Gopalakrishnan, Martin, and Demler Reply: In Ref. [1], we proposed a scheme for realizing quantum quasicrystals using ultracold atoms. We remarked that these quantum quasicrystals have more gapless phasons than classical quasicrystals. References [2,3] contest this. Two questions are at issue: first, whether the additional modes are gapless, and second, whether "phason" is the right term. The first question is substantive; the second is semantic. On the narrow definitions of the Comments [involving "local isomorphism (LI) classes" [2] or "indistinguishability" [3]], the additional modes are not phasons. However, the term phason is often used broadly [4,5] to refer to density-wave phase modulations. In this broad sense, it describes our additional modes. Regardless, we agree our terminology was potentially confusing.

Substantively, the Comments dispute that the additional modes are gapless. Sandbrink *et al.* [2] assert that as the modes change the LI class they *must* cost free energy. This depends on the form of the free energy; we explicitly showed [1] that *our* free energy is in fact invariant under the additional modes [6]. Lifshitz [3] writes down a free energy that differs from ours [1] (note that as Ref. [1] works at zero temperature it uses the term "Hamiltonian" instead [7]): the former [Eq. (2) of [3]] contains sixth- and higher-order terms in the Bose fields and the latter [Eq. (1) of [1]] does not. Naturally, Ref. [3] finds fewer continuous symmetries than Ref. [1]. We had anticipated this [1] and *explicitly noted* that higher-order terms in the Bose fields [*not* included in Ref. [1]] would break some of our symmetries.

The disagreement is thus about when one can treat a system as possessing a continuous symmetry. Such symmetries are never exact in nature: stray symmetry-breaking perturbations generically gap out all putative Goldstone modes [8]. Nevertheless, one regards a system as having a continuous symmetry when such perturbations are smaller than the physically relevant scales. Reference [1] treated higher-order terms in the Bose fields as a symmetry-breaking perturbation instead of including them in the Landau expansion; we now justify this by estimating the magnitude of the most important such term, which is sixth order in the fields.

The order parameters in Ref. [1] are the microscopic Bose fields themselves. Thus, the $(\phi^\dagger\phi)^2$ coefficient can be related to the microscopic two-body interaction, the $(\phi^\dagger\phi)^3$ coefficient to the three-body interaction, etc. These can be expressed in terms of contact and dipolar scattering lengths a_s and a_d, respectively; for the relevant parameters $a_s \approx a_d \approx 5$ nm [1]. As this is smaller than the confinement scale ($\gtrsim 250$ nm [1]), confinement does not affect the scattering. Thus, the two- and three-body contact interaction energy densities are [9,10]

$$E_{\text{two body}} \approx \frac{\hbar^2\rho_{2D}}{2m}\frac{a_s}{d_z}; \qquad E_{\text{three body}} \approx \frac{\hbar^2\rho_{2D}^2 a_s^4}{2md_z^2}, \qquad (1)$$

where ρ_{2D} is the density, m the mass, and d_z the transverse confinement scale. Similar results hold for dipolar interactions if one replaces a_s with a_d [11,12]. Thus, $E_{\text{three body}}/E_{\text{two body}} \sim \max(\rho_{2D}a_s^3/d_z, \rho_{2D}a_d^3/d_z)$. Typically, $\rho_{2D} \sim (250\text{ nm})^{-3}$ and $d_z \gtrsim 250$ nm. Therefore, $E_{\text{three body}}/E_{\text{two body}} \sim 10^{-5}$. This is much smaller than the temperature ($T \sim 0.1E_{\text{two body}}$), or the energy of harmonic confinement, which breaks translational symmetry and gaps out all phonon and phason modes. Therefore, the additional modes have as legitimate a claim as phonons to be described as gapless.

Finally, we address the claim [2] that similar modes exist classically. As these modes are density rearrangements, they can obviously be *imposed* classically; however, there is no reason to expect such modes to be nearly gapless. They are nearly gapless in Ref. [1] because (a) odd powers of the order parameter are forbidden by U(1) symmetry [1,3], and (b) higher-order terms in the Landau expansion are suppressed. Condition (a) does not hold in conventional classical quasicrystals, where the order parameters are Fourier components of the density. There might be classical quasicrystals with other [13] order parameters (perhaps magnetic [14]) satisfying these conditions, but we know of no physically realizable instances.

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