Non-Gaussian correlations imprinted by local dephasing in fermionic wires

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We study the behavior of an extended fermionic wire coupled to a local stochastic field. Since the quantum jump operator is Hermitian and quadratic in fermionic operators, it renders the model solvable, allowing investigation of the properties of the nonequilibrium steady state and the role of dissipation-induced fluctuations. We derive a closed set of equations of motion solely for the two-point correlator; on the other hand, we find, surprisingly, that the many-body state exhibits non-Gaussian correlations. Density-density correlation function demonstrates a crossover from a regime of weak dissipation characterized by moderate heating and stimulated fluctuations to a quantum Zeno regime ruled by strong dissipation, which tames quantum fluctuations. Instances of soluble dissipative impurities represent an experimentally viable platform to understand the interplay between dissipation and Hamiltonian dynamics in many-body quantum systems.

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The interplay of quantum many-body dynamics and decoherence is essential for understanding a broad range of physical phenomena: From suppression of weak localization of electrons due to coupling to phonons and electron-electron interactions [1–4], to the realization of light-induced topological phases [5–9], to the operation of quantum optical devices [10], and to the implementation of quantum computers [11] and simulators [12]. Competing effects of quantum entanglement and decoherence are also at the heart of questions of quantum control and quantum nondemolition measurements. One of the most surprising recent findings in this field is the effect of weak measurements on quantum fluctuations and on the decay rate of an excited state into a bosonic bath [13,14]. Depending on system parameters, this decay rate can be either inhibited or enhanced by weak measurements, with the two phenomena referred to as quantum Zeno and anti-Zeno effects [15–20], respectively. Several powerful techniques have been applied to the analysis of the interplay of quantum dynamics and decoherence in the many-body Zeno problem [22–26], including Lindblad quantum master equations, memory kernel formalism, Keldysh diagrammatic techniques, and renormalization group approaches [27–44]. Examples of quantum many-body systems with decoherence that allow exact theoretical solutions and can be realized experimentally are particularly valuable, since they enable a nonperturbative analysis of competing effects of dissipation. So far, such systems—including Bethe ansatz solution [45] of noisy tight-binding fermions [46], boundary driven quantum spin chains [47,48], and non-Hermitian Richardson-Gaudin magnets [49]—have been few and far between. On the experimental side, modern solid-state and cold-atom platforms—including atomic BEC with local losses, disordered trapped ion strings with dissipation facilitated transport properties, and the realization of dissipative scanning-gate microscopes with 6Li atoms—already enable investigation of effects of dissipation both in the form of losses and of dephasing [33,34,38,50–53].

In this work, we present an example of such a system, in which a local stochastic field couples to the electron density on a single site in a one-dimensional fermionic chain (see Fig. 1). This system has recently been realized experimentally [38]. Here we provide a theoretical analysis of this model and make predictions which can be tested with currently available experimental platforms.

Before entering the technical details of our work, we provide an overview of the key results. When a stochastic field couples locally to the electron density, it introduces two opposing effects on fluctuations in the difference of the number of particles between the left and right parts of the fermionic chain. The stochastic field provides local heating and thus enhances fluctuations; on the other hand, it performs “weak measurements” of the electron number, which hinders particle propagation across the site with decoherence, suppressing relative number fluctuations. We find that the competition between these two effects leads to the existence of two distinct regimes of dynamics: For weak dissipation, number fluctuations become enhanced with increasing decoherence; in contrast, for strong dissipation, fluctuations become suppressed. The two regimes are separated by a sharp crossover displayed in Figs. 5(b) and 5(c). A special feature of the local dephasing problem from the mathematical viewpoint is that the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy is closed, meaning that the equation of motion for a given n-point correlation function can be expressed through correlators whose order is n or less; for instance, Eq. (3) represents the evolution of the two-body correlation function. Such
a situation typically occurs for Gaussian systems [54], where high-order correlators factorize in terms of the two-body correlators. However, one does not expect the state to be Gaussian. Indeed, for a given realization of the fluctuating field, the state is Gaussian since the system evolves under a noninteracting Hamiltonian with time-dependent stochastic potential, while after averaging over different realizations of the noise, the state becomes non-Gaussian [55]. Figure 5(b) further supports such conclusion. This remarkable circumstance that the BBGKY hierarchy is closed, despite the emergence of non-Gaussian correlations, allows us to investigate the effects of dissipation nonperturbatively.

Throughout this Rapid Communication, we develop several complementary approaches for solving the model in Fig. 1. Let us start with the formalism of the quantum master equation (QME). The fermionic wire is described with the following Hamiltonian:

\[ \hat{H}_0 = \sum_k \xi_k \hat{c}_k \hat{c}_k^\dagger, \quad \xi_k = -2J \cos k - \mu. \]  

(1)

The operator \( \hat{c}_k^\dagger \) (an)ihilates (creates) a fermion with momentum \( k \) (the distance between neighboring sites is set to unity). \( \mu \) is the chemical potential. \( J \) is the hopping between nearby sites and it sets the unit of energy. The Planck constant \( \hbar = 1 \) throughout this Rapid Communication. The density matrix evolves according to (QME)

\[ \frac{d \hat{\rho} }{dt} = -i[\hat{H}_0, \hat{\rho}] + \gamma \left( \hat{L} \hat{\rho} \hat{L}^\dagger - \frac{1}{2} \{ \hat{L}^\dagger \hat{L}, \hat{\rho} \} \right), \]  

(2)

where \( \hat{L} = \hat{n}_0 = \frac{1}{\sqrt{N}} \sum_{p,q} \hat{c}_p \hat{c}_q^\dagger \) is the quantum jump operator representing the local dephasing. It is worth emphasizing that, although the Hamiltonian (1) is noninteracting, the complexity of the underlying evolution is due to local quartic “dissipative interactions” arising from the second term in Eq. (2). Significant simplification occurs by noting that \( \hat{L}^\dagger = \hat{L} \) and the dynamics of any observable \( \hat{O} \) can be written as

\[ \frac{d}{dt} \hat{O} = \frac{d}{dt} \text{tr} \hat{\rho} \hat{O} = i[\hat{H}_0, \hat{O}] + \frac{\gamma}{2} [\hat{L}, [\hat{O}, \hat{L}]]. \]

As long as the Hamiltonian and the quantum jump operator are both quadratic in the fermionic operators, it follows that the evolution of any \( n \)-point correlation function

\[ \Gamma_{\xi_1 \xi_2 \cdots \xi_n}^{kk'}(t) \equiv \langle \hat{c}_{k_1}^\dagger \hat{c}_{k_2}^\dagger \cdots \hat{c}_{k_n}^\dagger \hat{c}_{k_n} \rangle_{\xi} \]

can be expressed through operators whose order is \( n \) or less; in other words, we can write down a closed system of equations of motion for any given order of interest [56]. In particular, for \( \Gamma_{kk'}(t) \equiv \langle \hat{c}_{k_1}^\dagger \hat{c}_{k_1} \rangle_{\xi} \) we get

\[ \frac{d}{dt} \Gamma_{kk'}(t) = i(\xi_k - \xi_{k'}) \Gamma_{kk'} + \gamma / N \sum_{p \neq q} \Gamma_{pq}, \]

\[ -\frac{\gamma}{2N} \sum_q (\Gamma_{q,k} + \Gamma_{k,q}). \]  

(3)

Note that the total number of fermions \( N_0 = \sum \Gamma_{kk'} \) is conserved. \( N_{\text{tot}} = 0 \). Although we are able to write down a closed equation of motion for the two-body correlation function, we find that the many-body density matrix is genuinely non-Gaussian: Even if the initial state, such as a filled Fermi sea, is Gaussian, the local dephasing will imprint non-Gaussian correlations. A possible way to see this is to investigate higher-order correlation functions. For example, in Ref. [57] we demonstrate, by explicitly deriving the corresponding equation of motion, that the four-point correlator cannot be factorized (using Wick’s theorem) in terms of the two-body correlation functions. The mentioned equation turns out to be a challenge for numerical treatments of extended systems, motivating the development of an alternative approach for investigating higher-order correlators below.

We turn to explore Eq. (3) both numerically and analytically. In Fig. 2, we plot the evolution of the fermionic density profile \( n_{kk'}(t) \): The heater locally perturbs the system, resulting in the emission of a ballistic density front (its velocity equals to the maximum group velocity \( v_{\text{gm}} = 2J \)); at the same time, in the vicinity of the dissipative impurity, a.
nonequilibrium steady state (NESS) forms [58]. To investigate
NESS properties numerically, we let the system evolve up to
times $\sim N/2v_m$. In momentum space, since the total energy
is not conserved, we find that the initial Fermi sea becomes
redistributed towards large-momenta states, as shown in the
inset of Fig. 3(a). Interestingly, the system preserves the Fermi
device, and its distribution function becomes nonthermal. We
characterize the cumulative effect of the heater by the fraction
$f$ of fermions removed from the Fermi pocket. Figure 3(a)
shows that $f$, evaluated at the longest simulation time $t = N/2v_m$, exhibits a crossover as a function of $\gamma$, switching from the
anti-Zeno regime at weak dissipation to the Zeno regime
at strong dissipation.

We now discuss the approach towards the NESS. As shown
in Figs. 2(b) and 2(c), our numerical analysis indicates that
in regimes of both weak (I) and strong (II) dissipation, the
system exhibits $t^{-2}$ relaxation (on top of oscillatory behavior); see [57] for further details. Qualitative differences are present
in the intermediate-time dynamics, as one can inspect from
Figs. 2(b) and 2(c): For stronger dissipation, the density at
the origin demonstrates slower evolution. Indeed, strong noise
drives out of resonance hopping processes involving the dissipative site, implying a “trapping” of particles jumping to the
origin and a suppression of transport across this site.

Although the dynamics encoded in Eq. (3) can be
efficiently simulated numerically, they represent a challenge for
analytical solutions. Remarkably, diagrammatic field theory
provides an alternative derivation of Eq. (3), and allows one to
extract analytically the NESS properties. We start by noticing
that the Lindbladian evolution in Eq. (2) is equivalent to the
stochastic Schrödinger equation (SSE) with Hamiltonian:

$$\hat{H}_L(t) = \hat{H}_0 + \hat{V}(t), \quad \hat{V}(t) = \xi(t) \gamma \hat{n}_0, \quad (4)$$

where $\xi(t)$ is a white noise with $\langle \xi(t_1) \xi(t_2) \rangle \equiv \gamma \delta(t_1 - t_2)$. To compute the dynamics of any observable $\hat{O}$, one needs to perform averaging over the noise:

$$\langle O(t) \rangle = \langle \langle \hat{O} \rangle \rangle \xi \xi, \quad \langle \hat{O} \rangle = \text{tr}(\hat{O} \hat{n}_0),$$

where $\hat{O}_i$ is the density matrix for a given noise realization $\xi$). The conservation of the total number of fermions follows from $\langle \hat{n}_0, \hat{H}_L(t) \rangle = 0$. The initial density matrix at $t = 0$ is chosen to be a filled Fermi sea.

We now develop a nonequilibrium diagrammatic technique inspired by the treatment of disordered fermionic systems [59]. For the retarded Green’s function, defined as $G^{GR}_R(k, k') \equiv -i\theta(t - t') \langle \hat{c}_k(t) \hat{c}^\dagger_{k'}(t') \rangle$, the Dyson series [59] reads [see Fig. 4(a)]

$$\hat{G}^R = \sum_{m,n=0} \left( G^R_0 \circ \hat{V} \right)^m \circ \hat{G}^R_0 \circ \left( \hat{V} \circ G^R_0 \right)^n. \quad (5)$$

Here $G^R_0$ is the unperturbed retarded Green’s function. Because
the underlying problem is far from equilibrium, we will also need to compute the Keldysh Green’s function, $G^{K}_R(k, k') \equiv -i\theta(t - t') \langle [\hat{c}_k(t), \hat{c}^\dagger_{k'}(t')] \rangle$:

$$\hat{G}^K = \sum_{m,n=0} \left( G^R_0 \circ \hat{V} \right)^m \circ \hat{G}^K_0 \circ \left( \hat{V} \circ G^K_0 \right)^n. \quad (6)$$

An element of this series is schematically depicted in
Fig. 4(b). Because the noise is local in space and time, aver-
aging results in the self-energy known as a self-consistent
Born approximation (SCBA), shown in Fig. 4(c), which in
our case holds exactly. For further technical details we refer to
Ref. [57], where, in particular, we show that the equation for
the equal-time Keldysh Green’s function reduces to Eq. (3).
From (5), we compute the NESS retarded Green’s function in the frequency domain [Eq. (S12) in Ref. [57]], allowing one to extract, for example, the local density of states (DOS) at the impurity site \( \nu_0(\omega) \equiv -\frac{1}{\pi} \text{Im} G_{\omega}^{\text{nr}}(0, 0) \) [see Fig. 3(c)]. For \( \gamma \neq 0 \) it develops tails at high frequencies; the presence of low-energy modes confirms the aforementioned power-law dynamics nearby the NESS [see Figs. 2(b) and 2(c)]. This structure of the DOS can be directly measured in state-of-the-art solid-state experiments, which can access the dynamics of nonequilibrium quantum impurities [60]. Similarly, from Eq. (6), we derive an expression for the NESS Keldysh Green’s function [Eqs. (S15) and (S17) in Ref. [57]], from which we can compute, for instance, the spatial profile of the fermionic density. Figure 3(b) shows that the analytical expression for the density at the origin, \( n^{\text{nr}}_0 \), is in remarkable agreement with the result numerically computed from (3). For \( k_F = \pi/4 \) \( (k_F = 3\pi/4) \), \( n^{\text{nr}}_0 \) is a monotonically decreasing (increasing) function of \( \gamma \), and it remains finite even for very strong dissipation. It is compelling that different single-body observables, such as \( n^{\text{nr}}_0 \) and \( f \), exhibit qualitatively distinct behavior [see Figs. 3(a) and 3(b)]. Although dissipation occurs locally in space, we find that in the NESS, the fermionic density profile demonstrates long-range behavior following a \( x^{-1} \) fit with superimposed Friedel’s oscillations [see inset of Fig. 2(a)]. These two features are related to the fact that the system preserves the Fermi-edge singularity, shown in the inset of Fig. 3(a). This intrusive effect reminds one of the situation of a static impurity [61,62], and might be of relevance for experimental manipulations of quantum many-body systems subject to local dissipation.

We now turn to numerical simulation of the SSE which offers a complementary physical viewpoint. An infinitesimal time step is performed via “Trotterizing” the evolution operator: \( \hat{U}_{t+\delta t} = e^{-i\delta t \hat{H}_0} e^{-i\delta t \hat{W}} e^{-i\delta t \hat{H}_0} \) with a time step \( \delta t \ll \min(\gamma^{-1}, J^{-1}) \). By \( \delta W \) we denote a Wiener process [63] with \( \delta W \delta W = \gamma \delta t \). Figure 5(a) shows that the typical evolution of the density at the origin exhibits pronounced fluctuations and is far from the Linbladian result obtained from Eq. (3). Only after averaging over many \( N_t \approx 10^4 \) for \( \gamma = J \) noise realizations the two approaches start to match [57]. This physical picture suggests that these pronounced fluctuations dominate the aforementioned algebraic behavior and nonequilibrium crossover.

Simulations of the SSE allow direct investigation of correlation functions associated with density fluctuations—a formidable task for both the QME and diagrammatic methods. Specifically, we study two correlators:

\[
\Delta^{\xi}_{\text{LR}} \equiv \langle \hat{N}_L \hat{N}_R \rangle - \langle \hat{N}_L \rangle \langle \hat{N}_R \rangle, \quad \Delta^{\xi} \equiv \langle (\hat{N}_L - \hat{N}_R)^2 \rangle. \tag{7}
\]

\( \hat{N}_R \equiv \sum_{l=1}^{N} n_l \) is the total density of fermions on \( l \) sites on the right of the impurity (we fix \( l = 5 \)). Analogously, we define the total density of fermions on the left, \( \hat{N}_L \). Figure 5(b) shows the difference between the dynamics of the noise-averaged correlator \( \langle \Delta^{\xi}_{\text{LR}} \rangle_\xi \) and the evolution of the same quantity assuming the system in a Gaussian state. We find that non-Gaussian correlations, imprinted by the local dephasing channel, are strongest at intermediate times, when the system is already far from the filled Fermi sea, but did not yet reach the NESS, which turns out to also be non-Gaussian (a similar conclusion for the problem of global dephasing is discussed in Ref. [64]). The non-Gaussian correlations are also more pronounced near the site with decoherence, as we show in Ref. [57]. Deviations from Gaussianity are at reach in state-of-the-art cold-atom experiments, as recently demonstrated in the measurement of higher-point correlation functions of phase profiles in a pair of tunnel-coupled one-dimensional atomic superfluids [65].

We now focus on NESS properties. At equilibrium, these correlators depend monotonically on temperature [see inset of Fig. 5(d)]. After a relatively short time \( [t_0 \approx 30 J^{-1} \text{ for } \gamma = J; \text{see Fig. 5(b)}] \), both observables \( \Delta^{\xi}_{\text{LR}} \) and \( \Delta^{\xi} \) demonstrate saturation, indicating proximity to the NESS. This fact suggests that by averaging over both time and noise, \( \Delta_{\text{LR}} \equiv 1/T \int_{t_0}^{t_0+T} dt \langle \Delta^{\xi}_{\text{LR}} \rangle_\xi \), one probes the NESS correlations. Here we fix \( N_t = 10^3 \) and \( T = 60 J^{-1} \). Figures 5(c) and 5(d) show

![FIG. 5. Numerical simulations of the SSE. (a) Evolution of the density at the origin \( n^{\text{nr}}_0(t) \) for three different noise realizations \((N = 200, \gamma = J, \text{and } k_F = \pi/4)\). We find that the typical instance of dynamics is far from the average, \( n_0(t) \). (b) Deviation of the noise-averaged correlator \( \langle \Delta^{\xi}_{\text{LR}} \rangle_\xi \) [cf. Eq. (7)] from its expectation value assuming a Gaussian state, demonstrating the development of non-Gaussian corrections. (c) and (d) Two fluctuation correlation functions, \( \Delta_{LR} \) and \( \Delta \), exhibit the Zeno crossover in the NESS. Standard deviation in both observables is less than the “symbol” size. Inset of (c) The dependence of \( \Delta \) on temperature is monotonous in equilibrium, further indicating that the heating effect is suppressed at strong dissipation.](100301-4)
that in the NESS such correlations exhibit a crossover around $\gamma = J$. For small $\gamma \lesssim J$, the equilibrium cartoon in the inset of Fig. 5(d) suggests that the temperature of the system increases with the heater strength, in contrast to the case of strong $\gamma \gtrsim J$, where it starts to decrease. This inability to heat up the system for strong dissipation is the essence of the quantum Zeno effect. Our procedure also suggests the feasibility of experimental verification of this result (note that it is already accessible with cold-atom platforms [34] to probe correlations after $\sim 20/1^2$).

The Lindblad QME illustrates that the BBGKY hierarchy is closed, which mirrors at a diagrammatic level in the exactness of the SCBA. This circumstance allows one to extract analytically the NESS properties and, in particular, to demonstrate the onset of algebraic spatiotemporal correlations. On the other hand, the simulations of the SSE enable one to investigate the effects of fluctuations nonperturbatively. We have found that these fluctuations exhibit non-Gaussian correlations and behave nonmonotonically as a function of the dissipation strength (this manifests in the Zeno crossover discussed throughout this Rapid Communication). It would be interesting to extend this program to interacting systems both of fermionic and bosonic nature, in view of applications to solid-state and cold-atom experiments. As an example, the question of fluctuation statistics in the setup of coupled Josephson-junction arrays with local dissipation [39] is particularly promising.

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[57] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.102.100301 for (i) the development of the diagrammatic approach, (ii) further discussion of the four-point correlation function, and (iii) more details on the SSE.
[58] We assume that the system is infinite (more specifically, it means that $N/2v_m \gg J^{-1}, \gamma^{-1}$); hence energy produced locally by the dephasing channel is carried away to spatial infinity, without revivals. The steady state is defined only with respect to nearby the dissipative site, while far away from it the “density front” keeps spreading along the system.