Fermions and Bosons in Superconducting Amorphous Wires

YUVAL OREG[1] and EUGENE DEMLER[2]
[1] *Department of condensed matter physics, Weizmann Institute of Science, Rehovot 76100*
[2] *Lyman Laboratory of Physics, Harvard University, Cambridge MA 02138 USA*

We discuss the destruction of superconductivity in quasi-one-dimensional systems due to the interplay between disorder and Coulomb repulsion. We argue that to understand the behavior of the system one has to study both fermionic and bosonic mechanisms of suppression of superconductivity. The former describes reduction in the mean field critical temperature T_c, while the latter refers to thermal and quantum fluctuations in the order parameter. A change in parameters such as wire width and disorder strength significantly affects both mechanisms.

1 Introduction

The combination of disorder and Coulomb repulsion suppresses superconductivity in low dimensional samples, and may destroy it completely. Advanced technics for fabrication of superconducting wires allow a detailed experimental study [1,2,3] of this destruction.

Does R_N, the normal-state-resistance of the wire at high temperature, determine the destruction of superconductivity on it own? Or do parameters such as the amount of disorder, the width of the wire, its length, the introduction of a magnetic field, and the environment have additional effects, beside their direct effect on R_N? Can they lead to a transition in the ground state properties of the system at zero temperature, and to a qualitative change in the whole structure of the wire resistance curve as a function of the temperature T?

We argue below, that to understand the answers to these questions we have to follow both the physics of the electrons above the mean field T_c, and of the Cooper pairs (bosons) below it.

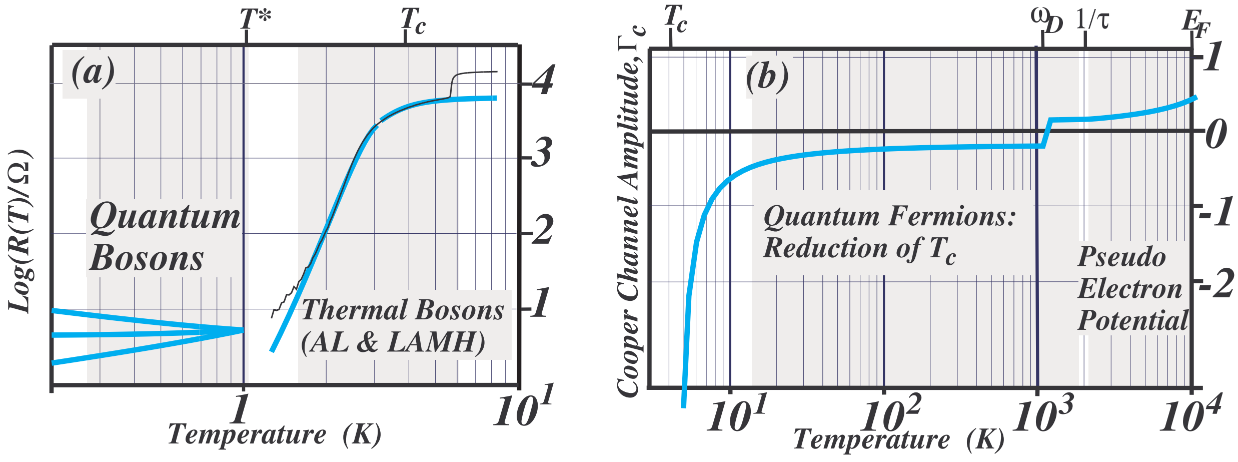

Figure 1: Theories of fermions and bosons, abbr. are explained in the text, notice the log scale of the temperature.

To follow the interplay between bosons and fermions we plot schematically the evolution of the resistance R (Fig. 1a), and of the vertex part of the Cooper channel amplitude, Γ_c (Fig. 1b), as T decreases. At T_c the superconducting instability occurs and Γ_c diverges. [4] It is useful to imagine a flow in the renormalization-group sense, and find the influence of disorder on it.

Let us first discuss Γ_c. Between the fermi energy E_F and the inverse of the mean free time between elastic collisions τ, Fermi-liquid parameters are established. In this temperature range the amplitude Γ_c describes the mutual Coulomb repulsion between electrons in the Cooper channel. In this region it is reduced by a logarithmic factor $\propto \log(E_F\tau)$, known as the Anderson-Morel-Tolmachev log, or *pseudo-electron potential* log. [5] Below $1/\tau$ the electrons' motion becomes diffusive, "dresses" the mutual Coulomb repulsion between them, and enhances it. Hence, the pseudo-electron logarithmic reduction is less effective.[6,7,8] At the Debye frequency[a] $\omega_D < 1/\tau$ we also have to include the attraction due to phonons. At yet lower T, Γ_c decreases until it diverges at T_c. Below ω_D the dynamically dressed Coulomb repulsion changes the whole evolution of Γ_c and reduces T_c. It appears that the dynamic change of the interaction in the Cooper channel arises from frequencies larger than T, we therefore use the phrase *quantum fermions* in Fig. 1b.[b]

Now we move on to discuss $R(T)$ (Fig 1a). Close to, but above T_c we can still describe the system by fermions and discuss Cooper pair fluctuations that enhance the conductance (the Aslamazov-Larkin (AL) correction). [9] Alternatively we can describe the system by fluctuations of a bosonic field (the complex-order-parameter) with zero average. [10] Below T_c the magnitude of the order parameter develops and it is more appropriate to describe the system in terms of an effective bosonic theory – the time dependant Ginzburg-Landau theory. Langer, Ambegaakor, McCumber and Halperin (LAMH) [11] discuss the way thermal activation of phase slips in the order parameter influence the resistance of the system below T_c.

At $T \ll T_c$ the amplitude of the order parameter is well developed and thermal activations of phase slips are very rare. One then has to consider, the quantized nature of the phase slips, at frequencies larger than T. [12] A variety of phase-only-models were suggested to describe the behavior of the system at low temperatures. We emphasize that the parameters in these models depend on fermionic physics which in turn depends on external parameters.

The resistance increases with the number of phase slips. Coupling to a dissipative environment increases the action of a single phase slip and can reduce their number, see for example Ref.[13,14]. The three curves in Fig. 1a at low T indicate the possible ways the resistance curve can behave: for strong coupling to a dissipative environment the number of phase slips (and hence resistivity) decreases when T decreases, and vice versa for weak coupling to the environment. The middle curve schematically describes a possible intermediate case.

For the most part the literature discusses the bosonic or the fermionic mechanisms separately. Larkin [15] addresses the necessity to discuss both of them together in the case of two dimensional systems. Here we suggest a scheme that combines fermionic and bosonic physics in quasi-1D.

2 Comparison with experiment

The reduction of T_c due to the enhanced Coulomb repulsion in dirty wires may be written, in terms of parameters that are experimentally accessible, as [6,7,8]

$$T_c = T_c^0 \exp\left[-b(4R_q)/R_\xi + O\left((R_q/R_\xi)^2\right)\right], \tag{1}$$

where T_c^0 is the temperature where Γ_c diverges in the absence of disorder, $R_q = 2\pi\hbar/(2e)^2 \approx 6.453k\Omega$ is the quantum resistance, $R_\xi = \rho\xi/A$ is the resistance for a wire of coherence length ξ,

[a] The case $\omega_D > 1/\tau$ can be discussed as well but to make the discussion simpler we assume $1/\tau > \omega_D$.

[b] Below $1/\tau$ there are also corrections to the system properties due to weak localization and interaction effects. For weak disorder, however, they are small compared to the correction to Γ_c and therefore we neglect them.

and A is the wire cross section. In the 1D limit where $\xi \gg \sqrt{A}$ the number $b = \sqrt{A}\Sigma_2/(\xi\pi)$ is $O(1)$ and depends weakly on T_c^0 and on A. In the intermediate region, where $\xi \sim \sqrt{A}$, b is very sensitive to microscopic parameters and boundary conditions. [8] Notice that the factor $4R_q$ in Eq. (1) suggests that we are dealing with the effects of fermions. The sum (over certain diffusion propagators) Σ_2, is defined precisely in Ref. [7]. To get a good estimate for T_c-reduction, we need to find b accurately since T_c depends on it exponentially, and to include effects of higher orders of R_q/R_ξ. This was performed for a few examples, [7,8] by a solution of an integral equation, .

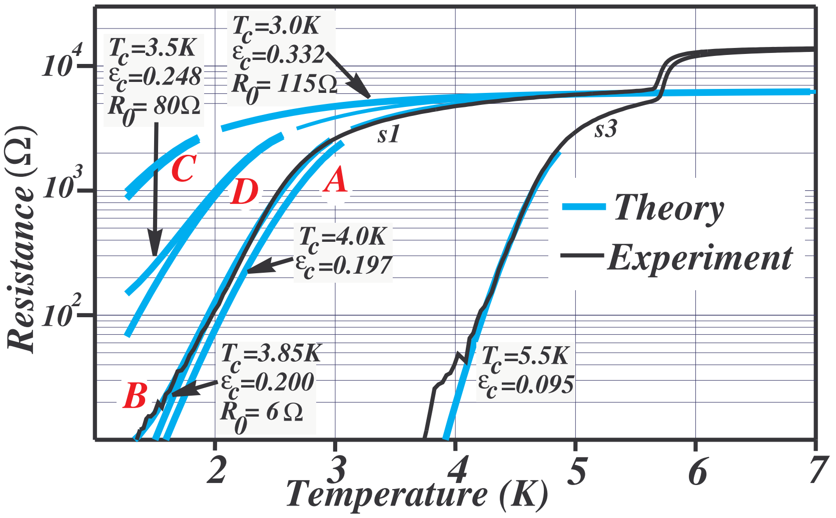

Figure 2: Comparison between theory and experiments. [16]

In Fig. 2 we plot experimental results taken from Ref. [1] in comparison with our theory. We use the expressions in Ref. [10] to plot the theoretical resistance curves (gray) based on LAMH-theory [11] ($T < T_c$), with the AL correction ($T > T_c$). LAMH-theory gives:

$$R(T) = R_N\, 2^{1/4}\sqrt{\pi}\alpha_0\, |\epsilon/\epsilon_c|^{9/4} \exp\left(-(4\sqrt{2}/3)\,|\epsilon/\epsilon_c|^{3/2}\right), \tag{2}$$

with $\alpha_0 \propto T_c\tau$, $\epsilon_c^{3/2} \propto 1/A, \epsilon \propto T - T_c$. To fit the experimental curves we use the two fitting parameters that enter LAMH-theory, namely the critical temperature T_c and the transition width ϵ_c. The agreement between theory and experiment appears to be good, and our fitting procedure is sensitive to T_c and ϵ_c. Indeed, when we plot a curve where ϵ_c and T_c deviate by less than 10% from the best fitting parameters (A in Fig. 2) it is far from the experimental curve $s1$.

At low T the experimental results deviate from the curve calculated from the thermal-bosonic-LAMH-theory combined with the quantum fermions T_c-reduction (B in Fig. 2). For $s1$ the deviation occurs below $R \approx 10\Omega$. This could be related to quantum phase slips. [2] We note that even for a perfect superconducting wire one should include the contact resistance in "two-probe measurements". [12] A rough estimate of the contact resistance for sample $s1$ in Ref. [1] shows that $R_{\rm contact} \approx R_q/(A/\lambda_F^2) \approx 11\Omega$ (we take $A \approx 50nm^2$ and $\lambda_F \approx 0.3nm$). In Fig. 2 the tail of the theoretical curve that fits $s1$ splits into two (near the point mark by B). One curve corresponds to LAMH theory and in the second we add a resistance R_0 in series. We choose $R_0 = 6\Omega$ to best fit the experiment. Since the phase slip core action is exponentially small in A and $R_{\rm contact} \propto 1/A$ we expect the former to be more relevant to R_0 for relatively narrow wires.

In addition, we show a curve that corresponds to a narrow long wire, so that R_N is similar to those in $s1$ and $s3$ (curves C and D) . Since the cross section A is smaller both the T_c-reduction is stronger and the contact resistance is higher. In particular note that, as Ref. [2] suggests, an addition of R_0 in series to LAMH expressions does not predict a negative curvature for $\log R(T)$ for all values of T. Due to the T_c-reduction $R(T)$ at $T \sim 1.5K$ is very sensitive to wire width.

Fig. 1 indicates that fermions determine T_c, and thermal activation of bosonic phase slips determines the resistance through the LAMH theory. The quantum bosons physics should be relevant at low T. Thus, in contrast to the previous theoretical[13] calculations and experimental[1] results, we do not expect R_N to be the only parameter that determines the quantum transition.

While preparing this manuscript we have learned of a wider set of experiments on superconducting wires of different lengths and cross sections.[2] They show, indeed, as was also observed in Ref.[3], that long wires become superconducting even when their resistance is much higher than the quantum resistance.[2] A bosonic theory[13] explains some features of this behavior. We believe, however, that to fully understand the nature of the phase transition in superconducting wires one should consider the combined effects of fermionic and bosonic mechanisms.

3 Conclusions.

We studied the interplay between fermions and bosons in superconducting wires. We argued that the disorder enhanced repulsion between the electrons – the quantum physics of fermions – reduces T_c in dirty wires, with larger reduction for narrower wires. This quantum physics of fermions controls the parameters of the effective bosonic physics (both classical and quantum). A modification in wire width affect the low temperature behavior directly through the core energy of quantum phase slips and the contact resistance, and indirectly through the extra reduction of T_c via the fermions mechanism. We showed that when analyzing the available experimental data it is important to include the reduction of T_c together with quantum phase slips. The contact resistance in two-probe-measurements may not be neglected in some cases.

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