Surface Cooper-Pair Spin Waves in Triplet Superconductors

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We study the electrodynamics of spin triplet superconductors including dipolar interactions, which give rise to an interplay between the collective spin dynamics of the condensate and orbital Meissner screening currents. Within this theory, we identify a class of spin waves that originate from the coupled dynamics of the spin-symmetry breaking triplet order parameter and the electromagnetic field. In particular, we study magnetostatic spin wave modes that are localized to the sample surface. We show that these surface modes can be excited and detected using experimental techniques such as microwave spin wave resonance spectroscopy or nitrogen-vacancy magnetometry, and propose that the detection of these modes offers a means for the identification of spin triplet superconductivity.

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Spin triplet superconductors are distinguished from their conventional spin singlet counterparts by the fact that they spontaneously break spin-rotation symmetry in addition to global U(1) symmetry. This additional symmetry breaking implies the existence of collective modes, analogous to spin waves in other spin-symmetry breaking systems such as ferromagnets and antiferromagnets [1], which originate from the coherent precession of the triplet order parameter. Therefore, the existence of these modes, manifested in the spin dynamics of the ordered state, can be used to experimentally identify spin triplet superconductors. In fact, this approach proved to be spectacularly successful in the study of superfluid ^{3}He, where probes of the spin dynamics, such as NMR, were crucial in the identification of the triplet order parameter [2–4]. Along these lines, there have been several proposals to probe spin waves in triplet superconductors via thermodynamic [5–7] and transport [8] measurements, but to date these modes have not been observed in any solid state system. Further, there has been extensive work concerning orbital collective modes in unconventional superconductors [9–16] and their (as yet unrealized) potential use as hallmarks of multi-component condensates.

In contrast, spectroscopic probes which rely on the coupling of spin waves to electromagnetic fields have proven to be a powerful tool in the study of magnetically ordered systems [17,18]. At the long wavelengths relevant to experiments, spin dynamics are dominated by dipolar interactions, which can lead to the emergence of collective modes corresponding to the coupled fluctuations of spin waves in the material and incident electromagnetic fields [19–21]. The most notable example is the Damon-Eshbach mode in ferromagnets [21] and antiferromagnets [22,23], which is localized to the sample surface.

In this Letter, we develop an electrodynamic theory of spin waves in triplet superconductors, including the effects of dipolar interactions which couple the spin and orbital dynamics of the condensate. Physically, the precession of the triplet order parameter generates a magnetic field that couples back onto the condensate magnetization, supporting collective dipolar modes. These modes remain stable even when coupled to the orbital motion of the condensate through Meissner screening. At the sample surface, this effect gives rise to a distinct set of surface modes, analagous to Damon-Eshbach modes, which we show are especially promising for experimental detection. The collective modes are identified from the solutions of Maxwell's equations in the triplet medium, characterized by a dynamical magnetic susceptibility $\underline{\chi}(\Omega)$. Thus, we must begin by developing the theory of the uncoupled spin dynamics of the triplet system.

Matter dynamics.—A spin triplet superconductor is characterized by an $S = 1$ order parameter which is a symmetric matrix in spin space. It is conventional to represent this matrix in terms of the so-called "**d**-vector" as [2,4]

$$\hat{\Delta} = \begin{pmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix}. \quad (1)$$

Physically, one may envision $\hat{\mathbf{d}}$ as the direction along which the condensate has an $m = 0$ spin projection. In rare cases, the condensate may be "nonunitary," and possess a net spin polarization given by $\mathbf{Q} = i\mathbf{d} \times \bar{\mathbf{d}}$ [24].

To describe the low energy behavior of a triplet superconductor, we may construct a phenomenological Ginzburg-Landau free energy,

$$\mathcal{F} = \int d^3r \left[-r\bar{\mathbf{d}}\cdot\mathbf{d} - \Gamma\bar{d}_z d_z + \frac{u}{2}(\bar{\mathbf{d}}\cdot\mathbf{d})^2 + \frac{\tilde{u}}{2}\mathbf{Q}^2 - g\mathbf{Q}\cdot\mathbf{h} \right], \tag{2}$$

where r, u, $\tilde{u}$, and g are coupling constants and we have neglected gradient terms [25]. We have introduced a phenomenological easy-axis spin anisotropy, $\Gamma > 0$ that pins the $\mathbf{d}$ vector along the $\hat{\mathbf{z}}$ axis [26]. The sign of $\tilde{u}$ determines whether or not an equilibrium condensate magnetization is favored, and in what follows we will take $\tilde{u} > 0$ to study unitary states, since spin-polarized nonunitary states are generally energetically unfavorable and thus exceedingly rare. Finally, we have coupled the condensate magnetic moment $g\mathbf{Q}$ to an external magnetic field, $\mathbf{h}$. Although $\mathbf{Q}$ vanishes in equilibrium for unitary states, the energetics of sustaining a fluctuating $\mathbf{Q}$ out of equilibrium affects the spin dynamics, as will be shown below. We consider only applying a weak field (which does not affect the equilibrium $\mathbf{d}$-vector) as a means to probe the system. We note that this model is essentially equivalent to that of a spin-one spinor condensate [27].

Considering a static system and minimizing Eq. (2), the equilibrium $\mathbf{d}$-vector is found to be $\mathbf{d}_0 = v\hat{\mathbf{z}}$, with $v = \sqrt{(r+\Gamma)/u}$. To study the order parameter fluctuations, we parametrize the six real collective modes of the triplet superconductor as

$$\mathbf{d} = \mathrm{e}^{i\varphi}[(v+h)\hat{\mathbf{z}} + \mathbf{a} + i\mathbf{b}], \tag{3}$$

where the global phase mode φ is lifted to the plasma frequency by the Anderson-Higgs mechanism [28], and the longitudinal fluctuation of the $\mathbf{d}$-vector is the usual amplitude Higgs mode which resides at the gap edge [29]. Additionally, there are four transverse modes $\mathbf{a} = (a_x, a_y, 0)$ and $\mathbf{b} = (b_x, b_y, 0)$ which are, respectively, in and out of phase with the equilibrium condensate. Physically, these modes are two species of spin waves associated with the coherent fluctuation of the superconducting condensate, i.e., Cooper-pair spin waves.

By applying the $S = 1$ representations of the generators of $SU(2)$ spin rotations to $\mathbf{d}_0$, we see that the $\mathbf{a}$ mode corresponds to long-wavelength rotations of the $\mathbf{d}$-vector. In the absence of anisotropy, $\mathbf{a}$ is a Goldstone mode, while a finite anisotropy Γ gaps the mode to $\Omega_a = \Gamma$. In contrast, the $\mathbf{b}$ mode gives rise to a fluctuating magnetization $\delta\mathbf{Q} = 2\mathbf{d}_0 \times \mathbf{b}$, and is gapped to a frequency $\Omega_b = 2\tilde{u}v^2 + \Gamma$ since magnetization fluctuations around the unitary ground state are energetically costly. The $\mathbf{a}$ and $\mathbf{b}$ modes are analogous to the fluctuations of the Neél vector and magnetization in an antiferromagnet, which can be thought of as in- and out-of-phase fluctuations of the sublattice magnetizations, respectively.

In order to study the collective dynamics of the triplet system, we employ a time-dependent Ginzburg-Landau formalism [30,31], which allows for both coherent and dissipative order parameter dynamics, with the equation of motion (taking $\hbar = 1$)

$$i\partial_t \mathbf{d} = \frac{\delta\mathcal{F}}{\delta\bar{\mathbf{d}}} + \alpha\partial_t\mathbf{d}, \tag{4}$$

where α is a dimensionless damping parameter that is analogous to the Gilbert damping parameter in the Landau-Lifshitz-Gilbert theory for magnetic dynamics. This phenomenological damping is meant to model the damping of spin waves by, e.g., nodal quasiparticles, which are otherwise absent in our purely bosonic theory. This Landau damping can be computed microscopically within weak coupling theory for a given model.

Solving the equations of motion, we can identify the magnetization $\mathbf{m} = -\delta\mathcal{F}/\delta\mathbf{h}$ and, correspondingly, the dynamic magnetic susceptibility $\mathbf{m} = \underline{\chi}\mathbf{h}$. Doing so, we find the transverse susceptibility $\chi_{xx} = \chi_{yy} = \chi_\perp$,

$$\chi_\perp(\Omega) = \chi_0 \frac{\Omega_b(\Gamma - i\alpha\Omega)}{(\Gamma - i\alpha\Omega)(\Omega_b - i\alpha\Omega) - \Omega^2}. \tag{5}$$

The susceptibility takes the simple form $\chi_\perp(\Omega) = \chi_0\Omega_0^2/(\Omega_0^2 - \Omega^2)$ in the absence of damping (with the resonant frequency $\Omega_0^2 = \Gamma\Omega_b$ and oscillator strength $\chi_0 = 2g^2v^2/\Omega_b$), and is plotted in Figs. 1(c) and 1(d). Like conventional magnetic systems, we take χ_0 to coincide with the normal state paramagnetic susceptibility.

This phenomenological model is meant to capture the key features of the low-frequency dynamics of a triplet condensate, in particular the pinning of the $\mathbf{d}$ vector along an easy axis by spin-orbit coupling. Given a specific material, one can construct a more detailed model taking into account the relevant point group symmetry, but we expect the main results of our analysis to hold for purely triplet superconductors on the general grounds that the transverse susceptibility exhibits a resonance (associated with a bare spin wave excitation) at finite frequency [32]. The extension of our results to nonunitary or mixed-parity states is left as the subject of future work.

This generic form of the susceptibility can be illustrated by a microscopic weak coupling calculation for a simple model system, which we describe in the Supplemental Material [36]. We consider a two-dimensional p-wave superconductor with an easy-axis spin anisotropy. At zero temperature, we find that the $\mathbf{a}$ mode disperses according to $\Omega^2 = \Omega_0^2 + v_F^2\mathbf{q}^2/2$, where v_F is the Fermi velocity and the Cooper-pair spin wave frequency $\Omega_0^2 = \gamma_{\mathrm{BCS}}(2\Delta_0)^2$ is set by a dimensionless measure of the spin anisotropy γ_{BCS} (see the Supplemental Material [36]) and the superconducting

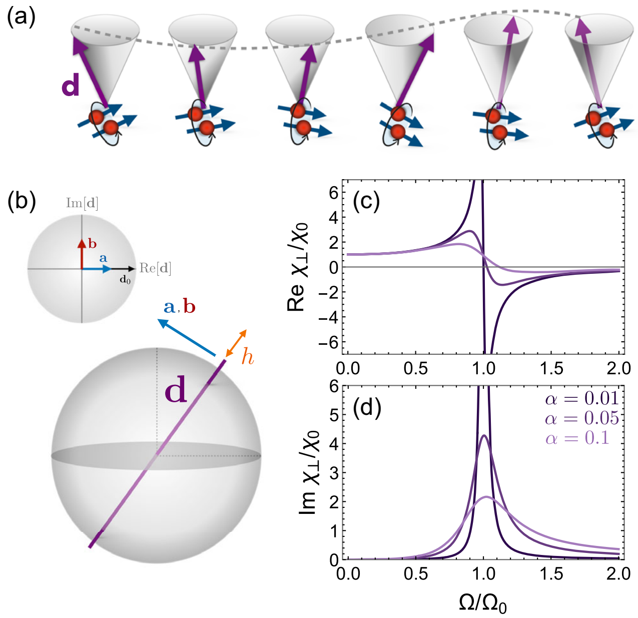

FIG. 1. Cooper-pair spin waves. (a) Illustration of Cooper-pair spin waves, represented as both the precession of the **d**-vector, and (schematically) of the associated triplet Cooper-pair spins that arise due to the time-dependent transverse fluctuations of the **d**-vector. (b) Illustration of the collective modes of a triplet superconductor: the longitudinal Higgs mode, h, and the transverse spin waves **a** and **b**. The **a** mode is in phase with the equilibrium **d**-vector, while the **b** mode is out of phase. (c) and (d) Dynamical transverse susceptibility $\chi_\perp(\Omega)$ for different values of the damping parameter α with $\chi_0 = 0.1$ and $\Gamma/\Omega_b = 0.05$. All curves exhibit resonant behavior at the Cooper-pair spin wave frequency $\Omega_0^2 = \Gamma\Omega_b$.

gap $2\Delta_0$. Meanwhile, the **b** mode has the dispersion $\Omega^2 = (4\Delta_0)^2 + \Omega_0^2 + v_F^2\mathbf{q}^2/2$, which shows that Ω_b coincides with the gap edge in the spin-rotation symmetric system, and is pushed up into the quasiparticle continuum by anisotropy. Intrinsically, these modes disperse over electronic length scales, which can be safely neglected when studying the long-wavelength behavior of the system. The transverse susceptibility [43] in this model is $\chi_\perp(\Omega) = \chi_0\Omega_0^2/(\Omega_0^2 - \Omega^2)$, consistent with the undamped susceptibility derived from time-dependent Ginzburg-Landau theory. This calculation also confirms that χ_0 should be identified as the normal-state paramagnetic susceptibility.

Electrodynamics.—Having derived the properties of the bare Cooper-pair spin waves, we may now consider their interaction and hybridization with classical electromagnetic fields. Since the system is a superconductor, the diamagnetic response of the condensate, originating from orbital screening currents, must be taken into account when studying the spin wave electrodynamics. That is, the Cooper-pair spin waves generate a fluctuating dipolar magnetic field, which must be screened by the Meissner currents. This unusual form of electrodynamics, where the spin dynamics and orbital currents are coupled at a classical level, distinguishes spin triplet superconductors from both ferromagnets and antiferromagnets as well as spinful, but uncharged, condensates such as superfluid ^{3}He or atomic spinor Bose-Einstein condensates.

In the quasimagnetostatic approximation, Maxwell's equations read (with $\mu_0 = 1$)

$$\nabla \cdot \mathbf{b} = 0, \tag{6a}$$

$$\nabla \times \mathbf{h} = \mathbf{j}_s, \tag{6b}$$

where $\mathbf{j}_s$ is the screening supercurrent. This current must be conserved, and is governed by the London equation

$$\nabla \cdot \mathbf{j}_s = 0, \tag{7a}$$

$$\nabla \times \mathbf{j}_s = -\lambda^{-2}\mathbf{b}, \tag{7b}$$

where λ is the London penetration depth. Finally, in the triplet system, **b** and **h** are related by the constitutive relation $\mathbf{b} = \underline{\mu}\mathbf{h}$ with $\underline{\mu} = \underline{\mathbb{1}} + \underline{\chi}$ where the susceptibility (5) encodes the bare spin wave spectrum. The magnetostatic collective modes are then the normal modes of the coupled Eqs. (6) and (7), comprising of intertwined fluctuations of the magnetic field, Cooper-pair spin waves, and induced supercurrent.

Bulk modes.—We begin by considering an infinite triplet system, where we can solve the magnetostatic equations in momentum space to find two branches of solutions. The first mode has the magnetic field polarized transverse to the **d**-vector and satisfies $\mu(\Omega) + \lambda^2 q^2 = 0$, where $\mu(\Omega) = 1 + \chi_\perp(\Omega)$ and q is the wave vector of the mode. This mode is dispersive even in an infinite system on account of the Meissner effect. In addition to this transverse, isotropically dispersing mode, the second mode is anisotropic and partially longitudinal, satisfying $(1 + \lambda^2 q^2 \sin^2\phi)\mu(\Omega) + \lambda^2 q^2 \cos^2\phi = 0$, where ϕ is the angle between **q** and the **d**-vector. The dispersions of the bulk modes are plotted in the Supplemental Material [36].

Surface modes.—Next, we consider a semi-infinite triplet superconductor occupying the half space $z < 0$, with vacuum above ($z > 0$). We take the **d**-vector to lie in the plane of the sample, along the x axis. The magnetostatic Eqs. (6),(7) must then be supplemented by the boundary conditions that b_z, h_x, and h_y be continuous across the interface, and that $j_{s,z}$ must vanish at the interface to ensure supercurrent conservation. The details of this calculation are summarized in the Supplemental Material [36], and the key results are shown in Fig. 2.

Figure 2(a) shows the dispersion of the surface mode for different directions of propagation with respect to the **d**-vector. Figure 2(b) shows the surface mode dispersion alongside the bulk spin wave bands projected onto the surface, which form two continua of scattering states as the out-of-plane wave vector q_z is varied. As the direction of

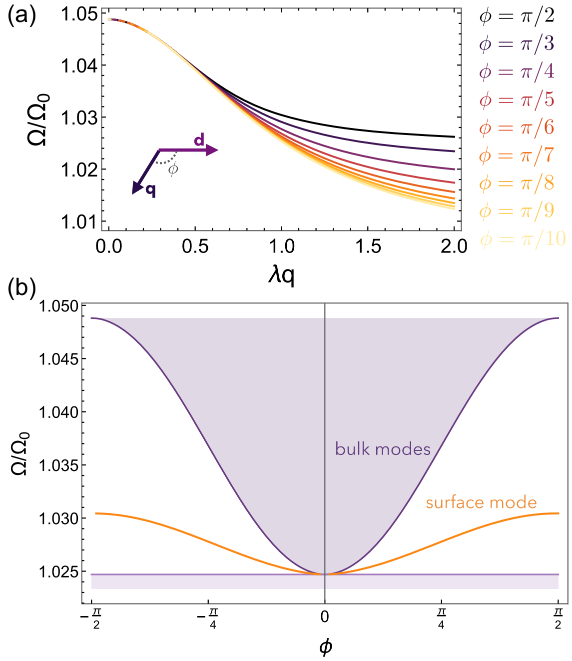

FIG. 2. Surface Cooper-pair spin waves. (a) Dispersion of the surface mode for different directions of in-plane propagation with respect to the **d**-vector. (b) Angular dependence of the bulk and surface mode frequencies for fixed in-plane wave vector $\lambda q = 1$. The surface-projected bulk spin wave bands form two continua, and the surface mode exists within the gap between them.

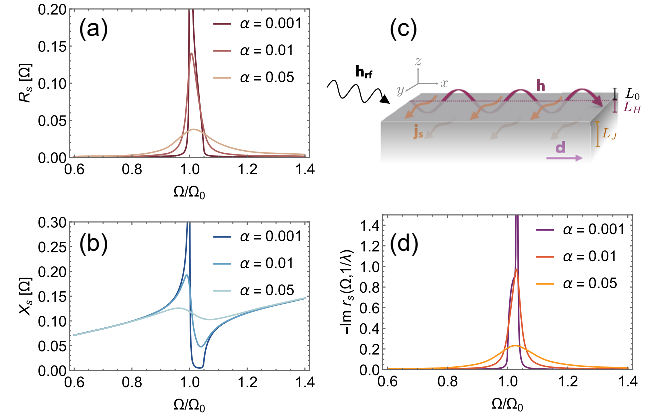

FIG. 3. Experimental signatures. (a) and (b) Surface resistance and reactance $Z_s = R_s - iX_s$ calculated within the two-fluid model. Both exhibit peaks at the bare Cooper-pair spin wave frequency Ω_0 and X_s features a dip at the bulk dipolar mode frequency. (c) Illustration of the surface mode in a semi-infinite sample with an in-plane **d**-vector. The magnetic field profile of the mode propagates in the plane of the interface, and decays over a length $L_H \sim \lambda$ inside the triplet superconductor and over a length L_0 in vacuum, while the supercurrent profile of the mode decays over a length L_J. **d**. Near-field reflection coefficient evaluated at $q = 1/\lambda$, which exhibits a peak at the surface mode frequency.

in-plane propagation becomes parallel to the **d**-vector ($\phi \to 0$), the surface mode intersects the bulk spin wave bands and ceases to be a distinct, surface-localized mode.

The Cooper-pair spin wave frequency can be estimated for the candidate triplet superconductor UPt_3 [44,45], where NMR [46] and neutron scattering experiments [47] show that the **d**-vector is depinned by a field of $\mu_0 H_{\text{pin}} = 230$ mT. We may then estimate the **d**-vector anisotropy to be $\Gamma \approx 2\mu_B \mu_0 H_{\text{pin}} \approx 300$ mK, and using the weak-coupling expression $2\Delta_0 \approx 3.5 k_B T_c \approx 1.75$ K, estimate the Cooper-pair spin wave frequency to be 14 GHz in this material. Broadly speaking, most candidate triplet superconductors have transition temperatures on the order of 1 K [48–50]. Estimating $\Gamma/2\Delta_0 \approx 5\% - 20\%$ [51,52], we expect Cooper-pair spin waves to generally reside at 15–30 GHz. However, these modes may be much lower lying in organic superconductors [5] or graphene-based systems [53–55], where the spin-orbit coupling responsible for pinning the **d**-vector is extremely weak.

Experimental detection.—Cooper-pair spin waves contribute to experimentally observable electrodynamic response functions such as the surface impedance and reflectance of the triplet medium. We calculate these functions within a two-fluid model of electrodynamics where the charge response is described via the conductivity $\sigma(\Omega) = \sigma_n(\Omega) + i/(\lambda^2 \Omega)$. The first term describes the dissipative conductivity due to quasiparticles, while the second describes the kinetic inductance of the condensate. The complex surface impedance, which can be measured using cavity resonator techniques, is then given by $Z_s(\Omega) = \sqrt{\mu(\Omega)/\varepsilon(\Omega)}$, where $\varepsilon(\Omega) = 1 + i\sigma(\Omega)/\Omega$ is the dielectric function of the triplet medium (see the Supplemental Material [36] for details). The surface resistance and reactance, $Z_s = R_s - iX_s$, are plotted in Figs. 3(a) and 3(b) and exhibit a strong resonance at the bare Cooper-pair spin wave frequency Ω_0, as well as a weaker feature at the frequency of the bulk dipolar mode.

The surface Cooper-pair spin wave lies outside of the light cone (i.e., disperses with a velocity much slower than the speed of light), and thus must be excited by a near-field source, such as an antenna or microwave strip line [56–58]. The same is true for dipolar spin waves in ferromagnets, and consequently near-field microwave spectroscopy techniques are well developed [19,59,60]. The appropriate response function to describe such a measurement is the near-field reflection coefficient $r_s(\Omega, q)$ [61,62] which is calculated in the Supplemental Material [36]. This reflection coefficient is presented in Fig. 3(d), and features a peak at the surface mode frequency. This demonstrates that surface Cooper-pair spin waves can be excited and detected much like other dipolar spin waves. For example, they can be excited by a microwave transmission line and detected either inductively by a second transmission line [59,60], or electrically via the inverse spin Hall effect [63,64].

Finally, recent advances in nitrogen-vacancy magnetometry [65,66] have enabled nitrogen-vacancy sensors to operate at the mK temperatures needed to access triplet superconductivity [67], allowing for the magnetic field profile of the surface modes to be directly imaged, just as Damon-Eshbach modes have recently been imaged in ferromagnets [56].

Conclusions.—We have identified a class of magnetostatic modes in triplet superconductors and demonstrated their coupling to experimental probes. The detection of these modes in a given material would constitute strong evidence for spin triplet superconductivity, just as the detection of various collective modes in ^{3}He proved crucial to identifying distinct superfluid phases [2,68]. Our results complement prior works on orbital collective modes in multicomponent superconductors [9–16], and tighten the analogies between unconventional superconductors, superfluid ^{3}He, and magnetic materials.

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[26] In general, $\mathbf{d}(\mathbf{r}, t)$ is a function of momentum. However, as a consequence of the **d**-vector pinning, we assume that $\mathbf{d}(\mathbf{r}, t)$ is uniformly oriented along $\hat{\mathbf{z}}$ at all points on the Fermi surface, and thus independent of the relative momentum of the triplet pairs.

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