Bilayer Quantum Hall Systems: Spin-Pseudospin Symmetry Breaking and Quantum Phase Transitions

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We discuss and review recent advances in our understaning of quantum Hall systems where additional quantum numbers associated with spin and/or layer (pseudospin) indices play crucial roles in creating exotic quantum phases. Among the novel quantum phases we discuss are the recently discovered canted antiferromagnetic phase, the spontaneous interlayer coherent phase, and various spin Bose glass phases. We describe the theoretical models used in studying these novel phases and the various experimental techniques being used to search for these phases. Both zero temperature quantum phase transitions and finite temperature phase transitions are discussed. Emphasis in this article is on the recent developments in novel quantum phases and quantum phase transitions in bilayer quantum Hall systems where nontrivial magnetic ground states associated with spontaneous spin symmetry breaking play central role.

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I. INTRODUCTION

In a seminal paper more than fifteen years ago Halperin pointed out [1] that if additional (i.e. in addition to the 2D orbital motion) degrees of freedom or quantum numbers (e.g. spin, layer, etc.) are important in quantized Hall systems, then intriguing novel phenomena associated with new quantum phases described by multicomponent (generalized Laughlin) wavefunctions, the so-called Halperin wavefunctions, become theoretically possible. The observation [2] and the theoretical understanding [3] of the $\nu = 1/2$ quantum Hall effect (where ν indicates the *total* Landau level filling throughout this paper, rather than the filling factor for each individual layer) was one of the spectacular early confirmations of Halperin's ideas. During the last decade there has been a great deal of theoretical and experimental work [4] on multicomponent quantum Hall systems with the primary emphasis on $\nu = 1/2$ and $\nu = 1$ bilayer structures (with the layer index serving as a pseudospin variable with $U(1)$ symmetry in the absence of interlayer tunneling) although there have also been some interesting developments in single layer composite spin states in higher Landau levels [4].

Since excellent and extensive reviews of bilayer (spin-polarized) $\nu = 1/2$ and $\nu = 1$ quantum Hall physics already exist [4] in the literature, we focus in this article on the recently discovered [5–15] rich and interesting physics associated with the bilayer $\nu = 2$ quantum Hall systems, where both spin and pseudospin dynamics compete in the quantum Hall phenomena leading to a number of novel magnetic quantum phases (and interesting possible $T = 0$ quantum phase transitions among them as physical parameters such as interlayer tunneling, Zeeman coupling, interlayer separation, etc. are tuned), some of which seem to have already been experimentally observed [16–20].

To introduce the concept of a spin symmetry breaking quantum phase transition in bilayer quantum Hall systems we start from the schematic non-interacting single particle energy level diagram shown in Fig. 1. Consider a bilayer system in an external magnetic field with a total filling factor $\nu = 2$ (i.e. the average filling in each layer is 1 in the balanced situation under consideration) assuming there is an interlayer tunneling induced symmetric-antisymmetric gap Δ_{SAS} between the orbital levels and a magnetic field induced Zeeman gap Δ_Z between the spin up and down levels. We restrict ourselves entirely to the lowest Landau orbital level for the time being, neglecting coupling to all higher orbital Landau levels assuming $\hbar\omega_c$ to be very large. Therefore tunneling- and spin-split energy levels are:

$$E_{s\alpha} = s\frac{\Delta_Z}{2} + \alpha\frac{\Delta_{SAS}}{2} \tag{1}$$

where we have ignored the ground state Landau level energy $\hbar\omega_c/2$. The spin (s) and the layer-pseudospin (α) quantum indices are each discreet and can take only two values $s = \pm 1$ (up/down) $\alpha = \mp 1$ (symmetric/antisymmetric). From now on we use notation 1 and 2 to denote symmetric (-1) and and antisymmetric (1) states. Equation (1) therefore desribes four possible single particle energy levels (as shown in Fig. 1) with the following energy values $E_{-1} = -\Delta_Z/2 - \Delta_{SAS}/2$; $E_{-2} = -\Delta_Z/2 + \Delta_{SAS}/2$; $E_{+1} = +\Delta_Z/2 - \Delta_{SAS}/2$; $E_{+2} = +\Delta_Z/2 + \Delta_{SAS}/2$ where we use the notation 1(2) and $-(+)$ to denote the symmetric (antisymmetric) tunneling-split and spin up (down) levels respectively. Each of these four single particle levels has a macroscopic Landau degeneracy associated with the ususal magnetic field induced Landau quantization which is not explcitly shown above. We emphasize that all four levels belong to the lowest orbital Landau level, and higher Landau levels are ignored in our consideration.

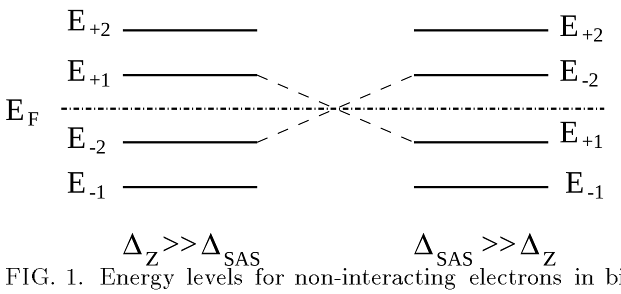

FIG. 1. Energy levels for non-interacting electrons in bilayer quantum Hall system in the lowest Landau level ($\nu = 2$) with E_F as a Fermi level

For total $\nu = 2$ only two of these energy levels are occupied (and the other two empty), as shown in Fig. 1 in the single particle picture. Depending on whether $\Delta_Z > \Delta_{SAS}$ (or $\Delta_{SAS} > \Delta_Z$) the weak (strong) tunneling situation, there is always a single particle excitation gap $\Delta_g = |\Delta_Z - \Delta_{SAS}|$ at the Fermi level. The $\nu = 2$ bilayer system is thus always in an incompressible quantized Hall state by virtue of the existence of the single particle excitation gap Δ_g. This is in sharp contrast to the corresponding extensively studied [4,21] zero tunneling $\nu = 1$ bilayer system in the fully spin polarized situation where the single particle gap is by definition zero ($\Delta_{SAS} = 0$ in the absence of tunneling), and any incompressibility must necessarily arise from interlayer many-body correlation in the pseudospin space. We thus already note two significant differences between $\nu = 2$ bilayer and (the better known and well-studied) $\nu = 1$ bilayer physics. In the $\nu = 2$ case, the system is always incompressible by virtue of the existence of the non-zero excitation gap $\Delta_g \neq 0$ whereas in the $\nu = 1$ zero-tunneling case there is never any single particle tunneling gap, but the $\nu = 1$ system may be driven incompressible (for suitable values of interlayer separation, etc.) by interlayer coherence effects as in Halperin $(1, 1, 1)$ state [1]- [4] or in related many-body incompressible states [21]; the other crucial difference between the two is that the interesting coherence and quantum phase transition physics in the $\nu = 1$ case [4,21] arise entirely from the pseudospin correlation effects in the absence of tunneling since the real electron spin is assumed to be completely frozen out by the large Zeeman splitting in the system (i.e. the electrons are are all completely spin-polarized) whereas the interesting quantum phase transition and symmetry breaking physics [5–8] in the $\nu = 2$ bilayer case arise from the interplay between the spin and pseudospin correlations (with both Δ_Z and Δ_{SAS} playing important roles, and the interesting regime is in fact $\Delta_{SAS}/\Delta_Z \sim 1$) and the actual symmetry breaking in the $\nu = 2$ bilayer case [5–15] is a peculiar breaking of the real spin symmetry (see Fig. 2) in the x-y plane of the layers (with the magnetic field orientation being along the z direction in the usual notation).

Of the four $E_{s\alpha}$ single particle states in the lowest Landau level (Fig. 1), arising from spin and pseudospin splitting (Zeeman and tunneling effects respectively), only two are filled with electrons for total $\nu = 2$. The level E_{-1} is always the lowest energy state and is therefore always filled, whereas the level E_{+2} is always the highest energy state and is therefore always empty. But the other two levels E_{-2} and E_{+1} could be filled or empty depending on the relative values of Δ_Z and Δ_{SAS}, and in fact in the single particle model there is a level crossing at $\Delta_Z = \Delta_{SAS}$ where these two levels (E_{-2} and E_{+1}) are degenerate. Within the single particle picture there will be a trivial first order phase transition in the system at the $\Delta_Z = \Delta_{SAS}$ level crossing point (as Δ_Z and/or Δ_{SAS} are being varied as tuning parameters) with the system making a transition from a fully spin-polarized ferromagnetic (F) for $\Delta_Z > \Delta_{SAS}$, with the occupation of both the spin-up E_{-1} and E_{-2} (symmetric and antisymmetric) levels (with E_+ levels being completely empty) in the high field (more precisely, the large Zeeman splitting) situation to a fully pseudospin polarized and spin paramagnetic singlet state (S) for $\Delta_{SAS} > \Delta_Z$, with the occupation of symmetric E_{-1} and E_{+1} spin up and down levels (with antisymmetric $E_{\pm 2}$ levels being completely empty). There is nothing interesting or noteworthy about this trivial $\Delta_Z = \Delta_{SAS}$ level crossing induced first order phase transition except that *it does not happen* – inclusion of many-body interaction effects, particularly interlayer correlations, pre-empts the trivial first order transition by introducing a novel spin symmetry breaking [5–8] which eliminates the $\Delta_Z = \Delta_{SAS}$ induced level crossing by producing a new symmetry-broken ground state which mixes the two relevant levels E_{+1} and E_{-2} generating a new ground state with the linear combination $\alpha|+1\rangle + \beta|-2\rangle$ (and $|\alpha|^2 + |\beta|^2 = 1$). The $\Delta_Z = \Delta_{SAS}$-induced level crossing of spin-up/symmetric $|+1\rangle$ and spin down/antisymmetric $|-2\rangle$ states and the associated first order transition, therefore, does not happen – instead a new purely interaction driven quantum phase, the so-called canted antiferromagnetic (C) phase [6], is stabilized around the $\Delta_Z = \Delta_{SAS}$ regime of the relevant parameter space in between the F phase ($\Delta_{SAS} >> \Delta_Z$) and the S phase $\Delta_{SAS} >> \Delta_Z$. This novel quantum phase diagram, where the phase transitions $F \leftrightarrow C \leftrightarrow S$ are all continuous, was first predicted on the basis of an unrestricted Hartree-Fock mean-field calculation in ref. [5], and was then further theoretically extended and confirmed in refs [6–15] using a variety of theoretical techniques including the spin-bond approach [8,10–12,14], more detailed Hartree-Fock calculations [7,9,10], an $O(3)$ quantum non-linear sigma model [6,7], a Chern-Simons field theory [13], and a small system direct numerical diagonalization calculation [15]. It is reasonable to state that the basic $\nu = 2$ bilayer quantum Hall phase diagram with three distinct spin quantum phases (F,C,S) as a function of interlayer tunneling, Zeeman splitting, and interlayer correlation is now well established. The spin orientations in the two layers in the C phase are depicted schematically in Fig. 2.

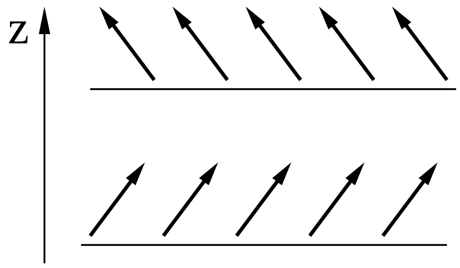

FIG. 2. Spin orientations in bilayer quantum Hall systems at $\nu = 2$ in the canted antiferromagnetic phase.

Our description of the C phase and the associated $\nu = 2$ bilayer quantum Hall phase diagram has so far utilized the single particle language emphasizing the competition between Δ_Z and Δ_{SAS}, and the interaction-induced avoided level crossing at the $\Delta_Z = \Delta_{SAS}$ degeneracy point leading to the spontaneous breaking of bilayer spin symmetry. This is however purely qualitative and is in fact quite simplistic since interaction strongly renormalizes the single particle energy levels in the system. One should interpret the energy levels of Fig. 1 as renormalized (for example, at the Hartree-Fock level) levels, and NOT purely single particle levels, i.e. the parameters Δ_Z and Δ_{SAS} should not be taken as the bare parameters, but as the renormalized effective spin and pseudospin splittings, respectively (remembering that the many-body renormalization by Coulomb interaction may be quite large in quantum Hall systems). One implication of this many-body renormalization is that the C phase may in fact extend all the way down to $\Delta_Z \propto \Delta_{SAS}^2$ instead of being around $\Delta_Z \propto \Delta_{SAS}$ as the simple level crossing qualitative argument above suggests. Another feature of the many-body nature of the phenomena manifests itself in the fact that the quantum phase transitions among the F,C, and S phases may be induced by tuning the interlayer separation d (instead of Δ_Z or Δ_{SAS} as discussed above) which automatically continuously renormalizes the energy levels producing the symmetry breaking. Thus, Δ_Z and Δ_{SAS} in the above discussion are not the bare single particle parameters, but are renormalized effective parameters of the system, which could, in principle, be very different from the bare parameters.

The spin symmetry breaking associated with the existence of the novel C phase (not present in the noninteracting theory) leads to interesting collective mode behavior [6–8] as one would expect for a continuous quantum phase transition. First, the lower $S = 1$ spin density excitation, the ω_- mode associated with the collective transition between E_{-2} and E_{+1}, becomes soft with its long wavelength spin gap vanishing at the boundaries between the C phase and the F or S phase. This theoretically predicted long wavelength softening of the appropriate spin density excitaion has been experimentally observed [16,17] via the inelastic light scattering spectroscopy, confirming the existence of the C phase. The second interesting collective mode behavior is the existence of a Goldstone mode in the C phase with a linear energy-wavevector dispersion in the long wavelength limit which arises from the spontaneous breaking of the symmetry of spin rotations in the XY plane in the CAF phase. This Goldstone mode, which exists only in the CAF phase, has not yet been experimentally observed perhaps because of the fact that the specific selection rules operating in the resonant Raman scattering experiments produces little spectral weight in the Goldstone mode making it unobservable in the usual inelastic light scattering spectroscopy [16,17]. The issue of the observation of the Goldstone mode, which is a characteristic of the CAF-phase, is an important open question in the subject. It has been argued theoretically [8] that disorder may modify the sharp Goldstone mode to a broad peak.

We close this introduction with a brief discussion of the nature of the canted antiferromagnetic phase and the associated symmetry breaking leading to it. In the absence of Zeeman splitting the spin symmetry of the problem is $SU(2)$ whereas the pseudospin/layer index has only a $U(1)$ symmetry in the absence of tunneling (the interlayer interaction is explicitly different from the interlayer interaction, the zero tunneling pseudospin symmetry becomes $SU(2)$ only in the limit of zero interlayer separation when the the intralayer and interlayer Coulomb interaction matrix elements are trivially equal). In the presence of tunneling, however, the $U(1)$ pseudospin symmetry is explicitly broken, and the only $SU(2)$ spin symmetry of the problem matters. The presence of the external magnetic field along the z direction in the quantum Hall problem now converts the whole problem to that of an XY quantum spin model. This XY spin symmetry is spontaneously broken in the C-phase as the electron spin chooses a particular orientation in the XY plane (opposite or antiferromagnetic in the two layers), making the total spin in each layer to be canted (in opposite directions) at some angle to the z axis (Fig. 2). The physical reason for this canting is simple: In the presence of finite tunneling the spontaneous symmetry breaking induced (antiferromagnetic) spin canting allows the $\nu = 2$ bilayer system to lower its total energy with respect to the normal F and S phases by exploiting the superexchange or virtual exchange mechanism – the effect is akin to (but more complex than) the conversion of a Hubbard type model to the t-J model with $J = 4t^2/U$, and the J term allows energy reduction via superexchange. The physical mechanism underlying the spontaneous symmetry breaking leading to the C phase is so transparent that one expects it to occur at the bilayer filling factors $\nu = 2/m$, where m is an odd integer, provided there is finite interlayer tunneling (and interlayer separation is neither too large nor too small). The reason for this expectation is that for m an odd integer, the $\nu = 2/m$ bilayer system supports two incompressible Laughlin states in each layer (when the layer separation is large) which should exploit the superexchange mechanism to lower its energy by creating the intermediate C-phase for intermediate values of Δ_{SAS}, Δ_Z and layer separation. More detailed considerations based on the Chern-Simons theory indicate that [13] C phases may exist even for general bilayer filling factors not satisfying $\nu = 2/m$ constraint.

The rest of this review is organized as follows: in section II we briefly describe the Hartree-Fock [5,6,10] and the spin bond mean field theories for the C-phase symmetry breaking; in section III we consider effects of disorder which, we argue theoretically [8], should give rise to Bose spin glass phase in the quantum phase diagram – there is a recent experimental claim of observing [19] the predicted [8] Bose glass phase in $\nu = 2$ bilayer transport experiments; in section IV we discuss microscopic wavefunctions for the CAF phase and the corresponding Chern-Simons field theory developed in ref [13]; in section V we conclude with a discussion of open issues and questions and related theoretical developments as well as a brief description of the experimental efforts in this problem [16–20].

II. MEAN FIELD HARTREE-FOCK AND SPIN BOND THEORIES

The Hartree-Fock calculations for the CAF phase in $\nu = 2$ bilayer quantum Hall systems follow the usual approach of the mean field theories. One starts with the Coulomb interaction Hamiltonian

$$\mathcal{H}_c = \frac{1}{2} \sum_{\sigma_1\sigma_2} \sum_{\mu_1\mu_2\mu_3\mu_4} \sum_{\alpha_1\alpha_2} \sum_{\mathbf{q}} V_{\mu_1\mu_2\mu_3\mu_4}(\mathbf{q}) e^{-q^2 l_0^2/2} \times e^{iq_x(\alpha_1-\alpha_2)l_0^2} C^\dagger_{\alpha_1+q_y\mu_1\sigma_1} C^\dagger_{\alpha_2\mu_2\sigma_2} C_{\alpha_2+q_y\mu_3\sigma_2} C_{\alpha_1\mu_4\sigma_1} \quad (2)$$

where $C^\dagger_{\alpha\mu\sigma}$ creates an electron in the lowest Landau level with the intra-Landau index α, pseudospin μ, and spin σ; and $V_{\mu_1\mu_2\mu_3\mu_4}$ are the intralayer and interlayer Coulomb interaction potentials (see [5–7,9] for details). Mean field approximation to a many-body problem is achieved by performing Hartree-Fock pairing through the expectation values $\langle C^\dagger_{\mu_1\sigma_1} C_{\mu_2\sigma_2}\rangle$, which should be found self-consistently. It is natural to assume expectation values of the fields that come into the non-interacting part of the Hamiltonian $\langle C^\dagger_{\mu\sigma} C_{\mu\sigma}\rangle$. Such terms generate effective tunneling and Zeeman splitting that strongly renormalize the bare parameters present in the non-interacting part of the Hamiltonian. They provide the Hund's rule tendency of quantum Hall systems: by generating large effective Zeeman or tunneling splitting all the electrons will be polarized in the spin or pseudospin sectors in phases F and S respectively. What is more surprising is that one also finds a non-trivial self-consistent solution for the fields $\langle C^\dagger_{1+} C_{2-}\rangle$ and its conjugate. This field breaks explicitly the S^z spin symmetry of the original Hamiltonian (2) and provides mixing between the states E_{-2} and E_{+1}. The existence of such non-trivial expectation values substantiates the statment of the existence of spontaneously broken spin symmetry in the CAF phase. By going back to the basis of layer indices for the single electron states (from the basis of symmetric/antisymmetric states) one can show that $\langle C^\dagger_{1+} C_{2-}\rangle$ describes the appearance of the spin expectation values that lie in the XY plane and are opposite in the two layers, i.e. a canted spin phase.

An apparent disadvantage of the Hartree-Fock approach is that it does not allow a consistent treatment of the quantum fluctuations of the Neel order parameter, so it overestimates the region of stability of the CAF phase and does not allow to address any of the interesting critical phenomena that take place in the vicinity of the transition lines. Non-linear sigma model introduced in [6,7] is a way to address both of these issues in a simple field theoretical approach. The two ingredients that come into this theory are: two well separated layers form fully polarized ferromagnets with a gap towards charge excitaions and the primary coupling between the layers is antiferromagnetic exchange. The disadvantage of the non-linear sigma model is that it treats the interlayer tunneling perturbatively and by concentrating on the spin it does not provide an equivalent treatment of the pseudospin degrees of freedom. So, for example, understanding the effects of an external gate voltage would be extremely difficult within this formalism. A spin bond approach suggested in [8] extends the non-linear sigma model by first treating non-perturbatively the inter-layer Coulomb interactions, interlayer tunneling, and the gate voltage to find the nature of the spin triplet and spin singlet states that provide the basis for the effective description of the system. In a nutshell the spin bond approach may be summarized as follows. Two electrons with the same intra-Landau level index but from the opposite layers may be combined into a spin singlet or spin triplet electron pairs. These combinations are then treated as hard core bosons, with the interaction between the two kinds of bosons coming from in-plane ferromagnetic exchange. When only the singlet or the triplet pairs are present in the ground state one finds the S and the F phases respectively, and when the two bosons are condensed simultaneously we find the CAF phase. There is a simple connection between the Hartree-Fock and the spin bond formalisms. In fact, if one considers a set of states that have the same expectation values as we discussed earlier, but allows these expectation values to be non-uniform, the energy functional for these states will coincide with the continuous limit of the spin-bond model. Reasonable quantitative agreement between the spin-bond model and the Hartree-Fock calculations has been demonstrated in [8,12], but computationally the first is significantly simpler.

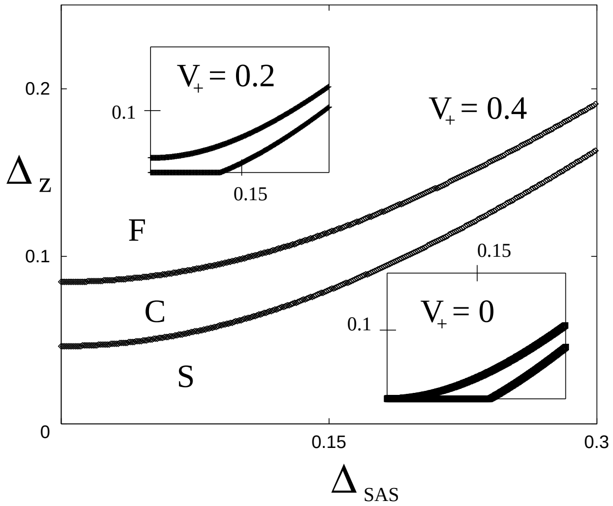

FIG. 3. Phase diagram for $\nu = 2$ bilayer quantum Hall system calculated using spin bond theory for different gate voltages. The length and the energy units are the magnetic length l_0 and the interlayer Coulomb energy $e^2/(\epsilon l_0)$. The interlayer separation is 1. Phase diagrams for different gate voltage V_+ are shown.

The phase diagram obtained from the spin-bond model for the case when the distance between the layers is equal to the magnetic length l_0 is shown for three values of the external gate voltage in Fig. 3. One can see that for large values of Δ_{SAS} and Δ_Z, the many body corrections to the effective tunneling and Zeeman energies are small and we find that the CAF phase is centered around $\Delta_{SAS} = \Delta_Z$ line as discussed earlier. However when Δ_{SAS} and Δ_Z are small, there are significant deviations from this line caused by Coulomb interactions. Spin-bond model or Hartree-Fock approaches allow one to calculate several experimentally testable properties of the CAF phase. Among them is the amplitude of the antiferromagnetic order parameter $|\mathbf{N}|$ that appears continuously on the phase boundaries between the C and S or F phases and reaches its maximum in the midddle of the C phase [10]. From $|\mathbf{N}|$ one can calculate the Goldstone mode velocity as well as find an estimate for the Kosterlitz-Thouless transition temperature. Another important probe of the CAF phase that has been discussed in refs [8,9] comes from applying a bias voltage (V_+) between the two layers. As shown in Fig. 3 the main effect of the charge imbalance between the layers is to shift the phase boundaries without altering their shape, so for example when V_+ is finite one can find C and S phase even when Δ_{SAS} is zero. In the context of bilayer $\nu = 1$ quantum Hall states a considerable emphasis has been given to the concept of the interlayer coherent states [23]. We emphasize that the concept of spontaneous interlayer coherence is only applicable when there is no tunneling between the layers. So, at $\nu = 2$ only when Δ_{SAS} is identically zero and there is finite bias voltage one can identify the CAF phase as interlayer coherent, qualitatively similar to the pseudospin coherent $(1, 1, 1)$ Halperin state for the $\nu = 1$ bilayer system [9,10].

III. DISORDER EFFECTS

Another obvious advantage of the spin bond model is that it gives a simple framework to understand the effects of disorder on the C phase [8]. Fluctuations in the distance between the wells or the presence of impurities give rise to random fluctuations in the energy of singlet and triplet bosons, and will stabilize the phase that may be visualized as consisting of domains of S, F, and CAF phases. By appealling to a similarity between this problem and a problem of charged bosons hopping in a random chemical potential this phase was identified as a spin Bose glass phase [8]. From the same analogy important conclusions may be drawn about the properties of this novel phase. Triplet and singlet bosons are localized, so there is no broken spin symmetry, however, there is infinite antiferromagnetic susceptibility, analogous to infinite superfluid susceptibilty of the charge Bose glass. Goldstone peaks, that were the dominant feature of the spin response function in the CAF phase, are replaced by finite longitudinal susceptibility at small frequencies. This spin Bose glass phase has a finite density of low energy excitations which provides another way to experimentally distinguish it. Finally the existence of the spin Bose glass phase separating the S, F, and CAF phases has important consequences in that it changes the critical exponents to those of the superconductor-insulator transition in the dirty boson system studied in [6,7]. It is worth pointing out that the spin Bose glass system may be a better experimental realization of a 2D superconductor-insulator transition in a boson system than the conventionally used 2D superconducting films in that it is free of long range forces and low energy fermionic excitations, and therefore allows one to vary the density of bosons by varying the magnetic field.

IV. HALPERIN WAVEFUNCTION AND THE CHERN-SIMONS FIELD THEORY

Another important perspective on the nature of the C phase comes from considering its microscopic (Halperin) wavefunction. Here again analogy with the spinless electron in a bilayer system at $\nu = 1$ (or $\nu = 1/m$ in general) is very useful. In the latter case Wen and Zee [22] discussed that the origin of the interlayer coherent wavefunction at this filling factor lies in the remarkable property of the Halperin (m, m, m) wavefunctions (neglecting electron spin) to fix the total filling factor but not the individual filling factor in each layer. This allows a construction of interlayer coherent states, i.e. the states that are a superposition of states with different numbers of particles in each layer but fixed total number of particles. Such states break spontaneously the $U(1)$ symmetry of the problem that in the absence of interlayer tunneling the number of electrons in each layer is a good quantum number, and that the system should have a specific number of electrons in each layer. For spinful electrons in bilayer quantum Hall systems the analogous Halperin-type construction leads naturally to a C phase [13]. In fact imagine making an analogue of the (m, m, m) wavefunction of the E_{-2} and E_{+1} states. In this case one is mixing states that have different S^z components of the spin and therefore creates a state that is not an irreducible represenatation of the S^z operator. This corresponds to spontaneously breaking the S^z symmetry of the system. From this reasoning one can also argue that the general fractional filling factor for which the CAF phase is possible is given by

$$\nu = \frac{n + m - 2l}{nm - l^2} \tag{3}$$

where n and m are odd integers and l is an arbitrary integer. This formula trivially includes the $\nu = 2/m$ originally suggested in [6,7]. The idea of the Halperin wavefunctions for the CAF phase can be extended to a full bosonic Chern-Simons theory that provides a unified picture of the gapless charge neutral Goldstone mode and charged excitations in the system [13]. The key ingredient of this approach is to represent the electrons as bosons with attached fluxe tubes and then describe the quantum Hall state as a state where the bosons have condensed. The main result that comes out of such calculation is that in the CAF phase the system has non-trivial topological vortex-like spin excitations, merons, that have a Neel order parameter winding by 2π on the periphery with S or F phases inside the meron core, and that these excitations carry an electric charge. The amount of charge carried by each meron depends on the exact position inside the quantum phase diagram, but the two merons with S and F cores always add up to a charge of one quasiparticle (i.e. 1 for the $\nu = 2$ case). This provides another demonstartion of the remarkable property of quantum Hall systems to mix charge and spin degrees of freedom [4]. Full experimental implications of the theory developed in [13] have not yet been worked out.

V. CONCLUSIONS

We have provided a brief qualitative review of recent developments in our understanding of the rich quantum phase diagram of bilayer quantum Hall systems at the total filling factor of $\nu = 2$ or more generally $\nu = 2/m$ where m is an odd integer. Similar considerations should also apply to bilayer systems with $\nu = 2m$, where m is odd, but Landau level coupling neglected here may be a significant issue for $\nu = 6$, 10. Interplay among interlayer tunneling, Zeeman spin splitting, intra- and inter-layer Coulomb interactions could produce novel spin symmetry breaking in the system, leading to a new class of magnetic ground states, the canted antiferromagnetic phase, nestled between the more usual spin-polarized ferromagnetic and symmetric (singlet) paramagnetic phases. The presence of disorder leads to interesting Bose glass regimes within the canted phase. The spontaneous spin XY symmetry breaking giving rise to the canted phase also produces softening of the appropriate collective spin density excitations in the F and S phases, which have presumably been experimentally observed [16,17] via the resonant Raman scattering spectroscopy. There have also been several reports of indirect observation of the canted and the Bose glass phase in transport experiments [18–20]. There are collective linearly dispersing Goldstone modes in the CAF phase which have not yet been experimentally observed – it has been argued [8] that the spectral weight carried by the Goldstone mode will be broadened into a weak broad peak in the Bose glass phase, which in fact is consistent with experimental light scattering measurements Further work along this line is necessary.

In addition to the $T = 0$ quantum phase transitions among the F, C, and S magnetic phases, there should be a finite temperature classical Kosterlitz-Thouless type phase transitions [5–7,14] within the CAF phase as the temperature is increased, and the usual vortex unbinding disordering transition of the $X - Y$ model takes place. The critical temperature for this Kosterlitz-Thouless type transition has been estimated [7] to be around 1K, and there is some experimental evidence in its support [16,17].

From a theoretical perspective the existence of the predicted canted phase and the associated continuous quantum phase transitions in bilayer quantum Hall systems is now well established. The original prediction [5] based on an unrestricted Hartree-Fock mean field theory has been well-confirmed and substantiated in subsequent theoretical analysis using O(3) nonlinear sigma model [6,7], spin bond theories [7,11,12], more detailed Hartree-Fock calculations [9,10], and most importantly, through the construction of explicit wavefunctions for the symmetry-broken phases associated with a Chern-Simons theory [13] and a direct diagonalization exact numerical calculations [15]. The direct diagonalization calculations [15] eliminates any lingering questions one may have about the existence of the C phase being an artifact of mean-field theories. The quantum phase diagram obtained in the exact diagonalization calculation in ref. [15] is essentially identical to that obtained [8] by the spin bond approach with the original Hartree-Fock theory [5–7,10] giving phase diagrams which are qualitatively identical except for the fact that the S phase is overemphasized in Hartree-Fock theories.

The experimental situation is somewhat less definitive although the inelastic light scattering experiments [16,17] convincingly demonstrate the mode softening and the quantum phase transition (and possibly even the classical Kosterlitz-Thouless transition) predicted [5–7] to occur in $\nu = 2$ bilayer quantum Hall systems. In addition, transport measurements of quantum Hall activation energies provide strong circumstantial evidence [18,19] for the existence of the canted phase. More detiled transport measurements on samples spanning wider regimes of parameter space (i.e. Δ_{SAS}, Δ_Z, d, etc.) would certainly be helpful in elucidating the complete quantum phase diagram. In principle, definitive experimental evidence confirming the canted phase could come from careful spin polarization and NMR measurements as suggested in ref. [10], but such magnetic moment measurements are extremely difficult in two dimensional semiconductor systems because of weak signal and background problems.

Finally we mention that there have been a number of related theoretical developments [23–27] in the field motivated by the prediction of the canted phase [5–7]. These theoretical developments explore several interesting features such as generalizing the C phase concept to multilayer superlattice [23] and to double quantum dot [27] structures as well as other topics in collective modes [25], the Kosterlitz-Thouless transition temperature [24], and the field theory [26] of the problem. A very recent preprint [28] considers the effects of spontaneous symmetry breaking in bilayer quantum Hall systems on the edge states.

We also point out that mean field calculations [29] suggest the possibility of an exchange-correlation-driven intersubband-spin-density-softening-induced ground state antiferromagnetic instability even for zero magnetic filelds in low density two-subband (bilayer or monolayer) two-dimensional electron systems, however, Raman scattering measurements indicate [30] no such zero field transitions for currently accessible 2D densities.

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