Floquet approach to $\mathbb{Z}_2$ lattice gauge theories with ultracold atoms in optical lattices

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Quantum simulation has the potential to investigate gauge theories in strongly-interacting regimes, which are up to now inaccessible through conventional numerical techniques. Here, we take a first step in this direction by implementing a Floquet-based method for studying $\mathbb{Z}_2$ lattice gauge theories using two-component ultracold atoms in a double-well potential. For resonant periodic driving at the on-site interaction strength and an appropriate choice of the modulation parameters, the effective Floquet Hamiltonian exhibits $\mathbb{Z}_2$ symmetry. We study the dynamics of the system for different initial states and critically contrast the observed evolution with a theoretical analysis of the full time-dependent Hamiltonian of the periodically-driven lattice model. We reveal challenges that arise due to symmetry-breaking terms and outline potential pathways to overcome these limitations. Our results provide important insights for future studies of lattice gauge theories based on Floquet techniques.

Lattice gauge theories (LGTs) [1, 2] are fundamental for our understanding of quantum many-body physics across different disciplines ranging from condensed matter [3–6] to high-energy physics [7]. However, theoretical studies of LGTs can be extremely challenging in particular in strongly-interacting regimes, where conventional computational methods are limited [8, 9]. To overcome these limitations alternative numerical tools are currently developed, which enable out-of-equilibrium and finite density computations [10–13]. In parallel, the rapid progress in the field of quantum simulation [14–17] has sparked a growing interest in designing experimental platforms to explore the rich physics of LGTs [18–25]. State-of-the-art experiments are now able to explore the physics of static [26] as well as density-dependent gauge fields [27] and have engineered controlled few-body interactions [28–30], which are the basis for many proposed schemes to realize LGTs. First studies of the Schwinger model have been performed with quantum-classical algorithms [31] and a digital quantum computer composed of four trapped ions [32]. The challenge for analog quantum simulators mainly lies in the complexity to engineer gauge-invariant interactions between matter and gauge fields.

Here, we explore the dynamics of a minimal model for $\mathbb{Z}_2$ LGTs coupled to matter with ultracold atoms in periodically-driven double-well potentials [33]. $\mathbb{Z}_2$ LGTs are of high interest in condensed matter physics [13, 34, 35] and topological quantum computation [36]. Our scheme is based on a mixture of bosonic atoms in two different internal states, which encode the matter and gauge field degrees of freedom. The interaction between the two components is engineered via resonant periodic modulation [37–40] of the on-site potential at the interspecies Hubbard interaction [41–44]. By choosing suitable modulation parameters, the effective Floquet model exhibits a $\mathbb{Z}_2$ symmetry. We present a detailed study of this effective Floquet model defined on a double well, which constitutes the basic building block of the LGT. We discuss the relation between the observed dynamics and the ideal model and we reveal the potential impact of symmetry-breaking terms.

In order to understand the observed phenomena, it is instructive to consider the properties of an extended one-dimensional (1D) $\mathbb{Z}_2$ LGT, as captured by the Hamiltonian

$$
\hat{H}_{\mathbb{Z}_2} = + \sum_j J_a \left( \hat{\tau}_j^{(j,j+1)} \hat{a}_j^\dagger \hat{a}_{j+1} + \text{h.c.} \right) $$

$$
- \sum_j J_f \hat{\tau}_j^{(j,j+1)}.
$$

Here $\hat{a}_j^\dagger$ describes the creation of a matter particle on lattice site $j$ and the Pauli operators $\hat{\tau}_j^{(j,j+1)}$, defined on the links between neighboring lattice sites, encode the gauge field degrees of freedom. The elementary ingredients of this $\mathbb{Z}_2$ LGT are illustrated in Fig. 1a. The matter field has a charge $\hat{Q}_j = e^{i\pi \hat{n}_j^z}$ on site $j$, which is given by the parity of the site occupation, with $\hat{n}_j^z = \hat{a}_j^\dagger \hat{a}_j$ the number operator. The dynamics of the matter field is coupled to the $\mathbb{Z}_2$ gauge field $\hat{\tau}_j^{(j,j+1)}$ with an amplitude $J_a$. The energy of the electric field $\hat{\tau}_j^{(j,j+1)}$ is $J_f$.

The model Hamiltonian (1) commutes with the lattice
### Figure 1. 1D $Z_2$ lattice gauge theory coupled to matter. Circles describe lattice sites, which are empty (gray) or occupied by a matter particle (blue). a) Elementary ingredients: $Z_2$ charge $\hat{Q}_j = e^{\pi n_j^g}$, $Z_2$ gauge field $\hat{G}_j = e^{\pi n_j^f}$, $Z_2$ electric field $\hat{Z}_j = e^{\pi n_j^2}$, $Z_2$ Gauss’s law $\hat{G}_j = \hat{Q}_j \prod_{i:(i,j)} \hat{Z}_{i,j}$, and local symmetry $\hat{\sigma} = \prod_{i:(i,j)} \hat{Z}_{i,j}$. Matter and gauge fields are implemented using two internal states $a$ (blue) and $f$ (red). Matter-gauge coupling occurs with strength $J_a$. b) Dynamics of the 1D model (1) for different values of $J_f/J_a$ with an initial state, where a single matter particle is located on site $j=0$ and the gauge field is in an eigenstate of the electric field. The evolution is calculated using exact diagonalization on a system with 13 sites based on Eq. (1).

### Gauge Transformations Defined by the Local Operators

$$\hat{G}_j = \hat{Q}_j \prod_{i:(i,j)} \hat{Z}_{i,j}, \quad \left[ \hat{H}, \hat{G}_j \right] = 0 \quad \forall j, \quad (2)$$

where $\prod_{i:(i,j)}$ denotes the product over all links connected to lattice site $j$. The eigenvalues of $\hat{G}_j$ are $g_j = \pm 1$. The dynamics of the model is constraint by the $Z_2$ Gauss’s law, $\hat{G}_j |\psi\rangle = g_j |\psi\rangle$, in analogy to electrodynamics. Gauss’s law effectively separates the Hilbert space into different sectors, which are characterized by the set of conserved quantities $\{g_j\}$. Lattice sites with $g_j = -1$ are interpreted as local static background charges. Different sectors can be explored by preparing suitable initial states. The two states sketched in Fig. 1a (lower right) belong to the same subsector and illustrate the basic matter-gauge coupling according to Gauss’s law, i.e., tunneling of the matter particle occurs in combination with a change of the electric field from $+1$ to $-1$.

In order to gain more insight into the physics of the 1D model (1), we consider a system initially prepared in an eigenstate of the electric field operator, with $\tau^x = +1$ on all links, and a single matter particle located on site $j = 0$. For this initial state $g_j = +1$, $\forall j \neq 0$ and $g_j = -1$ for $j = 0$ (Fig. 1b). In the limit of vanishing electric field strength $J_f \to 0$, the matter particle can tunnel freely along the 1D chain, thereby changing the electric field on all traversed links. For $J_f \neq 0$, tunneling of the matter particle is detuned due to the energy of the electric field and the matter particle is bound to the location of the static background charge at $j = 0$. In this regime the energy of the system scales linearly with the distance between the static charge and the matter particle, which we interpret as a signature of confinement.

Here, we engineer the elementary interactions of the $Z_2$ model on a two-site lattice following Ref. [33]. The matter and gauge fields are implemented using two different species denoted as $a$- and $f$-particles, which are realized by two Zeeman levels of the hyperfine ground-state manifold of $^{87}$Rb, $|a\rangle \equiv |F = 1, m_F = -1\rangle$ and $|f\rangle \equiv |F = 1, m_F = 1\rangle$. We prepare one $a$- and one $f$-particle in each two-site system. The matter field is associated with the $a$-particle. The $Z_2$ gauge field is the number imbalance $\hat{\tau}^z_{(j,j+1)} = \hat{n}^f_j - \hat{n}^f_{j+1}$ of the $f$-particle and the $Z_2$ electric field corresponds to tunneling of the $f$-particle, $\hat{\tau}^x_{(j,j+1)} = \hat{f}^\dagger_j \hat{f}_{j+1} + \hat{f}^\dagger_{j+1} \hat{f}_j$, where $\hat{f}^\dagger_j$ is the creation operator of an $f$-particle on site $j$ and $\hat{n}^f_j = \hat{f}^\dagger_j \hat{f}_j$ is the corresponding number-occupation operator.

The driving scheme is based on a species-dependent double-well potential with tunnel coupling $J$ between neighboring sites and an energy offset $\Delta \phi$ only seen by the $f$-particle. Experimentally the driving scheme is realized with a magnetic-field gradient, making use of the opposite magnetic moments of the two states $|a\rangle$ and $|f\rangle$ [45]. In the limit of strong on-site interactions $U \gg J$, first-order tunneling processes are suppressed but can be restored resonantly with a periodic modulation at the resonance frequency $\hbar \omega = \sqrt{U^2 + 4J^2} \approx U$. The full time-dependent Hamiltonian can be expressed as

$$\hat{H}(t) = -J \left( \hat{a}^\dagger_0 \hat{a}_1 + \hat{f}^\dagger_0 \hat{f}_1 + \text{h.c.} \right)$$

$$+ U \sum_{j=1,2} \hat{n}^a_j \hat{n}^f_j + \Delta \hat{\phi} \left( \hat{n}^a_0 + \hat{n}^f_0 \right),$$

where $A$ is the modulation amplitude and $\phi$ is the modulation phase. In the high-frequency limit $\hbar \omega \approx U \gg J$, the lowest order of the effective Floquet Hamiltonian contains renormalized tunneling matrix elements for both $a$- and $f$-particles [38–40], $\hat{J}_a e^{i\phi_0}$ and $\hat{J}_f e^{i\phi_f}$. For general modulation parameters, the amplitudes and phases are operator-valued and explicitly depend on the site-
occupations. For particular values of the modulation phase \( \phi \in \{0, \pi\} \), these expressions simplify: Tunneling of \( f \)-particles becomes real-valued \( \hat{\phi}_f = 0 \), with an amplitude that only weakly depends on the position of the \( a \)-particle. The tunneling amplitude of \( a \)-particles becomes density-independent \( \hat{J}_a \equiv \hat{J}_a \) and in addition this hopping process is accompanied by a phase shift of \( \hat{\phi}_a = 0 \) or \( \pi \), which depends on the position of the \( f \)-particle [33]. This non-trivial phase factor is central to our implementation of the \( \mathbb{Z}_2 \) symmetry and allows us to write the renormalized hopping of \( a \)-particles as \( \hat{J}_a \hat{\phi}_{\tilde{\tau}(1,2)} \). We drop the link-index from now on to simplify notations, \( \tilde{\tau} \equiv \tilde{\tau}_{(1,2)} \).

The individual tunneling processes of \( a \)- and \( f \)-particles are illustrated in Fig. 2a. For the \( a \)-particle the double-well potential is symmetric, but depending on the position of the \( f \)-particle, the on-site energy difference between neighboring sites is \( \pm U \). Resonant modulation at the frequency \( \hbar \omega \approx U \) restores coupling with a renormalized amplitude \( \hat{J}_a = J_0(\chi) \); here \( J_\nu \) denotes the \( \nu \)-th-order Bessel function of the first kind and \( \chi = \nu / \hbar \omega \) is the dimensionless driving parameter. The tunneling phase is either 0 or \( \pi \), which stems from the property of the Bessel function \( J_{-\nu}(\chi) = (-1)^\nu J_\nu(\chi) \) [45]. For the \( f \)-particle, the on-site energy difference between neighboring sites is additionally modified by the species-dependent tilt \( \Delta_f \) and thus the total energy difference for \( \Delta_f = U \) is either \( \Delta_f - U = 0 \) or \( \Delta_f + U = 2U \) (Fig. 2a). Therefore, tunneling is renormalized via zero- and two-photon processes, resulting in the real-valued tunneling matrix elements \( J_0(\chi) \) and \( J_2(\chi) \). To lowest order, the effective double-well Hamiltonian takes the form

\[
\hat{H}_{\text{eff}} = J_f \hat{\tau}^z \left( \hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2 \right) - \hat{J}_f \hat{\tau}^z, \tag{4}
\]

where \( \hat{J}_f \) depends on the position of the \( a \)-particle

\[
\hat{J}_f = J_0(\chi) \hat{n}_1^a + J_2(\chi) \hat{n}_2^a. \tag{5}
\]

The density dependence of \( \hat{J}_f \) can be avoided by choosing the dimensionless driving strength \( \chi \) such that \( J_0(\chi_0) = \chi_0 \), which occurs, e.g. at \( \chi_0 \approx 1.84 \). Then, Eq. (4) reduces to the two-site version of the \( \mathbb{Z}_2 \) LGT described by Hamiltonian (1). Note that the double-well model defined in Eq. (4) is \( \mathbb{Z}_2 \) symmetric for all values of the driving strength \( \chi \).

The experimental setup consists of a 3D optical lattice generated at wavelength \( \lambda_s = 767 \text{ nm} \). Along the \( x \)-axis an additional standing wave with wavelength \( 2\lambda_s = 1534 \text{ nm} \) is superimposed to create a superlattice potential. For deep transverse lattice potentials and suitable superlattice parameters, an array of isolated double-well potentials is realized, where all dynamics is restricted to the two double-well sites [45]. The periodic drive is generated by modulating the amplitude of an additional lattice with wavelength \( 2\lambda_s \), whose potential maxima are aligned relative to the double-well potential in order to modulate only one of the two sites. This enables the control of the modulation phase, which is set to \( \phi \in \{0, \pi\} \).

We first study the renormalization of the tunneling matrix elements for the relevant \( \nu \)-photon processes [43, 46–49] with a single atom on each double well (Fig. 2b).
Figure 3. Dynamics of the matter-gauge system prepared initially in an eigenstate of the electric field $\hat{\tau}^x$.

a) Illustration of the gauge invariant initial state $|\psi_0\rangle = |a, 0\rangle \otimes (|f, 0\rangle + |0, f\rangle)/\sqrt{2}$ and the expected dynamics according to Hamiltonian (4). b) Measured expectation values of the $Z_2$ charge $\langle Q_1 \rangle$ (blue points) and $Z_2$ gauge field $\langle \hat{\tau}^z \rangle$ (red points) for $\omega = 2\pi \times 4320$ Hz. Each data point represents the mean of at least three individual experimental results and the error bars denote the standard deviation. The blue and red lines and shadings show a numerical analysis using time-dependent exact-diagonalization, which includes averaging of the observables in the presence of an inhomogeneous tilt distribution $\Delta(x, y, z)$ approximated by a normal distribution with standard deviation $\sigma_{\Delta}/h = 0.44(2)$ kHz, which was independently calibrated [45]. The blue and red solid line is the median and the shading represents the 1σ-confidence interval obtained with a bootstrap analysis of 1000 repetitions. All calculations are performed using the independently calibrated experimental parameters, $J/h = 587(3)$ Hz, $\Delta_f/h = 4.19(3)$ kHz, $U/h = 3.85(7)$ kHz and taking into account additional terms that appear in the extended Bose-Hubbard model [45]. The gray solid line is the ideal dynamics according to Eq. (6).

For every measurement, the atom is initially localized on the lower-energy site with a potential energy difference $\Delta_\nu \approx \nu \hbar \omega$ to the higher-energy site, where $\nu \in \{0, 1, 2\}$. Then, the resonant modulation is switched on rapidly at frequency $\omega$ and we evaluate the imbalance $I = n_1 - n_2$ as a function of the evolution time, where $n_j$ is the density on site $j$. These densities were determined using site-resolved detection methods [29]. Note, in the experiment the average of this observable over the entire 3D array of double-well potentials is measured. Hence, an overall harmonic confinement and imperfect alignment of the lattice laser beams introduces an inhomogeneous tilt distribution $\Delta(x, y, z)$, which leads to dephasing of the averaged dynamics. The renormalized tunneling amplitude is obtained from the oscillation frequency of the imbalance and by numerically taking into account the tilt distribution $\Delta(x, y, z)$ [Fig. 2b]. We find that our data agrees well with the expected Bessel-type behavior for the $\nu$-photon processes [45]. Moreover, these measurements enable us to directly determine the value of the modulation amplitude, for which $\mathcal{J}_0(\chi_0) = \mathcal{J}_2(\chi_0)$, as indicated by the vertical line in Fig. 2b.

In order to study the dynamics of the $Z_2$ double-well model (4), we prepare two different kinds of initial states, where the gauge field particle is either prepared in an eigenstate of the electric field $\hat{\tau}^x$ (Fig. 3) or the gauge field operator $\hat{\tau}^z$ (Fig. 4a). In both cases the matter particle is initially localized on site $j = 1$.

First, we consider the state $|\psi_0\rangle = |a, 0\rangle \otimes (|f, 0\rangle + |0, f\rangle)/\sqrt{2}$ (Fig. 3a), where the gauge field particle is in a symmetric superposition between the two sites. This state is an eigenstate of $\hat{G}_j$ defined in Eq. (2). The corresponding eigenvalues are $g_1 = -1$ and $g_2 = +1$. After initiating the dynamics by suddenly turning on the resonant modulation, we expect that the matter particle starts to tunnel to the neighboring site ($j = 2$) according to the matter-gauge coupling. Depending on the energy of the electric field $J_f$, this process can be energetically detuned and the matter particle does not fully tunnel to the other site. Solving the dynamics according to Hamiltonian (4) analytically, gives:

$$
\langle \hat{Q}_1(t) \rangle = -\frac{J_f^2 + J_a^2 \cos \left(2t \sqrt{J_f^2 + J_a^2} \right)}{J_f^2 + J_a^2}.
$$

The maximum value of $\langle \hat{Q}_1(t) \rangle$ is limited to $(J_a^2 - J_f^2)/(J_a^2 + J_f^2)$. The experimental configuration is well suited to explore the regime $J_f/J_a = \mathcal{J}_0(\chi_0)/\mathcal{J}_1(\chi_0) \approx 0.54$, which corresponds to an intermediate regime between the two limiting cases discussed in Fig. 1c. These cases can also be understood at the level of the two-site model. In the weak electric field regime ($J_f/J_a \ll 1$) the matter particle tunnels freely between the two sites, while in the limit of a strong electric field ($J_f/J_a \gg 1$) the matter particle remains localized.

In the experiment we can directly access the value of the charge operator $\hat{Q}_j = e^{i\pi n_j}$ and the link operator $\hat{\tau}^z = n_j^f - n_j^s$ via site- and state-resolved detection techniques [29]. They provide direct access to the state-resolved density on each site of the double well $n_j^a$ and $n_j^f$, averaged over the entire 3D array of double-well realizations. The experimental results are shown in Fig. 3b for $U/J = 6.6$ and $\phi = 0$. As expected, we find that the charge oscillates, while the dynamics of the $f$-particle is strongly...
suppressed. We observe a larger characteristic oscillation frequency for the a-particle compared to the prediction of Eq. (6) (gray line, Fig. 3b). This is predominantly caused by an inhomogeneous tilt distribution $\Delta(x, y, z)$ present in our system. Taking the inhomogeneity into account, the numerical analysis of the full time-dependence according to Eq. (3) (solid blue line, Fig. 3b) shows good agreement with the experimental results. The fast oscillations both in the data and the numerics is due to the micromotion at non-stroboscopic times.

The $f$-particle is initially prepared in an eigenstate of the electric field operator $\hat{\tau}^z$, which corresponds to an equal superposition of the particle on both sites of the double-well potential, i.e. $\langle \hat{\tau}^z(t=0) \rangle = 0$. The $\mathbb{Z}_2$ electric field follows the oscillation of the matter particle in a correlated manner to conserve the local quantities $g_j$. At the same time the expectation value of the gauge field $\langle \hat{\tau}^+ (t) \rangle$ is expected to remain zero at all times. This is a non-trivial result, which is a direct consequence of the $\mathbb{Z}_2$-symmetry constraints. In contrast, a resonantly driven double-well system with $\Delta_f = 0$, which does not exhibit $\mathbb{Z}_2$ symmetry, would show dynamics with equal oscillation amplitudes for the $a$- and $f$-particle. In the experiment we clearly observe suppressed dynamics for the $f$-particle, which is a signature of the experimental realization of the $\mathbb{Z}_2$ symmetry (Fig. 3b). Deviations between the time-dependent numerical analysis and the experimental results are most likely due to an imperfect initial state, residual energy offsets, and finite ramp times.

In a second set of experiments we study the dynamics where the gauge field particle is initialized in an eigenstate of the gauge field operator $\hat{\tau}^z$, while the matter particle is again localized on site $j = 1$, $|\psi_0\rangle = |a, 0\rangle \otimes |0, f\rangle$ (Fig. 4a). Here, the system is in a coherent superposition of the two subsectors with $g_1 = -g_2 = \pm 1$ and the expectation value of the locally conserved operators are $\langle G_1 \rangle = \langle G_2 \rangle = 0$. Note that there is no coupling between different subsectors according to Hamiltonian (4). The basic dynamics can be understood in the two limiting cases of the model. For $J_f \ll J_a$ the electric field vanishes and the system is dominated by the gauge field $\hat{\tau}^z$. In this limit, a system prepared in an eigenstate of $\hat{\tau}^z$ will remain in this eigenstate because $\hat{\tau}^z$ commutes with Hamiltonian (4) for $J_f = 0$. In the opposite regime ($J_f \gg J_a$), where the electric field dominates, $\langle \hat{\tau}^z \rangle$ oscillates between the two eigenvalues. The dynamics of the $\mathbb{Z}_2$ charge on the other hand is still determined by Eq. (6). In the experiment we probe the intermediate regime at $J_f/J_a \approx 0.54$ for $U/J = 6.7$ and $\phi = \pi$ (Fig. 4b). The dynamics agrees with the ideal evolution (gray line, Fig. 4b) for short times. For longer times it deviates due to the averaging over the inhomogeneous tilt distribution $\Delta(x, y, z)$, which is well captured by the full time-dynamics according to Hamiltonian (3) (red and blue lines, Fig. 4b). Notably, $\langle \hat{\tau}^z \rangle$ exhibits a non-zero average value.

An important requirement for quantum simulations of gauge theories is the exact implementation of the local symmetry constraints in order to assure that $\langle G_j \rangle$ is conserved for all times. Since we do not have direct access to $\langle G_j \rangle$ experimentally, we study the implications of symmetry-breaking terms numerically. The dominant contribution stems from the inhomogeneous tilt distribution $\Delta(x, y, z)$, which, however, can be avoided in future experiments by generating homogeneous box potentials. The second type of gauge-variant terms are coupling processes that do not fulfill the constraints of Gauss’s law. These are correlated two-particle tunneling and nearest-neighbor interactions, which are known to exist in interacting lattice models [50]. For our experimental parameters these terms are on the order of $0.03 J$ and can

Figure 4. Dynamics of the matter-gauge system prepared initially in an eigenstate of the gauge field $\hat{\tau}^z$. a Schematic of the initial state $|\psi_0\rangle = |a, 0\rangle \otimes |0, f\rangle$ and the expected dynamics. b Measured expectation values of the $\mathbb{Z}_2$ charge $\langle Q_1 \rangle$ (blue points) and the $\mathbb{Z}_2$ gauge field $\langle \hat{\tau}^z \rangle$ (red points) for $\omega = 2\pi \times 4314$ Hz. Each data point represents the mean of at least three individual experimental results and the error bars denote the standard deviation. The blue and red solid line shows a numerical analysis using time-dependent exact-diagonalization, with $J/h = 578(3)$ Hz, $\Delta_f/h = 4.19(3)$ kHz, $U/h = 3.85(7)$ kHz and $\Delta_s/h = 0.46(2)$ kHz as explained in the caption of Fig. (3b) and Ref. [45]. The blue and red solid line is the median and the shading represents the 1σ-confidence interval obtained with a bootstrap analysis of 1000 repetitions. The gray solid line is the ideal dynamics according to Eq. (6).
be neglected for the timescale of the observed dynamics [45]. The same gauge-variant processes appear in the higher-order terms of the Floquet expansion for finite $U/J$ [38–40]. We study them analytically by calculating the first-order terms of the expansion and compare them to numerics [45]. In Fig. 5 we show the numerically-calculated dynamics for the initial state $|\psi_0^g\rangle$ (Fig. 3a) according to the full time-dependent Hamiltonian [Eq. (3)] for $U/J = 7$, similar to the experimental values (Fig. 5). We find that in this regime the driving frequency is crucial and defines the timescale for which the $Z_2$ symmetry of the model remains. In particular there is an optimal value around $\hbar \omega \approx 1.01 U$, where the value of $\langle G_1 \rangle$ deviates by $< 10\%$ even for long evolution times.

In summary, we have studied the dynamics of a minimal model for $Z_2$ LGTs. Our observations are well described by a full time-dependent analysis of the 3D system. Moreover, we find non-trivial dynamics of the matter and gauge field in agreement with predictions from the ideal $Z_2$ LGT. We further reached a good understanding of relevant symmetry-breaking terms. The dominant processes we identified are species-independent energy offsets between neighboring sites and correlated two-particle tunneling terms [50], which can be suppressed in future experiments. We have further provided important insights into the applicability of Floquet schemes. While the Floquet parameters can be fine-tuned in certain cases to ensure gauge invariance, this complication can be avoided by reaching the high-frequency limit [38–40] and minimize finite-frequency corrections. In experiments this could be achieved using Feshbach resonances to increase the inter-species scattering length, which, however, comes at the cost of enhanced correlated tunneling processes. On the other hand, numerical studies indicate that certain experimental observables are robust to imperfections, hence, relaxing experimental constraints regarding the precise implementation of the local gauge invariance [51, 52], whose investigation we leave for future work. We anticipate that the double-well model demonstrated in this work serves as a stepping stone for future experimental studies of $Z_2$ LGTs coupled to matter in extended 1D and 2D systems, which can be realized by coupling many double-well links along a 1D chain or in a ladder configuration [33]. Finally, the use of state-dependent optical lattices could further enable an independent tunability of the matter- and gauge-particle tunneling terms.

Figure 5. **Finite-frequency corrections to the effective Floquet Hamiltonian (4).** Stroboscopic dynamics of the expectation value of the local $Z_2$-symmetry operator $\langle G_1 \rangle$ for $|\psi_0^g\rangle$, with $g_1 = -1$, based on Hamiltonian (3) for different driving frequencies $\omega$. The panels on the right show examples of the time traces for $\hbar \omega \approx 1.04 U$ and $\hbar \omega = 1.01 U$.

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[39] Marin Bukov, Luca D’Alessio, and Anatoli Polkovnikov, “Universal high-frequency behavior of periodically driven...


[43] See Supplementary Material which includes Ref. [53–55], not cited in the main text.


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EFFECTIVE HAMILTONIAN

Floquet expansion

The $\mathbb{Z}_2$ double-well model realized in this work consists of a two-site potential with one $a$- and one $f$-particle implementing the matter and gauge field, respectively. Such a two-site two-particle model can be represented using the four basis states: $|1\rangle = |a, 0\rangle \otimes |f, 0\rangle$, $|2\rangle = |a, 0\rangle \otimes |0, f\rangle$, $|3\rangle = |0, a\rangle \otimes |f, 0\rangle$ and $|4\rangle = |0, a\rangle \otimes |0, f\rangle$, where the labels $a$ and $f$ before or after the comma mark the particle occupation on the left and right site. As described in the main text, a species-dependent energy offset $\Delta_f = U$ between the two sites and a species-independent resonant driving $A \cos(\omega t + \phi)$ of the on-site potential are applied, which leads to the time-dependent Hamiltonian (3) in the main text. In the new basis defined above, this Hamiltonian reads

$$\hat{H}(t) = -J (|3\rangle \langle 1| + |4\rangle \langle 2| + |2\rangle \langle 3| + |1\rangle \langle 4|) + U (|2\rangle \langle 1| + |3\rangle \langle 3| + |4\rangle \langle 4|) + A \cos(\omega t + \phi) (|2\rangle \langle 1| + |2\rangle \langle 2| + |3\rangle \langle 3|),$$

(S.1)

where the tunneling rate of both $a$- and $f$-particles is $J$ and the intra-species interaction energy is $U$.

The stroboscopic dynamics of such a time-dependent system can be described by an effective Floquet Hamiltonian represented by a series of time-independent terms in powers of $1/\omega$. The series can be truncated to lowest order in the high-frequency limit $\omega \to \infty$. Following the method presented in [S1, S2], we calculate the Floquet Hamiltonian $\hat{H}_F$ up to first order. To this end, Eq. (S.1) is transformed by a unitary transformation $|\psi\rangle \to |\psi'\rangle = \hat{R}(t)|\psi\rangle$, such that the new Hamiltonian $\hat{H}(t)$ does not contain divergent terms in the high-frequency limit:

$$\hat{H}(t) = \hat{R}(t)\hat{H}_F^\dagger - i\hat{R}\partial_t \hat{R}^\dagger = \sum_{k \in \mathbb{Z}} \hat{H}^{(k)} e^{ik\omega t}. \quad \text{(S.2)}$$

We express this transformed Hamiltonian in a Fourier-series with time-independent components $\hat{H}^{(k)}$. For our model (S.1), we choose the transformation

$$\hat{R}(t) = \exp\left\{+i\omega t (|2\rangle \langle 1| + |3\rangle \langle 3| + |4\rangle \langle 4|) + \frac{iA}{\hbar \omega} \sin(\omega t + \phi) (|2\rangle \langle 1| + |2\rangle \langle 2| + |3\rangle \langle 3|)\right\}$$

(S.3)

and obtain the following Fourier components

$$\hat{H}^{(k)} = -J \left\{J_{-k-1}(\chi) |3\rangle \langle 1| e^{i(k+1)\phi} + J_{k-1}(\chi) |1\rangle \langle 3| e^{i(k-1)\phi} + J_{-k+1}(\chi) |4\rangle \langle 2| e^{i(k-1)\phi} + J_{k+1}(\chi) |2\rangle \langle 4| e^{i(k+1)\phi} + J_{-k}(\chi) |4\rangle \langle 3| e^{i(k+1)\phi} + J_{k}(\chi) |3\rangle \langle 4| e^{i(k+1)\phi} + J_{-k-2}(\chi) |2\rangle \langle 1| e^{i(k+2)\phi} + J_{k+2}(\chi) |1\rangle \langle 2| e^{i(k-2)\phi}\right\}.$$  

(S.4)

Here $J_{\nu}(\chi)$ is the $\nu$-th Bessel function of the first kind with the dimensionless driving strength $\chi = A/\hbar \omega$ and $\delta_{ij}$ the Kronecker delta. Note that in the finite-frequency regime the resonant driving frequency for a double well tilted by the energy difference $U$ is equal to the energy gap, $\hbar \omega = \sqrt{U^2 + 4J^2} \neq U$. In the high-frequency limit $U \gg J$, however, $\hbar \omega \approx U$. Using the components $\hat{H}^{(k)}$ [Eq. (S.4)] the lowest orders of the Floquet Hamiltonian
are
\[ \hat{H}_F = \hat{H}^{(0)} + \frac{1}{\hbar \omega} \sum_{k > 0} \frac{1}{k} \left[ \hat{H}^{(k)}, \hat{H}^{(-k)} \right] + \mathcal{O} \left( \frac{1}{\omega^2} \right) . \] (S.5)

Floquet model in the infinite-frequency limit

In the limit $U/J \to \infty$, the resonant driving frequency is $\hbar \omega \approx U$ and the diagonal terms in Eq. (S.4) vanish. The Floquet Hamiltonian (S.5) reduces to the lowest order
\[ \hat{H}^{(0)}_F = -J \left( J_1(\chi) |3\rangle \langle 2| + J_1(\chi) |2\rangle \langle 4| \right) e^{-i\phi} + J_0(\chi) |4\rangle \langle 3| + J_2(\chi) |2\rangle \langle 1| e^{2i\phi} + \text{h.c.}, \]

referred to as zeroth-order Floquet Hamiltonian in the following. As discussed in the main text, different tunnel processes have different renormalizations with an additional phase factor depending on the modulation phase $\phi$. The tunnel processes $|3\rangle \langle 1|$ and $|4\rangle \langle 2|$ correspond to $a$-particle tunneling and $|4\rangle \langle 3|$ and $|2\rangle \langle 1|$ to $f$-particle tunneling. The $a$-particle’s tunneling rate is renormalized with the first-order Bessel function of the first kind $J_1(\chi)$, while on the other hand the $f$-particle’s tunneling rate depends on the $a$-particle’s position. The latter asymmetry can be resolved by choosing the driving strength such that $J_0(\chi_0) = J_2(\chi_0)$. This happens the first time at $\chi_0 \approx 1.84$ (Fig. 2 in main text).

When choosing the modulation phase $\phi = \{0, \pi\}$, the Floquet Hamiltonian (S.6) directly realizes the $\mathbb{Z}_2$ double well, where the $a$-particle’s tunnel phase is either $0$ or $\pi$ depending on the $f$-particle’s position, while tunneling of the $f$-particle is real-valued. This density-dependent phase shift reflects the sign of the effective energy offset $\pm U$ between neighboring sites experienced by the $a$-particle because of the reflection properties of the Bessel function $J_1(\chi) = e^{i\pi} J_1(\chi)$.

Floquet model including the first-order correction for finite-frequency drive

In experiments it is generally challenging to work deep in the high-frequency limit especially without the availability of Feshbach resonances; here, we work at $U/J \approx 6.6$. Consequently, higher order corrections become relevant. In order to study their impact on the dynamics we calculate the first-order terms of the Floquet expansion and include them in our calculations (Fig. S1). We restrict the derivation to stroboscopic time points $nT$ with $T = 2\pi/\omega$, $n \in \mathbb{N}$ and set the driving phase to $\phi = 0$.

The first-order term of the Floquet expansion can be calculated following Eq. (S.5). Note, the resonant driving frequency is now $\hbar \omega = \sqrt{U^2 + 4J^2} \neq U$ and the diagonal terms in $\hat{H}^{(0)}_F$ become nonzero.

\[ \hat{H}_F = \hat{H}^{(0)} + \hat{H}^{(1)}_{\text{pair-hopping}} + \hat{H}^{(1)}_{\text{detuning}} \] (S.7)

The first-order correction contains two types of terms: a pair-tunneling term $\hat{H}^{(1)}_{\text{pair-hopping}}$, which corresponds to tunneling of both particles, and a detuning term $\hat{H}^{(1)}_{\text{detuning}}$, describing the deviation from the exact resonance condition.

When calculating the time evolution including higher order terms, the time evolution operator $\hat{U}$ gets modified in general by both the transformation $\hat{R}$ and the kick operator $\hat{K}$ [S1]. However, for stroboscopic times the time-evolution operator reduces to
\[ \hat{U}(0) \rightarrow nT) = e^{-i\hat{K} e^{i\phi} nT} \]
and only depends on the stroboscopic kick operator
\[ \hat{K} = \frac{1}{\hbar \omega} \sum_{k > 0} \frac{1}{k} \left( \hat{H}^{(k)} - \hat{H}^{(-k)} \right) + \mathcal{O} \left( \frac{1}{\omega^2} \right) . \] (S.9)

In Fig. S1 the dynamics of the effective Floquet model including first-order corrections is compared to a full time-dependent analysis of Hamiltonian (3) from the
main text. We find that for the experimental timescales and parameters, i.e. $U/J=7$, the effective model up to first order is sufficient to describe the dynamics of the system.

DESCRIPTION OF THE EXPERIMENT

Lattice setup

In the experiment ultracold $^{87}$Rb atoms in a three-dimensional (3D) optical lattice potential are used. The lattice consists of three mutually orthogonal standing waves with wavelength $\lambda_s = 767\,\text{nm}$ and an additional lattice with $\lambda_1 = 2\lambda_s$ along the $x$-direction, which generates the superlattice potential

$$V_{SL}(x) = V_{x,s} \cos^2(k_s x) + V_{x,1} \cos^2(k_1 x + \varphi_{SL}),$$

(S.10)

where $V_{x,\mu}$ denotes the lattice depth and $k_\mu = 2\pi/\lambda_\mu$ is the wave number, with $\mu \in \{s, 1\}$. In addition, a standing wave with wavelength $\lambda_1$ is superimposed, where the relative phase between the potentials is chosen such that the potential maxima affect only one of the double-well sites (Fig. S2). The overall potential is then given by

$$V(x) = V_{SL}(x) + V_{\text{mod}} \cos^2(k_1 x - \pi/4),$$

(S.11)

with $V_{\text{mod}}$ the lattice depth of the additional modulation lattice. A time-dependent modulation $V_{\text{mod}} = V_{\text{mod}}^{(0)} + A_{\text{mod}} \cos(\omega t + \phi) \langle \mu \rangle$ around a mean value $V_{\text{mod}}^{(0)}$ in combination with a suitable static superlattice phase $\varphi_{SL}$ then generates the time-dependent Hamiltonian (S.1).

Measurement of the renormalization of the tunnel coupling in a driven double well

We experimentally determine the renormalization of the tunnel coupling of a single particle in a driven double well. The time-dependent Hamiltonian can be written in the basis, where the particle occupies the left $|L\rangle$ or the right $|R\rangle$ site of the double well:

$$\hat{H}_{1P}(t) = - J' (|L\rangle \langle R| + |R\rangle \langle L|) + \{\Delta + A \cos(\omega t + \phi)\} |L\rangle \langle L| + \{\Delta + A \cos(\omega t + \phi)\} |R\rangle \langle R|,$$

(S.12)

where $J'$ is the single-particle tunneling rate, $\Delta$ the energy offset between neighboring sites, $A$ the modulation amplitude, $\omega$ the modulation frequency and $\phi$ the modulation phase. In the high-frequency limit with resonant multi-frequency driving $\Delta = \nu \hbar \omega$, with $\nu \in \mathbb{Z}$, we find the Fourier components of the Floquet expansion analogous to Eq. (S.4)

$$\hat{H}_{1P}^{(k)} = - J' \left\{ J_{-k-\nu}(\chi) e^{ik \nu \phi} |R\rangle \langle L| + J_{k-\nu}(\chi) e^{ik \nu \phi} |L\rangle \langle R| \right\}.$$

(S.13)

In the infinite frequency limit $\omega \to \infty$, we can extract the renormalization of the tunnel coupling $J_\nu = J' J_{\nu}(\chi)$, which scales with the $\nu$-th order Bessel function of the first kind.

We load atoms in the $|F=1, m_F=1\rangle$ state into a 3D optical lattice by performing an exponential-shaped ramp during a time segment of 50 ms; the final lattice depths $V_{z,1} = 30(1) E_r$, $V_y = 35(1) E_r$, are reached within the $1/e$-time $\tau = 5$ ms, while the vertical lattice $V_z = 50(1) E_r$, is ramped up within $\tau_z = 1.5$ ms. The lattice depth is given in units of their respective recoil energy $E_{r,\mu} = \hbar^2 k_\mu^2 / 2m$ with $m$ the mass of a rubidium atom and $\mu = \{s, 1\}$, then, we use a filtering sequence to remove doubly-occupied sites by light assisted-collisions between atoms in the $|F=2, m_F=-1\rangle$ state. The subsequent part of the sequence is in principle not needed for this measurement but it is part of the final experimental sequence and it is kept in order to reach a comparable experimental situation. In this part the long-period lattice site is split within 15 ms by increasing $V_{x,s} = 30(1) E_r$, together

Figure S1. Finite-frequency corrections to the zeroth-order Floquet Hamiltonian. Solid lines are the time evolution of the charge $\langle \hat{Q}_1 \rangle$ (blue) and gauge field $\langle \hat{\mathbf{x}} \rangle$ (red) according to (S.8) with the two lowest-order terms of the effective Floquet Hamiltonian (S.7). Diamonds show the stroboscopic dynamics of the full time-dependent Hamiltonian (3) from the main text for resonant driving at $h \omega = \sqrt{U^2 + 4 J^2} \approx 1.04\,U$ and $U/J = 7$. Gray lines show the ideal solution Eq. (6) of the zeroth-order effective Hamiltonian (S.6) for $\phi = 0$.

Figure S2. Lattice setup. A superlattice (black) is composed of a lattice with wavelength $\lambda_s$ (blue) and a lattice with wavelength $\lambda_1 = 2\lambda_s$ (red) with a relative phase of $\varphi_{SL}$. In addition a second lattice with wavelength $\lambda_l$ is superimposed with a relative phase of $\varphi_{\text{mod}} = -\pi/4$, such that the on-site energy of the left double-well site can be modified by a change in the lattice depth $V_{\text{mod}}$ [Eq. (S.11)] illustrated by the red and gray shadings. The dashed lines show the superlattice for $V_{\text{mod}} = 0$. 

In the infinite frequency limit $\omega \to \infty$, we can extract the renormalization of the tunnel coupling $J_\nu = J' J_{\nu}(\chi)$, which scales with the $\nu$-th order Bessel function of the first kind.
with the transversal lattices $V_{x,s} = V_{z,s} = 60(2) E_{r,s}$ and merged again in 15 ms. Then, the modulation lattice is turned on to the mean value $V^{(0)}_{\text{mod}} = 15.0(2) E_{r,1}$ in 20 ms and the superlattice phase $\varphi_{\text{SL}}$ is tuned to a value such that $\Delta \gg J$ and the particle localizes to one site during the subsequent splitting process, where the short lattice is ramped to $V_{x,s} = 40(1) E_{r,s}$ in 10 ms. Then, $\varphi_{\text{SL}}$ is non-adiabatically raised to reach the final energy offset between the two sites $\Delta$, in 10 ms. The time evolution begins with a rapid coupling of the two sites by decreasing $V_{x,s} = 9.5(1) E_{r,s}$ in 100 $\mu$s and starting the modulation with frequency $\omega = 2\pi \times 4122$ Hz. This procedure was repeated for various modulation amplitudes $A_{\text{mod}}$. For the detection, we freeze the motion in 100 $\mu$s and use site-resolved band mapping to determine the site occupations $N_L$ and $N_R$ from which we determine the site imbalance $I = (N_L - N_R)/(N_L + N_R)$ [S3, S4].

For finite driving frequencies, corrections to the resonant driving condition have to be taken into account $\nu \hbar \omega = \sqrt{\Delta^2 + 4J^2}$ with $\nu \neq 0$. Thus, we have experimentally determined $\Delta$, using a spectroscopic measurement by varying the energy offset at constant driving frequency $\omega$. Measurements of a time-trace for different modulation amplitudes $A_{\text{mod}}$ were performed for $\nu \in \{0,1,2\}$. Each resulting imbalance time-trace represents an average over the time-traces of all double wells in a 3D array. For technical reasons each double well in this 3D array experiences different energy offsets between neighboring sites. To model this effect, we assume that these energy offsets are distributed according to

$$G_\Delta(\delta) = \frac{1}{\sqrt{2\pi} \Delta^2} \exp\left(-\frac{\delta^2}{2\Delta^2}\right), \quad (S.14)$$

with a standard deviation $\Delta$. Taking this effect into account, the renormalized tunneling rates $J_\nu$ were extracted by fitting an average of $S = 10$ sinusoidal functions

$$I_\nu(t) = \frac{A}{S} \sum_{n=1}^S \sin \left(\sqrt{\delta_n^2 + 4J_\nu^2} t/\hbar + \xi\right) + I_0 \quad (S.15)$$

to the data, where $A$ is the oscillation amplitude, $\delta_n$ a random sample of the tilt distribution (S.14), $J_\nu = J_\nu(\chi)$ is the renormalized tunnel coupling, $\xi$ is an initial phase due to the finite initialization ramp time, and $I_0$ the imbalance offset of the oscillation. The fit uses five free fit parameters: $J_\nu$, $A$, $\Delta$, $\xi$, and $I_0$. To estimate the confidence interval we performed a bootstrap with 1000 realizations for different sets of $\{\delta_n\}$, a Gaussian error in the detection of the imbalance of 0.05 and randomly-guessed fit start values for $J_\nu$. In Fig. 2 (main text), $J_\nu$ is normalized to its respective bare tunnel coupling strength $J_{\nu}'$ calculated from the double-well Wannier functions based on the calibrated lattice parameters: $J_{\nu}'/h = 490$ Hz, $J_{\nu}'/h = 520$ Hz and $J_{\nu}'/h = 563$ Hz, which where slightly different for the three data sets, $\nu = \{0,1,2\}$. The data points and error bars in Fig. 2 show the median and the $1\sigma$-confidence interval from the bootstrap analysis. The conversion $\alpha$ of the driving amplitude $A = \alpha A_{\text{mod}}$ is calibrated by a fit of the function $J_{\nu}(\alpha \text{fit}/A_{\text{mod}}/\omega)$ to the experimental results for $\nu = 0$. A comparison of the fitted to the calculated value results in $\alpha_{\text{fit}}/\alpha_{\text{calc}} \approx 1.09(2)$. The order of magnitude of this value is consistent for all measurements and is most likely attributed to the non-linear behavior of $\Delta(V_{\text{mod}})$. In addition, also the tunneling rate depends on the value of the modulation lattice depth $J_{\nu}'(V_{\text{mod}})$ and changes therefore slightly during a driving period. These effects are especially important for large values $V_{\text{mod}} \sim V_{x,1}$, because the modulation lattice changes not only the on-site energy, but also the combined lattice potential’s shape $V(x)$.

Species-dependent energy offset for the $Z_2$ two-site model

The species-dependent energy offset is realized by a combination of a magnetic gradient and a superlattice with a relative phase $\varphi_{\text{SL}}$. The magnetic gradient induces opposite energy offsets $\Delta_M$ for the two hyperfine-states that encode the $\omega$- and $f$-particles, i.e. $|F = 1, m_F = \pm 1\rangle$, because these states have opposite magnetic moments. The experimental implementation requires an energy offset of zero between neighboring sites for the $\alpha$-particles. Thus, the species-independent energy offset from the superlattice potential is chosen, such that it compensates the tilt $\Delta_M = -\Delta_{\text{SL}}$. The energy offset between neighboring sites for the $f$-particles is then $\Delta_f = \Delta_M + \Delta_{\text{SL}}$, which needs to be matched with the interaction energy $U$.

Tight binding description of the two-site two-particle model

The working principle of the experimental implementation for the $Z_2$ two-site model is discussed in detail in the main text. Here, we present a tight-binding description including the full set of experimental parameters and terms from the extended Bose-Hubbard model, which are known to exist for interacting particles in lattices [S5]. The extended Bose-Hubbard model takes into account that the Wannier functions $w_{\mu}(r)$ of particles on different sites $\mu$ overlap. Hence, this leads to nearest-neighbor interactions $u_{LR} = g \int |w_L(r)|^2 |w_R(r)|^2 \, dr$ and density-assisted tunneling, where the tunnel matrix element $J = J_{\nu} + \delta J$ gets modified by $\delta J = g \int w_{\mu}^2(r) w_L(r) w_R(r) \, dr$. In addition, two-particle hopping processes arise with a strength $u_{LR}$, where either two particles on the same site tunnel simultaneously to a neighboring site, or two neighboring particles exchange their positions. Note, the two-particle hopping processes directly break the $Z_2$ symme-
try but for our parameters the value is small, \( u_{LR}/J \approx 0.03 \), compared to the experimental timescales. All terms of the extended Bose-Hubbard model are proportional to the effective interaction strength \( g = 4\pi h^2 a_s/m \), with \( a_s \) the inter-species s-wave scattering length and \( m \) the mass of a rubidium atom. In summary, the time-dependent Hamiltonian (3) generalizes to

\[
H(t) = -J (|3\rangle \langle 1| + |4\rangle \langle 2| + |4\rangle \langle 3| + |2\rangle \langle 1| + \text{h.c.}) \\
+ U (|1\rangle \langle 1| + |4\rangle \langle 4|) \\
+ A \cos(\omega t) (|1\rangle \langle 1| + |2\rangle \langle 2| + |3\rangle \langle 3|) \\
+ \Delta_{SL} (|1\rangle \langle 1| + |2\rangle \langle 2| + |3\rangle \langle 3|) \\
+ \Delta_M (|1\rangle \langle 1| - |2\rangle \langle 2| + |3\rangle \langle 3|) \\
+ u_{LR} (|1\rangle \langle 4| + |2\rangle \langle 3| + |4\rangle \langle 1| + |3\rangle \langle 2| + |2\rangle \langle 2| + |3\rangle \langle 3|). \tag{S.16}
\]

To realize the \( Z_2 \) two-site model, all parameters in Hamiltonian (S.16) need to be calibrated separately. Especially, the magnetic gradient needs to be matched with the superlattice energy offset \( \Delta_M = -\Delta_{SL} \) to obtain zero tilt for the \( a \)-particles. This leads to a tilt \( \Delta_f = \Delta_{SL} + \Delta_M \) for the \( f \)-particles, which needs to be matched with the interaction energy \( \Delta_f = U \). Furthermore, we drive the system resonantly at \( \hbar \omega = \sqrt{\Delta_f^2 + 4J^2} \). In the following we describe how the parameters are calibrated.

### Parameter calibrations

#### Tunnel coupling in a symmetric double well

A single particle is localized on one site of a double well with superlattice parameters \( V_{x,1} = 35(1) E_{r,1} \) and \( V_{x,s} = 9.5(1) E_{r,s} \). From the measured tunnel oscillation frequency (Fig. S3a) we infer a precise value of the lattice depth \( V_{x,s} \). The measurement sequence is analogous to the sequence for the renormalized tunnel coupling discussed before, where we measure the site-imbalance at various times. This time trace was then modeled using a two-site Bose-Hubbard Hamiltonian, including Gaussian distributed energy offsets \( \mathcal{G}_\Delta(\delta) \) [see Eq. (S.14)]. To this end, time traces for 256 randomly-sampled values \( \delta_n \) were calculated and its median least-square fitted to the measured data set. The results for the free parameters of this fit are \( V_{x,s} = 9.5(1) E_{r,s} \) and a tilt distribution with a standard deviation of \( \Delta_\pi/h = 0.44(4) \) kHz.

#### On-site interaction energy

The on-site interaction energy \( U \) was calibrated by a measurement of superexchange oscillations. To this end, we localize two distinguishable particles to the left and right site of a double well. Then we couple the two sites and observe the oscillations of individual species. For a known single-particle tunneling rate, the interaction energy can be inferred from the measured oscillation frequency, which scales as \( J^2/U \). In the experiment, the on-site interaction energy depends strongly on the chosen lattice parameters, thus a measurement in the final configuration is preferred. Directly using \( a \)- and \( f \)-particles is not possible as they have
opposite magnetic moments and the $f$-particle experiences an energy offset between neighboring sites. However, we can use a microwave-driven adiabatic passage to transfer the $f$-particle to the state $|F=2, m_F=+1\rangle$ in the $F=2$ manifold, such that it has the same magnetic moment as the $a$-particle. In this configuration we can measure site-imbalance traces of the superexchange oscillations by $F$-state selective imaging (Fig. S3b) and calibrate the interaction energy $U = 3.85(7)\text{kHz}$.

**Magnetic and superlattice tilts** — The energy offset between neighboring sites of the double well is given by a combination of the magnetic gradient, the tilted superlattice potential and the modulation lattice. The modulation lattice is static at $V^{(0)}_{\text{mod}} = 15.0(2)E_r,1$ and the resulting tilt can be fully compensated by the superlattice phase. Starting form this situation, we introduce a magnetic gradient and measure the imbalance $I_m$ of a single $a$-particle. At the superlattice phase where $I_m = 0$ the tilts are identical $\Delta_{\text{SL}} = \Delta_M$. Then we measure the energy offset $\Delta_f = \Delta_{\text{SL}} + \Delta_M$ introduced for the $f$-particles by modulation spectroscopy with the modulation lattice (Fig. S3c). From the measurement we fit the resonance frequency $\omega$. This procedure was iteratively applied until the condition $\Omega = \sqrt{\omega^2 - 4T^2}$ was fulfilled.

**Characterization of the modulation amplitude** — First, $V_{\text{mod}}$ was determined by a spectroscopic measurement of the energy difference between neighboring sites induced exclusively by the modulation lattice. Therefore, a single particle was loaded to the lower site of a double well with $\varphi_{\text{SL}} = 0$ and $V_{\text{mod}} \approx 15E_r,1$ at a fixed modulation lattice phase of $-\pi/4$ (Fig. S3d). From this outcome a first estimate of the modulation amplitude can be made, however, more accurate results can be gained by determining the oscillation frequency of driven tunnel oscillations. We choose $\nu = 0$ as in this case the driving is always resonant and the observable depends strongly on the driving strength. We start with zero driving $A = 0$ and verify that the oscillation frequency in the combined potential agrees with the theoretical expectations (Fig. S3e). Then, we measure a set of driven tunnel oscillations at $A \neq 0$ (Fig. S3f-h). The driving amplitude was then fitted to match the measured oscillation frequencies. In this model we also take inhomogeneous energy offsets into account, which are modeled to be Gaussian distributed [Eq. (S.14)].

**Initial state preparation** — As discussed in the main text, we first prepare an initial state $|\psi_0^a\rangle = (|1\rangle + |2\rangle)/\sqrt{2} = |a,0\rangle \otimes (|f,0\rangle + |0,f\rangle)/\sqrt{2}$, where the $f$-particle is in an eigenstate of the electric field operator $\hat{z}$ and the $a$-particle is localized to the left site of the double well. This initial state probes a single subsector of the model. Experimentally we achieve this by realizing the groundstate of a specially-designed static model. To this end, we start with two distinguishable atoms $a$ and $f$ on a single long period lattice site (sequence of the Z$_2$ double-well model, see below). Then, the magnetic gra-

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**Experimental sequence for the Z$_2$ two-site model**

The experimental sequence for the data presented in Fig. 3 and 4 in the main text starts by loading atoms in the $|F=1, m_F=-1\rangle$ state into a three-dimensional (3D) optical lattice with lattice depths $V_{x,1} = 30(1)E_r,1$, $V_y = 35(1)E_r,s$ and $V_z = 50(2)E_r,s$. These initial ramps are exponentially shaped and carried out in a time-segment of 50 ms length. The 1/e-time of the exponential ramp is $\tau = 5\text{ms}$ for the horizontal lattices ($x$, $y$) and $\tau_z = 1.5\text{ms}$ for the vertical lattice ($z$). This loading procedure was optimized for a maximal amount of doubly-occupied sites, while loading a negligible amount of tripply- and a small amount of singly-occupied sites. Later, each doubly-occupied site will realize a $\text{ doubloonsinglon}$.

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**Figure S4. Experimental sequence.** a Separate illustrations of the individual experimental steps for singly- and doubly-occupied double wells. The dots denote atoms in the $F=1$ manifold of $^{85}\text{Rb}$ in the $m_F=-1$ (blue), $m_F=0$ (gray) and $m_F=+1$ (red) state. The red and blue triangles illustrate the respective potential induced by the magnetic gradient. b Parameter ramps for the numbered steps in a. c Measured modulation lattice intensity, with sudden jump at $t=0$.

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<th>$\Delta_M$</th>
<th>$V_{\text{SL}}$</th>
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<td>$+1.5(2)\text{kHz}$</td>
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**Table S1.** Experimental parameters used in the measurement, with $E_r,s$ and $E_r,1$ representing the lattice depths of the two lattices, respectively.
first apply a series of microwave-driven adiabatic passages to transfer the atoms to the \(|F=1, m_F=0\) state. Subsequently, the short lattice in x-direction was ramped up to \(V_{x,x} = 40(1) E_{r,s}\) within 15 ms with a superlattice phase \(\varphi_{\text{SL}}\), such that both particles are localized to a single site of the double well. At the same time the lattice depths along the orthogonal directions are raised to \(V_y = 100(5) E_{r,s}\) and \(V_z = 120(6) E_{r,s}\) to further increase the on-site interaction energy. Finally, an adiabatic passage of microwave-mediated SCC was performed within 100 ms. Note, single atoms on a double well are not affected by SCC and remain in the \(|F=1, m_F=0\) state, which allows us to independently detect singly- and doubly-occupied sites. After the SCC sequence, the double-well sites are merged again in 15 ms by adiabatically switching off \(V_{x,s} = 0\). Then, a magnetic field gradient of \(B'/\hbar \approx 5.9(3) \text{kHz}/\lambda_n\) was applied within 120 ms. Simultaneously, the modulation lattice is turned on to \(V_{\text{mod}} = 6.0(1) E_{r,1}\) and the superlattice phase ramped up to the final value. The initial state is prepared by ramping up \(V_{x,s} = 9.50(1) E_{r,s}\) in 10 ms as described above. Then, the modulation is started with a sudden jump (50 μs) of the modulation intensity to obtain the correct initial driving phase of \(\phi = 0\) (Fig. S4c). The modulation is performed with an amplitude of \(A_{\text{mod}} = 9.0(1) E_{r,1}\) around a mean value of \(V_{\text{mod}} = 15.0(1) E_{r,1}\) and a driving frequency of \(\omega = 2\pi \times 4320 \text{Hz}\). The initial state evolves in this driven model, for different times \(t\), approximating the time-dynamics in the \(Z_2\) two-site model. Subsequently, the tunneling dynamics was inhibited by rapidly increasing the potential barrier between the two sites \(V_{x,s} = 40(1) E_{r,s}\) in 50 μs, followed by a site-resolved band-mapping detection in combination with a Stern-Gerlach species separation [S3, S4, S7].

**NUMERIC TIME EVOLUTION**

The analysis of the Floquet model is contrasted by a full, time-dependent numeric analysis of the two-site two-particle model. We use formulation (S.16), which is very close to the experimental realization including the superlattice, magnetic gradient and first-order corrections to the Bose-Hubbard model.

We use the experimentally calibrated parameters \(J', U, \Delta_M, \Delta_{\text{SL}}, \omega\) together with calculated extended Bose-Hubbard parameters \(\delta_j\) and \(u_{\text{LR}}\) to perform a numerical time evolution. The time evolution is performed using the Trotter method by applying the quasi-static time evolution operator \(U_n = \exp\{-i H(t_n) \Delta t / h\}\) for each time point \(t_n = n \Delta t\) with \(n \in \mathbb{N}\). The time step \(\Delta t = 2\pi/(\omega \Delta t)\) is chosen to subsample the driving frequency with \(s \sim 50\). For the initial state \(|\psi_{\text{init}}\rangle\), the groundstate of the configuration for the initial state preparation is chosen as explained above. This leads to an initial state of \(|\psi_{\text{init}}\rangle \approx 0.70|1\rangle + 0.71|2\rangle + 0.087|3\rangle + 0.036|4\rangle\).

A single calculated time-trace shows strong oscillatory behavior (Fig. S5), which is not present in the stroboscopic calculation of the Floquet Hamiltonian and can be attributed to the micromotion. The micromotion is an additional dynamics with a period equal to the driving frequency.

The experimentally measured observables are averaged over many realizations of double wells. Due to inhomogeneities in the energy offsets between neighboring sites, the traces of individual double wells are different. We model these inhomogeneities with a Gaussian distribution of tilts as described in Eq. (S.14) with a standard deviation of \(\Delta_{\varphi}\), extracted from the measurement of tunnel oscillations. From this distribution we randomly draw 1000 tilt values and average the observables.

**SYMMETRY-BREAKING CORRECTION TERMS IN THE EXPERIMENTAL REALIZATION**

It is essential for experimental realizations of LGTs that the underlying gauge symmetry is sufficiently conserved during the experiment. This puts high requirements on experimental implementations to suppress symmetry-breaking terms. In the presented scheme, we find two main sources for symmetry breaking: finite-frequency corrections to the effective Floquet Hamiltonian and first-order corrections to the Bose-Hubbard model.

The first-order correction to the Floquet Hamiltonian for finite-frequency drive was derived above [Eq. (S.7)]. The corrections contain diagonal detuning, correlated hopping and direct exchange coupling terms, which scale as \(J^2/\hbar \omega\). The driving frequency is connected to the interaction energy in this realization. Therefore the corrections can be reduced by either increasing the interaction.
energy or decreasing the tunnel coupling \( J \). The effect on the \( \mathbb{Z}_2 \) symmetry is studied in Fig. S6a for \( U/J = 7 \), similar to the experimental parameters, and an initial state in the \( g_1 = -1 \) subsector.

The additional terms from the extended Bose-Hubbard model were introduced in Eq. (S.16) and are nearest-neighbor interactions, correlated tunneling and a direct exchange coupling. The terms scale with the square overlap of neighboring Wannier functions, \( u_{LR} = \int |w_L(r)|^2 |w_R(r)|^2 \, dr^3 \), and vanish therefore faster than the tunnel coupling \( J \), when reducing the Wannier overlap. In Fig. S6b the correction terms were added to the infinite-frequency Floquet model with strength \( u_{LR} = 0.03 \, J \) and the time evolution of the expectation value of \( \hat{G} \) was calculated numerically.

**\( \mathbb{Z}_2 \) LGT IN 1D**

The \( \mathbb{Z}_2 \) double well is a minimal two-site model, which consists of one link and a single matter particle. A direct generalization of this model arises when extending the double well to a chain by adding more links. We discuss the dynamics for the resulting \( \mathbb{Z}_2 \) chain described by Eq. (1) in the main text in different regimes \( J_f/J_a \) and for two initial states analogous to the ones used in the main text to study the two-site model. The dynamics of the system is analyzed by a calculation of the numeric time evolution with a Hamiltonian for 13 sites. Initially, the matter particle is always in the center of the system labeled \( j = 0 \).

We first start with an initial state, where all links are in the groundstate of the electric field operator \( -\hat{\tau}_f^{j,j+1} \). Therefore \( \tau_{f,j+1}^- = 1 \) for all sites \( j \). Together with the matter particle at site \( j = 0 \), this leads to \( g_j = +1 \) for all \( j \neq 0 \) and a local static charges at \( j = 0 \) described by \( g_0 = -1 \). As \( \hat{G} \) commutes with the Hamiltonian (1), the static charges \( g_j \) are conserved. Thus, when the matter particle traverses a link its electric field value needs to change. This process is associated with an energy cost proportional to \( J_f \). Figure S7a summarizes the result for two different regimes. In the regime \( J_f \ll J_a \), the energy cost of flipping a traversed link, changing the value of \( \tau_f \), is small and the matter particle can freely expand, which is clearly visible in the expectation value of \( \hat{Q}_f \). The same cone shape is visible in the expectation value of the electric field operator \( \langle \hat{\tau}_f \rangle \). Inside the cone \( \langle \hat{\tau}_f \rangle \approx 0 \) because the particle either traversed the link or not. In the opposite regime, where \( J_f > J_a \), the energy cost for flipping \( \tau_f \) is high, and also linearly increases with the number of traversed links. Therefore, the matter particle stays confined to the local static charge at site \( j = 0 \).

Analyzing the results on the two-site model, an initial state is examined, where each link is in an eigenstate of the gauge field operator \( \hat{\tau}_f \). In this situation, instead of a single subsector a superposition of two subsectors is probed. We again investigate the two different regimes and present the results in Fig. S7b. In the regime \( J_f \ll J_a \), the matter particle still expands freely, while we again observe confinement of the matter particle in the regime \( J_f > J_a \). The expectation value of the electric field operator \( \langle \hat{\tau}_f \rangle = 0 \) for all sites and times. The expectation value of the gauge field operator \( \langle \hat{\tau}_f \rangle \), on the other hand, shows dynamics. This dynamics can be explained on the level of a single link prepared in one of the eigenstates of \( \hat{\tau}_f \). The coupling \( -J_f \hat{\tau}_f \) leads to Rabi os-
cillations between $\tau^z = \pm 1$. In the presence of the matter particle the Rabi oscillations are detuned, which leads to faster oscillations with a reduced amplitude.