Dynamical interplay between a single hole and a Hubbard antiferromagnet

Geoffrey Ji,1 Muqing Xu,1 Lev Haldar Kendrick,1 Christie S. Chiu,1 Justus C. Brüggenjürgen,1 Daniel Greif,1 Annabelle Bohrdt,2,3 Fabian Grusdt,3,4 Eugene Demler,1 Martin Lebrat,1 and Markus Greiner1

1Department of Physics, Harvard University, 17 Oxford St., Cambridge, MA 02138, USA
2Department of Physics and Institute for Advanced Study, Technical University of Munich, James-Franck-Str. 1, 85748 Garching, Germany
3Munich Center for Quantum Science and Technology, Schellingstr. 4, 80799 München, Germany
4Department of Physics and Arnold Sommerfeld Center for Theoretical Physics, Ludwig Maximilian University of Munich, Theresienstr. 37, 80333 München, Germany

(Dated: June 12, 2020)

In strongly correlated quantum materials, the intricate interplay between charge carriers and spins leads to a breakdown of the conventional notion of charge transport. A prime example of such behaviour can be found in the two-dimensional Hubbard model, where doping an antiferromagnetic insulator with charge carriers is believed to give rise to intriguing behaviours such as $d$-wave superfluidity and bad metallic phases. Studying the motion of individual dopants is a fundamental step towards understanding these emergent phenomena and has been a long-standing problem in non-equilibrium quantum many-body physics. Here we report on the time- and position-resolved dynamics of a single hole in a 2D Hubbard insulator using a cold-atom quantum simulator of about 400 sites. We observe an initial hole expansion determined by the tunnelling rate, featuring a slowdown from a non-interacting quantum walk that is explained by quantum interference effects. At later times, hole motion strongly depends on the spin exchange energy instead; this two-stage process is mirrored in the spin sector, where spin correlations show a fast disruption by the hole followed by a prolonged restoration. These results reveal a dynamical dressing of the hole by its spin environment, indicating the formation and spreading of a magnetic polaron. Our work allows for the study of emergent structures in the Fermi-Hubbard model one dopant at a time, highlighting the possibilities offered by local control and readout in a large-scale quantum simulator.
EXAMINING A SINGLE DOPANT

To investigate the dynamics of a single dopant, we prepare individual holes in a Mott insulator (Fig. 1) and suddenly shut off the potential holding them in place (Fig. 1). We prepare the initial system of fermionic Lithium-6 in a half-filled 2D square lattice with Hubbard parameters $t/\hbar = 2 \pi \times 744(12)$ Hz and $U/t = 8.72(28)$ ($t/J = 2.18(7)$) [20] at a temperature of $T/t = 0.340(19)$ and with correlation length $\xi = 1.695(11)$ (expressing lengths in units of the lattice spacing $a = 569$ nm and setting $k_B = 1$ here and subsequently). Simultaneously, we project repulsive light from two digital micromirror devices (DMDs). The first DMD performs entropy redistribution and removes residual harmonic confinement (Extended Data Fig. 1) over a rounded-square-shaped area of 31 sites in diameter [21]. The second DMD acts to selectively project one or several repulsive potentials that are localised to single lattice sites. During the adiabatic loading process atoms will preferentially not populate these sites, resulting in pinned holes. After adiabatic loading, we rapidly shut off the light illuminating the second DMD within $0.03 \hbar/t$, releasing the pinned holes. We then allow the system to time-evolve for a variable time $\tau$, resulting in delocalisation of the holes. Finally, we freeze the dynamics by rapidly increasing the lattice depth, and make a projective site-resolved measurement of either the parity-projected (singles) population or a

single spin state by removing the other via a resonant spin-removal laser [22].

SHORT-TIME DENSITY DYNAMICS

We first examine the short-time dynamics of the system. For an enhanced data collection rate, we si-
multaneously create four holes arranged in a 7-site-wide square pattern and study each independently for times \( \tau < 2 \hbar/t \) (Fig. 4). The average hole density on these four sites is 0.81(2) initially, without significantly affecting the adjacent sites (Extended Data Fig. 2). Due to charge fluctuations in the form of doublon-hole pairs, we are unable to exactly track where the holes are in each experimental realisation. Instead, we compute the average singles density distribution and subtract it from a background single density obtained from experimental realisations where no hole is present (see Methods).

We plot the background-subtracted density distributions for selected times, averaged across all four holes and the dihedral symmetries of the square lattice in Fig. 2. Within half a tunnelling period, the hole tunnels to the four neighbouring sites \((\tau = 0.47(1) \hbar/t)\); its subsequent propagation retains clear coherent features such as the loss and revival of the hole density at the origin \((\tau = 0.93(2) \hbar/t\) and 1.40(2) \hbar/t\) that sets it apart from a classical diffusion process. To accurately model the hole propagation, we must use quantum mechanical models that consider the spin background through which the hole moves.

In Fig. 2, we plot the hole densities on the central site and its diagonal neighbours. We also show the predictions of three models that feature different magnetic phases: a non-interacting quantum walk, equivalent to the propagation of a hole in a spin-polarised background; a quantum Monte-Carlo (QMC) simulation of a disordered spin background \((T = \infty, J = 0)\) [11, 23]; and a time-dependent density matrix renormalisation group (TD-DMRG) simulation of the \(t-J\) model with \(t/J = 2\) on a 18\(\times\)4 system, initially in the antiferromagnetic ground state [24]. All three models quantitatively describe the experimental results during the first tunnelling event up to \(\tau \approx 0.5 \hbar/t\); a detailed comparison is presented in Extended Data Fig. 3. Afterwards, the spinful simulations correctly predict the revival time of the central density at \(\tau \approx 1.2 \hbar/t\), though finite-size effects magnify the revival in the TD-DMRG density. The amplitude of the oscillation in the diagonal density is directly related to the indistinguishability of spin backgrounds after two tunnelling events ending at a given site, as sketched in Fig. 1a and b: quantum interference is maximal in a ferromagnet (free quantum walk), reduced in a antiferromagnet (ground-state TD-DMRG) and between these two extremes in a paramagnet (infinite-temperature QMC). The suppressed diagonal density seen experimentally at \(\tau \approx 1 \hbar/t\) therefore hints at the quantum statistical role of the antiferromagnetic background on the hole motion over short timescales.

**LONG-TIME HOLE DELOCALISATION**

To quantitatively describe the global delocalisation dynamics, we can calculate the root-mean-squared (RMS) hole distance directly from the experimentally-measured hole density:

\[
d_{\text{RMS}} = \sqrt{\frac{\sum_{d_x,d_y} (d_x^2 + d_y^2) \rho_d}{\sum_{d_x,d_y} \rho_d}},
\]

where \(\rho_d\) is the hole density at coordinate \(d = (d_x, d_y)\) relative to the initial hole position. However, statistical fluctuations in the hole density at large distances lead to large uncertainties in the hole position. Instead, we fit the experimental data in an analysis region containing most of the hole density.
(Extended Data Fig. 4) to a Gaussian density distribution (Extended Data Fig. 5), which empirically matches the hole distribution at longer times (Extended Data Fig. 6). For these data, we extend the hole propagation to later times where the four holes have overlapping density distributions. As a result, for data after $\tau = 1 \, h/t$ and all subsequent presented data, we prepare a single hole in the centre of the system. Here, the hole density on the initial site before release is $0.867(12)$.

We plot the RMS distance of the fitted distribution in Fig. 3a. At short times, its linear growth indicates a ballistic expansion that matches the analytical expression for a free quantum walk, $d_{\text{RMS}} = (2t/\hbar)\tau$. The delocalisation then clearly slows after $\tau = 1 \, h/t$; the hole eventually leaves our analysis region of 11 sites in radius (Fig. 3b) at times greater than $\tau = 10 \, h/t$ (Extended Data Fig. 4).

A minimal theoretical model for motion in a spinful background that solely includes the tunnelling energy as a free parameter relies on the Bethe lattice, a fractal graph that can encode sequences of non-retracing hole moves as distinct nodes [23, 25]. We perform a quantum walk on such a graph and model path-distinguishability effects stemming from the spin background by incoherently summing probabilities on nodes leading to the same hole location. This mapping results in a crossover to diffusive hole motion at times consistent with the downward bend of the measured distance, confirming the qualitative importance of quantum interference at timescales comparable with the tunnelling time.

To further test the validity of a diffusive assumption, we directly compare the RMS distance to a diffusive square-root law $d_{\text{RMS}} = \sqrt{4D\tau}$ with the diffusion constant $D = t(2t/\hbar)$ set by the Mott-Ioffe-Regel (MIR) limit. This limit classically corresponds to Brownian motion with a mean-free-path of one lattice constant and a hole velocity set by the tunnelling energy $t$, and was experimentally shown to be a lower bound for diffusion at larger doping [15]. We find that hole motion violates this lower bound, but satisfies a similar lower bound $D = J t\gamma/\hbar$, which suggests that long-time diffusive behaviour would occur with velocities related to the superexchange rather than the bare tunnelling.

To directly compare the finite-size, interacting TD-DMRG simulation introduced in Fig. 2, with our experimental data obtained on a larger system, we calculate its one-dimensional RMS distance and scale it to a two-dimensional Euclidian norm (see Methods). We observe good agreement over our experimental timescales, although it may be coincidental as the anisotropy and ground-state character of the numerical simulation may have opposite effects on the computed hole distance: reducing the system’s dimensionality favours faster ballistic motion, while increasing magnetic order may enhance the interaction-induced slowdown of the hole.

FIG. 3. Long-time dynamics and dependence on superexchange. a, Root-mean-squared distance from the origin, obtained from a Gaussian fit to the 2D hole density distribution. The initial linear increase indicates a ballistic expansion compatible with a non-interacting quantum walk (dashed black line). The hole slows down around $\tau = 1 \, h/t$, an effect qualitatively predicted by a Bethe-lattice model capturing a complete loss of path interference (solid blue line). At later times, the hole is more confined than predictions from the Bethe-lattice model or a diffusion model based on the Mott-Ioffe-Regel (MIR) limit (dashed red line). The MIR limit is satisfied when setting $J$ as its energy scale (dashed grey line) instead of $t$. The data agree with a ground-state TD-DMRG simulation (dash-dotted green line) with adjustments for finite-size effects (see Methods). b, Comparison of propagation speeds at different interaction energies. Increasing the interaction energy decreases the superexchange and the long-time hole propagation speed, suggesting that the hole motion is related to the superexchange. Linear fits to the behaviour for $\tau \geq 0.8 \, h/J$ result in different slopes. The inset shows the same data with time rescaled in units of $J$, along with a prediction for a magnetic polaron which is offset to account for finite polaron extent as a guide to the eye (dashed black line).
We further verify the role of the superexchange on hole motion by doubling the on-site interaction to $U/t = 17.2(6)$ ($t/J = 4.30(15)$) at fixed tunnelling using a Feshbach resonance. We thereby halve the superexchange energy $J$ while slightly decreasing the temperature $T/t = 0.241(18)$ and increasing the correlation length $\xi = 2.48(25)$. The experimental RMS distance shown in Fig. 3 agrees with the previous data at short times, apart from small initial deviations due to a larger hole preparation infidelity (Extended Data Fig. 2). The hole velocity is then visibly reduced for smaller $J$: a linear fit excluding data at $\tau < 0.8 \hbar/J$ yields a velocity of $0.15(17) \sqrt{\hbar/J}$ down from a value of $0.40(10) \sqrt{\hbar/J}$ at $U/t = 8.72(28)$. These results suggest that at long times the hole becomes dressed by the magnetic background and forms a magnetic polaron. Indeed, in the inset of Fig. 3b we show the long-time dynamics rescaled in units of the superexchange time, which agrees with the theoretically predicted velocity of a ballistic polaron (see Methods[26]). In light of this agreement, the initial expansion of the hole RMS can be seen as the fast dynamical formation of a polaron. Eventually, at time and distance scales beyond what we observe, finite-temperature effects should result in diffusive polaron motion.

**SPIN RECOVERY DYNAMICS**

The dressing of the hole motion by the spin background can be conversely investigated by observing the dynamical rearrangement of the spin correlations by the hole and their subsequent evolution. To do so, we measure the sign-corrected spin correlation function $C_{\tau}(d) = (-1)^{d_x + d_y} 4 \langle S_r^z S_{r+d}^z \rangle - \langle S_r^z \rangle \langle S_{r+d}^z \rangle$ in an independent dataset (see Methods). We plot a map of the correlations from the initial hole location $C_0(d)$ averaged over all spatial symmetries for select times, see Fig. 3a. At $\tau = 0$, these correlations are vanishing due to the presence of the hole; within one tunnelling time, the hole hops to a neighbouring site and the correlations become those of the exchanged neighbouring spin. This swapping of correlations results in a reversal of the global antiferromagnetic correlation pattern (red color in Fig. 3a); for instance, the negative sign of the diagonal correlation $C_0(|d| = \sqrt{2})$ at $\tau = 0.467(8) \hbar/J$ and $\tau = 0.935(15) \hbar/J$ is at odds with its positive equilibrium value (Fig. 3b). One exception is the adjacent spin correlator $C_0(|d| = 1)$, which does not become ferromagnetic at short times as it results

![FIG. 4. Reversal and recovery of antiferromagnetic correlations. a. Sign-corrected spin correlations $C_0(d)$ from the initial hole location (black circle) at select times, symmetrised across reflections. Within one tunnelling time, the hole swaps a neighbouring spin to the origin, producing antiferromagnetic correlations but with the incorrect sign (red) apart from the nearest-neighbour correlator. The correct AFM pattern (blue) then slowly restores itself, although the correlations have not fully equilibrated even at our longest measured times ($\tau = 23.4(4) \ h/J = 10.7(4) \ h/J$; Extended Data Fig. 7). b, c. Sign-corrected spin correlations between the initial hole location and its diagonal and adjacent neighbours ($C_0(|d| = \sqrt{2}, 1)$) as a function of time. The insets highlight how the $\tau = 0$ correlations encircling the hole are mixed into the correlations from the centre site at later times. These short-time dynamics are captured by a Bethe-lattice model (purple-grey band), while spin exchange must be taken explicitly into account (here as a polaronic model, blue band) to quantitatively describe the long-time relaxation of the correlations towards their equilibrium values (blue-green line). This correlation relaxation is slower than the density relaxation on a $3 \times 3$ area around the centre, shown in green (green line to guide the eye; equilibrium at blue-green line).]
from a mixture of diagonal correlations weakened by the presence of the hole before the initial quench (Fig. 4). The reversal of the antiferromagnetic pattern extends here up to 3 sites away from the centre and is a dynamical analogue to the short-range polaronic behaviour seen at equilibrium in [18].

At later times, we see a slow return of the spin correlators to their equilibrium values at timescales consistent with the superexchange time. These slow dynamics are also visible in adjacent and diagonal correlations away from the centre, which remain non-uniform even for the last measured time of $\tau = 23.4(4) \ h/t$ (Extended Data Fig. 7). Since the adjacent correlations $C_r(|d| = 1)$ are directly proportional to magnetic energy in the $t - J$ approximation, their non-uniformity implies that the magnetic energy imparted by the hole motion has not equilibrated even though the hole itself has left the system.

We can quantitatively describe the swapping of correlations resulting from fast hole dynamics on the spin background, highlighted in the insets of Figs. 4b and c. To do so, we take experimental pictures at $\tau = 0$ and displace spins according to the distribution of hole trajectories predicted by the Bethe-lattice model (see Methods). The numerically averaged correlations (purple-grey band in Figs. 4b and c) agree well with the experimental data at short times only, indicating that the local correlation swapping is an accurate picture in that regime.

A more complete model for the hole dynamics is one considering the energy cost of correlation swapping, which at first binds the hole to around its initial location [27]. Spin exchange can then enable the restoration of the disrupted spin background, leading to a polaron delocalising with a long-time velocity set by the superexchange energy. An example of such a model is presented in [28], where the polaron is described as a composite object formed by a holon and spinon connected by a string of displaced correlations. We include the spinon dynamics by shifting the experimental pictures used in our model (and therefore the effective hole origin) according to the time-dependent spinon distribution whose offset RMS is plotted in the inset of Fig. 3 (see Methods). Including these dynamics results in an accurate prediction of the long-time behaviour of the spin correlations, capturing their slow return to equilibrium (blue band in Figs. 4b, c). We observe a discrepancy between the spinon-holon model and the experiment data around 1–2 $h/t$ for the predicted value of $C_0(|d| = 1)$, which may be caused by spin relaxation mechanisms not captured in the model such as magnon emission. Furthermore we note that the model overestimates the RMS distance of the hole measured experimentally (Extended Data Fig. 8), possibly because it neglects spin fluctuations that effectively lead to a disordered magnetic background and inhibit hole motion.

In this work, we demonstrate the intricate link- age between charge and spin degrees of freedom in the two-dimensional Fermi-Hubbard model by examining the quench dynamics of a single hole at the tunnelling and superexchange timescales. The size and homogeneity of this cold-atom quantum simulator were key to benchmarking non-equilibrium theories of magnetic polaron formation and numerical techniques away from the linear regime. Our work could be extended to investigate the properties of magnetic polarons created deterministically (such as their effective mass or lifetime), in particular when approaching the strange metal phase, where the quasi-particle picture is expected to break down, or the pseudo-gap phase, where precursors of hole pairing may be visible microscopically. Our site-resolved manipulation techniques could finally be applied to study the stability of presumptive exotic phases such as striped phases.

Acknowledgements We thank J. Carlström, M. Kanássz-Nagy, J. Léonard, M. Knap, A. Lukin, and Z.-X. Shen for insightful discussions. We acknowledge support from the Gordon and Betty Moore Foundation grant no. 6791; NSF grant nos. PHY-1734011, OAC-1934598 and OAC-1934714; ONR grants no. W911NF-11-1-0400 and N00014-18-1-2863; ARO grant no. W911NF-20-1-0163; ARO/AFOSR/ONR DURIP; the DoD through the NDSEG program (G.J.), the NSF GRFP (L.H.K. and C.S.C.), the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy – EXC-2111 – 390814868 (A.B. and F.G.), the Swiss National Science Foundation and the Max Planck/Harvard Research Center for Quantum Optics (M.L.).

Author contributions All authors contributed to the conceptualization of the work, interpretation of the data, and editing of the manuscript. G.J., M.X., L.H.K, M.L, and M.G, performed the experimental data acquisition, analysis, and writing of the initial manuscript. They in addition with C.S.C., J.B., and D.G. performed the experimental investigation and developed the methodology. A.B., F.G., and E.D. performed the numerical simulations, which G.J. and L.H.K. applied to experimental data.

Competing Interests The authors declare that they have no competing financial interests.

Correspondence Please address correspondence and
requests for materials to: mgreiner@g.harvard.edu.


[37] See Supplementary Material.
METHODS

Experimental Stability

To ensure consistent lattice position, atom number, and alignment of the hole DMD potential to the lattice, we perform active feedback on the experiment. The feedback corrects for drifts on the timescale of hours. We maintain a running average of the mean atom position and atom number and feedback on the angle of the lattice in-coupling mirrors and the final evaporation power. This ensures a total atom number of $1094 \pm 29$ for the short-time $U/t = 8.7(3)$ dataset over 1099 realisations ($99 - 102$ per evolution time), $1098 \pm 22$ for the long-time $U/t = 8.7(3)$ dataset over 7590 realisations ($754 - 762$ per evolution time), $919 \pm 27$ for the $U/t = 17.2(6)$ dataset over 2873 realisations ($190 - 194$ per evolution time), and $1099 \pm 25$ for the correlation dataset over 13255 realisations ($585 - 764$ per evolution time and per spin or no spin removal).

To ensure alignment of the hole DMD potential, we periodically display a superlattice potential on the DMD and measure the positional phase of the atomic distribution. We then offset the position of the DMD pattern accordingly.

Uniformity of the Potential Background

The overall Gaussian envelope of the two radial lattice beams results in an overall harmonic confinement that is corrected by projecting an anti-harmonic potential with a Digital Micromirror Device.

We ensure that the residual potential curvature is negligible and the anti-confining potential does not introduce additional disorder over short distances. To do so, we load the central region with atoms in the compressible, metallic regime at an average singles density of $\rho_s = 0.46(6)$ over 90 realisations (Extended Data Fig. 1b). We then convert the density to chemical potential $\mu(r)$ in the local density approximation, using the equation of state of the Fermi-Hubbard model numerically computed at $U/t = 8.7(3)$ using non-linear cluster expansion (NLCE) [29], and extrapolated down to experimental temperatures $T/t = 0.340(19)$. Assuming that the system is in thermal equilibrium at a global chemical potential $\bar{\mu}$, we can infer the local on-site potential $V(r) = \bar{\mu} - \mu(r)$. The variations of density correspond to a site-to-site RMS deviation of $V(r)$ of about $0.4 t$ across the filled region, which is comparable to the statistical error on each point.

Extended Data Figure 1. a, Left: Site-resolved singles density in the metallic regime, averaged over 90 realisations. Right: Local chemical potential along the dashed lines, inferred from NLCE [29] and defined with respect to the chemical potential at half-filling. Within local density approximation, this corresponds to a site-to-site RMS deviation of the underlying potential of about 0.4 $t$ across the filled region. b, Singles density in the Mott-insulating regime upon pinning about 20 holes on a square grid. Nearest neighbours to the same initial hole with statistically significant density differences (with a 95% confidence level) are highlighted by red links at $\tau = 0.7(1) h/t$. We then image the system after a release time of $\tau = 0.7(1) h/t$, which is about the time where the hole density is expected to be maximal on the four sites directly adjacent to the initial hole locations. We do not observe any long-range density variation from hole to hole other than inhomogeneities resulting from the decrease of the initial hole fidelity away from the centre. Among lattice sites adjacent to the
Extended Data Figure 2. Initial hole probability. a, Four-hole dataset used in Figs. 2 and 3, averaged over 100 realisations. b, Single-hole dataset used in Fig. 3, at $U/t = 8.72(28)$ (762 realisations) and c, $U/t = 17.2(6)$ (193 realisations). d, Single-hole dataset used in Fig. 4 (764 realisations).

Hole Preparation

We initially create holes inside the two-dimensional Mott-insulating region by exposing one or several lattice sites to localised blue-detuned light created by a Digital Micromirror Device (DMD). This initial state is obtained by first ramping up the DMD potential in 30 ms and then ramping up the lattice potentials in 60 ms to prepare a system in equilibrium with the presence of the repulsive potential. The choice of the DMD potential height is a tradeoff between high hole probability on the target sites and unwanted atom depletion of neighbouring sites due to an imperfect point spread function. We show in Extended Data Fig. 2 the initial hole density for the different datasets used in this work.

To measure short-time site densities (Fig. 2), we decrease the initial density of four sites to hole fidelities ranging from 0.64(5) to 0.802(34), with variations caused by the non-uniform illumination of the DMD. To combine the data from each hole, we normalise each hole density distribution separately after background subtraction (see below) such that at $t = 0$ the hole density at the pinned site is unity. The four hole density distributions are then averaged together.

To measure the time evolution of the average hole distance (Fig. 3) and the correlations of the spin background (Fig. 4), we use a stronger pinning potential that leads to a higher hole fidelity at the origin and an increase of the average hole density on the neighbouring sites. This depletion is associated to an excess hole number of about 0.15 in Extended Data Fig. 2, 0.25 in Extended Data Fig. 3 and 0.3 in Extended Data Fig. 4. This additional density away from the central site leads at $U/t = 17.2(6)$ to an increased RMS distance of 0.26(4) as shown in Fig. 3. This hole excess is however quickly reduced during the initial ballistic expansion as it is diluted over an area growing as the square of the propagation time: at $\tau = 0.47(2) \hbar/t$, the fitted distance already agrees within errorbars with the prediction for a free quantum walk.

Error on the Evolution Time

We obtain a normalised evolution time by multiplying the experimental elapsed time by the tunnelling energy in units of $\hbar$ obtained by lattice modulation spectroscopy [22]. The stated error includes the error in the spectroscopic measurement and the systematic error on the elapsed time as a result of the finite duration of the DMD shutoff time. We neglect error arising from the finite time of the lattice freezing ramp. We find that the hole distributions before and after releasing the hole are consistent, indicating that the freeze negligibly affects the hole distribution.

Background Subtraction

To determine the location of the prepared hole, we must measure and remove the equilibrium contribution from double-hole pairs due to quantum fluctuations and thermal effects. By loading the system with no hole potential and immediately imaging the system, we measure the equilibrium singles density where no hole is present $\rho_{eq}^s = 1 - (\rho^h + \rho^d)_{eq}$.
Extended Data Figure 3. **Full comparison between experimental and simulated densities.** a, Two-dimensional hole density around the origin, clipped to \(5 \times 5\) quadrants. The experimental data is averaged over horizontal, vertical and diagonal reflections. The DMRG simulation is performed on a \(18 \times 4\) system with periodic boundary conditions along the short vertical direction; the rows at \(d_y = \pm 2\) are equivalent and are duplicated for clarity. b Hole density averaged over sites at distances \(|d| = 0, 1, \sqrt{2}, 2\) and \(\sqrt{5}\).
Extended Data Figure 4. Plot of the total hole density inside the analysis window. For both values of U/t, we find that the hole density is consistent with 0 at the last measured time of $\tau t/\hbar = 23.4$. The blue dashed line is the hole density at $t = 0$.

where $\rho^h (\rho^d)$ is the average probability of an empty (doubly-occupied) site. The image is then convolved with a $3 \times 3$ kernel with a normalised weight of 0.15 on the nearest-neighbour sites to remove high-frequency statistical noise. The hole density during hole propagation is estimated by subtracting the imaged singles density to the equilibrium one, $\rho = \rho_{eq} - \rho^h$. This subtraction process can cause statistical fluctuations to result in negative hole densities, which are nonetheless included in the Gaussian fit to estimate the average hole distance.

In principle the effective background may change at different evolution times due to heating. We measure the heating rate to be $0.0009(3) t/\text{ms}$, which has a negligible effect on the density even at the longest evolution time of 5 ms.

Distance Fits

To fit the hole density distribution, we first truncate the experimental data to a circular window of 4 or 11 sites in radius around the initial hole location, containing 49 or 377 sites, respectively. The smaller window is used for the low U/t dataset where four holes are prepared, to avoid including the effect of the neighbouring holes. The window contains most of the integrated hole density over the experimental timescales, as plotted in Extended Data Fig. 4.

We fit to the experimental distribution a discrete Gaussian probability density function, where the probability on each site is equal to the integral of a continuous Gaussian function over the 1-site square corresponding to each lattice site. The discrete density distribution is centred on the initial hole location and normalised to the initial hole fidelity, and we use the width of the continuous Gaussian distribution as the only fit parameter. We numerically compute the RMS value of this distribution, and propagate the error from the fit assuming normally distributed errors.

We plot the fits as well as the reduced chi-squared test statistic versus the continuous fit parameter in Extended Data Fig. 5. We observe good quantitative agreement with the Gaussian functional form. We note that at $\tau t/\hbar = 7.01$ for the high U/t data we fit to a local minimum; the global minimum at large radius is a result of the low total density at that time. We believe this effect is a statistical fluctuation as the later data point at $\tau t/\hbar = 9.35$ does not exhibit this issue. We do not show the fitted RMS at $\tau t/\hbar = 9.35$ in the main text as the fit at low U/t exhibits sensitivity to the exact window used. For the total density, we also plot the value at $\tau t/\hbar = 23.4$, where we find that for both U/t values the value is consistent with 0, i.e. the hole having left the window. As a result, we do not fit the hole density distribution here.

Agreement of Gaussian Model

For a hole undergoing a diffusive random walk process, the spatial hole probability distribution is expected to be Gaussian. This distribution is a result of the central limit theorem and the fact that hole motion at each time interval is independent. However, if the hole propagation is not a memoryless process (therefore motion at each time interval may be correlated), we may not expect a Gaussian distribution. As a result, we examine the suitability of the Gaussian fit on both the Bethe-lattice model and the TD-DMRG data by comparing the fitted RMS with the true RMS, see Extended Data Fig. 6a.
Extended Data Figure 5. **Gaussian fits to experimental hole density distributions.** Experimental (yellow and green points) hole density and its Gaussian fit (grey line) as a function of Euclidian distance, and resulting reduced \( \chi^2 \) test statistics (black line) as a function of the width of the Gaussian function. The number of experimental points displayed is reduced for clarity by binning them according to the floor or ceiling of their distance. The last measured distance for each value of \( U/t \) is not included in the main text due to the sensitivity of the fit to the exact window size used.
Extended Data Figure 6. **Suitability of the Gaussian fits.** a, RMS extracted from Gaussian fits including and excluding the centre point (rung) for the Bethe-lattice model and TD-DMRG simulations, along with the RMS calculated from the bare distributions. Excluding the centre point improves the TD-DMRG fit, which we believe is due to a finite-size effect. b, c, RMS extracted from Gaussian fits on the experimental data for different-sized fitting windows and with and without the centre point. The fit results are not dramatically affected by these changes.

The fit has excellent agreement with the Bethe-lattice model, especially at longer times. We believe this agreement is because hole motion in the Bethe lattice only displays interference effects when it retracts its path, which becomes increasingly unlikely for longer lengths. As a result, the hole motion appears diffusive and with a Gaussian shape at longer times.

The fit does not agree as well with the TD-DMRG data with system size 18 × 4, which has been integrated along the short dimension with periodic boundary conditions to alleviate finite-size effects. The TD-DMRG data has long-lived coherent oscillations along the 4-site rung including and surrounding the initial hole location, leading to an integrated distribution peaked at the centre. Excluding the central rung from the fit results in a quantitatively accurate RMS. When excluding the central point from the Bethe and experimental fits, we see minimal deviation at longer times, further supporting the conclusion that importance of the centre point is a finite-size effect. At short times the fit can fail as the central point represents a substantial part of the probability density.

We also investigate changing the window in which we fit the hole density distribution to, see Extended Data Fig. 6b and c. We see minimal deviation on the fitted RMS distance, with the exception of the last data point for the lower interaction energy data. There, the agreement is only statistical. We note that the two largest window sizes agree, however.

**Quantum Walk, Bethe Lattice, QMC, and TD-DMRG Calculations**

The free quantum walk is calculated from the analytic formula in [30], scaled to a two-dimensional system:

\[
\rho_{ij}(\tau) = |J_i(2\tau t)J_j(2\tau t)|^2,
\]

where \( J \) is the Bessel function of the first kind and \( i, j \) are the site coordinates relative to the initial location.

Hole propagation in the Bethe lattice and in a paramagnetic spin-\( \frac{1}{2} \) environment are calculated as in [23]. In the latter case, we perform QMC in a 40 × 40 system with 100 runs and 1 × 10^7 paths, sampling both the forward and backward paths. We use the code provided at: https://github.com/MartonKN/Dynamical-spin-correlations-at-infinite-temperature.

The DMRG simulations are performed using the TeNPy package [31] on a 18 × 4 system with periodic boundary conditions. The time evolution is performed using the matrix product operator based \( W^H \) method [32, 34]. The TD-DMRG simulation has finite-size effects due to the short dimension being only 4 lattice sites. To ameliorate these effects...
when calculating the RMS distance, we first integrate the hole density distribution along the short dimension. This integration will result in the same distribution as a system with a larger short dimension as long as the configurations do not add coherently. We then calculate the RMS distance along the long direction, and multiply this result by $\sqrt{2}$, the scaling factor from one-dimensional to two-dimensional RMS for a symmetric system. Related tensor network simulations have been performed in [35].

Spinon-Holon Model

We predict the spin correlations near the system centre and the RMS hole distance using a spinon-holon model, described extensively in [24] [28] [36]. The wave function separates into a spinon part and a string-holon part. For the spinon we follow the procedure described in [24]: We equate the spinon dispersion relation with the zero-temperature magnetic polaron dispersion. The latter can be fitted by the functional form $\omega_{\text{sp}}(\vec{k}) = A[\cos(2k_x) + \cos(2k_y)] + B[\cos(k_x + k_y) + \cos(k_x - k_y)]$, with parameters $A = 0.25J$ and $B = 0.36J$ at $t/J = 2$ [29]. These values change very weakly when $t/J$ is increased, e.g. $A = 0.28J$ and $B = 0.32J$ for $t/J = 3.5$. Using $\omega_{\text{sp}}(\vec{k})$ and starting from a spinon localised on the central site, we simulate a quantum random walk. For the string-holon part, we follow [36] and solve the Schrödinger equation on the Bethe lattice in a linear potential, truncated at a depth $\ell_{\text{trunc}}$. We take the initial state fully localised on the root node, reducing the problem to a 1D problem on $\ell_{\text{trunc}}$ sites, which we solve by exact diagonalisation.

The parameters of the linear potential (the string tension and the $\ell = 0$ offset) are computed from experimental correlations as indicated in [28]. For the $U/t = 8.72$ data, we use the $\tau = 0$ dataset, excluding the $3 \times 3$-site region surrounding the hole in the tension computation to eliminate the suppressed correlations around the hole. At $U/t = 17.2$, we lack correlation measurements at $\tau = 0$, so we use a separate dataset taken under experimental conditions but with no hole initialised in the centre. This likely overestimates the offset, as it ignores the suppression of correlations around the hole, but the simulation results in [37] are quite insensitive to the
offset, and therefore still reliable. We find string tensions of 0.217(7)\(t\) (0.101(4)\(t\)) and offsets of 0.002(3)\(t\) (0.046(2)\(t\)) at \(U/t = 8.72\) (17.2). These values justify using truncation lengths of \(\ell_{\text{trunc}} = 33\) (100) for \(U/t = 8.72\) (17.2), guaranteeing that >99% of the wave function remains with \(\ell < \ell_{\text{trunc}}\) at all times.

From the separable wave function, we compute probability distributions \(p_s(\mathbf{r}_s, \tau)\) and \(p_h(\ell, \tau)\) for the spinon position and the path length on the Bethe lattice, and from the latter compute the probability distribution \(p_{h, \text{rel}}(\mathbf{r}_h - \mathbf{r}_s, \tau)\) of the hole position relative to the spinon. Summing the RMSes of \(p_s\) and \(p_{h, \text{rel}}\) in quadrature yields the hole RMS distance. We use a \(21 \times 21\) (23 \\times 23) site ROI for the spinon (hole) in this computation. This guarantees \(>95\%\) (>99\%) of the wave function remains within the ROI.

To compute spin correlations, we randomly sample nodes on the Bethe lattice at every depth \(\ell < \ell_{\text{trunc}}\) and use each node to shuffle the values in the binarized atom matrices measured at \(\tau = 0\) by moving the initial central matrix value along the path corresponding to the Bethe lattice node. We only use those matrices with 0 measured in the central site. At every \(\ell < \ell_{\text{trunc}}\), we randomly sample a set of these shuffled matrices, and compute \(pp\) and \(p\) averages (defined in [22]), denoted \(\langle pp\rangle^\ell\) and \(\langle p\rangle^\ell\), within each set. We then use the separable wave function to predict these averages in the spinon-holon theory, e.g., as \(\langle pp\rangle_{\mathbf{r}, \mathbf{r}+\mathbf{d}}^\text{pred}(\tau) = \sum_{\mathbf{r}, \ell} p_s(\ell, \tau)p_h(\mathbf{r}_s, \tau)(pp)_{\mathbf{r}, \mathbf{r}+\mathbf{d}}^\ell\). Once these spinon-holon averages are computed, we assemble them into spin correlators as in [22]. For every \(\ell < \ell_{\text{trunc}}\), we sample 350 paths on the Bethe lattice and 20,000 shuffled atom matrices. Increasing these numbers does not change the results. We assign errors to the \(p\) and \(pp\) averages by bootstrapping on the sampled ensembles at each depth \(\ell\), and to \(p_h\) and \(p_s\) by varying their input parameters (string tension, \(J\), etc.) by one standard deviation. The error bands on the predicted correlators in Fig. 4 are then obtained by linear error propagation. We account for the finite spinon escape probability in the predicted correlators by mixing in half the equilibrium correlator value with an inflated error bar; see [37]. For the ‘Bethe’ predictions in Fig. 4 we repeat this procedure with \(J = 0\) and a truncation depth of \(\ell_{\text{trunc}} = 100\).

**Acknowledgements** We thank J. Carlström, M. Kanász-Nagy, J. Léonard, M. Knapp, A. Lukin, and Z.-X. Shen for insightful discussions. We acknowledge support from the Gordon and Betty Moore Foundation grant no. 6791; NSF grant nos. PHY-1734011, OAC-1934598 and OAC-1934714; ONR grants no. W911NF-11-1-0400 and N00014-18-1-2863; ARO grant no. W911NF-20-1-0163; ARO/AFOSR/ONR DURIP; the DoD through the NDSEG program (G.J.), the NSF GRFP (L.H.K. and C.S.C.), the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy – EXC-2111 – 390814868 (A.B. and F.G.), the Swiss National Science Foundation and the Max Planck/Harvard Research Center for Quantum Optics (M.L.).

**Author contributions** All authors contributed to the conceptualization of the work, interpretation of the data, and editing of the manuscript. G.J., M.X., L.H.K., M.L., and M.G. performed the experimental data acquisition, analysis, and writing of the initial manuscript. They in addition with C.S.C., J.B., and D.G. performed the experimental investigation and developed the methodology. A.B., F.G., and E.D. performed the numerical simulations, which G.J. and L.H.K. applied to experimental data.

**Competing Interests** The authors declare that they have no competing financial interests.

**Correspondence** Please address correspondence and requests for materials to: mgreiner@g.harvard.edu.
SUPPLEMENTARY

Extended Discussion on Spinon-Holon Model

We generate predictions for the spin correlations near the system centre and the RMS hole distance as a function of time by applying a spinon-holon model of the dynamics to experimental data from $\tau = 0$. This model has been described extensively in [24, 28, 36], so we will only outline it here. This theory predicts the correlation curves shown in Fig. 4 and the RMS distance curves in Extended Data Fig. 8.

The model assumes that the hole dynamics are much faster than the spin dynamics, so that the hole experiences an effectively static spin background (the frozen spin approximation). Under this assumption, the hole must displace the spins that lie on the trajectory it takes across the lattice, leaving a trail of spins that are misaligned with the AFM background. This trail, called a geometric string, extends from the hole to the site from which it was released, where the spin correlations are disturbed in a characteristic pattern called a spinon. Spin dynamics are then introduced into this picture by passing from the strict frozen spin approximation to a Born-Oppenheimer approximation, in which spin dynamics are merely assumed to be adiabatic relative to the hole motion. This allows the spinon to hop on the lattice via spin exchange.

The disturbance of the adjacent spin correlations $C_r(|d| = 1)$ along the geometric string associates a magnetic energy to the string. In a system with local AFM correlations, this energy is usually positive (because, e.g., negative adjacent correlations get replaced with positive diagonal correlations $C_r(|d| = \sqrt{2})$) and usually increases with string length, so that the hole is bound to the spinon by a confining string potential. The spinon-hole system is thus a composite object, the motion of which is controlled by the slow spinon motion. Since the spinon carries spin and no charge, while the hole carries charge and no spin, the hole is sometimes called a chargon or holon.

To implement the Born-Oppenheimer dynamics in our simulation, we assume that the system begins and remains in a separable state, $\psi(\tau) = \psi_s(\tau)\psi_h(\tau)$, where $\psi_s$ and $\psi_h$ are spinon and string-hole wave functions (that is, $\psi_h$ describes not just the hole, but also the string that links it to the spinon). The spinon wave function is obtained as in [24] by initialising the wave function fully localised at the origin, and then time-evolving under the assumption that the spinon dispersion is given by the zero-temperature magnetic polaron dispersion from [26]. This procedure is motivated by the observation that the spinon motion controls the motion of the bound spinon-holon pair (i.e., the magnetic polaron), so that the two objects should have equal dispersion relations. Since the magnetic polaron dispersion from [26] is well-fit near $t/J = 2$ by a tight-binding dispersion $\omega(k) = A[\cos(2k_x) + \cos(2k_y)] + B[\cos(k_x + k_y) + \cos(k_x - k_y)]$ with $A = 0.25 J$ and $B = 0.36 J$, this time-evolution procedure is equivalent to evolving under a tight-binding Hamiltonian with diagonal and straight next-nearest-neighbour hopping, and therefore tractable by exact diagonalisation.

To compute the string-hole wave function, we first make the Bethe-lattice approximation to the string-hole Hilbert space (as in [36]). That is, we assume that all of the trajectories the hole can take on the square lattice that do not contain self-retracing components correspond to different states in the string-hole Hilbert space. This approximation, which is also made for the ‘Bethe’ curves in Figs. 2 and 3, is motivated by the observation that, due to the AFM background, most of the trajectories the hole can take result in different spin configurations and are therefore distinguishable.

Solving for the string-hole wave function then reduces to solving for the dynamics of an initially localised wavepacket on the Bethe lattice in the string potential. We make the approximation of linear string theory (LST), in which the magnetic energy of a string is assumed to increase linearly with its length. Specifically, we assume that the potential energy of a string $\Sigma$ is $V(\Sigma) = (dE_\Sigma/dt)\ell(\Sigma) - g\delta_{\ell(\Sigma)}$, where $\ell(\Sigma)$ is the length of $\Sigma$, the string tension $dE_\Sigma/dt$ gives the increase in string energy per unit length, and $g$ is an offset that captures the devia-
tension of \( V|_{\ell(S)}=1 - V|_{\ell(S)=0} \) from the string tension. Roughly speaking, this deviation occurs because the first step the hole takes away from its initial site breaks more AFM bonds than do its later steps.

This problem is solved in [36], where it is shown that the eigenstates of the problem correspond to vibrational and rotational excitations of the string. Rotationally excited states have nontrivial phase windings as one circles the root node of the Bethe lattice. We assume that our initial state is perfectly localized on the root node of the Bethe lattice, that is, \( \psi_h(\Sigma, \tau) = \delta_{0,\ell(S)} \). This wave function has no overlap with the rotationally excited states, due to its isotropy, which in turn implies that \( \psi_h(\Sigma, \tau) \) is isotropic at all times – that is, \( \psi_h(\Sigma, \tau) \) depends only on \( \ell(S) \) and \( \tau \).

Together with the LST potential, this reduces the string-hole problem to a problem on a half-infinite one-dimensional chain of sites, labelled by \( \ell = 0, 1, 2, \ldots \), where site \( \ell \) corresponds to the \( \ell \)-th layer of the Bethe lattice. The initial wave function is \( \psi_{1D}(\ell, \tau = 0) = \delta_{0,\ell} \). It should be noted that this one-dimensional wave function is rescaled from the wave function on the Bethe lattice by the multiplicity of a layer of the Bethe lattice, that is, \( \psi_h(\Sigma, \tau) = \psi_{1D}(\ell(S), \tau) / \sqrt{4 \cdot 3^{(S)} - 1} \). The one-dimensional Hamiltonian is a sum of \( V(\ell) = \frac{dE}{d\ell} \ell - g\delta_m \) and a hopping term \( T_{\ell \ell'} \), originating from the hopping term in the Hubbard model, which connects different sites of the chain. As shown in [36],

\[
T_{\ell \ell'} = \begin{cases} 
-2t, & \ell = 0, \ell' = 1 \text{ or } \ell = 1, \ell' = 0 \\
-t\sqrt{3}, & |\ell - \ell'| = 1, \ell > 0, \ell' > 0 \\
0, & \text{else}
\end{cases}
\]

The factors of 2 and \( \sqrt{3} \) are due to the connectivity between adjacent layers of the Bethe lattice.

We solve the string-hole problem by diagonalising the Hamiltonian \( H_{1D} \) as \( T_{\ell \ell'} + V(\ell)\delta_{\ell,\ell'} \), decomposing the initial wavepacket into its eigenstates, and time-evolving. We truncate the Hamiltonian at \( \ell = 33 \) for the data in Fig. 4 and \( \ell = 100 \) for the data in Fig. 8 (these truncation \( \ell \) values are exclusive endpoints). These truncation lengths are sufficient to retain nearly all of the wave function’s probability amplitude in the truncation region at all times in the simulations (>99% in Fig. 4 and 100% to within machine precision in Fig. 8). It should be noted that the retained probability for a given truncation length depends on the string tension; our truncation lengths were chosen based on our measured string tensions (see Methods).

Together with the spinon wavefunction, this yields the full Born-Oppenheimer wave function \( \psi(r_s, \Sigma, \tau) = \psi_s(r_s, \tau)\psi_{1D}(\ell(S), \tau) / \sqrt{4 \cdot 3^{(S)} - 1} \), where \( r_s \) is the spinon position and \( \Sigma \) is a string (or equivalently, a node on the Bethe lattice). With these wave functions, we may also immediately calculate \( p_h(\ell, \tau) \) and \( p_h(\ell, \tau) \), respectively the probabilities of finding the spinon at \( r_s \) and the holon at depth \( \ell \) on the Bethe lattice at time \( \tau \). This wave function is used to generate the spinon-holon curves in Fig. 4 and Fig. 8 as follows.

To compute the RMS hole distance, we first convert the string-hole wave function to a probability distribution \( p_{h,rel}(r_h - r_s, \tau) \), where \( r_h \) is the hole position. This is possible because every string \( \Sigma \) can be mapped (by following the path \( \Sigma \) takes on the square lattice) to a relative position \( r_h - r_s \), so that converting \( p_h(\ell, \tau) \) to \( p_{h,rel}(r_h - r_s, \tau) \) only requires solving the combinatorial problem of how many paths at each layer \( \ell \) of the Bethe lattice arrive at each relative position \( r_h - r_s \). The RMS distance of the hole is then given by \( d_{RMS}(\tau)^2 = \sum_{r_h-r_s} p_{h,rel}(r_h-r_s, \ell) r_h^2 \), which can be computed either directly or by summing the RMS distances of \( p_{h,rel} \) and \( p_s \) in quadrature.

To compute the spin correlations, we shuffle the experimental snapshots measured at \( \tau = 0 \) (just before the hole is released) according to the Born-Oppenheimer wave function, and compute the spin correlations of these shuffled ensembles of snapshots as we do with normal experimental data (see [22]). More specifically, for every string length \( \ell \) up to the truncation length, we sample a random set \( r_h \) of strings (uniformly with replacement) from all possible strings of length \( \ell \). For every string \( \Sigma \) in \( r_h \) and every experimental snapshot \( M \) at \( \tau = 0 \), we generate the shuffled snapshot \( M' \) that results from rearranging the values in the snapshot as the hole would spin background in the frozen spin approximation. We then randomly sample sets \( M_{\ell} \) (uniformly and with replacement) from the set of all \( M_{\Sigma} \) generated by all \( \Sigma \in r_h \) (we also enforce the constraint that each \( M_{\ell} \) has an equal number of snapshots with no spin removal and with each type of spin removal). Then, treating each \( M_{\ell} \) as an independent ensemble of snapshots, we compute the \( pp \) and \( p \) averages (defined in [22]) in each \( M_{\ell} \) across a \( 23 \times 23 \) site ROI centred on the initial hole site. Denote these averages as \( \langle pp \rangle_{r_h+r_s+d}^{\ell} \) and \( \langle p \rangle_{r_h}^{\ell} \). We then compute the corresponding averages in the spinon-holon theory as \( \langle pp \rangle_{r_h+r_s+d}^{\ell} \) and \( \langle p \rangle_{r_h}^{\ell} \). We
then assemble the predicted $p$ and $pp$ averages at each time into the sign-corrected, polarisation-corrected spin correlator $C_r(d)$ defined in \cite{22}. For the ‘Bethe’ curves in Fig. 4, we repeat this procedure with $J = 0$ and $\ell_{\text{trunc}} = 100$.

This procedure is equivalent to associating the configuration with spinon position $r_s$ and string $\Sigma$ with the snapshots $M_{\Sigma, r_s}$ that result from translating $M_{\Sigma}$ by $r_s$. This is the most natural way to handle the hopping of the spinon in our simulation, as it is based on the translational invariance of the system at $\tau = 0$ away from the initial hole site. The functions $C_r(d)$ at time $\tau$ are then computed to equal the correlations that would result from the ensemble of snapshots $\{M_{\Sigma, r_s}\}$, weighted by the probability distribution $|\psi(r_s, \Sigma, t)|^2$. Random sampling is only introduced to the procedure to deal with the exponentially large number of terms that would otherwise comprise the correlator. For the curves plotted in Fig. 4, each $\tau$ had size 350 and each $M_\ell$ had size 20000. These values are sufficiently large that the resulting curves in Fig. 4 do not appreciably change upon repeating the random sampling or increasing the number of samples.

Errors are calculated as follows. For the $p$ and $pp$ averages, we bootstrap on each ensemble $M_\ell$ to generate error bars. For the probability distributions $p_h(\ell, \tau)$ and $p_s(r_s, \tau)$, we first increase, then decrease the experimentally measured input parameters (string tension, offset, $J$, and $t$) by one standard deviation, and recalculate the probability distributions to generate distributions $p^+ h(\ell, \tau)$ and $p^+ s(r_s, \tau)$. The error bars we assign to $p_h(\ell, \tau)$ and $p_s(r_s, \tau)$ are then $1/2|p^+ h(\ell, \tau) - p_h(\ell, \tau)|$ and $1/2|p^+ s(r_s, \tau) - p_s(r_s, \tau)|$. We then propagate errors through to the quantities $\langle pp \rangle_{r, r+d}(\tau)$ and $\langle p \rangle_{r, r+d}(\tau)$ by standard linear error propagation.

Finally, after assembling the predicted $p$ and $pp$ averages into the correlators $C_r(d)$, we account for error induced by the finite probability $p_{\text{esc}}(\tau) = 1 - \sum r_s p_s(r_s, \tau)$ that the spinon escapes its ROI. We do this by adding a quantity $p_{\text{esc}} C_{\text{eq}}(d)/2$ with error bar $p_{\text{esc}} C_{\text{eq}}(d)/2$ onto the predicted $C_r(d)$, where $C_{\text{eq}}(d)$ is the background value of $C_r(d)$ at $\tau = 0$ (obtained by averaging $C_r(d)$ with $r$ in a $23 \times 23$ ROI with a $3 \times 3$ ROI about the origin excluded to avoid the reduced correlations around the hole). The motivation for this procedure is to allow ‘missing’ snapshots, where the spinon escapes its ROI, to contribute a random value between 0 and $C_{\text{eq}}(d)$ to the predicted correlator.