Full quantum distribution of contrast in interference experiments between interacting one-dimensional Bose liquids

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The existence of quantum noise is the most direct evidence for the probabilistic nature of quantum mechanics. In strongly interacting systems we expect quantum noise to reveal non-local correlations of the underlying many-body states. Here, we show that quantum noise in interference experiments with cold atoms can be used to investigate the unusual character of one-dimensional interacting systems. We analyse interference experiments for a pair of independent bosonic one-dimensional condensates and explicitly calculate the distribution function of fringe amplitudes using methods of conformal field theory. Moreover, we point out interesting relations between interference experiments with cold atoms, the problem of a quantum impurity in a one-dimensional Luttinger liquid and a variety of statistical models ranging from stochastic growth models to two-dimensional quantum gravity. Such connections can be exploited to design a quantum simulator of unusual two-dimensional models described by non-unitary conformal field theories with negative central charges.

The hallmark of a Bose–Einstein condensate is the existence of a well-defined macroscopic phase. Indeed, experiments with large condensates exhibit robust matter-wave interference with negligible fluctuations in the fringe contrast[1]. From this point of view, large three-dimensional condensates may be thought of as classical objects. However, there is a continuous range of possibilities intermediate between perfect condensates on one side and systems that do not exhibit an interference pattern, such as high-temperature thermal gases, on the other[2]. For example, one-dimensional interacting bosons exhibit interference patterns with a reduced contrast and with non-negligible shot-to-shot fluctuations of the fringe contrast. In this paper, we analyse the full distribution associated with this quantum noise, and discuss what it can tell us about the underlying strongly correlated state.

By virtue of its direct connection to the concepts of quantum measurement, the study of quantum noise has deepened our understanding in a variety of areas. Understanding the noise in photodetection[3] prompted the creation of non-classical states of light and led to the development of quantum optics. In mesoscopic electron systems, current fluctuations contain information that is not available in simple transport measurements. For example, they can be used to distinguish electrical resistance owing to diffusive scattering from the resistance resulting from point contact tunnelling (see ref. 4 for a review). Single-atom detectors have recently been used to carry out Hanbury–Brown–Twiss experiments with cold atoms[5,6]. Finally, analysis of noise correlations in time-of-flight experiments of ultracold atoms has been proposed[7] and tested[8,9], promising a powerful new technique to access many-body correlations in such systems. In these applications, the quantities of direct interest are contained in the first few moments of the noise distribution, such as the noise power spectrum. However, deeper insights into quantum systems may be gained by obtaining the full statistics of the fluctuations.

One of the central problems in the field of ultracold atoms is finding new ways to characterize many-body quantum states. In this paper, we demonstrate that analysis of the distribution function of contrast in interference experiments between interacting one-dimensional Bose liquids provides a novel probe of non-local

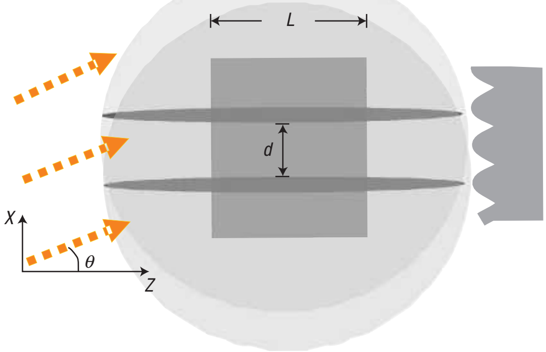

Figure 1 Typical experimental scheme. Schematic view of the possible experimental setup, which produces an interference pattern between two independent one-dimensional condensates.

correlations and entanglement present in these systems. It is known that the average amplitude of interference fringes can be used to extract two-point correlation functions in fluctuating condensates[2]. This idea has been successfully used by Hadzibabic *et al.* to measure the Kosterlitz–Thouless transition in two-dimensional systems[10]. Interference experiments, however, contain more information than the average value of the contrast. Each observation of the interference pattern is a classical measurement of a quantum mechanical state, so the result of each individual measurement will be different from the average value. As we discuss below, higher moments of the distribution function of interference amplitudes correspond to high-order correlation functions. Hence, the knowledge of the entire distribution function reveals global properties of the system that depend on non-local correlation functions of arbitrarily high order. So far, the discussion of full counting statistics has been limited mainly to systems of non-interacting particles[11]. The main result of this paper, in contrast, is an expression for the full distribution of amplitudes of interference fringes arising from one-dimensional interacting Bose liquids, that can be described by a Luttinger liquid.

We start our analysis by establishing the relation between the probability distribution function of the interference amplitude and the partition function of a boundary sine–Gordon model. Using methods of conformal field theory (CFT), we reduce this problem to that of finding a spectral determinant of a simple one-dimensional, single-particle Schrödinger equation. We solve this problem numerically, as well as analytically using the Wentzel–Kramer–Brillouin (WKB) approximation, to obtain the desired fringe distribution for any value of the Luttinger parameter.

It is interesting to point out that the alternative point of view on the relation between interference experiments with a Bose–Einstein condensate and the quantum impurity problem can be taken. By measuring the distribution function of the interference amplitudes experimentally the full partition function of the boundary sine–Gordon problem (see equation (5) below) is obtained. This model describes a range of interesting problems such as an impurity in a Luttinger liquid[12], the asymmetric Kondo model[13] and the tunnelling of a particle in the presence of dissipation within a Caldeira–Leggett approach[14]. As we discuss below, interference experiments can be used to obtain the partition function of these systems not only in the ground state but also in the non-equilibrium regimes (for example, in the presence of a finite voltage). Hence, interference experiments can be considered as a quantum solver of these non-trivial many-body problems.

Another surprising implication of our analysis is that interference experiments with one-dimensional cold atoms can be used as quantum simulators of several fundamental problems in physics. This connection relies on the fact that many models of systems with critical behaviour in two-dimensional statistical mechanics, one-dimensional field theories and many-body quantum systems can be described by continuum theories with conformal invariance. Basic ingredients of any CFT are the central charge and the conformal dimensions of the operators. One-dimensional quantum systems of interacting particles have positive central charges and correspond to the so-called unitary class. On the other hand, there is a non-unitary class of models that have negative central charges. Such models appear in contexts as different as two-dimensional quantum gravity and stochastic growth models and they have very unusual properties (see, for example, ref. 15). In this paper, we demonstrate that interference experiments with one-dimensional condensates can be used to analyse models in the non-unitary universality class by virtue of the exact mathematical relation between the distribution function of the interference amplitude and the so-called **Q** operators of CFTs.

FULL QUANTUM DISTRIBUTION OF THE INTERFERENCE CONTRAST

The setup for the interference experiments we consider is shown in Fig. 1. Two independent quasi-condensates are allowed to expand in the transverse direction. After sufficient expansion time, the integrated density profile is measured by the imaging beam, which is sent at an angle θ to the condensate axis. This setup is quite common for the interference experiments in cold atoms and was already realized by several groups[10,16].

Throughout this paper we consider the two condensates to be identical, although our analysis can be generalized to the case of different condensates (see also ref. 17). We assume that before the expansion, atoms are confined to the lowest transverse channels of their respective traps and that the optical imaging length L (which is smaller than or equal to the size of the system in the axial direction) is much larger than the coherence length of the condensates. This enables us to use an effective Luttinger liquid description of the interacting bosons[18]. The operator corresponding to the interference signal of the two condensates[2] is given by

$$A_{\tilde{p}} = \int_0^L \mathrm{d}z a_1^\dagger(z) a_2(z) \mathrm{e}^{i\tilde{p}z}. \tag{1}$$

Here z is a one-dimensional coordinate, a_1 and a_2 are the bosonic operators in the two systems before the expansion, and the integrals are taken along the condensates. The seeming winding of the relative phase between the two systems, described by the exponential term in equation (1), can either come from the measurement process itself or from the actual motion of the condensates[2]. If the condensates are at rest, we have $\tilde{p} = (md/\hbar t)\tan\theta$, where m is the atom's mass, d is the separation between the two condensates, $\hbar$ is the reduced Planck's constant and t is the time when the measurement was made after the free expansion started. With some abuse of terminology we will call $\tilde{p}$ the relative momentum. When condensates one and two are independent, the expectation value of $\langle A_{\tilde{p}} \rangle$ vanishes, however $\langle |A_{\tilde{p}}|^2 \rangle$ is finite. This means that individual measurements show a finite amplitude of interference fringes, however, their phase is completely unpredictable. Higher moments of the interference fringe amplitude are given by[2]

$$\langle |A_{\tilde{p}}|^{2n} \rangle = A_0^{2n} Z_{2n}^{(p)}, \quad \text{where } A_0 = \sqrt{C\rho\, \xi_h^{1/K} L^{2-1/K}}, \tag{2}$$

where C is a constant of order unity, ρ is the particle density in each condensate, ξ_h is the short range cutoff equal to the healing length and K is the Luttinger parameter describing the interaction strength. Although, in general, even purely repulsive bosons in one dimension can have effective positive scattering length and thus $K < 1$ (see ref. 19), here we assume that $K \geq 1$, which is always the case for bosons with δ-function-type repulsive interactions. Coefficients Z_{2n} in equation (2) are given by:

$$Z_{2n}^{(p)}(K) = \int_0^{2\pi} \cdots \int_0^{2\pi} \prod_{i=1}^{n} \frac{\mathrm{d}u_i}{2\pi} \frac{\mathrm{d}v_i}{2\pi} \mathrm{e}^{i2p\sum_i (u_i - v_i)} \left| \frac{\prod_{i<j} 2\sin\left(\frac{u_i - u_j}{2}\right) \prod_{k<l} 2\sin\left(\frac{v_k - v_l}{2}\right)}{\prod_{i,k} 2\sin\left(\frac{u_i - v_k}{2}\right)} \right|^{1/K}, \quad (3)$$

where p is the relative momentum measured in units of $2\pi/L$: $p = \tilde{p}L/2\pi$ and we assumed periodic boundary conditions.

Coefficients $Z_{2n}^{(p)}$ originally appeared in the grand canonical partition function of a neutral two-component Coulomb gas on a circle

$$Z_p(K, x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(n!)^2} Z_{2n}^{(p)}(K). \quad (4)$$

Here x is the fugacity of Coulomb charges and Z_{2n} describes contributions from configurations with $2n$ charges (that is, canonical partition functions). The partition function, equation (4), with $K > 1$ and p being half integer describes several problems in statistical physics (see ref. 20). In particular, it describes an impurity in a one-dimensional interacting electron liquid. At low energies this problem is described by a Luttinger liquid with an additional local nonlinear term due to backscattering from the impurity:

$$S = \frac{\pi K}{2} \int_{-\infty}^{\infty} \mathrm{d}y \int_0^{\beta} \mathrm{d}\tau \left[(\partial_\tau \phi)^2 + (\partial_y \phi)^2 \right] + 2g \int_0^{\beta} \mathrm{d}\tau \cos\left[2\pi\phi(0, \tau) + \frac{p}{\beta}\tau \right],$$

where g is the amplitude of backscattering on the impurity, τ is imaginary time, β is the inverse temperature and $\phi(x)$ is a bosonic phase field associated with the electron field operator $\psi(x) \sim \mathrm{e}^{i\phi}$. In the bosonized form the electron–electron interaction becomes quadratic and is effectively described by the Luttinger parameter K. Perturbative expansion of the corresponding partition function in powers of g produces the series equation (4) with the fugacity given by $x = g\beta(2\pi/\beta\kappa)^{1/2K}$. Here κ is a non-universal renormalization factor, which sets the scale for the long-distance asymptotics of the correlation functions: $\langle \exp[2i(\phi(x, \tau) - \phi(0, 0))] \rangle \sim (\kappa\sqrt{x^2 + \tau^2})^{-1/K}$. Finally, the single-impurity Kondo model is related to $Z_p(K, x)$ as well[13].

It is easy to understand the origin of the relation between interference experiments and a quantum impurity problem. Moments of fringe amplitudes are determined by high-order correlation functions computed at the same time but in different points in space. On the other hand, expansion of the partition function for a quantum impurity contains correlation functions computed at the same spatial point but at different times. Lorentz invariance of the Luttinger liquid ensures that the two are the same. Note that the analogue of the finite imaging angle θ in the interference experiments is a finite voltage in the quantum impurity problem[21]. This analogy can also be understood from the interchanged roles of space and time in the two systems.

When describing interference experiments it is convenient to define the normalized amplitude of interference fringes $\alpha = A_{\tilde{p}}^2/A_0^2$. From equation (2) we find that $\langle \alpha^n \rangle = Z_{2n}^{(p)}$, so by measuring

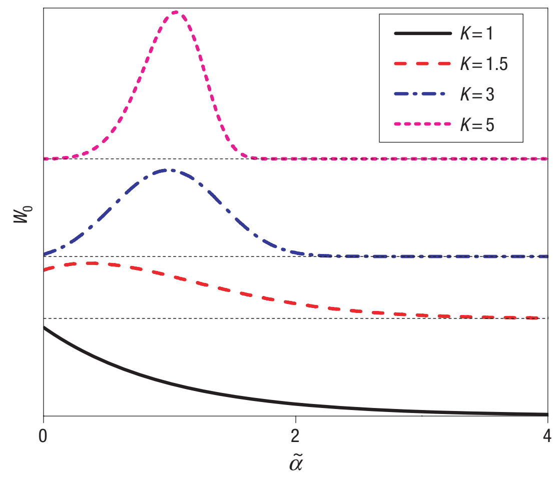

Figure 2 Small K distribution function. Evolution of the distribution function $W_0(\tilde{\alpha})$ for different values of K at $p = 0$. At larger values of K the function W_p tends to the δ function (see text for details).

the distribution function $W_p(\alpha)$ experimentally, we get direct access to the partition function, equation (4). We point out that $W_p(\alpha)$ can be used to compute all moments of $A_{\tilde{p}}^2$, and therefore contains information about high-order correlation functions of the interacting Bose liquids.

Using Taylor expansion of the modified Bessel function as well as equation (4) and the fact that $\langle \alpha^n \rangle = Z_{2n}^p$ we find

$$Z_p(K, x) = \int_0^{\infty} W_p(\alpha)\, I_0(2x\sqrt{\alpha})\, \mathrm{d}\alpha. \quad (5)$$

Inverting equation (5) we can express the probability $W_p(\alpha)$ through the partition function $Z_p(K, x)$. Noting that the Bessel function $I_0(ix) = J_0(x)$ and using the completeness relation for Bessel functions, $\int_0^\infty J_0(\lambda x) J_0(\lambda y) |x| \lambda \mathrm{d}\lambda = \delta(|x| - |y|)$, we obtain

$$W_p(\alpha) = 2 \int_0^{\infty} Z_p(K, ix) J_0(2x\sqrt{\alpha}) x \mathrm{d}x. \quad (6)$$

It is important that the last equation has the partition function at an imaginary value of the coupling constant. This should be understood as an analytic continuation of $Z_p(K, x)$.

There are several ways to compute $W_p(\alpha)$. The most straightforward approach is to evaluate the coefficients $Z_{2n}^{(p)}$ and thus determine all the moments of the distribution. Explicit expressions for $Z_{2n}^{(p)}$ can be obtained using orthogonal Jack polynomials[20]. However, each of these coefficients is given as a series of products of Γ functions and their evaluation becomes extremely cumbersome for $n \geq 3$. Another approach to finding $W_p(\alpha)$ is to compute $Z_p(K, x)$ using the thermodynamic Bethe ansatz for the quantum impurity problem[20]. This method works only for half-integer K and requires the solution of coupled integral equations. Besides, relating $Z_p(K, x)$ to the distribution function of interference amplitudes requires analytic continuation of $Z_p(K, x)$ into the complex plane, $x \to ix$ (see equation (6)), which introduces additional complications. In this paper we will use a different method, which is based on studies of the integrable structure of CFTs[22]. In particular, it was shown that the vacuum expectation value of Baxter's $\mathbf{Q}$ operator, central to the integrable structure of the models, coincides with the grand partition function of interest up to an overall prefactor[22] (see Supplementary Information for more details):

$$Q_p^{\mathrm{vac}}(c, \lambda) = \lambda^{4p/K} Z_p(K, -ix), \quad (7)$$

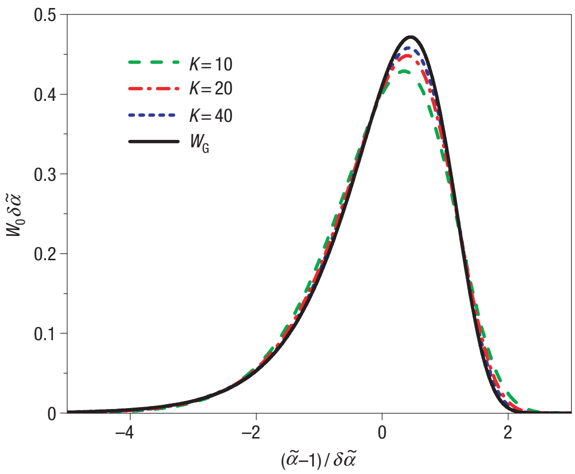

Figure 3 Large *K* distribution function. Scaled distribution function $\delta\tilde{\alpha}\, W_0((\tilde{\alpha}-1)/\delta\tilde{\alpha})$, where $\delta\tilde{\alpha}$ is the width of the distribution, for large K. The function W_0 is multiplied by $\delta\tilde{\alpha}$ to preserve the total probability, which must be equal to unity. The dashed and dotted lines correspond to different values of K. The solid line corresponds to the conjectured Gumbel distribution.

where x is related to the spectral parameter λ as $x=\pi\lambda/\sin(\pi/2K)$ and the central charge $c=1-6(2K-1/(2K))^2$. It was conjectured in refs 23,24 that the vacuum expectation value $Q_p^{\rm vac}(\lambda)$ is proportional to the spectral determinant of the single-particle Schrödinger equation

$$-\partial_x^2\Psi(x)+\left(x^{4K-2}+\frac{l(l+1)}{x^2}\right)\Psi(x)=E\Psi(x), \qquad (8)$$

where $l=4pK-(1/2)$. So, $Q^{\rm vac}(\lambda)=\lambda^{4p/K}D(\rho\lambda^2)$, where $\rho=(4K)^{2-1/K}\Gamma^2(1-1/2K)$, $D(E)$ is the spectral determinant defined as $D(E)=\prod_{n=1}^{\infty}(1-E/E_n)$ and E_n are the eigenvalues of equation (8). Thus, we get

$$Z_p(K,ix)=\prod_{n=1}^{\infty}\left(1-\frac{\rho\lambda^2}{E_n}\right). \qquad (9)$$

To evaluate the distribution function we solve the Schrödinger equation (8) numerically. We checked the accuracy of the numerics as well as the conjecture in equation (9) by comparing coefficients $Z_2^{(p)}(K)$ and $Z_4^{(p)}(K)$ evaluated for various K using (1) the spectral determinant (for example, $Z_2^{(p)}\sim\sum 1/E_n$, $Z_4^{(p)}\sim(\sum 1/E_n)^2-\sum 1/E_n^2$)) and equation (2) the exact expressions of ref. 20 based on Jack polynomials. We found perfect agreement between the two methods.

To compare distributions at different K with each other it is convenient to use a normalized interference amplitude $\tilde{\alpha}=\alpha/\langle\alpha\rangle=A^2/\langle A^2\rangle$ instead of α. This change of variable is also convenient for comparison with experiments. The distribution function $W_0(\tilde{\alpha})$ is shown in Fig. 2 for several values of K. For K close to 1 (Tonks–Girardeau limit) W_0 is a wide poissonian function, which gradually narrows as K increases, finally becoming a narrow δ function at $K\to\infty$ (the limit of non-interacting bosons). Interestingly, the distribution function remains asymmetric for arbitrarily large K. In fact, we find that $W_0(\tilde{\alpha}-1)$ tends to a universal scaling form, parameterized by a single number characterizing the width of the distribution: $\delta\tilde{\alpha}\equiv\sqrt{\langle\tilde{\alpha}^2\rangle-1}\approx\pi/\sqrt{6}K$ (see ref. 2). We conjecture that this limiting form of W_0 is the Gumbel distribution W_G, which frequently appears in problems of extreme-value statistics[25]: $W_0(\tilde{\alpha}-1)\to W_G(\tilde{\alpha},K,\gamma)$, where $\gamma\approx 0.577$ is the Euler gamma-constant and

$$W_G(x,a,b)=a\exp\big(ax-b-\exp(ax-b)\big).$$

We plot the scaled distribution functions: $\delta\tilde{\alpha}\,W_0\big((\tilde{\alpha}-1)/\delta\tilde{\alpha}\big)$. Note that W_0 was multiplied by $\delta\tilde{\alpha}$ to preserve the normalization condition (so that the total probability is equal to unity) for $K=10,20,40$. For comparison, in Fig. 3 we also present the scaled Gumbel distribution. It can be seen that as K increases, the function W_0 indeed approaches W_G. Gumbel distributions are frequently associated with random walks in strongly correlated systems[25]. Indeed, the interference signal in equation (1) can be viewed as a sum of contributions coming from different points along the condensates. For weak interactions (large K) these contributions are strongly correlated because the phases of each of the condensates only weakly fluctuate along z. Thus, there is no surprise that W_0 approaches the Gumbel distribution. This result can also be understood noting that for $K\gg 1$ the distribution function of the interference amplitude is dominated by rare events that reduce the contrast. The Gumbel distribution was introduced precisely to describe rare events such as stock-market crashes or earthquakes. In the Supplementary Information we also discuss distribution functions for finite values of the observation angle (that is, finite p).

Interestingly, the distribution function $W_p(\alpha)$ provides a very simple and convenient framework for describing both the partition function, equation (4), and the expectation values of the **Q** operator. Indeed, $W_p(\alpha)$ is a smooth well-behaved function at all values of K and it can be easily approximated by simple analytic expressions.

QUANTUM SIMULATION

As we mentioned earlier, the distribution function $W_p(\alpha)$ can be used to obtain the partition function $Z_p(K,x)$ (see equation (5)) describing a range of various problems such as quantum impurity in a one-dimensional electron liquid, the asymmetric Kondo problem and dissipative tunnelling. It is easy to see that the momentum p in the interference experiments corresponds to the external applied voltage in the impurity problem ($ip\sim V$). This follows from the interchanged roles of space and time in the two problems. Thus, measuring $W(\alpha)$ experimentally and taking its integral transform directly simulates these problems in or out of equilibrium. Moreover, after substituting $I_0(2x\sqrt{\alpha})\to J_0(2x\sqrt{\alpha})$ in equation (5) the partition functions of the above models with imaginary coupling constants are obtained. Such models have been actively investigated recently in the context of theories with parity-time-symmetric rather than hermitean hamiltonians[26].

The partition function with the imaginary coupling also gives the expectation values of the Baxter **Q** operator (see equation (7)) corresponding to various CFTs. In general, such theories are particularly important because many models of two-dimensional statistical mechanics, field theory and many-body quantum systems at critical points can be described by some continuum theories having a property of conformal invariance. This leads to a description of critical systems on the basis of CFT, the basic ingredients of which are the central charge and conformal dimensions. This set of data classify different universality classes, which often describe very different physical models. The property of positivity of the central charge leads to a unitary theory. Physically, the central charge determines the vacuum (Casimir) energy of the system and governs the finite-size scaling effects. On the other hand, there is a class of models whose universality classes

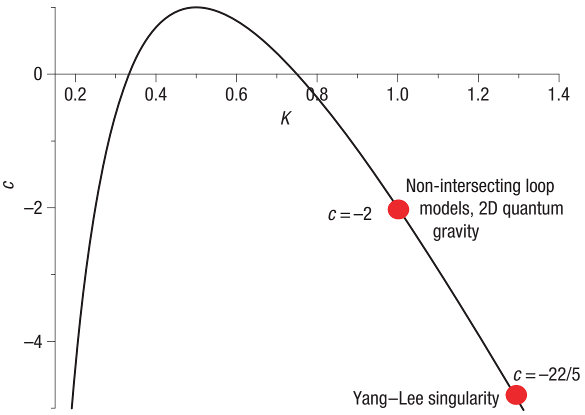

Figure 4 Central charges. Dependence of the central charge of CFTs corresponding to interference experiments on the Luttinger liquid parameter *K*. The red circles correspond to some particular examples discussed in the text.

of critical behaviour are described by the conformal-invariant models with negative central charges. These theories are non-unitary and their properties are thus very different from those described by positive central charges.

CFTs corresponding to equation (7) are characterized by negative central charge $c = 1 - 6(2K - 1/(2K))^2$ and the highest weight $\Delta = (2pK)^2 + (c-1)/24$ (see ref. 22). These relations give $c \leq -2$ for $K \geq 1$. Theories with negative central charges appear in different contexts of statistical mechanics, stochastic growth models, two-dimensional quantum gravity, models of two-dimensional turbulence and even high-energy quantum chromodynamics. In particular, $c = -2$ CFT has the field-theoretical representation in terms of the ghost (anticommuting) fields and also corresponds to the critical behaviour of the non-intersecting loop model on a two-dimensional lattice[27] as well as to the special case of the stochastic Loewner evolution equation, describing the growth of a random fractal stochastic conformal-invariant interface (see, for example, ref. 28). The classic example of CFT with negative c is the Yang–Lee singularity, $c = -22/5$, describing the critical behaviour of the Ising model in an imaginary magnetic field. Models of two-dimensional quantum gravity described by the fluctuating lattice geometry are related to negative c as well. Possibly, the high-energy limit of multicolour quantum chromodynamics is described by the integrable CFT with negative (or zero) central charge[29,30]. We illustrate the dependence of c on K as well as particular examples of models corresponding to different values of c in Fig. 4. The spectrum of the **Q** operator can be used to reconstruct the transfer matrices of the above-mentioned negative c models. A particular example of such a procedure for the $c = -2$ universality class was explicitly constructed in ref. 22. In this case, only the vacuum expectation value was needed to reconstruct the whole transfer matrix. The transfer matrices contain all the information about the properties of underlying models. In this sense interference experiments simulate these models. Experimentally, the central charge can be tuned by varying the interaction K and the scaling dimensions Δ of corresponding physical operators can be manipulated by changing the observation angle p. An interesting challenge here is the experimental determination of non-vacuum values of the **Q** and **T** operators. This is an open question.

Needless to say the range of models mentioned above, which belong to non-unitary universality classes, is difficult (if possible at all) to realize by another way. The interference of condensates provides a possible and a plausible way to explore the interesting physics of various models ranging from statistical to high-energy physics.

We considered only a particular example of interference between two one-dimensional condensates and showed the connection between the distribution function of fringe amplitudes and the properties of various models. This analysis can be extended to other systems with quasi-long-range order, for example, to two-dimensional Bose systems at finite temperature. It is expected that there will be analogous connections to different classes of problems, some of which might not be exactly solvable. The interference experiments open new ways of solving these problems by direct simulation of the underlying models.

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Competing financial interests

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