Reply to comment by S.-K. Yip cond-mat/0611426

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We respond to S.-K. Yip's criticism of our work on the classification of spinor condensates. We explain why his criticism is unfounded, emphasizing that the phases he mentions have been addressed in our paper cond-mat/0611230. To provide a constructive aspect to this response, we use it as an opportunity to show how our classification scheme makes explicit not only spin rotations which leave spinor states invariant, but also the phase factors which need to accompany them.

In a comment [1] S.-K. Yip criticized our recent work on the classification of spinor condensates [2] and claimed that when discussing residual symmetries of various ground states, we did not understand the importance of phase factors and their role in determining the nature of vortex excitations. This criticism is unfounded. Since the focus of [2] was on classifying the symmetries of the phases, with topological excitations presented as an application, it was unnecessary to give a thorough treatment of such phases. We described only the spin structure of the defects, which does give a complete classification of defects in the Mott insulating phase since there is no phase coherence. On the other hand, in the follow-up paper [3], the focus was on vortices in condensates. We applied our approach to classify spinors for $S = 3$ condensates and explicitly discussed such phase factors (see, for example, Eq. (8)). This is the information needed to determine the quantization rule for the phase change of different vortices. Properties of the isotropy group fully determine the fundamental group and the nature of line defects [4]. This paper therefore addresses the issues that Yip claims we disregarded. This work was not cited in [1], although it had been available on the archive before he posted his comment.

To provide a constructive aspect to this response, we will use it as an opportunity to illustrate that our classification scheme makes explicit not only spin rotations which leave spinor states invariant but also the phase factors which, as Yip emphasizes, need to accompany them. We will show that our classification scheme allows one to calculate these phases as simple geometrical factors related to properties of polyhedra representing various spinor states.

The result (which will be proved elsewhere [5]) is as follows. Consider a spinor $|\psi\rangle$, and a rotation $R = e^{-i\mathbf{F}\cdot\hat{\mathbf{n}}\alpha}$ which is a symmetry of the set of the $2F$ points on the unit sphere consisting of coherent states $|\zeta\rangle$ such that $\langle\psi|\zeta\rangle = 0$ (which we refer to as "spin roots"). Then $R|\psi\rangle$ has the same set of spin roots. Since the spin roots determine $|\psi\rangle$ up to phase we have that,

$$R|\psi\rangle = e^{i\lambda}|\psi\rangle. \tag{1}$$

where λ is a real number which may be determined by considering the axis of rotation $\hat{\mathbf{n}}$. Let r be the multiplicity of the spin root which coincides with the direction of this axis ($r = 0$ corresponds to no root on the axis). Then

$$\lambda = \alpha(r - F), \tag{2}$$

where α is the angle of the rotation (counterclockwise when viewed facing from the end of $\hat{\mathbf{n}}$ towards the origin).

We will now proceed to apply Eq. (2) to the examples that Yip considers [1]. For the first example of the spin-two ferromagnetic state we have a continuous symmetry of rotations by α about the z-axis. The spin roots consist of four degenerate points on the negative z-axis. Then for arbitrary rotation by α counterclockwise about the positive z-axis we accumulate the phase $\lambda = \alpha(0 - 2) = -2\alpha$ (as could have been obtained directly by applying the rotation R to the spinor). The second example is the spin-two tetrahedratic (i.e. cyclic) state which has the spin roots residing at the vertices of a regular tetrahedron. This will have as one of its symmetries a rotation by $2\pi/3$ about an axis going through any of the vertices. Such a rotation will result in the phase $\lambda = \frac{2\pi}{3}(1 - 2) = -\frac{2\pi}{3}$. As the last example, we take the (spin-three) hexagonal state. This state has a symmetry of a rotation by $2\pi/6$ which results in the phase $\lambda = \frac{2\pi}{6}(0 - 3) = -\pi$.

[1] S. K. Yip, cond-mat/0611426.
[2] R. Barnett, A. Turner, and E. Demler, Phys. Rev. Lett. **97**, 180412 (2006).
[3] R. Barnett, A. Turner, and E. Demler, cond-mat/0611230.
[4] N. D. Mermin, Rev. Modern Phys. **51**, 591 (1979).
[5] A. Turner, R. Barnett, and E. Demler, to be published.