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Dicke phase transition without total spin conservation

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We develop a fermionic path-integral formalism to analyze the phase diagram of open nonequilibrium systems. The formalism is applied to analyze an ensemble of two-level atoms interacting with a single-mode optical cavity, described by the Dicke model. While this model is often used as the paradigmatic example of a phase transition in driven-dissipative systems, earlier theoretical studies were limited to the special case when the total spin of the atomic ensemble is conserved. This assumption is not justified in most experimental realizations. Our approach allows us to analyze the problem in a more general case, including the experimentally relevant case of dissipative processes that act on each atom individually and do not conserve the total spin. We obtain a general expression for the position of the transition, which contains as special cases the two previously known regimes: (i) nonequilibrium systems with losses and conserved spin and (ii) closed systems in thermal equilibrium and with the Gibbs-ensemble averaging over the values of the total spin. We perform a detailed study of different types of baths and point out the possibility of a surprising nonmonotonic dependence of the transition on the baths' parameters.

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I. INTRODUCTION

Understanding phase transitions in open quantum systems is a challenging problem at the interface of quantum optics, condensed matter, and atomic physics. In contrast to equilibrium phase transitions, which have been well understood using powerful theoretical tools such as renormalization-group approaches and conformal field theories, we still lack reliable theoretical tools for analyzing nonequilibrium open systems. This makes it particularly important to analyze systems with known experimental realizations that allow direct comparison between theoretical predictions and experimental measurements. Two important examples of such systems are the directed percolation and the driven dissipative Dicke model, which have been respectively realized in liquid crystals [1,2] and quantum optics [3–9]. In the case of the Dicke model, theoretical approaches that have been developed so far rely on the existence of an integral of motion, the total spin, which significantly reduces the complexity of the problem [10–16]. In contrast, actual experiments involve dissipative processes that do not respect this conservation law, such as dissipative baths coupled to each individual atom. Their description requires more advanced theoretical tools.

The effects of single-atom baths on the Dicke model were considered in Refs. [17,18], using approximate methods based on effective *bosonic* field theories. These approaches map the two-level systems to continuous variables and are valid only if all the atoms are strongly polarized in a given direction [18]. In this paper we instead employ an exact mapping to a *fermionic* path-integral representation, which allows us to obtain an exact expression for the location of the Dicke transition. In the limit of a large number of atoms we recast our result in terms of single-atom correlation functions, which can be computed using standard master equations. The present approach reproduces the known position of the equilibrium phase transition and additionally allows us to systematically describe single-atom dephasing and decay (see Fig. 1). As we will show, these processes renormalize the position of the Dicke transition and in some cases completely destroy it.

II. MODEL

The Dicke model describes the interaction of N two-level atoms (or spins) $\sigma_j^z = \pm 1/2$ with a single bosonic degree of freedom a,

$$H = \omega_0 a^\dagger a + \omega_z \sum_{j=1,N} \sigma_j^z + \frac{2g}{\sqrt{N}} \sum_{j=1}^{N} \sigma_j^x (a + a^\dagger). \qquad (1)$$

Here ω_0 and ω_z are, respectively, the detuning of the cavity and of the atoms, g is the photon-atom coupling, $[a,a^\dagger] = 1$, and $[\sigma_j^x,\sigma_j^y] = i\sigma_j^z$. For simplicity we assume that all the atoms are identical, although the present approach can be immediately generalized to the inhomogeneous case.

The Hamiltonian (1) commutes with the total spin operator $S = (S^x)^2 + (S^y)^2 + (S^z)^2$, where $S^\alpha = \sum_{j=1}^N \sigma_j^\alpha$. Due to this symmetry it is possible to decouple the 2^N spin states into block-diagonal Dicke manifolds with a well-defined total spin $S \leqslant N/2$. This analysis reveals that the equilibrium Dicke model presents a continuous phase transition between a normal and a superradiant phase, both at zero and finite temperatures [10–16]. The Dicke transition signals the spontaneous symmetry breaking of a discrete Z_2 symmetry ($\sigma^x \to -\sigma^x$ and $a \to -a$) and belongs to the mean-field Ising universality class [17,19,20].

Following the theoretical proposal of Refs. [21–23], the Dicke transition was recently realized in driven dissipative quantum optical systems [3–9]. The theoretical description of this transition [24–28] considered the effect of the cavity decay κ, modeled as a Markovian bath coupled to the cavity field a. This dissipative channel conserves the total spin and can be described through a semiclassical Holstein-Primakoff [15,29] approximation in which the total-spin operators are substituted by the

Dissipative bath	Rate	Lindblad
Cavity decay	κ	a
Single-atom(i) Dephasing	γ_ϕ	σ_i^z
(ii) Thermal bath	γ_T	σ_i^- and σ_i^+
(iii) Generalized Markovian bath	γ_λ	$\sigma_i^- + \lambda\sigma_i^+$

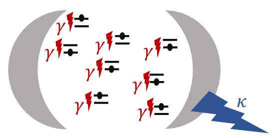

FIG. 1. Dissipative processes considered in the present study. Previous studies mostly focused on the cavity decay, which conserves the total spin.

bosonic operators b and $b^\dagger$, according to $S^z \to -N/2 + b^\dagger b$ and $S^x \to \sqrt{N}(b + b^\dagger)$. This analysis leads to the critical coupling

$$g_c = \frac{1}{2}\sqrt{\omega_z \frac{\omega_0^2 + \kappa^2}{\omega_0}}. \tag{2}$$

For $\kappa \to 0$, Eq. (2) recovers the known equilibrium result. This semiclassical approach relies on the conservation of the total spin and cannot be generalized to the case of single-atom dissipative processes.

III. MAJORANA FERMIONS

To describe the atomic dephasing and decay we employ a fermionic path-integral approach that allows us to expand the Dicke model in a $1/N$ series and resum all the leading terms.[1] We specifically consider the Majorana fermion representation of spin-1/2 systems[2] [31–33] $\sigma_j^z = f_j^\dagger f_j - 1/2$ and $\sigma_j^- = \eta_j f_j$. Here f_j are Dirac fermions whose occupied (unoccupied) states correspond to spin-up (spin-down) states of the jth atom and η_j are Majorana fermions satisfying $\eta_j^\dagger = \eta_j$ and $\eta_j^2 = 1$. The role of the latter operators is essentially to map the commutation relations of the spins to the anticommutation relations of the fermions. In terms of Majorana fermions, the Dicke model (1) becomes

$$H = \omega_0 a^\dagger a - \omega_z \sum_{j=1}^N f_j^\dagger f_j + \frac{g}{\sqrt{N}} \sum_{j=1}^N \eta_j (f_j - f_j^\dagger)(a^\dagger + a). \tag{3}$$

Following the usual path-integral prescription, we first introduce the bare Green's functions describing the cavity and the fermions and then derive Feynman rules for their coupling.[3] In this study we focus on the *long-time* steady state in which all decay processes have time to stabilize and the Green's functions depend on the time difference only. The bare (retarded) Green's function of the cavity is then given in Ref. [17] and is equal to a 2×2 diagonal matrix $G_a^{-1,R}(\omega) = (\omega + i\kappa)\tau_z - \omega_0 1_z$. Here τ_z is a Pauli matrix whose entries correspond to particles $a^\dagger$ and holes a and 1_z is the unit matrix. The bare Green's functions of the atoms describe their dynamics in the absence of the photon-atom coupling. We assume that each atom is coupled to an independent dissipative channel, leading to Green's functions that do not couple different atoms and will be denoted by $G_{f_j}(\omega)$ and $G_{\eta_j}(\omega)$. Note that our analysis is only valid as long as the atoms can be effectively described as (spin-1/2) two-level systems. Thus, for example, we cannot account for processes that bring the atoms to hyperfine states not included in the double-lambda scheme of Ref. [22] or to farther momentum states in the realization of Ref. [23].

We next introduce the Rabi coupling as a vertex connecting the cavity field a, a fermionic field f_j, and a Majorana field η_j, with coefficient $g/\sqrt{N}$. This coupling generates a self-energy for the cavity field of the form $\Sigma_a^R(\omega)(1_z + \tau_z)$. The Dicke transition corresponds to a diverging response function at $\omega = 0$ or, equivalently, to a zero-frequency pole and is set by

$$\det\left[G_a^{-1,R}(0) + \Sigma_a^R(0)\right] = 0. \tag{4}$$

Substituting the expression for $G_a^{-1,R}$, we obtain

$$\det\left[\begin{pmatrix} i\kappa - \omega_0 + \Sigma_a^R(0) & \Sigma_a^R(0) \\ \Sigma_a^R(0) & -i\kappa - \omega_0 + \Sigma_a^R(0) \end{pmatrix}\right] = 0,$$

leading to the critical condition

$$\omega_0^2 + \kappa^2 + 2\omega_0 \Sigma_a^R(0) = 0. \tag{5}$$

In general, the self-energy Σ_a^R depends on the photon-atom coupling g and Eq. (5) sets its critical value g_c.

IV. THE 1/N EXPANSION

To compute the self-energy $\Sigma_a^R(\omega)$ we need to consider all possible diagrams that start and end with a cavity field (see Fig. 2 for details). A one-loop diagram is plotted in Fig. 2(c)

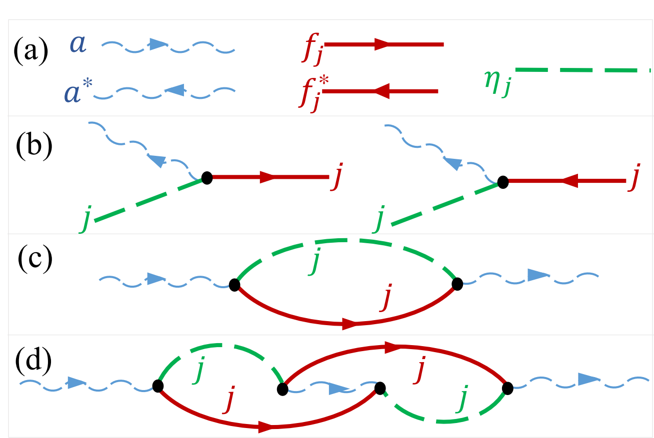

FIG. 2. (a) Bare Green's functions. (b) Bare vertices, proportional to $g/\sqrt{N}$. (c) One-loop and (d) two-loop self-energies for the cavity field Σ_a^R. The former contribution does not scale with N, while the latter scales as $1/N$ and can be neglected in the limit of $N \to \infty$.

[1] Path integrals offer a simple method to organize time-dependent perturbation theory. The same results can be alternatively obtained using, for example, the Nakajima-Zwanzig approach (see Ref. [30] for an introduction).

[2] Not to be confused with the Majorana representation of spins.

[3] See Ref. [34] for an introduction to Keldysh path integrals in the context of quantum optics.

and is equal to[4]

$$\Sigma_a^R(\omega) = \frac{g^2}{N}\sum_{j=1}^{N}\int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}\big[G_{f_j}^K(\Omega)G_{\eta_j}^R(\omega-\Omega) + G_{f_j}^K(-\Omega)G_{\eta_j}^R(\omega-\Omega)\big]. \tag{6}$$

Here the second term is generated by a diagram analogous to Fig. 2(c), but with an inverse direction of the fermionic arrow. Note that the resulting integral does not depend on N: Each vertex introduces a $1/\sqrt{N}$ factor, balanced by the sum over all atoms. Figure 2(d) shows an irreducible two-loop integral that contributes to the self-energy of the cavity field. This diagram contains four vertices and a single sum over j and is therefore suppressed as $1/N$. (See also Refs. [35,36] for a similar result in the case of atoms with motional degrees of freedom.) In the limit of $N \to \infty$ only a series of one-loop irreducible diagrams does not vanish. This series is exactly resummed by the above-mentioned self-energy approach.

The self-energy (6) has a simple interpretation in terms of spin-spin correlation functions. To see this mapping it is convenient to transform the integral expression appearing in Eq. (6) to the time domain

$$\Sigma_a^R(\omega) = \frac{ig^2}{N}\sum_{j=1}^{N}\int_0^{\infty} dt\langle[f_j(t)\eta_j(t), f_j^\dagger(0)\eta_j(0)]\rangle e^{i\omega t} + \langle[f_j^\dagger(t)\eta_j(t), f_j(0)\eta_j(0)]\rangle e^{i\omega t} \tag{7}$$

$$= \frac{4ig^2}{N}\sum_{j=1}^{N}\int_0^{\infty} dt\big\langle\big[\sigma_j^x(t),\sigma_j^x(0)\big]\big\rangle e^{i\omega t} \tag{8}$$

$$= -\frac{8g^2}{N}\sum_{j=1}^{N}\int_0^{\infty} dt\,\mathrm{Im}\big[\big\langle\sigma_j^x(t)\sigma_j^x(0)\big\rangle\big]e^{i\omega t}. \tag{9}$$

Here the average $\langle\cdots\rangle$ refers to the bare theory in which the atoms are decoupled from the cavity, in analogy to the Lamb theory of the lasing transition [37–40]: Eq. (9) involves a sum over j, indicating that in the limit of $N \to \infty$, the cavity experiences each atom independently.

Equations (5) and (9) express the position of the Dicke transition in terms of the correlation functions of individual dissipative spins. These correlations can be computed using either the Majorana fermion representation [31–33] or more conventional methods of quantum optics such as master equations in the Lindblad form. For the sake of brevity, we employ here the latter method and leave the corresponding calculations using Majorana fermions for a future longer study. The introduction of Majorana fermions in the present work is nevertheless necessary to develop the $1/N$ expansion leading to Eq. (9).

We specifically consider three distinct types of single-atom baths, listed in Fig. 1 along with their corresponding Lindblad operators.

(i) Dephasing. Dephasing processes preserve the spin polarization of the atoms and can be mathematically described by the Lindblad operators σ_j^z. In the presence of this type of dissipation, the spin-spin correlation functions can be computed using the appropriate Lindblad master equation: For any $t > 0$ one finds (see the Appendix)

$$\big\langle\sigma_j^x(t)\sigma_j^x(0)\big\rangle = e^{-\gamma_\phi t}[\cos(\omega_z t) + i\langle\sigma_j^z\rangle\sin(\omega_z t)]. \tag{10}$$

Combining this expression with Eqs. (5) and (9) we find

$$\Sigma_a^R(0) = 4g^2\frac{\langle\sigma_j^z\rangle\omega_z}{\omega_z^2+\gamma_\phi^2}, \quad g_c = \frac{1}{2}\sqrt{\frac{\omega_z^2+\gamma_\phi^2}{-2\langle\sigma_j^z\rangle\omega_z}\frac{\omega_0^2+\kappa^2}{\omega_0}}. \tag{11}$$

In the limit of $N \to \infty$, this specific type of bath preserves σ_j^z, and the expectation value $\langle\sigma_j^z\rangle$ is determined by the initial condition of the atoms. In contrast, for finite values of N it is necessary to consider $1/N$ corrections, which can modify the steady-state value of $\langle\sigma_j^z\rangle$. As shown in Ref. [41], these terms generically lead to a steady state with $\langle\sigma_j^z\rangle = 0$, where the Dicke transition does not occur ($g_c \to \infty$). On the other hand, repumping schemes leading to a steady state with $\langle\sigma_j^z\rangle \neq 0$ can guarantee the observation of the Dicke transition [41]. Note that the spins do not need to form a coherent or entangled state to support this transition [42,43].

(ii) Thermal bath. Let us now consider a decay channel induced by a thermal bath at temperature T, with decay rate γ_T. This situation is equivalent to having two Lindblad baths respectively coupled to σ_j^- and σ_j^+ with rates $(1+n_T)\gamma_T$ and $n_T\gamma_T$, where n_T is the Bose-Einstein distribution (see the Appendix). Equation (11) is modified according to $\langle\sigma^z\rangle \to 0.5\tanh(\omega_z/2T)$ and $\gamma_\phi \to \gamma_T/\tanh(\omega_z/2T)$. The critical coupling is then given by

$$g_c = \frac{1}{2}\sqrt{\frac{\omega_z^2\tanh^2(\omega_z/2T)+\gamma_T^2}{\omega_z\tanh^3(\omega_z/2T)}\frac{\omega_0^2+\kappa^2}{\omega_0}}. \tag{12}$$

In the limit of $\gamma_T \to 0$ and $\kappa \to 0$, Eq. (12) reproduces the critical temperature of the equilibrium closed system [10,11,13,14], given by $\tanh(\omega_z/2T) = \omega_z\omega_0/4g_c^2$.

In general, Eq. (12) is a monotonic increasing function of the temperature indicating that, as expected, the superradiant transition is suppressed by the temperature of the spins. Interestingly, Eq. (12) shows that the critical temperature is affected by the decay rates κ and γ_T. This result is in striking contrast to the common classical equilibrium case, where the strength of the coupling to a dissipative bath is not expected to affect the critical temperature [44].

(iii) Generalized Markovian bath. We finally consider a Markovian bath that couples coherently to both σ_j^- and σ_j^+ and is described by the Lindblad operator $L_j = \sigma_j^- + \lambda\sigma_j^+$, where λ is a fixed unitless parameter. This situation might be relevant to some implementations of Dicke-type models using the four-level scheme of Ref. [22] (see Ref. [45] for details). A straightforward calculation (see the Appendix) shows that the critical coupling is given by Eq. (11) with $\langle\sigma_j^z\rangle = 0.5(1-\lambda^2)/(1+\lambda^2)$ and $\gamma_\phi \to \gamma_{\text{eff}} = \gamma_\lambda(1-\lambda)^2$,

[4]For simplicity here we assume that $G_f^K(\omega)$ is diagonal in Nambu space and that $G_\eta^K(\omega) = 0$. The final expression (9) does not rely on these assumptions.

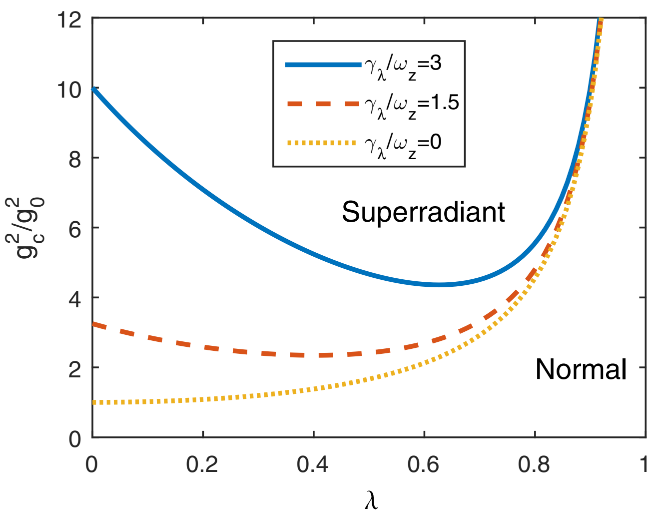

FIG. 3. Critical coupling of the Dicke model in the presence of a Markovian single-atom decay channel described by the Lindblad operator $L = \sigma^- + \lambda\sigma^+$. Here g_0 is the critical coupling for a fully polarized system (2). The critical coupling diverges in the limit of $\lambda \to 1$, where $\langle\sigma_j^z\rangle = 0$.

leading to

$$g_c = \frac{1}{2}\sqrt{\frac{(1+\lambda^2)\left[\omega_z^2 + \gamma_\lambda^2(1-\lambda)^2\right]}{(1-\lambda^2)\omega_z}\,\frac{\omega_0^2+\kappa^2}{\omega_0}}. \qquad (13)$$

In the limit $\lambda \to 0$ we recover the semiclassical result of Ref. [18]: In this case the steady states coincides with the fully polarized state $\prod_j |\downarrow_z\rangle_j$ and the Holstein-Primakoff approximation becomes exact. In the opposite limit $\lambda \to 1$, the Dicke transition does not occur because the steady state is characterized by $\langle\sigma_j^z\rangle = 0$, in contrast to the result of the nonlinear sigma model of Ref. [17].

For intermediate $0 < \lambda < 1$, the interplay between $\gamma_{\rm eff}$ and $\langle\sigma_j^z\rangle$ leads to the nontrivial behavior depicted in Fig. 3. Note in particular that $\gamma_{\rm eff}$ is a decreasing function of λ and tends to 0 at $\lambda = 1$, in analogy to the spontaneous-emission-induced coherence of Ref. [46]. Indeed, in this limit the Lindblad operator is σ_j^x and does not directly affect the correlator $\langle\sigma_j^x(t)\sigma_j^x(0)\rangle$. As a consequence, for small $\lambda \ll 1$, $g_c(\lambda)$ has a negative slope due to the linear decrease of $\gamma_{\rm eff}$ as a function of λ. In contrast, for $\lambda \lesssim 1$, $g_c(\lambda)$ has a positive slope due to the decrease of $\langle\sigma_j^z\rangle \sim (1-\lambda^2)$. The resulting nonmonotonic behavior differs from the previously studied collective decay channels, where the critical coupling depends on one effective decay rate only.

V. CONCLUSION

In summary, we studied the effects of atomic decay channels on the Dicke transition. Employing a fermionic path-integral analysis, we derived a closed expression for the critical coupling in terms of single-atom correlations (5) and (9). We considered several types of dissipative channels and computed the correspondent value of the critical photon-atom coupling g_c. We found that in general the critical coupling does not depend on the total spin of the system S, but rather on the average spin polarization $\langle\sigma_j^z\rangle$ [see Eqs. (5) and (11)]. If the dissipative channel leads to a depolarized steady state with $\langle\sigma_j^z\rangle = 0$ the Dicke transition disappears.

In the present discussion we considered nonequilibrium steady states, in which all correlation functions depend on the time difference only. The present analysis can nevertheless be directly extended to the study of the real-time dynamics, by considering the *retarded* Green's function $\theta(t-t')\langle[\sigma_j^x(t),\sigma_j^x(t')]\rangle$. In analogy to the steady-state situation, it is sufficient to first solve for the dynamics of each atom independently and then use this result to compute the response of the cavity. This approach might be useful to describe the transient Dicke transition observed in the experiments [7–9,47].

It is also possible to extend the present analysis to realistic experimental situations including, for example, the coexistence of thermal and Markovian baths, nonsymmetric Dicke models where the rotating and counterrotating terms of the Dicke Hamiltonian are different, and multimode cavities where glassy transitions are expected [48–51]. Furthermore, it would be interesting to study the critical exponents of the nonequilibrium transition and compare them with the equilibrium case, following the lines of Ref. [17]. Finally, it is desirable to extend the present analysis to include the decay of individual atoms to additional states that were neglected in their present two-level description.

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APPENDIX: METHODS

Master equations for a single spin coupled to a dissipative bath

In this appendix we use the Lindblad master equation [30] to compute the correlations of a single spin in the presence of dissipation. We then apply Eq. (9) and compute the cavity self-energy.

We consider an isolated spin described by the Hamiltonian $H = \omega_z\sigma^z$ and the Lindblad operator L. To compute $S_x(t) = \langle\sigma^x(t)\sigma^x(x)\rangle$ we first derive the time evolution of the operator $\sigma^x(t)$ from the master equation

$$\frac{d\sigma^x}{dt} = i[H,\sigma^x] - \gamma_\alpha(L_\alpha^\dagger L_\alpha\sigma^x + \sigma^x L_\alpha^\dagger L_\alpha - 2L_\alpha^\dagger\sigma^x L). \qquad (A1)$$

(i) Dephasing. For $L = \sigma^z$, Eq. (A1) becomes

$$\frac{d\sigma^x(t)}{dt} = \omega_z\sigma^y - \gamma_\phi\sigma^x, \qquad (A2)$$

$$\frac{d\sigma^y(t)}{dt} = -\omega_z\sigma^x - \gamma_\phi\sigma^y. \qquad (A3)$$

These equations are solved by

$$\sigma^x(t) = e^{-\gamma_\phi t}[\cos(\omega_z t)\sigma^x(0) + \sin(\omega_z t)\sigma^y(0)], \tag{A4}$$

$$\sigma^y(t) = e^{-\gamma_\phi t}[\cos(\omega_z t)\sigma^y(0) - \sin(\omega_z t)\sigma^x(0)]. \tag{A5}$$

Using $(\sigma_j^x)^2 = 1/4$, we find that for any $t > 0$,

$$S_x(t) = \langle \sigma^x(t)\sigma^x(0)\rangle \tag{A6}$$

$$= \tfrac{1}{4}e^{-\gamma_\phi t}[\cos(\omega_z t) - 2i\langle\sigma^z\rangle \sin(\omega_z t)]. \tag{A7}$$

A straightforward integration gives

$$\Sigma_a^R = 4g^2 \int_0^\infty dt\, S_x(t) - S_x(-t) = \frac{4g^2\omega_z\langle\sigma^z\rangle}{\omega_z^2 + \gamma^2}. \tag{A8}$$

(ii) Thermal bath. We now consider the decay process due to the coupling to a finite-temperature bath. The correspondent master equation is

$$\frac{d\sigma^x}{dt} = -i[H,\sigma^x] - \sum_{\alpha=\pm} \gamma_\alpha(\sigma^\alpha\sigma^{-\alpha}\sigma^x + \sigma^x\sigma^\alpha\sigma^{-\alpha} - 2\sigma^\alpha\sigma^x\sigma^{-\alpha}), \tag{A9}$$

where $\gamma_- = \gamma_T(n_T+1)$, $\gamma_+ = \gamma_T n_T$, and $n_T = (e^{\omega_z/T} - 1)^{-1}$ is the Bose-Einstein distribution [52]. A direct evaluation demonstrates that Eqs. (A2) and (A3) are modified by $\gamma_\phi \to (1+2n_T)\gamma_T = \gamma_T/\tanh(\omega_z/2T)$. Using the corresponding master equation for σ^z, one finds

$$\frac{d\sigma^z}{dt} = -\gamma_T[(1+n_T)(2\sigma^z - 1) + n_T(2\sigma^z + 1)] \tag{A10}$$

$$= -\gamma_T[-1 + (2+4n_T)\sigma^z]. \tag{A11}$$

Thus, in the steady state $\langle\sigma^z\rangle = 1/(2+4n_T) = 0.5\tanh(\omega_z/T)$, in agreement with the equilibrium result.

(iii) Generalized Markovian bath. We now consider the Lindblad operator $L = \sigma^- + \lambda\sigma^+$. A direct evaluation leads to

$$L^\dagger L\sigma^x + \sigma^x L^\dagger L - 2L^\dagger\sigma^x L = (1-\lambda)^2\sigma^x. \tag{A12}$$

As a consequence, the equations of motion of σ^x are the same as (A2) and (A3), with $\gamma_\phi \to \gamma_\lambda(1-\lambda)^2$. Note in particular that for $\lambda = 1$, $L = \sigma^x$ and the correlator of σ^x does not decay over time. We deduce that

$$\Sigma_a^R(0) = \frac{4g^2\omega_z\langle\sigma^z\rangle}{\omega_z^2 + \gamma_\lambda^2(1-\lambda^2)}. \tag{A13}$$

We next need to find the steady-state expectation value of σ^z. For this purpose we use the master equation (A1) with $\sigma^x \to \sigma^z$ and obtain

$$\frac{d\sigma^z}{dt} = -\gamma_\lambda[(1-\lambda^2) - 2(1+\lambda^2)\sigma^z]. \tag{A14}$$

In the steady state the expectation value of the left-hand side is zero and $\langle\sigma^z\rangle = 0.5(1-\lambda^2)/(1+\lambda^2)$. Combining this expression with Eq. (A13), we obtain Eq. (13).

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