Interferometric Approach to Measuring Band Topology in 2D Optical Lattices

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Recently, optical lattices with nonzero Berry's phases of Bloch bands have been realized. New approaches for measuring Berry's phases and topological properties of bands with experimental tools appropriate for ultracold atoms need to be developed. In this Letter, we propose an interferometric method for measuring Berry's phases of two-dimensional Bloch bands. The key idea is to use a combination of Ramsey interference and Bloch oscillations to measure Zak phases, i.e., Berry's phases for closed trajectories corresponding to reciprocal lattice vectors. We demonstrate that this technique can be used to measure the Berry curvature of Bloch bands, the π Berry's phase of Dirac points, and the first Chern number of topological bands. We discuss several experimentally feasible realizations of this technique, which make it robust against low-frequency magnetic noise.

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Introduction.—Topology underlies many fundamental physical phenomena in two-dimensional (2D) materials, most notably the quantum and anomalous Hall effects [1–4]. The topological features of a 2D Bloch band are determined by the Berry's (geometrical) phases [5]—the phases picked up during an adiabatic motion of a particle along closed trajectories in quasimomentum space. The energy bands are classified by an integer-valued topological invariant $\mathcal{C}$, the first Chern number, which is proportional to the Berry's phase for a trajectory enclosing a full Brillouin zone (BZ). A filled band with the Chern number $\mathcal{C}$ is characterized by the quantized Hall conductivity $\mathcal{C}e^2/h$ [1,2].

Over the past few years, the interest in topological properties of 2D systems has been strongly revived, following the discovery of new materials, including graphene [6] and topological insulators [7], where the Berry's phases play an important role in defining the transport properties. For example, the band structure of graphene hosts two massless Dirac points at the corners of the BZ. A trajectory enclosing a Dirac point is characterized by the Berry's phase π, and it is this π Berry's phase that lies at the heart of new phenomena observed in graphene, such as the half-integer quantum Hall effect and weak antilocalization [6].

There is currently a strong interest in realizing topological band structures in cold atomic systems [8–13]. Very recently, highly tunable 2D optical lattices with nontrivial Berry's phases have been demonstrated experimentally [8–11]. In particular, a staggered flux lattice [8] as well as brick-wall and honeycomb optical lattices with massless Dirac points have been engineered [9,10]. Moreover, several intriguing theoretical proposals of how to realize optical lattices with nonzero Chern numbers were put forward [14–23].

However, the traditional transport measurements, which can be used to determine the Chern number of a band via the quantized Hall effect, are very challenging in cold atomic systems. Thus, novel methods of probing the topology of bands in optical lattices are needed. One possible route to studying the topological structure of optical lattices relies on the fact that the Berry curvature gives rise to an anomalous velocity in semiclassical dynamics that can be monitored through *in situ* images of the atom cloud [2,24]. Another possibility that has been pointed out recently is that time-of-flight images could be used to reveal topological invariants [25,26].

Here we propose an alternative, interferometric method for measuring Berry's phases, the Berry curvature, and the Chern number of bands in 2D optical lattices. The idea is to combine Bloch oscillations with Ramsey interferometry for particles with two internal states, $|\uparrow\rangle$, $|\downarrow\rangle$, loaded into the optical lattice. The BZ has the topology of a torus, and therefore during the Bloch oscillations particles follow closed trajectories corresponding to the cycles of the BZ torus. The Berry's phases of such trajectories are known as Zak phases [27].

We note that the two main ingredients required for the implementation of our proposal—Bloch oscillations and Ramsey interferometry—have become standard techniques in cold atoms experiments [28–33]; thus, we expect that our proposal can be used to measure topological properties of 2D lattices in the near future. Very recently we have successfully applied a related approach to measure the Zak phases of topological Bloch bands in one dimension [34].

We show how the measurements of the Zak phase allow one to obtain the more familiar topological characteristics of 2D Bloch bands. First, measuring the change of the Zak phase in the BZ, one can determine the distribution of the

Berry curvature. This allows one to measure the Chern number of the band, given by the winding number of the Zak phase across the BZ. Second, we show that the difference of Zak phases measured along certain trajectories can be linked to the Berry's phase of Dirac fermions. To be concrete, we consider the example of the brick-wall lattice [10]. However, all protocols that we propose can be extended to the cases of the staggered flux lattice [8] and the hexagonal lattice [9,11].

The scheme for measuring the Zak phase is as follows (see Fig. 1): Initially, a spin-up state with a given quasimomentum $\mathbf{k_0}$ is prepared. The proposed protocol consists of three steps. (1) A $\pi/2$ pulse is used to create a coherent superposition of two spin states, $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. (2) Next, opposite forces $\pm\mathbf{F}$ on the spin-up and spin-down states are applied, e.g., though a magnetic field gradient. It is required that the force $\mathbf{F}$ is parallel to some reciprocal lattice vector $\mathbf{G_1}$. (3) After half a period of the Bloch oscillations, when the two spins meet in quasimomentum space, another $\pi/2$ pulse is applied and the z component of the spin is measured. During such an evolution, the up and down states pick up a geometric contribution, equal to the Zak phase, which can be determined from the Ramsey phase.

Bloch oscillations and Zak phase.—We start our analysis by relating the Ramsey phase measured in the setup described above to the Zak phase. Consider a 2D lattice with the lattice vectors $\mathbf{a_1}$, $\mathbf{a_2}$, and the unit cell $\mathbf{r} \in x_1\mathbf{a_1} + x_2\mathbf{a_2}$, where $x_i \in [0, 1)$. We denote the two primitive reciprocal lattice vectors by $\mathbf{G_1}$, $\mathbf{G_2}$. According to the Bloch theorem, the eigenfunctions in the nth band can be represented in the following form

$$\psi_{\mathbf{k}n}(\mathbf{r}) = e^{i\mathbf{kr}} u_{\mathbf{k}n}(\mathbf{r}), \tag{1}$$

where $u_{\mathbf{k}n}$ is the cell-periodic Bloch function, satisfying $u_{\mathbf{k}n}(\mathbf{r} + \mathbf{a_i}) = u_{\mathbf{k}n}(\mathbf{r})$, $i = 1, 2$. Notice that the $u_{\mathbf{k}n}$ is not periodic in the momentum space, but obeys the following condition:

$$u_{\mathbf{k}+\mathbf{G_i}n}(\mathbf{r}) = e^{-i\mathbf{G_i r}} u_{\mathbf{k}n}(\mathbf{r}), \tag{2}$$

which originates from the periodicity of the full Bloch function $\psi_{\mathbf{k}n}$.

We will assume that during the Bloch oscillations the particle motion remains adiabatic, such that the probability for the particle to get excited to another band is negligible. Following the first $\pi/2$ pulse, the particles are in the state $\psi_{\mathbf{k_0}n}(\mathbf{r}) \otimes \frac{|\uparrow\rangle+|\downarrow\rangle}{\sqrt{2}}$. The evolution under the application of the force $\pm\mathbf{F}$ (opposite signs for opposite spins) is described by the time-dependent wave function $\Psi_\uparrow(\mathbf{r}, \mathbf{t}) \otimes \frac{|\uparrow\rangle}{\sqrt{2}} + \Psi_\downarrow(\mathbf{r}, \mathbf{t}) \otimes \frac{|\downarrow\rangle}{\sqrt{2}}$. The wave functions $\Psi_{\uparrow(\downarrow)}(\mathbf{r}, \mathbf{t})$ obey the Schrödinger equation

$$i\hbar \frac{\partial \Psi_\uparrow(\mathbf{r}, t)}{\partial t} = H_{\uparrow(\downarrow)} \Psi_{\uparrow(\downarrow)}(\mathbf{r}, \mathbf{t}), \tag{3}$$

with the Hamiltonian

$$H_{\uparrow(\downarrow)} = H_0 \mp \mathbf{Fr} \pm E_Z, \qquad H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}). \tag{4}$$

$V(\mathbf{r})$ is the lattice potential, and E_Z is the Zeeman energy.

As long as the adiabaticity condition is fulfilled, the particles remain within one band, and the solution of Eq. (3) has the following form

$$\Psi_{\uparrow(\downarrow)}(\mathbf{r}, \mathbf{t}) = e^{i\xi_{\uparrow(\downarrow)}(t)} \psi_{\mathbf{k}_\pm(\mathbf{t}),n}(\mathbf{r}), \tag{5}$$

where $\mathbf{k}_\pm(t) = \mathbf{k_0} \pm \mathbf{f}t$, $\mathbf{f} = \mathbf{F}/\hbar$, and the phase $\xi_{\uparrow(\downarrow)}(t)$ is given by

$$\xi_{\uparrow(\downarrow)}(t) = i\int_{\mathbf{k_0}}^{\mathbf{k}_\pm(t)} \langle u_{\mathbf{k}'n}|\nabla_{\mathbf{k}'} u_{\mathbf{k}'n}\rangle d\mathbf{k}' - \frac{1}{\hbar}\int_0^t \varepsilon_n(\mathbf{k}_\pm(t'))dt' \mp \frac{E_Z t}{\hbar}. \tag{6}$$

The first term in the above equation describes the geometrical phase, while the second and third correspond to the dynamical phase, which depends on the speed of motion through the band.

The Ramsey interferometry, performed after half a period of the Bloch oscillations (the period is given by $T = G/|\mathbf{f}|$), measures the phase difference picked up by the two spin species $\xi_\uparrow(T/2) - \xi_\downarrow(T/2)$. Using Eq. (6), we obtain the Ramsey phase

$$\varphi_{\text{tot}} = \varphi_{\text{Zak}} + \varphi_{\text{dyn}} + \varphi_{\text{Zeeman}}, \tag{7}$$

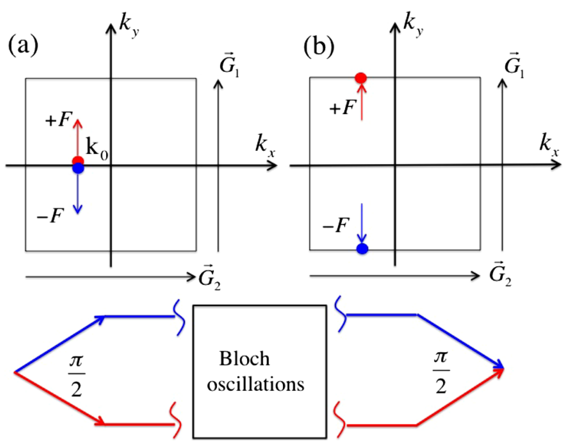

FIG. 1 (color online). Experimental setup for measuring the Zak phase. A cloud of ultracold atoms with a well-defined quasimomentum $\mathbf{k_0}$ is loaded into a 2D optical lattice. $\mathbf{G_1}$, $\mathbf{G_2}$ denote reciprocal lattice vectors. Initially, atoms have spin up. (a) A $\pi/2$ pulse creates a coherent superposition of $|\uparrow\rangle$ and $|\downarrow\rangle$ states [marked by blue (dark gray) and red (gray)]. After that, spin-selective forces $\pm\mathbf{F}$ parallel to $\mathbf{G_1}$ are applied. (b) After half a period of Bloch oscillations, when the two spins meet in the quasimomentum space, another $\pi/2$ pulse is applied. The accumulated phase difference between the two states, which contains the Zak phase contribution, is measured by reading out the phase of the resulting Ramsey fringe.

where the Zak phase is given by [27]

$$\varphi_{\text{Zak}} = i\int_{\mathbf{k_0}-\mathbf{G_1}/2}^{\mathbf{k_0}+\mathbf{G_1}/2}\langle u_{\mathbf{k}'n}|\nabla_{\mathbf{k}'}u_{\mathbf{k}'n}\rangle d\mathbf{k}', \tag{8}$$

and the dynamical phase and Zeeman phase are given by

$$\begin{aligned}\varphi_{\text{dyn}} &= -\frac{1}{\hbar}\int_{-T/2}^{T/2}\operatorname{sgn}(t')\varepsilon_n(\mathbf{k_0}+\mathbf{f}t')dt',\\ \varphi_{\text{Zeeman}} &= -\frac{E_Z T}{\hbar}.\end{aligned} \tag{9}$$

For the case of a band structure with the symmetric dispersion relation $\varepsilon_n(\mathbf{k_0}+\mathbf{f}t') = \varepsilon_n(\mathbf{k_0}-\mathbf{f}t')$, the dynamical phase vanishes [35], and the Ramsey interferometry directly gives the Zak phase. This is the case for special choices of $\mathbf{k_0}$ and $\mathbf{G_1}$ in the experimentally relevant case of the brick-wall lattice which we will discuss below.

Measuring the Berry curvature and Chern number of a generic band.—Let us now turn to the discussion of how Ramsey interferometry can be used to determine the Berry curvature and the Chern number (and therefore the topological class) of a gapped band; no special symmetries are assumed, except for the symmetry of dispersion which guarantees the cancellation of the dynamical phase, and allows the separation of the Zak phase.

We choose the primitive cell in quasimomentum space to be a torus defined by $\mathbf{k} = \mathbf{K_0} + \alpha_1\mathbf{G_1} + \alpha_2\mathbf{G_2}$, where $\alpha_i \in [0;1)$ and $\mathbf{K_0}$ is an arbitrary quasimomentum (as shown in Fig. 2). We notice that the Chern number cannot be determined by measuring the Zak phases along the four sides of the torus, essentially, because the Zak phase is only defined modulo 2π. However, the Chern number $\mathcal{C}$ can be related to the winding number of the Zak phase across the BZ. This fact is well known in the context of adiabatic pumping (see Ref. [2] for a review); below we provide a brief derivation for the reader's convenience.

We consider an experiment in which the Zak phase is measured for torus cycles defined by $\mathbf{G_1}$ as a function of α_2 (see Fig. 2). Experimentally, this would be achieved by

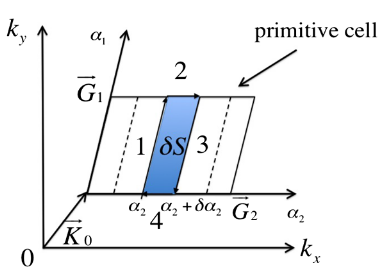

FIG. 2 (color online). Primitive momentum-space cell of a generic 2D lattice. The Chern number of the band is related to the winding number of the Zak phase across the primitive cell [see Eq. (11)]. The Zak phase can be measured using a combination of Bloch oscillations and Ramsey interferometry as described in the text.

preparing the initial state $\mathbf{k_0} = \mathbf{K_0} + \alpha_2\mathbf{G_2}$ for different values of α_2.

First, notice that the small change of the Zak phase as α_2 is increased by $\delta\alpha_2$ is equal to the integral of the Berry curvature over the region δS defined by the corresponding trajectories (see Fig. 2), or equivalently, equal to the Berry's phase for the contour 1234. It is easiest to see this by choosing a smooth gauge (a gauge without discontinuities) for the periodic Bloch function in δS (this can be done since the region δS is small; in general, no smooth gauge can be chosen in the whole BZ). The Berry's phase γ can be represented as the sum of the Berry's phases for the four sides of the rectangle $\gamma = \sum_{i=1}^4 \gamma_i$. Since the sides 2 and 4 are equivalent, but are traversed in the opposite direction, their contributions cancel out, $\gamma_2 + \gamma_4 = 0$, while $\gamma_3 + \gamma_1$ is equal to the difference of the Zak phases for trajectories 3 and 1. Thus, the change of the Zak phase is related to the Berry's phase, which can be written as an integral of the Berry curvature Ω_{12}, $\gamma = \int_{\delta S} d^2k\Omega_{12}(\mathbf{k})$:

$$\int_{\delta S} d^2k\Omega_{12}(\mathbf{k}) = -ie^{-i\varphi_{\text{Zak}}(\alpha_2)}\partial_{\alpha_2}e^{i\varphi_{\text{Zak}}(\alpha_2)}\delta\alpha_2. \tag{10}$$

Summing Eq. (10) over different regions, and using the definition of the Chern number $\mathcal{C} = \frac{1}{2\pi}\int_{\text{BZ}} d^2k\Omega_{12}(\mathbf{k})$, we then obtain

$$\mathcal{C} = -\frac{i}{2\pi}\int_0^1 d\alpha_2 e^{-i\varphi_{\text{Zak}}(\alpha_2)}\partial_{\alpha_2}e^{i\varphi_{\text{Zak}}(\alpha_2)}. \tag{11}$$

This relation implies that the interferometric measurements of the Zak phase across the BZ allow the extraction of the Chern number, and can be used to detect the topological nature of the bands and topological phase transitions.

Brick-wall lattice: measuring π Berry's phase and detecting topological phase transitions.—Our approach is also suitable for measuring the π Berry's phase of the massless Dirac fermions in nontrivial lattices [8–10]. For definiteness, we consider the brick-wall lattice [10], illustrated in Fig. 3.

The particles in the brick-wall lattice with nearest-neighbor hopping t are described by the Hamiltonian $H = -t\sum_{\langle ij\rangle}c_i^\dagger c_j$ (the analysis below can be straightforwardly generalized to the case of anisotropic hopping in the x and y directions). Assume that an A site coincides with the origin [this choice is important, because it fixes the periodicity condition of Eq. (2) (see Fig. 3)]. The cell-periodic wave function, given by a two-component vector $(v_{A\mathbf{k}}, v_{B\mathbf{k}})$ (corresponding to the amplitudes on the sublattices A, B), can be found from the Schrödinger equation with an effective Hamiltonian

$$H_{\mathbf{k}} = -\begin{pmatrix} 0 & t_{\mathbf{k}} \\ t_{\mathbf{k}}^* & 0\end{pmatrix}, \tag{12}$$

$$t_{\mathbf{k}} = t(2\cos k_x d + e^{-ik_y d}) = \varepsilon_{\mathbf{k}}e^{i\theta_{\mathbf{k}}}, \tag{13}$$

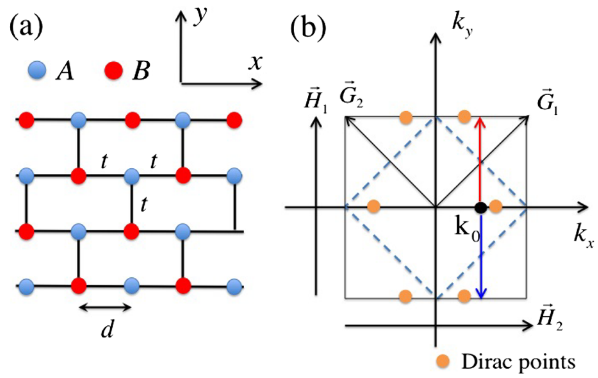

FIG. 3 (color online). (a) Brick-wall lattice. A and B sites are marked by blue (light gray) and red (gray) circles, and nearest-neighbor hopping is assumed. (b) The Brillouin zone of the brick-wall lattice model (dashed square). The band structure exhibits two Dirac points marked by orange (light gray) circles. Owing to the symmetry of the dispersion, it is convenient to measure the Zak phase with initial quasimomentum $\mathbf{k_0} = (k_0, 0)$ lying on the x axis, and applying a force in the y direction. Measuring the variation of the Zak phase as a function of k_0, it is possible to (i) measure the π Berry's phase of Dirac particles, and (ii) measure the Chern number of the bands when they are separated by energy gaps.

where we have chosen the vector connecting the A site to its neighbors to be equal to d, and the energy of the Bloch states is given by $\varepsilon = \pm\varepsilon_{\mathbf{k}}$,

$$\varepsilon_{\mathbf{k}} = t\sqrt{4\cos^2 k_x d + 4\cos k_x d \cos k_y d + 1}. \tag{14}$$

The eigenfunctions for the valence and conduction bands are given by $(v_{A\mathbf{k}}, v_{B\mathbf{k}}) = \frac{1}{\sqrt{2}}(1, \pm e^{-i\theta_{\mathbf{k}}})$. In principle, the wave functions are defined up to a gauge transformation; here the gauge is fixed such that the wave functions satisfy the periodicity condition of Eq. (2).

The BZ, illustrated in Fig. 3, is defined by the primitive reciprocal lattice vectors $\mathbf{G_1} = (\pi/d, \pi/d)$ and $\mathbf{G_2} = (-\pi/d, \pi/d)$. However, measuring the Zak phases for the cycles defined by $\mathbf{G_{1,2}}$ is complicated by the lack of corresponding symmetry of the energy dispersion; thus, it would not be feasible to cancel the dynamical phase.

To overcome this difficulty, we propose to measure the Zak phases for $\mathbf{k_0}$ lying on the x axis, with the force applied in the y direction. This allows for a measurement of the Zak phases corresponding to the reciprocal lattice vector $\mathbf{H_1} = \mathbf{G_1} + \mathbf{G_2}$ (see Fig. 3), while canceling the dynamical phase {as guaranteed by the $k_y \to -k_y$ symmetry of the dispersion [Eq. (14)]}. Similarly, it is possible to measure the Zak phases for trajectories corresponding to the reciprocal vector $\mathbf{H_2} = \mathbf{G_1} - \mathbf{G_2}$, when the initial momentum $\mathbf{k}_0$ lies on the y axis. In this case, the dynamical phase cancels out due to the $k_x \to -k_x$ symmetry of the dispersion.

Measuring the Zak phases across the BZ in this case allows direct detection of the π Berry's phase of the Dirac fermions. As the initial momentum $\mathbf{k_0}$ passes either of the two Dirac points, situated at $(\pm 2\pi/3d, 0)$, the Zak phase must jump by π.

Furthermore, the method described above can be used to determine the topological nature of gapped bands in the brick-wall lattice. The winding number of the Zak phase measured as a function of initial momentum $\mathbf{k_0}$ on the x axis gives twice the Chern number (repeating the argument above, one can show that Berry's phase winding is equal to the integral of the Berry curvature over a region $-\pi/d < k_x, k_y < \pi/d$, which contains two Brillouin zones). We note that both topologically trivial and nontrivial gaps can be introduced by adding extra terms in the Hamiltonian [Eq. (12)]. The topologically trivial gaps arise, e.g., when a staggered potential between A and B sites is turned on. In this case, the Chern number is zero. The Haldane model, in which next-nearest-neighbor hoppings with nontrivial phases $\pm\eta$ are turned on [36], provides a realization of a nontrivial band, with the Chern number ± 1. Importantly, both models still preserve the symmetry of dispersion with respect to $k_x \to -k_x$, $k_y \to -k_y$ (the Haldane model obeys this condition when $\eta = \pm\pi/2$). Thus, the problem of dynamical phases does not occur when the Zak phases (and Chern numbers) are measured.

Protocols insensitive to fluctuating magnetic fields.—Now we would like to address an important experimental concern related to the presence of slowly fluctuating magnetic fields. The fluctuations of the magnetic field induce shot-to-shot variations in E_Z and $\varphi_{\mathrm{Zeeman}}$, complicating the reliable extraction of the Zak phase. We propose various protocols which are insensitive to the magnetic noise. Here we briefly mention the main ideas, and provide a detailed description in the Supplemental Material [37].

The first protocol allows measurements of the variations of the Zak phase across the BZ (and in fact, only the difference of the Zak phase is physically meaningful, while the Zak phase itself depends on the choice of the real-space unit cell (see, e.g., Ref. [34]). The idea is to prepare the system in two or more different quasimomentum states, and to carry out the sequence described above for each initial state. Assuming that the dispersion is symmetric, the difference of the phases extracted from Ramsey fringes will be equal to the difference of the Zak phases. This protocol therefore allows one to obtain the Berry curvature and the Chern number of the band. It is insensitive to the shot-to-shot fluctuations of the magnetic field because the Zeeman phases picked up by different quasimomentum states are equal. A more detailed discussion of this protocol for the brick-wall lattice can be found in the Supplemental Material [37]. Another approach is to design protocols that combine Bloch oscillations with a spin echo sequence (see the Supplemental Material [37]). Such protocols are naturally insensitive to the fluctuating magnetic fields.

Summary.—In conclusion, we have presented an approach to studying topological properties of 2D optical lattices. This approach allows one to measure the Berry's phase of the Dirac fermions, as well as the local Berry curvature and the Chern number of the band. The latter measurement can be used to study the topological structure of the bands in optical lattices and to detect the topological phase transitions that occur as a function of the lattice parameters.

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[1] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Phys. Rev. Lett. **49**, 405 (1982).
[2] D. Xiao, M. C. Chang, and Q. Niu, Rev. Mod. Phys. **82**, 1959 (2010).
[3] M.-C. Chang and Q. Niu, Phys. Rev. Lett. **75**, 1348 (1995).
[4] F. D. M. Haldane, Phys. Rev. Lett. **93**, 206602 (2004).
[5] M. V. Berry, Proc. R. Soc. A **392**, 451 (1984).
[6] A. H. C. Neto, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, Rev. Mod. Phys. **81**, 109 (2009).
[7] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
[8] M. Aidelsburger, M. Atala, S. Nascimbène, S. Trotzky, Y.-A. Chen, and I. Bloch, Phys. Rev. Lett. **107**, 255301 (2011).
[9] P. Soltan-Panahi, J. Struck, P. Hauke, A. Bick, W. Plenkers, G. Meineke, C. Becker, P. Windpassinger, M. Lewenstein, and K. Sengstock, Nat. Phys. **7**, 434 (2011).
[10] L. Tarruell, D. Greif, T. Uehlinger, G. Jotzu, and T. Esslinger, Nature (London) **483**, 302 (2012).
[11] C. Becker, P. Soltan-Panahi, J. Kronjäger, S. Dörscher, K. Bongs, and K. Sengstock, New J. Phys. **12**, 065025 (2010).
[12] Y.-J. Lin, K. Jimenez-Garcia, and I. B. Spielman, Nature (London) **471**, 83 (2011).
[13] L. W. Cheuk, A. T. Sommer, Z. Hadzibabic, T. Yefsah, W. S. Bakr, and M. W. Zwierlein, Phys. Rev. Lett. **109**, 095302 (2012).
[14] D. Jaksch and P. Zoller, New J. Phys. **5**, 56 (2003).
[15] E. J. Mueller, Phys. Rev. A **70**, 041603 (2004).
[16] K. Osterloh, M. Baig, L. Santos, P. Zoller, and M. Lewenstein, Phys. Rev. Lett. **95**, 010403 (2005).
[17] L.-K. Lim, C. M. Smith, and A. Hemmerich, Phys. Rev. Lett. **100**, 130402 (2008).
[18] F. Gerbier and J. Dalibard, New J. Phys. **12**, 033007 (2010).
[19] L.-K. Lim, A. Lazarides, A. Hemmerich, and C. M. Smith, Phys. Rev. A **82**, 013616 (2010).
[20] N. R. Cooper, Phys. Rev. Lett. **106**, 175301 (2011).
[21] N. R. Cooper and J. Dalibard, Europhys. Lett. **95**, 66004 (2011).
[22] A. Kolovsky, Europhys. Lett. **93**, 20 003 (2011).
[23] T. Kitagawa, E. Berg, M. Rudner, and E. Demler, Phys. Rev. B **82**, 235114 (2010).
[24] H. M. Price and N. R. Cooper, Phys. Rev. A **85**, 033620 (2012).
[25] E. Alba, X. Fernandez-Gonzalvo, J. Mur-Petit, J. K. Pachos, and J. J. Garcia-Ripoll, Phys. Rev. Lett. **107**, 235301 (2011).
[26] E. Zhao, N. Bray-Ali, C. J. Williams, I. B. Spielman, and I. I. Satija, Phys. Rev. A **84**, 063629 (2011).
[27] J. Zak, Phys. Rev. Lett. **62**, 2747 (1989).
[28] M. B. Dahan, E. Peik, J. Reichel, Y. Castin, and C. Salomon, Phys. Rev. Lett. **76**, 4508 (1996).
[29] G. Roati, E. de Mirandes, F. Ferlaino, H. Ott, G. Modugno, and M. Inguscio, Phys. Rev. Lett. **92**, 230402 (2004).
[30] Y. Shin, M. Saba, T. Pasquini, W. Ketterle, D. Pritchard, and A. Leanhardt, Phys. Rev. Lett. **92**, 050405 (2004).
[31] T. Schumm, S. Hofferberth, L. M. Andersson, S. Wildermuth, S. Groth, I. Bar-Joseph, J. Schmiedmayer, and P. Krüger, Nat. Phys. **1**, 57 (2005).
[32] C. Gross, T. Zibold, E. Nicklas, J. Estève, and M. K. Oberthaler, Nature (London) **464**, 1165 (2010).
[33] A. Widera, S. Trotzky, P. Cheinet, S. Fölling, F. Gerbier, I. Bloch, V. Gritsev, M. Lukin, and E. Demler, Phys. Rev. Lett. **100**, 140401 (2008).
[34] M. Atala *et al.* (to be published).
[35] We note that a similar trick was previously employed in neutron interference experiments, see, e.g., Y. Hasegawa, R. Loidl, G. Badurek, M. Baron, N. Manini, F. Pistolesi, and H. Rauch, Phys. Rev. A **65**, 052111 (2002).
[36] F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988).
[37] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.110.165304 for the description of implementations that are robust against magnetic field fluctuations.