do not have identical microscopic Hamiltonians. We know that in equilibrium many-body systems typically fall within certain universality classes. Basic examples are states with spontaneously broken symmetries such as BEC of bosons and superfluid paired phases of fermions [1, 2], Fermi liquid states of electrons and interacting ultracold fermions [3, 4]. We have numerous examples of emergent universal

properties of classical systems driven out of equilibrium (see [5] for a review). However very few examples of collective behavior of coherent quantum evolution of many-body systems, which can be classified as exhibiting emergent universality, are known (several recently studied examples can be found in [6–12]) The main result of this paper is demonstration of the universality of semiclassical dynamics of lattice spin systems and strongly interacting bosons in two and three dimensions, summarized in the phase diagram in figure 1.

In addition to fundamental conceptual importance, our study has strong experimental motivation. Rapid progress of experiments with ultracold atoms in optical lattices makes it possible to create well controlled realizations of systems that we discuss [14, 28]. Tunability of such systems, nearly

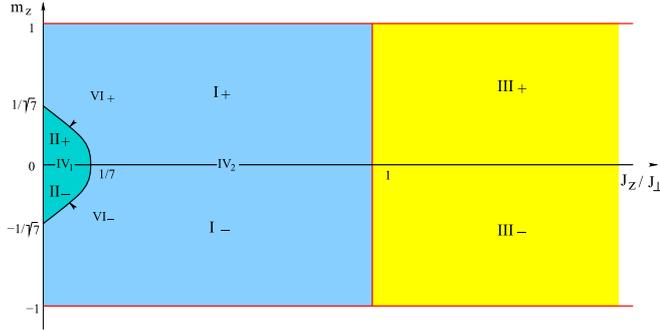


Figure 1. Dynamical phase diagram of the anisotropic Heisenberg model (1) for initial states with finite magnetization in the XY plane. Vertical axis labels magnetization for spin models and density for hard-core bosons on a lattice. Regions I, II correspond to easy plane anisotropy of the Hamiltonian, linearized collective modes are of the hyperbolic type. Regions III correspond to easy axis anisotropy, linearized collective modes are of the elliptic type. Special lines $J_z/J_\perp = 1$ and $m_z = \pm 1$ all have linearized collective modes of parabolic type. Region I_+ has particle solitons unstable to transverse modulation. Region I_- has hole solitons unstable to transverse modulation. Region II_+ has hole solitons stable to transverse modulation. Region I_- has particle solitons stable to transverse modulation.

perfect isolation from the environment, and a rich toolbox of experimental probes, including the possibility of single site resolution [15–18], makes them excellent candidates for exploring nonequilibrium quantum dynamics (see [10, 19] and references therein). Several intriguing nonequilibrium phenomena have been demonstrated recently in ensembles of ultracold atoms, including collapse and revival of coherence [20], absence of relaxation in nearly integrable systems [6], exponential slowdown of relaxation in systems with strong mismatch of excitation energies [21], anomalous diffusion [11], prethermalization [12], many-body Landau–Zener transitions [22], light-cone spreading of correlations in a many-body system [23].

Model

In this paper we study non-equilibrium dynamics of lattice spin models and strongly correlated bosons in optical lattices. Our starting point is the anisotropic Heisenberg model

$$\mathcal{H}_{\text{AH}} = -J_\perp \sum_{\langle ij \rangle} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) - J_z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z, \quad (1)$$

where σ^a are Pauli matrices. Anisotropic Heisenberg models with tunable interactions can be realized with two component Bose mixtures in spin dependent optical lattices [24–30]. Hamiltonian (1) also describes spinless bosons in the regime of infinitely strong on-site repulsion [31]. In this case states $|\downarrow\rangle$ and $|\uparrow\rangle$ correspond to states with zero and one boson per site respectively, J_\perp is the tunneling strength, and J_z is the strength of the nearest neighbor repulsion. For atoms with contact interaction and confined to the lowest Bloch band $J_z = 0$. Non-local interactions can be present for atoms in higher Bloch bands [32], polar molecules [33–35], Rydberg atoms [36, 37], and ion crystals [38].

Semiclassical equations of motion can be understood either as the lattice version of Landau–Lifshitz equations or as dynamics in the manifold of variational states $|\Psi(t)\rangle = \prod_i [\sin \frac{\theta_i}{2} e^{-i\varphi_i/2} |\downarrow\rangle_i + \cos \frac{\theta_i}{2} e^{i\varphi_i/2} |\uparrow\rangle_i]$, where all θ_i and φ_i are independent functions of time (see supplementary material and [39–41]). It is convenient to introduce parametrization $\sin \frac{\theta_i}{2} = \left(\frac{1-z_i}{2}\right)^{1/2}$, $\cos \frac{\theta_i}{2} = \left(\frac{1+z_i}{2}\right)^{1/2}$, $e^{i\varphi_i} = \frac{x_i + iy_i}{(x_i^2 + y_i^2)^{1/2}}$, then semiclassical dynamical equations are given by

$$\begin{aligned} \dot{x}_i &= 4 J_\perp z_i (y_{i+1} + y_{i-1}) - 4 J_z y_i (z_{i+1} + z_{i-1}) \\ \dot{y}_i &= -4 J_\perp z_i (x_{i+1} + x_{i-1}) + 4 J_z x_i (z_{i+1} + z_{i-1}) \\ \dot{z}_i &= 4 J_\perp (y_i x_{i+1} - x_i y_{i+1} + y_i x_{i-1} - x_i y_{i-1}). \end{aligned} \quad (2)$$

Note that variables (x_i, y_i, z_i) reside on a sphere. Specifying $x_i^2 + y_i^2 + z_i^2 = 1$ in the beginning of the evolution will preserve this condition at all times. For clarity of notations we wrote equation (2) for a one dimensional system. Generalization to the case of several dimensions is straightforward.

We consider dynamics in a state with finite magnetization in the XY plane (superfluid phase for hard core bosons), where close to equilibrium dynamics is determined by the Bogoliubov (Goldstone) mode. Considerable difference between the nonlinear hydrodynamics of hard core bosons on a lattice and the more familiar GP model was observed in the numerical study of solitary waves in [42] (for a discussion of solitons in systems of ultracold atoms in the regime where GP equation applies see [43–54]). Additional evidence for the special character of solitons in systems of strongly correlated lattice bosons came from the analytical study in [39], which extended linear hydrodynamics to include the first nonlinear term and the first dispersion correction to the long wavelength expansion and showed that solitons were of the KdV type. In this paper we derive a full phase diagram of semiclassical dynamics of (1) for initial states with finite magnetization in the XY plane. Our approach relies on combining analytical results describing small density perturbations [39] with numerical studies of equation (2) describing dynamics of large amplitude modulations.

Phase diagram

Hyperbolic, elliptic, and parabolic regimes

The main results of our analysis are summarized in figure 1. Firstly there is fundamental difference between elliptic and hyperbolic regions in the character of linearized equations of motion. The hyperbolic regime (hyperbolic regions I and II in figure 1) corresponds to the easy plane anisotropy of the Hamiltonian. If we start with initial state that has finite magnetization in the XY plane, we find collective modes with linear dispersion, $\omega \propto |q|$, which describe Bogoliubov (Goldstone) modes. In the elliptic regime we have easy axis anisotropy: lowest energy configuration should have full polarization along the z -axis, $m = \pm 1$. However our initial state has magnetic order in the XY plane. Thus we find unstable collective modes with imaginary frequency $\text{Im} \omega \propto \pm |q|$. This corresponds to the dynamical instability in

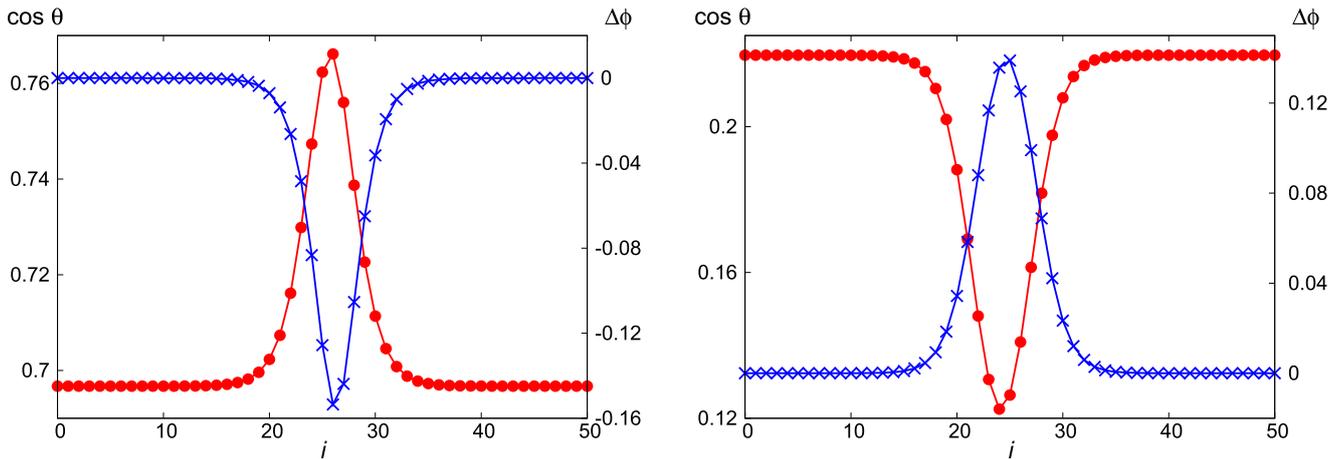


Figure 2. Left figure: particle soliton in region I_+ . Right figure: hole soliton in region II_+ . Red line shows magnetization (density) $\cos \theta_i$. Blue line shows superfluid velocity $\varphi_{i+1} - \varphi_i$. Results obtained from numerical solutions of lattice equation (2). Transverse modulation was not included.

which small density fluctuations grow in time exponentially at short time scales. There are also several special lines on the phase diagram 1: the $SO(3)$ symmetric Heisenberg model with $J_z = J_\perp$ and fully polarized regimes $m = \pm 1$ (fully occupied or empty regimes for spinless bosons). In all of these cases collective modes have quadratic dispersion, i.e. linearized equations of motion are of the parabolic type.

Hyperbolic regime

Solitons

Hyperbolic regions I and II are differentiated further according to the character of solitonic excitations. In region I_+ we find particle-like one dimensional solitons, which are unstable to modulation in the transverse directions. In region I_- we find hole-like one dimensional solitons unstable to transverse modulations. In regions II_\pm we find particle and hole-like solitons stable to transverse modulation. Lines IV_1 and IV_2 correspond to two different regimes of the mKdV equation. Both hole and particle-type soliton solutions can be found on the line IV_2 . Both types of solitons are suppressed close to the line IV_1 .

In the limit of small amplitude and long wavelength inhomogeneities non-linear hydrodynamic equations (2) can be mapped to KdV type equations and solitons can be studied analytically (see supplementary material and [39]). This analysis can be used to obtain explicit expressions for boundaries of different regimes in figure 1. In the general case of large amplitude inhomogeneities one needs to solve lattice equations (2) numerically. Surprisingly we find that even for large amplitude solitons, their properties, including stability to transverse fluctuations, are consistent with small amplitude solitons. Hence the phase diagram in figure 1 is generic.

While we can not present all of our numerical results, we provide examples, which demonstrate properties of large

amplitude solitons. Figure 2 shows particle- and hole-type solitons in regions I_+ and II_+ respectively. These specific solutions were chosen arbitrarily from a plethora of solitons with different amplitudes and velocities, which can be found in the system for the same values of microscopic parameters. In these specific examples only one dimensional variation of parameters were considered when solving lattice equation (2). The possibility of transverse modulation of one-dimensional solitons in two dimensional systems is considered in figures 3 and 4. These figures contrast stable and unstable regimes. In the stable regime II_+ transverse modulation does not lead to any dramatic change of the soliton solution. Analogous dynamics takes place in the other stable regime, II_- , except for a change from hole solitons to particle ones. In the unstable regimes I_+ (the same behavior is observed in I_-) we observe that transverse modulation leads to the formation of two-dimensional ‘lump’ solutions. For small amplitude modulations stability to transverse modulation can be studied analytically using Kadomtsev–Petviashvili equation [39, 55]. Our analysis of the stability to transverse modulation is not limited to solitons but applies to all inhomogeneous states.

Parabolic and elliptic regimes

A rapidly growing oscillation zone characterizes elliptic instability for system (2) in region III on the phase diagram. It is shown in figure 5. In supplementary material we show solution of (2) in the parabolic regime separating the elliptic and hyperbolic regions on the phase diagram 1.

Decay of magnetization step

We now discuss the problem of the decay of magnetization step in a system of type (1). Configurations of this type have been recently realized in 3d magnetic systems using tunable magnetic field gradient [56]. They can be created for strongly

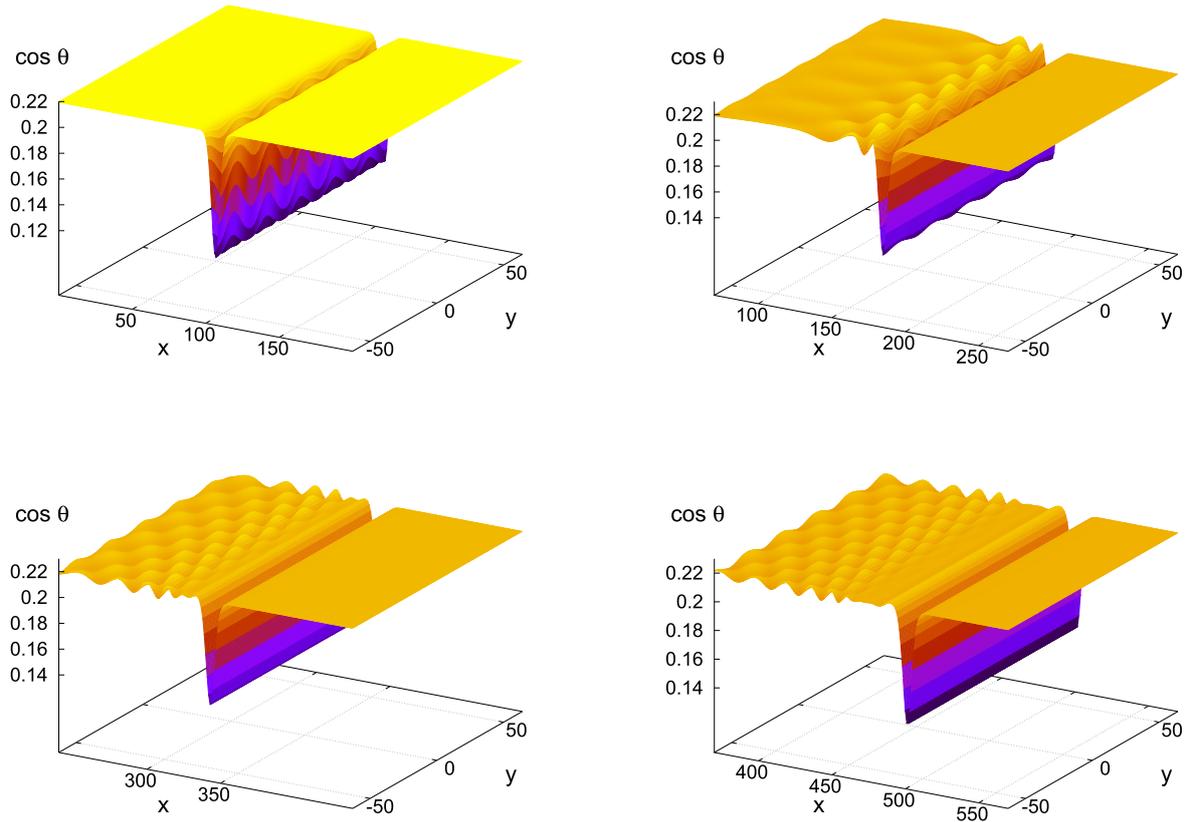


Figure 3. Time evolution of a soliton for $J_z/J_{\perp} = 0$ in region II₊ in a 2d system. Horizontal axis are spatial coordinates. Vertical axis is magnetization (density). Different plots correspond to $tJ_{\perp} = 0, 10, 30, 50$. This soliton is stable to small transverse modulation introduced explicitly in the initial state.

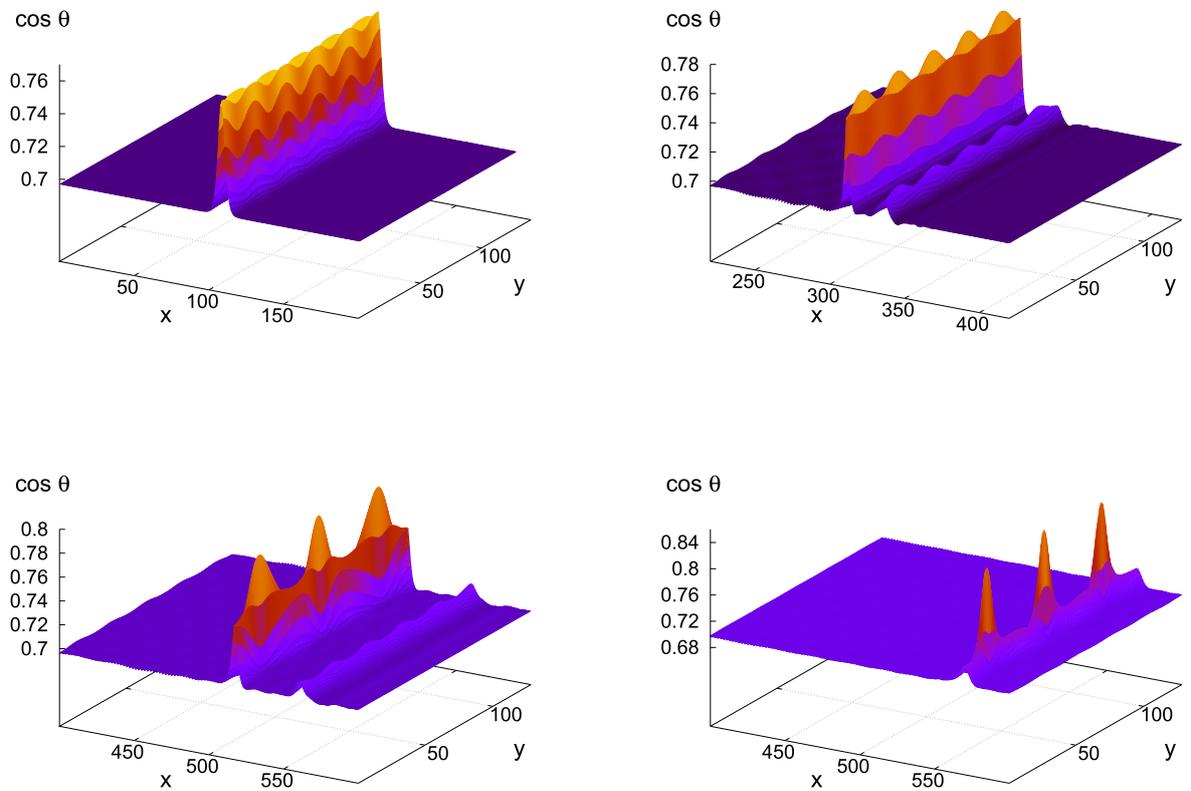


Figure 4. Time evolution of a soliton for $J_z/J_{\perp} = 0$ in region I₊ in a 2d system. Axis are the same as in figure 3. Different plots correspond to $tJ_{\perp} = 0, 40, 80, 90$. This soliton is *unstable* to small transverse modulation introduced explicitly in the initial state.

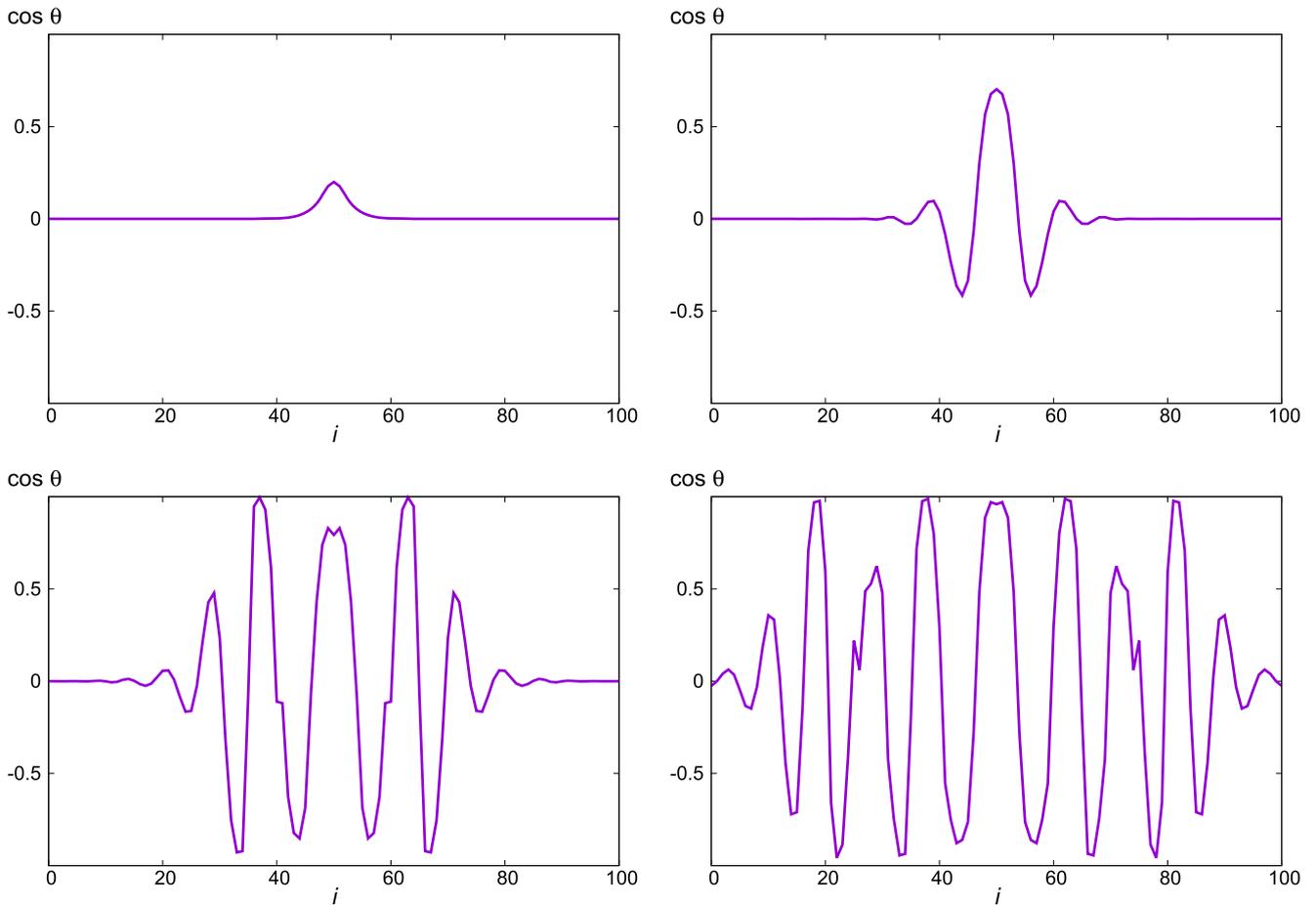


Figure 5. Time evolution of a small density perturbation in the elliptic region. Vertical axis is magnetization (density). Note the rapidly developing oscillation zone typical to modulation instability. Different plots correspond to $tJ_{\perp} = 0, 2, 4, 6$.

correlated spinless bosons in 2d optical lattices using local addressability [17, 18]. Besides this experimental motivation, relaxation of the magnetization (density) step is an important methodological problem. Dynamics starting from this state allows extended unattenuated propagation of the magnetization (density) modulation, which should amplify the role of nonlinearities. This may be contrasted to e.g. dynamics starting with a ring type inhomogeneity, where already at the level of linear hydrodynamics there is a decrease of the magnetization (density) modulation in time. In classical hydrodynamics decay of the density step is one of the canonical problems considered in [57–59].

Figure 6 shows the main stages of the magnetization step decay in regions I_{\pm} , II_{\pm} , IV: separation of left- and right-moving parts, steepening of one of the moving edges and formation of the oscillatory front. While there is general agreement between small amplitude limit analyzed in [39] and results of numerical analysis of lattice equations presented in figure 6, there is one important difference. In the oscillatory region appearing from the decay of a large amplitude magnetization step we observe pairing of solitons, which can be understood as a result of interaction between solitons through the background of small amplitude waves.

This is manifestation of the absence of exact integrability of lattice equation (2).

Experimental considerations

Direct experimental tests of theoretical results presented in this paper can be obtained by analyzing dynamics starting from soliton configurations of ultracold atoms in optical lattices, such as shown in figures 2–4. These figures show that lattices of the order of tens of sites should be sufficient for such studies. Experimentally soliton like initial states can be created and their dynamics can be studied using single site resolution and addressability available in current experiments with ultracold atoms in optical lattices. Density modulation can be achieved by letting the system reach equilibrium with a specified inhomogeneous potential. Phase modulation can be done by applying a short potential pulse. We note that clear signatures of soliton dynamics can be achieved by starting with initial states that have density modulation and no phase imprinting. In this case a pair of solitons appears and propagates in the opposite directions. This process takes place on relatively short time scales

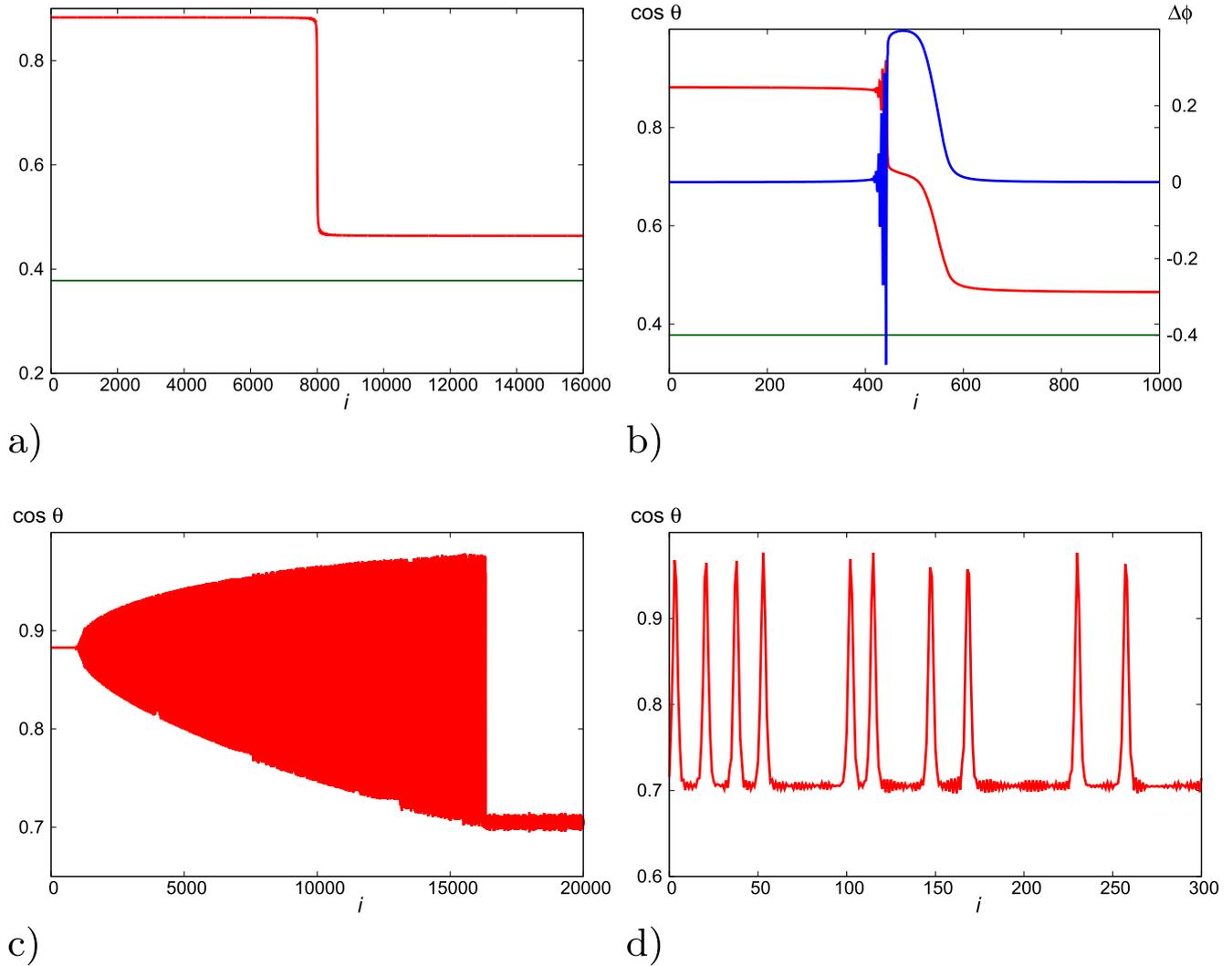


Figure 6. Decay of the density step in region I_+ . Red lines describe density, blue line corresponds to the phase difference. (a) Initial state $t = 0$. (b) After $tJ_{\perp} = 25$ of the evolution. Note the separation of the right and left moving parts and formation of the oscillatory region on the left moving part. (c) Expanded view of the left front after $tJ_{\perp} = 5000$ showing formation of the leading edge of the oscillation zone. (d) Left front after $tJ_{\perp} = 15000$. Note soliton pairing mediated by background waves with small amplitude.

accessible to experiments. Additional details about this procedure are discussed in the supplementary material. We also note that to observe soliton dynamics one does not need to create initial configuration, which match theoretically calculated solitonic solutions exactly. Numerical analysis of (2) shows that initial conditions with spatially localized excess or deficit of magnetization (density) separate reasonably fast into solitons and the wave background. On the other hand observing all stages of the magnetization (density) step decay requires longer times and larger system sizes. Another important experimental consideration, which we did not address so far, is the presence of the parabolic confining potential in all of the currently available experimental set-ups. In figure 7 we show that as long as the chemical potential change is not taking the system across one of the boundaries in figure 1, it has no strong effect on

the soliton propagation. However crossing any of the boundaries leads to the soliton break-up.

Summary

We analyzed semiclassical dynamics of anisotropic Heisenberg models in dimensions higher than one. Combining the analytical study of small amplitude, long wavelength excitations with numerical studies of large amplitude, short wavelength excitations we demonstrated the existence of a universal dynamical phase diagram, in which different regions can be distinguished based on the character of one-dimensional solitonic excitations, and stability of such solitons to transverse modulation. Universality of dynamics, which we find from direct solution of lattice equations, is very

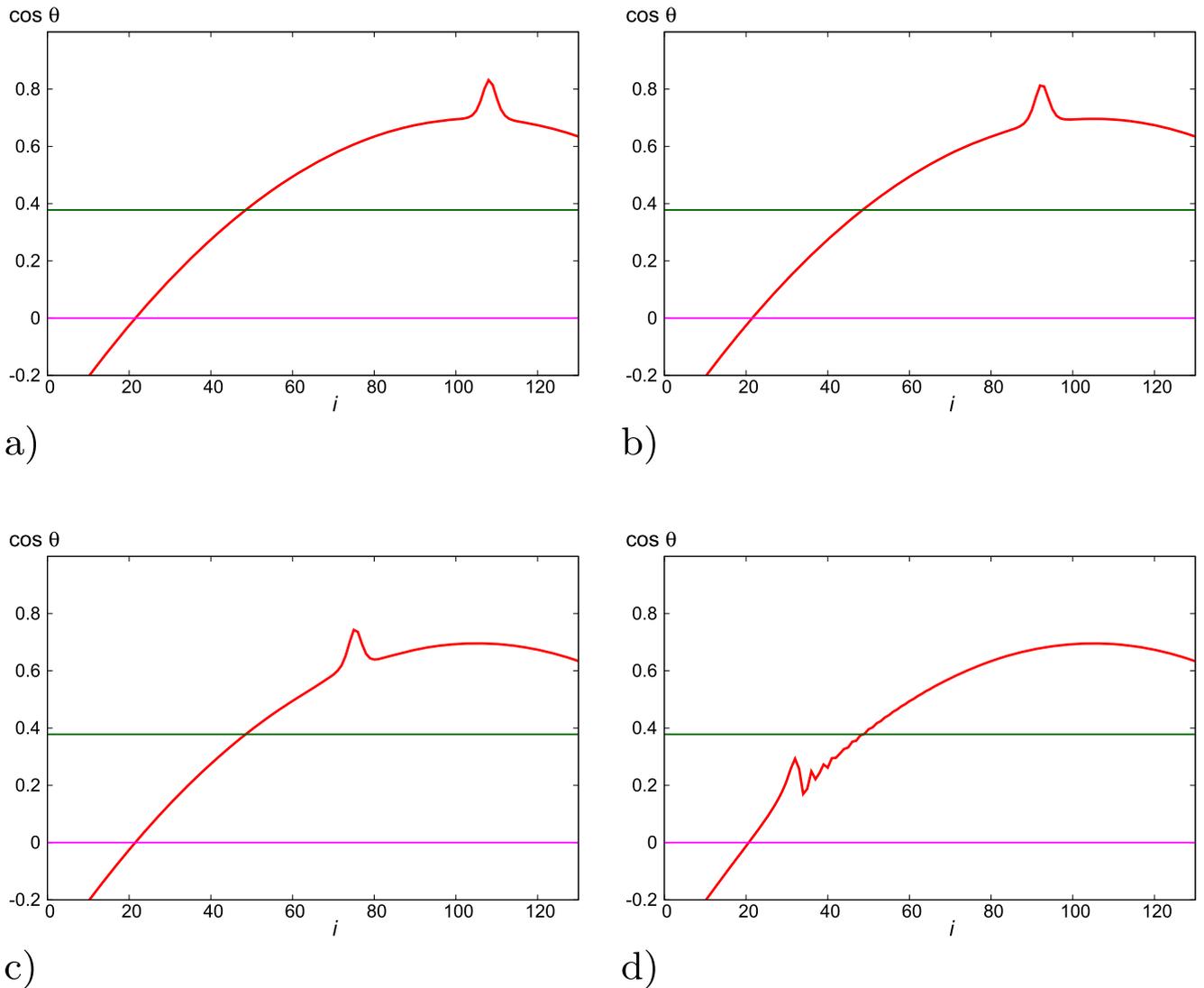


Figure 7. Soliton dynamics in the presence of parabolic potential. Soliton propagation is stable when parabolic potential is not taking the system across boundaries of different regions (figures (a)–(c)). Crossing the phase boundary causes the soliton break-up (figure (d)). Green line marks the boundary between regions I_+ and II_+ .

intriguing. Our model is not a special lattice regularization of an integrable continuum system. We analyze lattice model describing real physical systems which, in principle, can contain many terms breaking universality.

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